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Extra dimensions in physics and astrophysics/Dimensions supplémentaires en physique et astrophysique

Brane cosmology

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Abstract

We summarize the main ideas underlying brane cosmology, or the cosmology of a Universe considered as a submanifold of a higher-dimensional spacetime and where ordinary matter is supposed to be confined. This new scenario, motivated by recent developments in string theory, leads to several specific features that could allow, via forthcoming high precision cosmological observations, to distinguish it from the traditional cosmological scenario. ***To cite this article: P. Binétruy et al., C. R. Physique 4 (2003).***

Résumé

Cosmologie branaire. Nous récapitulons les idées principales de la cosmologie branaire, ou cosmologie d'un univers considéré comme sous-variété d'un espace-temps de plus grande dimension et où la matière ordinaire est censée être confinée. Ce nouveau scénario, motivé par des développements récents dans la théorie de cordes, mène à plusieurs propriétés spécifiques qui pourraient permettre, par l'intermédiaire de prochaines observations cosmologiques de précision élevée, de le distinguer du scénario cosmologique traditionnel. ***Pour citer cet article : P. Binétruy et al., C. R. Physique 4 (2003).***

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1. Introduction

The possibility which arises in string theories that matter is localized on a brane, whereas extra dimensions are accessible to gravity, leads to interesting cosmological scenarios. What is now known as brane cosmology has thus attracted a lot of attention in the last few years. One way to distinguish between the corresponding scenarios is to look at their motivations: this somehow parallels a similar classification that can be drawn for works on brane worlds motivated by particle physics.

A first approach to brane cosmology, which one could call 'perturbative', studies possible deviations from the standard cosmological scenario. The non-observation of such deviations leads to constraints on the parameters of the models (or possibly, in the case of a deviation, to a cosmological 'discovery' of large extra-dimensions!). The reason to expect such deviations in braneworld models comes from a dramatic change in the way one is thinking about the gravitational interaction and its relation to the other fundamental interactions of high energy physics. This change of perspective, in particular, is leading to the fact that in braneworld models gravity can behave very differently in regimes beyond the regimes already explored in laboratory

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experiments. Namely, most braneworld models predict deviations from standard gravity at small length scales (but there are also models giving a deviation at large length scales). This is true, e.g., in the string-inspired brane worlds models of Arkani-Hamed, Dimopoulos and Dvali [1], where the recovery of gravity at usual length scales is ensured by the compacity of extra dimensions. It is also true in the model of Randall and Sundrum [2], where gravity gets localized in the vicinity of a 3-brane (seen as our 4-dimensional space–time), so that it behaves at large distances as in an ordinary 4-dimensional theory, because of the geometry of the bulk spacetime, which is a slice of anti de Sitter spacetime.[2]

Since the cosmological evolution of the Universe is driven by gravity, this change of perspective on gravity obviously leads to changes in the way one may envisage cosmology. Namely one expects the behavior of the Universe to be drastically modified in a primordial era corresponding to the energy scale associated with the above mentioned modification of gravity at short distance. This era can be very close to (indirectly) observable periods of the history of the Universe. This brings up cosmological signatures which could be observed in the near future, especially with the advent of so-called precision cosmology aiming at measuring cosmological parameters with an unprecedented accuracy.

In a second approach, one tries to use the brane set up to propose new ways to address old cosmological problems. One is often exploiting the freedom offered by the brane world construction to put various objects (such as other branes, black holes) or fields in the bulk. Some of the new scenarii, for example, depict dark matter as being placed on another brane, a sort of parallel world, in the bulk; others address in a novel way the cosmological constant problem by considering the effects of a scalar field in the bulk; a last example of this is given by constructions in which the Big Bang is seen as the collision of two branes, one being the brane where we live. In a usual seesaw motion, some of these scenarios, advocated by phenomenologists, have in fact increased the interest of researchers pursuing more formal developpements of string theory for issues such as the study of superstring theories in time dependent backgrounds.

A last category of works that we can mention here are less motivated by phenomenology or by solving cosmological puzzles, but are rather using the cosmology (or cosmological solutions) as theoretical tools for addressing more abstract questions. This concerns, e.g., works dealing with certain aspects of the so-called AdS-CFT correspondence, or works addressing certain aspects of ‘massive gravity’.

