Available online at www.sciencedirect.com
SCIENCE DIRECT®
C. R. Physique 4 (2003) 355–362
COMPTES RENDUS PHYSIQUE

Extra dimensions in physics and astrophysics/Dimensions supplémentaires en physique et astrophysique

Unification and physics with extra dimensions

Keith R. Dienes [a], Emilian Dudas [b,c,*], Tony Gherghetta [d]

[a] *Department of Physics, University of Arizona, Tucson, AZ 85721, USA*
[b] *LPT (Unité Mixte de Recherche du CNRS-UMR 8627), Université de Paris-Sud, bâtiment 210, 91405 Orsay cedex, France*
[c] *CPT, École Polytechnique, 91128 Palaiseau, France*
[d] *School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA*

Presented by Guy Laval

Abstract

The physical motivations and consequences of large extra dimensions are reviewed in light of recent developments, in particular mm-sized extra dimensions, gauge coupling unification and neutrino masses. ***To cite this article: K.R. Dienes et al., C. R. Physique 4 (2003).***

Résumé

Unification et physique des dimensions supplémentaires. Nous passons en revue les motivations physiques et les conséquences des grandes dimensions d'espace supplémentaires, en particulier l'unification des couplages et les masses des neutrinos. ***Pour citer cet article : K.R. Dienes et al., C. R. Physique 4 (2003).***

1. Introduction

It was noticed [1,2] that in Type I strings the string scale can be lowered all the way down to the TeV range. Similar ideas appeared for lowering the fundamental Planck scale in theories with (sub)millimeter gravitational dimensions [3,4], as an alternative solution to the gauge hierarchy problem, and, simultaneously, a new way for lowering the GUT scale in theories with large (TeV) dimensions [5,6] was proposed. The new emerging picture found a simple realization in a perturbative Type I setting [7] with low string scale (in the TeV range) and became the subject of an intense activity, mostly on the phenomenological side, but also on the theoretical side. We review here some of the salient features of this class of models and their phenomenological and experimental consequences.

2. Millimeter and TeV^{-1} large extra dimensions

The presence of branes in String Theory opens new perspectives for particle physics phenomenology. Indeed, in Type I strings the string scale is not necessarily tied to the Planck scale. In view of the new D-brane picture, let us take a closer look at the simplest example of compactified Type I string, with only D9 branes present. The string scale can be in the TeV range if the

* Corresponding author.
E-mail address: Emilian.Dudas@th.u-psud.fr (E. Dudas).

1631-0705/03/$ – see front matter
doi:10.1016/S1631-0705(03)00041-0

string coupling is extremely small, $\lambda_I \sim 10^{-32}$. One can see that in this case the compact volume is very small $VM_I^6 \sim 10^{-32}$. Let us split the compact volume into two parts, $V = V^{(1)}V^{(2)}$, where $V^{(1)}$, of dimension $6 - n$, is of order one in string units and $V^{(2)}$, of dimension n, is very small. The Kaluza–Klein states of the brane fields along $V^{(2)}$ are much heavier than the string scale and therefore are difficult to excite. The physics is then better captured in this case performing T-dualities along $V^{(2)}$, which can be written:

$$\lambda_I' = \frac{\lambda_I}{V^{(2)}M_I^n}, \qquad V_\perp = \frac{1}{V^{(2)}M_I^{2n}}. \tag{1}$$

In the T-dual picture, neglecting numerical factors, we find

$$M_P^2 \sim \frac{1}{\alpha_{\mathrm{GUT}}\lambda_I'} V_\perp M_I^{2+n}, \qquad \frac{1}{\alpha_{\mathrm{GUT}}} \sim \frac{V_\parallel M_I^{6-n}}{\lambda_I'}, \tag{2}$$

where for transparency of notation we redefined $V^{(1)} \equiv V_\parallel$. After the n T-dualities, the D9 brane becomes a D($9 - n$) brane, since the T-dual winding modes of the bulk (orthogonal) compact space are very heavy and therefore the brane fields cannot propagate in the bulk. As seen from (2), for a very large bulk volume the string scale can be very low $M_I \ll M_P$. The geometric picture here is that we have a D-brane with some compact radii parallel to it, of order M_I^{-1}, and some very large, orthogonal compact radii. In particular, if the full compact space is orthogonal to the brane ($n = 6$), from (2) the T-dual string coupling is fixed by the unified coupling $\lambda_I' \sim \alpha_{\mathrm{GUT}}$, and therefore we find [7]

$$M_P^2 \sim \frac{1}{\alpha_{\mathrm{GUT}}^2} V_\perp M_I^{2+n}, \tag{3}$$

a relation similar to that proposed in the field-theoretical scenario of [3,4].