The above classification is of course very schematic and many works are belonging to several of its categories. One could also have distinguished between works trying to keep a close link to a very well circumscribed string construction, and others trying to capture some general properties of brane world cosmologies in the frameworks of toy models, further away from explicit string constructions, but reaching a larger scope.

2. Brane cosmology: a toy model

We will illustrate here brane cosmology (Fig. 1) with a 5-dimensional toy model consisting of a 3-brane with tension σ embedded in a 5-dimensional spacetime (from now on referred to as the ‘bulk’), empty but endowed with a cosmological constant, or vacuum energy, Λ_B. Matter with pressure p and energy density ρ is localized on the brane.

The fact that the brane is a hypersurface of codimension 1 allows to solve the system completely. Using the 5-dimensional Einstein equations and the Israel junction conditions which relate the discontinuities of the metric coefficients (or more precisely of their derivatives in the fifth direction orthogonal to the brane) to the matter localized on the brane, one may obtain a generalized Friedmann equation on the brane. This equation [3–6] gives the evolution of the cosmic scale factor $a_0(t)$ on the brane through the Hubble parameter $H \equiv \dot{a}_0(t)/a_0(t)$:

$$H^2 = \frac{\Lambda_B}{6}\Lambda_B + \frac{1}{36M_5^6}\sigma^2 + \frac{1}{18M_5^6}\sigma\rho + \frac{1}{36M_5^6}\rho^2 + \frac{\mathcal{C}}{a_0^4} - \frac{k}{a_0^2} \tag{1}$$

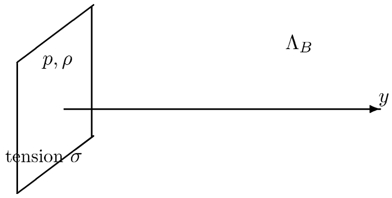

Fig. 1. A 3-brane with tension σ embedded in a 5-dimensional spacetime.

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[2] The anti de Sitter spacetime, AdS in short, is a maximally symmetric space time (with as many symmetries as Minkowski spacetime) with a negative cosmological constant.

with the 5-dimensional Newton's constant given by $8\pi G_5 \equiv M_5^{-3}$, and where k is the sign of the three-dimensional spatial curvature ($k = 0, \pm 1$ for a flat, elliptic or hyperbolic space respectively).

This should be compared with the standard Friedmann equation

$$H^2 = \frac{\lambda}{3} + \frac{1}{3M_P^2}\,\rho - \frac{k}{a^2}, \tag{2}$$

where λ is the cosmological constant and k the spatial curvature ($k = 0, \pm 1$ if space is flat, closed or open). Comparing (1) with (2), one may make several remarks.

First, the 4-dimensional cosmological constant λ receives contributions both from the bulk cosmological constant Λ_B and from the string tension σ (squared).

Comparing the linear terms in ρ allows a cosmological determination of the 4-dimensional Planck scale M_P

$$M_P^2 = 6\frac{M_5^6}{\sigma}. \tag{3}$$

In the brane universe case (1), there is a new contribution which is quadratic in ρ, the energy density of matter on the brane. This shows that the cosmology of a 4-dimensional brane universe is generically different from the cosmology of a 4-dimensional universe: an important difference is the notion of extrinsic curvature, i.e., the way the brane is curved within the higher-dimensional spacetime; the presence of localized energy induces extrinsic curvature. We note that, because the energy density of the universe increases as one goes back in time, the importance of the non-conventional ρ^2 term increases as well. Such a term may have an important role in the early universe, for example during or at the end of inflation.

Finally, the dark radiation term $\mathcal{C}/a_0^4$ represents the (cosmologically averaged) effect of bulk gravitons on the cosmological evolution of the brane. Its presence is a clear sign that the evolution on the brane cannot be decoupled from the evolution in the bulk. The brane does not represent a closed system.