Let us now imagine a 'brane-world' picture,[1] in which the Standard Model gauge group and charged fields are confined to the D-brane under consideration. We can then ask a very important question: what are the present experimental limits on parallel $R_\parallel$ and perpendicular $R_\perp$ type radii? The Standard Model fields have light KK states in the parallel directions $R_\parallel$. Their possible effects in accelerators were studied in detail [15,16] and the present limits are $R_\parallel^{-1} \geqslant$ 4–5 TeV. On the other hand, Standard Model excitations related to $R_\perp$ are very heavy and are thus irrelevant at low energy. The main constraints on $R_\perp$ come from the presence of very light KK gravitational excitations, which can therefore generate effects in colliders [17–19] and deviations from the Newton law of gravitational attraction. The actual experimental limits on such deviations are limited to the mm range and experiments in the near future are planned to improve them [20]. There are also collider effects coming from string physics at energies close to M_I [21–23]. For $M_I \sim$ TeV in (3), the case of only one extra dimension is clearly excluded, since it asks for $R_\perp^{-1} \sim 10^8$ Km. However, for two extra dimensions, we find $R_\perp^{-1} \sim 1$ mm, only marginally excluded by the present experimental data. On the other hand, if all compact dimensions are perpendicular and large, one finds $R_\perp^{-1} \sim$ fm, a distance scale completely inaccessible for Newton law measurements. Such a physical picture with $M_I \sim$ TeV provides in principle a new solution to the gauge hierarchy problem, i.e., of why the Higgs mass M_h is much lower than the 4d Planck mass M_P, provided the physical cutoff M_I has similar values $M_I \simeq M_h$.

In Type I strings [24–26], the brane we considered can be a D9 or a D5 brane, up to T-dualities. Our brane world can live on any of the branes; let us choose for concreteness that our Standard Model gauge group be on a D9 brane. Notice that, while D9 branes fill (before T-dualities) the full 10d space, D5 branes fill only six dimensions. The D5 degrees of freedom can of course propagate in what we called previously bulk space, and can change slightly our previous picture. The relation between the corresponding D9 and D5 gauge couplings is:

$$\frac{g_9^2}{g_5^2} = V_\perp, \tag{4}$$

where $V_\perp$ denotes here (before T-dualities) the compact volume perpendicular to the D5 brane. If $V_\perp \gg 1$ in string units, then D5 branes live in (at least part of) the bulk and, by (4) their gauge coupling is very suppressed compared to our (D9) gauge coupling. In particular, if $V_\perp$ in (4) is as in (3), the D5 gauge couplings are of gravitational strength. The fields in mixed 95 representations are charged under both gauge groups. Then, due to their very small gauge couplings, the D5 gauge groups manifest themselves as global symmetries on our D-brane, and could be used for protecting baryon and lepton number non-conservation processes. Indeed, global symmetries are presumably violated by non-renormalizable operators suppressed by the fundamental scale M_I and, since M_I can be very low, we need suppression of many higher-dimensional operators.

[1] For earlier proposals of such a 'brane-world' picture, see [8–14].

There are clearly many challenging questions that such a scenario must answer in order to be seriously considered as an alternative to the conventional 'desert picture' of supersymmetric unification at energies of order 10^{16} GeV. The gauge hierarchy problem still has a counterpart here, understanding the possible mm size of the compact dimensions (perpendicular to our brane) in a theory with a fundamental length (energy) in the 10^{-16} mm (TeV) range. There are several ideas concerning this issue in the literature, which, however, need further studies in realistic models in order to prove their viability. A serious theoretical question concerns gauge coupling unification, that in this case, if it exists, must be completely different from the conventional MSSM (Minimal Supersymmetric Standard Model) one. Moreover, there is more and more convincing evidence for neutrino masses and mixings, and the conventional picture provides an elegant explanation of their pattern via the seesaw mechanism with a mass scale of the order of the 10^{12}–10^{15} GeV, surprisingly close to the usual GUT scale. Cosmology, astrophysics, accelerator physics and flavor physics put additional strong constraints on the low-scale string scenario.

3. Gauge coupling unification

Models with gauge-coupling unification at low energy triggered by Kaluza–Klein states were proposed in [5,6]. It is clear that low-scale string models are the natural framework for this fast-driven unification.

The essential ingredient in this proposal are the KK excitations of the Standard Model gauge bosons and matter multiplets and their contribution to the energy evolution of the physical gauge couplings. In the early paper [27], Taylor and Veneziano pointed out that the KK excitations give power-law corrections that at low energy can be interpreted as threshold corrections. Actually, as shown in [5,6], if the energy is higher than the KK compactification scale $1/R$, these corrections should be interpreted as power-law accelerated evolutions of gauge couplings that, under some reasonable assumptions, can bring these couplings together at low energies.

Let us start, for reasons to be explained later on, with the MSSM in 4d and try to extend it to 5d, where the fifth dimension is a circle of radius $R_\parallel$, in the notation introduced in the previous section. In this case the evolution of gauge couplings is governed by [5,6]

$$\frac{1}{\alpha_a(\mu)} = \frac{1}{\alpha_a(\mu_0)} + \frac{1}{2\pi}\sum_r \mathrm{Str}\int_{1/\mu^2}^{1/\mu_0^2} \frac{\mathrm{d}t}{t} Q_{a,r}^2\left(\frac{1}{12}-\chi_r^2\right)\left(\sum_n \mathrm{e}^{-tm_{n,r}^2(R_\parallel)} + \mathrm{e}^{-tm_r^2}\right), \tag{5}$$

where we separated the mass operator into a part containing fields with KK modes and a part containing fields without KK modes. In (5), $Q_{a,r}$ is the gauge group generator in the representation r of the gauge group, m_r^2 is the mass operator and χ_r is the helicity of various charged particles contributing in the loop. Indeed, consider again for concreteness gauge couplings of a D9 brane and consider δ large compact dimensions $R_\parallel M_I \gg 1$ parallel to D9 and orthogonal to D5. Then the 99 states will have associated KK states, but 95 states will not. Evaluating (5) with $\mu_0 = M_Z$, one finds

$$\begin{aligned}\frac{1}{\alpha_a(\mu)} &= \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} - \frac{\tilde{b}_a}{2\pi}\int_{1/\mu^2}^{1/M_Z^2}\frac{\mathrm{d}t}{t}\theta_3^\delta\left(\frac{\mathrm{i}t}{\pi R_\parallel^2}\right)\\ &\simeq \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} + \frac{\tilde{b}_a}{2\pi}\ln(\mu R_\parallel) - \frac{\tilde{b}_a}{2\pi}\big[(\mu R_\parallel)^\delta - 1\big].\end{aligned} \tag{6}$$

The coefficients $\tilde{b}_a$ in (6) denote one-loop beta-function coefficients of the massive KK modes, to be computed in each specific model. The important term contained in (6) is the power-like term $(\mu R_\parallel)^\delta \gg 1$, which overtakes the logarithmic terms in the higher-dimensional regime and governs the eventual unification pattern.

The power-like term is proportional to the coefficients $\tilde{b}_a$, that *are not* the usual 4d MSSM ones which successfully predict unification. Let us, however, go on and find the *minimal* possible embedding of the MSSM in a 5d spacetime. Before doing so, notice that compactifying on a circle a supersymmetric theory in 5d gives a 4d theory with at least $\mathcal{N}=2$ supersymmetries. The simplest way to avoid this is to compactify on an *orbifold*. We consider as an example the case of a Z_2 orbifold which breaks supersymmetry down to $\mathcal{N}=1$. 5d fields can be even or odd under this operation, in particular 5d Dirac fermions in 4d truncate into one even Weyl fermion containing a zero mode and its KK tower and one odd Weyl fermion, with no associated zero mode, and its KK tower. It is easy to realize that a 4d chiral multiplet (ψ_1, ϕ_1) can arise from a 5d hypermultiplet containing KK modes $(\psi_1^{(n)}, \psi_2^{(n)}, \phi_1^{(n)}, \phi_2^{(n)})$ or from a 5d vector multiplet. Similarly, a 4d massless vector multiplet (λ, A_μ) arises from a 5d vector multiplet containing the KK modes $(\lambda^{(n)}, \psi_3^{(n)}, A_\mu^{(n)}, a^{(n)})$, where $\psi_i^{(n)}$, $i=1,2,3$, are 4d Weyl fermions and $\phi_i^{(n)}$,

$a^{(n)}$ are complex scalars. The massive KK representations are clearly nonchiral, while chirality is generated at the level of zero modes.

The simplest embedding of the MSSM in 5d is the following [5,6]. The gauge bosons and the two Higgs multiplets of the MSSM are already in real representations of the gauge group and naturally extend to KK representations $(\lambda^{(n)}, \psi_3^{(n)}, A_\mu^{(n)}, a^{(n)})$ and $(\psi_1^{(n)}, \psi_2^{(n)}, H_1^{(n)}, H_2^{(n)})$, respectively.[2] The matter fermions of MSSM, being chiral, can either contain only zero modes or, alternatively, can have associated mirror fermions and KK excitations for $\eta = 0, 1, 2, 3$ families. The unification pattern does not depend on η (the value of the unified coupling, on the other hand, does), since each family forms a complete SU(5) representation. The massive beta-function coefficients for this simple 5d extension of the MSSM are