Because of the cosmological symmetries (homogeneity and isotropy along three of the spatial directions), the five-dimensional vacuum Einstein equations can be solved easily in the bulk and yield a static solution, usually known as the AdS–Schwarzschild metric,

$$\mathrm{d}s^2 = -h(r)\,\mathrm{d}t^2 + r^2\,\mathrm{d}\Sigma_k^2 + \frac{1}{h(r)}\,\mathrm{d}r^2, \quad h(r) = k - \frac{\mathcal{C}}{r^2} + \frac{r^2}{\ell^2}, \tag{4}$$

where

$$\ell \equiv \sqrt{\frac{6}{|\Lambda_B|}} \tag{5}$$

is called the AdS_5 curvature radius. This simple result can be seen as a generalization of Birkhoff's theorem, which states that a spherically symmetric gravitational field in vacuum must be static, the metric being given by the Schwarzschild solution. With the static metric (4), the cosmological evolution given by (1) is simply due to the *motion* of the brane [7]. The metric also shows that the parameter $\mathcal{C}$ can be interpreted as the five-dimensional Schwarschild mass. Although $\mathcal{C}$ is constant when the bulk is strictly empty, the emission of gravitational radiation, due to brane matter fluctuations, is so strong during the high energy regime $\rho \gg \sigma$ that the corresponding energy outflow cannot be neglected and leads to an important growth of $\mathcal{C}$ [8].

But when do we expect to recover on the brane the 4-dimensional evolution described by (2)? In principle, whenever the physics on the brane is 4-dimensional. One may distinguish three different situations:

- The extra dimension is compact and its radius is stabilized. An example is Hořava–Witten supergravity where the 11th dimension radius is fixed [9]. Another is a two brane system with stabilized distance between them [10].
- The extra dimension is noncompact but the 4-dimensional graviton is localized. For example, in the Randall–Sundrum (RS-II) model [2], the extra dimension is warped, i.e., the cosmic scale factor has a dependence in the fifth coordinate:[3]

  $$a(y) = \mathrm{e}^{-|y|/\ell}. \tag{6}$$

  Spacetime is Minkowski (M_4) under the constraint that the cosmological constant vanishes, i.e., using (1) and (2),

  $$\frac{1}{3}\lambda = \frac{\Lambda_B}{6} + \frac{1}{36M_5^6}\sigma^2 = 0. \tag{7}$$

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[3] This may be seen from (4) with $k = \mathcal{C} = 0$ and $r = \ell\,\mathrm{e}^{-y/\ell}$.

This is known as the Randall–Sundrum constraint. Then the Planck scale of the effective theory is computed to be

$$M_P^2 = M_5^3 \int_{-\infty}^{+\infty} \mathrm{e}^{-2|y|/\ell}\,\mathrm{d}y = M_5^3 \ell. \tag{8}$$

The fact that the integral converges is directly connected with the presence of a normalizable graviton zero mode (the 4-dimensional graviton). We note that, even though the distance from the brane is allowed to be infinitely large in the bulk, the volume of the extra dimension is finite (and equal to ℓ). Indeed, (8) agrees with the standard expression for the Planck scale of the low energy 4-dimensional effective theory:

$$M_P^2 = M_5^3 L, \tag{9}$$

for a 5-dimensional theory with one compact dimension of size L.
The expression (8) agrees with the cosmological evaluation (3): using the Randall–Sundrum condition (7) we have

$$M_P^2 = M_5^3 \ell = M_5^3 \left(\frac{6}{|\Lambda_B|}\right)^{1/2} = \frac{6M_5^6}{\sigma}.$$

It should be stressed however that the determination of the Planck scale using the effective low energy and the Friedmann equation may differ [11]. This is for example what happens in the case of dS_4 or AdS_4 branes.