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = \left(\frac{3}{5}, -3, -6\right) + \eta(4, 4, 4), \tag{7}$$

where, as usual, we use the SU(5) embedding $\tilde{b}_1 \equiv (3/5)\tilde{b}_Y$. These coefficients in the case $\eta = 3$ are not the same as the MSSM ones $(b_1, b_2, b_3) = (33/5, 1, -3)$. However, interestingly enough, as seen from Fig. 1, the couplings unify with a surprisingly good precision, for any compact radius 10^3 GeV $\leqslant R_\parallel^{-1} \leqslant 10^{15}$ GeV, at a energy scale roughly a factor of 20 above the compactification scale $R_\parallel^{-1}$. The algebraic reason for this is that, in order to have MSSM unification, the conditions that must be fulfilled are:

$$\frac{B_{12}}{B_{13}} = \frac{B_{13}}{B_{23}} = 1, \quad \text{where } B_{ac} \equiv \frac{\tilde{b}_a - \tilde{b}_c}{b_a - b_c}. \tag{8}$$

Although these relations are not satisfied exactly in our case, they are nonetheless approximately satisfied $B_{12}/B_{13} = 72/77 \simeq 0.94$, $B_{13}/B_{23} = 11/12 \simeq 0.92$.

This fast unification with KK states is another numerical miracle, similar to the MSSM unification and may be regarded as one serious hint pointing into the possible relevance of extra dimensions in our world. The usual MSSM unification is a limiting case of our more general scenario in the limit where R^{-1} approaches $10^{16}y$ GeV (the usual GUT scale). There are clearly many questions that this scenario can raise, which were discussed in detail in the literature [5,6,28–30] and will not be discussed here.

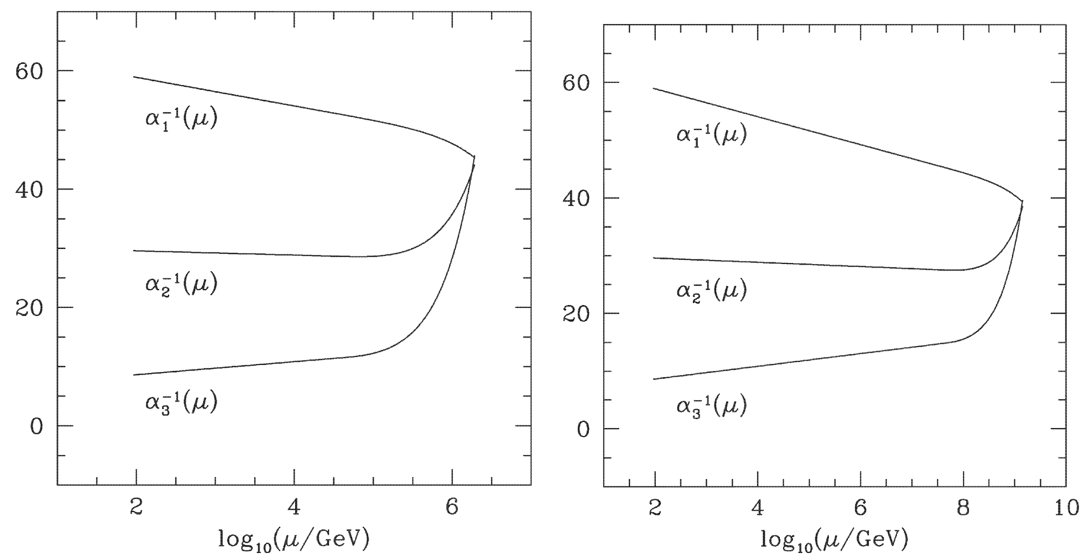

Fig. 1. Unification of gauge couplings in the presence of extra spacetime dimensions. We consider two representative cases: $R^{-1} = 10^5$ GeV (left), $R^{-1} = 10^8$ GeV (right). In both cases we have taken $\delta = 1$ and $\eta = 0$.

[2] As one of the two Higgses in a hypermultiplet is odd under Z_2, the simplest extension actually has one KK Higgs hypermultiplet and one Higgs without KK excitations.

4. Bulk physics: neutrino masses with large extra dimensions

There is more and more convincing evidence for the existence of neutrino masses and mixings. Any extension of the Standard Model should therefore address this question, at least at a qualitative level. The most elegant mechanism for explaining the smallness of neutrino masses postulates the existence of right-handed neutrinos with very large Majorana masses 10^{11} GeV $\leqslant M \leqslant 10^{15}$ GeV. Via the seesaw mechanism, very small neutrino masses, of the order of $m_\nu \sim v^2/M$, are generated, where $v \simeq 246$ GeV is the vev of the Higgs field. This suggests the presence of a large (intermediate or GUT) scale in the theory, related to new physics. On the other hand, low-scale string models do not have such a large scale and therefore, superficially, have problems to accommodate neutrino masses.

The scenario we present here is based on the observation that right-handed neutrinos can be put in the bulk of a very large (mm size) compact space [31–33], perpendicular to the brane where we live. Consider for simplicity the case of one family of neutrinos. The model consists of our brane with the left-handed neutrino ν_L and Higgs field confined to it and one bulk Dirac neutrino, $\Psi = (\psi_1, \bar\psi_2)^{\mathrm{T}}$ in Weyl notation, invading a space with (again for simplicity) one compact perpendicular direction y. The compact direction is taken here to be an orbifold S^1/Z_2, since as is well known circle compactifications are not phenomenologically realistic. The Z_2 orbifold acts on the spinors as $Z_2\Psi(y) = \pm\gamma_5\Psi(-y)$, so that one of the two-component Weyl spinors, ψ_1, is even under the Z_2 action $y \to -y$, while the other spinor ψ_2 is odd. If the left-handed neutrino ν_L is restricted to a brane located at the orbifold fixed point $y = 0$, ψ_2 vanishes at this point and so ν_L couples only to ψ_1. This then results in a Lagrangian of the form

$$\mathcal{L} = -\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}y\, M_s\big\{\overline{\Psi}\,\mathrm{i}\gamma^M\partial_M\Psi - \partial_M\overline{\Psi}\,\mathrm{i}\gamma^M\Psi\big\} - \int \mathrm{d}^4x\,\big\{\bar\nu_L\,\mathrm{i}\bar\sigma^\mu D_\mu\nu_L + (\widehat{m}\nu_L\psi_1|_{y=0} + \text{h.c.})\big\}. \tag{9}$$

Here M_s is the mass scale of the higher-dimensional fundamental theory (a reduced Type I string scale) and the space–time index M runs over all five dimensions: $x^M \equiv (x^\mu, y)$. The first line describes the kinetic-energy term for the 5d Ψ field, while the second line describes the kinetic energy of the 4d two-component neutrino field ν_L, as well as the coupling between ν_L and ψ_1. Note that in 5d, an even (under Z_2) bare Dirac mass term for Ψ would not have been invariant under the action of the Z_2 orbifold, since $\overline{\Psi}\Psi \sim \psi_1\psi_2 +$ h.c.

Now compactify the Lagrangian (9) down to 4d, expanding the 5d Ψ field in Kaluza–Klein modes. The orbifold relations $\psi_{1,2}(-y) = \pm\psi_{1,2}(y)$ imply that the Kaluza–Klein decomposition takes the form

$$\psi_1(x,y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=0}^{\infty}\psi_1^{(n)}(x)\cos\left(\frac{ny}{R}\right), \qquad \psi_2(x,y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=1}^{\infty}\psi_2^{(n)}(x)\sin\left(\frac{ny}{R}\right). \tag{10}$$

However, a more general possibility emerges naturally from the Scherk–Schwarz compactification. Recall that our original 5d Dirac spinor field Ψ is decomposed in the Weyl basis as $\Psi = (\psi_1, \bar\psi_2)^{\mathrm{T}}$, where ψ_1 and ψ_2 have the mode expansions given in (10). Let us consider performing a local rotation in (ψ_1, ψ_2) space of the form

$$\begin{pmatrix}\hat\psi_1\\ \hat\psi_2\end{pmatrix} \equiv U\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \quad \text{where } U \equiv \begin{pmatrix}\cos(\omega y/R) & -\sin(\omega y/R)\\ \sin(\omega y/R) & \cos(\omega y/R)\end{pmatrix}, \tag{11}$$

with ω an (for the moment) arbitrary real number. The effect of the matrix U in (11) is to twist the fermions after a $2\pi R$ rotation on y. Such twisted boundary conditions, as we have seen, are allowed in field and in string theory if the higher-dimensional theory has a suitable U(1) symmetry. The 4d Lagrangian of the component fields coming from the 5d Lagrangian is found from (9) by replacing everywhere $\psi_i \to \hat\psi_i$.