- The extra dimension has infinite size. In this case, (9) can no longer apply, even in an approximate way. Concurrently, there is no normalizable graviton zero mode. It is however possible [12,13] that the continuum of bulk zero modes produces a metastable state which reproduces the 4-dimensional graviton at small distances. However, since this state is metastable, one finds that gravity is modified at large (cosmological) distances, namely the metastable state decays and one recovers the 5-dimensional law of gravity. This type of models stays a bit aside the main stream of brane world models, e.g., the fact that gravity is modified at large distances constrasts with the usual brane world picture where gravity is only modified at small distances. However it can lead to interesting cosmological consequences exploiting the large distance modification of gravity. For example the model of [13] (for which the Friedmann's equation (1) does not apply) has the ability to produce a late time acceleration of the Universe (as seems to be required by recent observations of type Ia Supernovae) without the need for a nonvanishing cosmological constant [14]. This model has also been used [15] to investigate some aspects of "massive gravity" related to the so-called van Dam–Veltman–Zakharov discontinuity [16].

3. Cosmological perturbations

We have so far discussed only *homogeneous* brane cosmology. Realistic cosmology requires however to take into account deviations from strict homogeneity and isotropy. The analysis of the birth and evolution of cosmological perturbations has become an essential step for any scenario since the confrontation with cosmological observations is usually a drastic test. In the last years the observation of cosmological perturbations has entered a precision era with the measurement of the temperature fluctuations of the cosmic microwave background (first discovered in 1992 by the satellite COBE). These measurements will reach a remarkable level of precision with current or planned experiments in the next few years, in particular the WMAP and Planck satellites.

This opens the fascinating possibility to use these fluctuations as a window on the very early history of the Universe, but also as a probe of the very large scale behavior of gravity [17]. For this reason, a crucial program for the brane models is to quantify possible deviations from the standard picture that could leave a specific signature in those fluctuations. It is thus necessary to reconsider the theory of cosmological perturbations in the context of brane cosmology (see, e.g., [18] for a list of relevant references).

In the standard (4D with no brane) picture, the behaviour of cosmological pertubations follows from the perturbed 4D Einstein equations which can be written:

$$\delta G_{\mu\nu} = 8\pi G_N \delta T_{\mu\nu}, \tag{10}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ the matter energy momentum tensor. In the above equation, the perturbation are defined with respect to an isotropic and homogeneous cosmological background.

In the case of brane cosmology, the projection on the brane of the linearized Einstein's equations governing the evolution of cosmological perturbations can be written in the form [19]

$$\delta G_{\mu\nu}^{(4)} = \frac{\sigma}{6M_5^6}\delta T_{\mu\nu} + \delta \Pi_{\mu\nu} - \delta E_{\mu\nu}, \tag{11}$$

where the tensor $\Pi_{\mu\nu}$ is constructed solely from the brane energy-momentum tensor $T_{\mu\nu}$, and whose perturbations therefore depend only on the usual matter (and brane geometry) perturbations, whereas the tensor $\delta E_{\mu\nu}$ (the perturbation of the so-called projected Weyl tensor) embodies the effect of five-dimensional gravitational waves on the brane.

It is then clear that the modifications to the standard theory of cosmological perturbations can arise from two possible sources. The term $\delta\Pi_{\mu\nu}$ is negligible in the low energy regime and thus can affect the cosmological perturbations only in the very early universe, when the matter energy density is of the order or higher than the brane tension. The last term $\delta E_{\mu\nu}$ however must a priori be taken into account also in the low energy regime, and thus for recent cosmology. It can be interpreted as an effective energy-momentum tensor and thus means that five-dimensional gravitational waves will be seen by a brane cosmological observer, as some effective but invisible matter at the perturbative level with energy density, pressure (the equation of state being that of radiation) and anisotropic stress [20]. The main difficulty, which has prevented so far to give definite predictions, is that the brane projected equation (11), as such, does not provide the evolution equations on the brane for all the degrees of freedom of $\delta E_{\mu\nu}$. This means that one must solve for the bulk dynamics in order to know what is going on on the brane. This presumably needs to be done numerically.

The study of cosmological perturbations also requires some prescriptions about the initial conditions. In standard cosmology, these are given usually by some inflationary model. Similar attempts, but only partial until now, have been considered in brane cosmology. One idea for instance is to consider inflation induced by a scalar field confined in the brane leading to a quasi-de Sitter brane [21,22] which leads to scale-invariant initial conditions for the scalar and tensor perturbations [23,24], with relative amplitudes different from the standard inflation case if inflation takes place in the high energy regime. However, the general question of initial conditions for brane cosmology is still an open question.

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