For convenience, let us define $M_0 = \omega/R$ and the linear combinations $N^{(n)} \equiv (\psi_1^{(n)} + \psi_2^{(n)})/\sqrt{2}$ and $M^{(n)} \equiv (\psi_1^{(n)} - \psi_2^{(n)})/\sqrt{2}$ for all $n > 0$. We also define in the following the effective Dirac neutrino mass couplings

$$m \equiv \frac{\widehat{m}}{\sqrt{2}\sqrt{\pi M_s R}}. \tag{12}$$

In the Lagrangian (9), the Standard Model neutrino ν_L mixes with the entire tower of Kaluza–Klein states of the higher-dimensional Ψ field. Indeed, if for simplicity we restrict our attention to the case of only one extra dimension, define

$$\mathcal{N}^{\mathrm{T}} \equiv \big(\nu_L, \psi_1^{(0)}, N^{(1)}, M^{(1)}, N^{(2)}, M^{(2)}, \ldots\big), \tag{13}$$

and integrate over the compactified dimension, we see that the mass terms in the Lagrangian (9) take the form $(1/2)(\mathcal{N}^{\mathrm{T}}\mathcal{M}\mathcal{N} +$ h.c.), where the mass matrix is

$$\mathcal{M} = \begin{pmatrix} 0 & m & m & m & m & m & \cdots \\ m & M_0 & 0 & 0 & 0 & 0 & \cdots \\ m & 0 & M_0 + 1/R & 0 & 0 & 0 & \cdots \\ m & 0 & 0 & M_0 - 1/R & 0 & 0 & \cdots \\ m & 0 & 0 & 0 & M_0 + 2/R & 0 & \cdots \\ m & 0 & 0 & 0 & 0 & M_0 - 2/R & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{14}$$

Let us start for simplicity by disregarding the possible bare Majorana mass term, setting $M_0 = 0$. In this case, the characteristic polynomial which determines the eigenvalues λ of the mass matrix (14) can be worked out exactly and takes the form:

$$\left[\prod_{k=1}^{\infty}\left(\frac{k^2}{R^2} - \lambda^2\right)\right]\left[\lambda^2 - m^2 + 2\lambda^2 m^2 R^2 \sum_{k=1}^{\infty} \frac{1}{k^2 - \lambda^2 R^2}\right] = 0, \tag{15}$$

which is clearly invariant under $\lambda \to -\lambda$. From this we immediately see that all eigenvalues fall into *degenerate*, pairs of opposite sign. In order to solve this eigenvalue equation, it is convenient to note that $\lambda = k/R$ is never a solution (unless of course $m = 0$), as the cancellation that would occur in the first factor in (15) is offset by the divergence of the second factor. We are therefore free to disregard the first factor entirely, and focus on solutions for which the second factor vanishes. The summation in the second factor can be performed exactly, resulting in the transcendental equation

$$\lambda R = \pi (mR)^2 \cot(\pi \lambda R). \tag{16}$$

All the eigenvalues can be determined from this equation, as functions of the product mR. This equation can be analyzed graphically [31], and in the limit $mR \to 0$ (corresponding to $m \to 0$), the eigenvalues are k/R, $k \in Z$, with a double eigenvalue at $k = 0$. Conversely, in the limit $mR \to \infty$, the eigenvalues with $k > 0$ shift smoothly toward $(k + 1/2)/R$, while those with $k < 0$ shift smoothly toward $(k - 1/2)/R$. Finally, the double zero eigenvalue splits toward the values $\pm 1/(2R)$. The overlap between the light mass eigenstates and the neutrino gauge eigenstate is generically less than half in this scenario. The important prediction of this scenario is that the gauge neutrino and the (lightest) sterile neutrino are degenerate in mass, a possibility that can be experimentally tested.

Let us now return to the more general case $M_0 \neq 0$. To this end, it is useful to define

$$k_0 \equiv [M_0 R], \qquad \varepsilon \equiv M_0 - \frac{k_0}{R}, \tag{17}$$

where $[x]$ denotes here the integer nearest to x. Thus, ε is the smallest diagonal entry in the mass matrix (14), corresponding to the excited Kaluza–Klein state $M^{(k_0)}$. In other words, $\varepsilon \equiv M_0$ (modulo R^{-1}) satisfies $-1/(2R) < \varepsilon \leqslant 1/(2R)$. The remaining diagonal entries in the mass matrix can then be expressed as $\varepsilon \pm k'/R$, where $k' \in Z^+$. Reordering the rows and columns of our mass matrix, we can therefore cast it into the form

$$\mathcal{M} = \begin{pmatrix} 0 & m & m & m & m & m & \cdots \\ m & \varepsilon & 0 & 0 & 0 & 0 & \cdots \\ m & 0 & \varepsilon + 1/R & 0 & 0 & 0 & \cdots \\ m & 0 & 0 & \varepsilon - 1/R & 0 & 0 & \cdots \\ m & 0 & 0 & 0 & \varepsilon + 2/R & 0 & \cdots \\ m & 0 & 0 & 0 & 0 & \varepsilon - 2/R & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{18}$$

While this is of course nothing but the original mass matrix (14), the important consequence of this rearrangement is that the *heavy* mass scale M_0 has been replaced by the *light* mass scale ε. Unlike M_0, we see that $|\varepsilon| \sim \mathcal{O}(R^{-1})$. Thus, the heavy Majorana mass scale M_0 completely *decouples* from the physics. Indeed, the value of M_0 enters the results only through its determinations of k_0 and the precise value of ε. Therefore, interestingly enough, the presence of the infinite tower of regularly-spaced Kaluza–Klein states ensures that only the value of M_0 modulo R^{-1} plays a role.

The easiest way to solve (18) for the eigenvalues $\lambda_\pm$ is to integrate out the Kaluza–Klein modes. It turns out that there are two relevant cases to consider, depending on the value of ε. If $|\varepsilon| \gg m$ (which can arise when $mR \ll 1$), *all* of the Kaluza–Klein modes are extremely massive relative to m, and we can integrate them out to obtain an effective $\nu_L \nu_L$ mass term of size:

$$|\varepsilon| \gg m\colon\ m_\nu = \frac{m^2}{\varepsilon} + m^2 \sum_{k'=1}^{\infty}\left(\frac{1}{\varepsilon + k'/R} + \frac{1}{\varepsilon - k'/R}\right) = \pi m^2 R \cot(\pi R \varepsilon). \tag{19}$$

We shall discuss the special case $\varepsilon = 1/(2R)$ later on. Alternatively, if $|\varepsilon| \ggg m$, the lightest Kaluza–Klein mode $M^{(k_0)}$ should not be integrated out, and the end result is an effective $\nu_L \nu_L$ mass term of size μ, where

$$|\varepsilon| \ggg m\colon\ \mu \equiv -m^2 \sum_{k'=1}^{\infty} \left(\frac{1}{\varepsilon + k'/R} + \frac{1}{\varepsilon - k'/R} \right) = \frac{m^2}{\varepsilon} - \pi m^2 R \cot(\pi R \varepsilon). \tag{20}$$

Note that $\mu \to 0$ smoothly as $\varepsilon \to 0$, with μ otherwise of size $\mathcal{O}(m^2 R)$. Diagonalizing the final 2×2 mass matrix mixing ν_L and $M^{(k_0)}$ in the presence of this mass term then yields

$$|\varepsilon| \ggg m\colon\ \lambda_\pm = \frac{1}{2}\Big[(\mu + \varepsilon) \pm \sqrt{(\mu - \varepsilon)^2 + 4m^2}\Big]. \tag{21}$$

We therefore conclude that, although we may have started with a bare Majorana mass $M_0 \gg R^{-1}$, in all cases the final neutrino mass remains of order $m^2 R$. Even though we might have expected a neutrino mass of order m^2/M_0 from the mixing between ν_L and the original zero-mode $\psi_1^{(0)}$, the contribution m^2/M_0 from the zero-mode is completely canceled by the summation over the Kaluza–Klein tower, while the seesaw between ν_L and $M^{(k_0)}$ becomes dominant.

In string theory, however, there are additional topological constraints that permit only *discrete* values of M_0. Taking $M_0 = 1/(2R)$ then implies $\psi_{1,2}(2\pi R) = -\psi_{1,2}(0)$, which shows that lepton number is broken globally (although not locally) as the spinor is taken around the compactified space. It is straightforward to show that when $\varepsilon = 1/2R$, the characteristic eigenvalue equation $\det(\mathcal{M} - \lambda I) = 0$ for the mass matrix (14), (18) becomes

$$\lambda R \left[\prod_{k=1}^{\infty} \left(\lambda^2 R^2 - \left(k - \frac{1}{2} \right)^2 \right) \right] \left[1 - 2m^2 R^2 \sum_{k=1}^{\infty} \frac{1}{\lambda^2 R^2 - (k - 1/2)^2} \right] = 0. \tag{22}$$

This has an exact trivial solution $\lambda = 0$, corresponding to an exactly massless neutrino. Indeed, the characteristic polynomial for the mass matrix in this case has the form

$$\lambda R = -\pi (mR)^2 \tan(\pi \lambda R). \tag{23}$$

It is then clear than the zero eigenvalue is always present, irrespective of the value of the radius. In fact, by changing the value of M_0, we see that it is possible to smoothly *interpolate* between the scenario with $M_0 = 0$ and the scenario we are discussing here [31]. This also provides another explanation of why only the value $\varepsilon \sim M_0$ (modulo R^{-1}) is relevant physically. The regular, repeating aspect of the infinite towers of Kaluza–Klein states is now manifested graphically in the periodic nature of the cotangent function.

The scenario(s) presented have also other interesting consequences. The neutrino eigenstate can now oscillate into an infinite tower of right-handed KK neutrinos with a probability that can be reliably estimated and experimentally tested. Moreover, even if in the last scenario presented the physical neutrino is massless, its probability of oscillation into the tower of KK states is nonvanishing. In particular, a neutrino mass difference $\Delta m \sim 10^{-2}$ eV, that fits the experimental data, could well be explained by an oscillation of the massless neutrino into the first KK state, for a radius $R^{-1} \sim 10^{-2}$ eV, precisely in the mm region we are interested in! Thus, we see that neutrino oscillations do not require neutrino masses in higher dimensions. Finally, let us mention that recent SNO data strongly disfavors oscillations of an active neutrino into a sterile one. The scenario mentioned above involves oscillations into several sterile neutrinos and, as such, is not directly excluded by the recent data. More effort is needed, however, in order to check the viability of this scenario in light of the new solar neutrino data.

Another possible explanation for neutrino masses in low scale string models, without invoking bulk sterile neutrinos, was recently proposed in [34], by producing a neutrino mass of the order $m_\nu \sim M_Z^4/M_I^3$. Finally, it should be mentioned that a similar formalism, but with axions in the bulk, can potentially be used to provide possible new mechanisms for axion invisibility [3,4,35,36].

References

[1] E. Witten, Nucl. Phys. B 471 (1996) 135.
[2] J.D. Lykken, Phys. Rev. D 54 (1996) 3693.
[3] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B 429 (1998) 263.
[4] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Rev. D 59 (1999) 086004.
[5] K.R. Dienes, E. Dudas, T. Gherghetta, Phys. Lett. B 436 (1998) 55.
[6] K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B 537 (1999) 47.
[7] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B 436 (1998) 257.
[8] V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 125 (1983) 139.

[9] V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 125 (1985) 372.
[10] K. Akama, in: N. Nakanishi, H. Nariai (Eds.), Lecture Notes in Phys., Vol. 176, Springer-Verlag, 1982, p. 267.
[11] K. Akama, Prog. Theor. Phys. 78 (1987) 184.
[12] M. Visser, Phys. Lett. B 159 (1985) 22.
[13] P. Horava, E. Witten, Nucl. Phys. B 460 (1996) 506.
[14] P. Horava, E. Witten, Nucl. Phys. B 475 (1996) 94.
[15] I. Antoniadis, Phys. Lett. B 246 (1990) 377.
[16] I. Antoniadis, K. Benakli, M. Quiros, Phys. Lett. B 331 (1994), hep-ph/9403290.
[17] G.F. Giudice, R. Rattazzi, J.D. Wells, Nucl. Phys. B 544 (1999) 3.
[18] E.A. Mirabelli, M. Perelstein, M.E. Peskin, Phys. Rev. Lett. 82 (1999) 2236.
[19] T. Han, J.D. Lykken, R.J. Zhang, Phys. Rev. D 59 (1999) 105006.
[20] J.C. Long, H.W. Chan, J.C. Price, Nucl. Phys. B 539 (1999) 23.
[21] E. Dudas, J. Mourad, Nucl. Phys. B 575 (2000) 3, hep-th/9911019.
[22] S. Cullen, M. Perelstein, M.E. Peskin, Phys. Rev. D 62 (2000) 055012, hep-ph/0001166.
[23] I. Antoniadis, K. Benakli, A. Laugier, JHEP 0105 (2001) 044, hep-th/0011281.
[24] A. Sagnotti, in: G. Mack, et al. (Eds.), Non-Perturbative Quantum Field Theory, Cargese ’87, Pergamon Press, Oxford, 1988, p. 521.
[25] G. Pradisi, A. Sagnotti, Phys. Lett. B 216 (1989) 59.
[26] M. Bianchi, G. Pradisi, A. Sagnotti, Nucl. Phys. B 376 (1992) 365.
[27] T.R. Taylor, G. Veneziano, Phys. Lett. B 212 (1988) 147.
[28] K.R. Dienes, E. Dudas, T. Gherghetta, hep-ph/9807522.
[29] Z. Kakushadze, Nucl. Phys. B 548 (1999) 205.
[30] Z. Kakushadze, T.R. Taylor, Nucl. Phys. B 562 (1999) 78.
[31] K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B 557 (1999) 25.
[32] S. Dimopoulos, Talk at SUSY98 conference, July 1998.
[33] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, J. March-Russell, Phys. Rev. D 65 (2002) 024032.
[34] E. Dudas, C.A. Savoy, hep-ph/0205264.
[35] S. Chang, S. Tazawa, M. Yamaguchi, Phys. Rev. D 61 (2000) 084005, hep-ph/9908515.
[36] K.R. Dienes, E. Dudas, T. Gherghetta, Phys. Rev. D 62 (2000) 105023, hep-ph/9912455.