

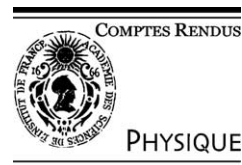


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Exotic nuclei/Les noyaux exotiques

## A personal view of some issues involved in the structure of nuclei with large neutron excess

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### Abstract

Large neutron excess poses new issues concerning the competition of deformation and pairing as well as changes in nuclear shell structure. **To cite this article:** *I. Hamamoto, B.R. Mottelson, C. R. Physique 4 (2003).*

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### Résumé

**Les points-clef de la structure des noyaux exotiques possédant un fort excès de neutrons.** Les noyaux possédant un fort excès de neutrons posent de nouveaux challenges : quel est l'effet de la compétition entre l'appariement et la déformation ? Comment la structure en couche se transforme-t-elle quand le déséquilibre entre protons et neutrons est grand ? **Pour citer cet article :** *I. Hamamoto, B.R. Mottelson, C. R. Physique 4 (2003).*

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One of the most active and challenging areas of current study in nuclear structure concerns the properties of nuclei far from the line of  $\beta$  stability. In this brief personal view of the subject we shall confine our attention to the neutron rich side despite the significant interest that is also associated with the proton side (nuclear astrophysical importance, neutron–proton correlations in the  $N \approx Z$  region, new opportunities provided by  $p$  and  $2p$  decay, among others). We concentrate on the neutron side because of its special importance for nuclear synthesis, and the challenging shell structure issues provided by the weakly bound neutrons which may move outside the proton core for an appreciable fraction of the time. In addition there is the hope that the study of neutron rich nuclei may provide information relevant to the structure of neutron stars.

A special problem with the weakly bound neutron systems involves the ever present and nearby continuum of unbound states. The low-lying states of the continuum are of course of immediate importance for pairing, deformations, and threshold strength functions. In addition the higher energy states of the continuum are displaced with respect to the corresponding states of the proton continuum and thus the iso-spin structure of giant resonances are significantly affected.

The spectroscopy of nuclei close to the  $\beta$ -stable region reveals two sharply distinguished coupling schemes reflecting the symmetry of the mean field potential. For configurations close to closed shells, the quasiparticles are labeled by quantum numbers reflecting the spherical symmetry of the mean field potential. For configurations more removed from closed shells the mean field potential may acquire a non-spherical shape and the quasiparticle spectrum is significantly modified. The quasiparticles are labeled by quantum numbers that reflect the remaining symmetries of the system (for example,  $\Omega\pi$ , for the states in an axially symmetric quadrupole deformation as in the Nilsson diagram [1]). The presence of the broken symmetry is in these cases strikingly apparent in the rotational band structure appearing in the low energy spectrum. The deformations

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result from the degeneracies in the spherical one-particle potential which make possible highly anisotropic basis states. The gain in energy from the attractive interactions between the valence particles can thus be increased even when the system is described by a single Slater determinant, by including the anisotropic terms in the average potential. Thus in the absence of other correlations, configurations involving even a few particles outside closed shells would be expected to exhibit significant deformations. However, it is observed that around the closed shell configurations of  $\beta$ -stable nuclei there are appreciable regions in which the mean field retains spherical symmetry as shown by the quantum numbers of the low energy excitations, despite the presence of a number of particles outside closed shells. This effect is especially marked for nuclei in which either the neutrons (or protons) form closed shells while the protons (or neutrons) may have many particles outside closed shells. The occurrence of these surviving islands of spherical symmetry are attributed to the effect of the special correlations which tend to build like particles into correlated pairs with angular momentum  $J = 0$  (as in the correlated wave functions employed by Bardeen, Cooper and Schrieffer [2] in their description of superconductivity of electrons in metals). In such a pair the anisotropies of the two one-particle states exactly cancel and the resulting system retains spherical symmetry. Thus, the pairing and deformations can be seen as two different and competing modes of correlation for particles moving in open shell configurations, and in particular the survival of the islands of spherical nuclei can be seen as a result of the dominance of the pair correlation over the tendency to deformation of the mean field. For configurations in which the equilibrium shape is not spherical, this circumstance is manifested by the occurrence of rotational band structure. Pair correlation may still be present, though reduced in strength, and is seen from its effect in reducing the moment of inertia below the rigid-body value. The competition between pairing and deformation is expected to be an important theme also in the regions far from  $\beta$  stability, with however, the modifications implied by changes in the shell structure, the radial wave functions, and the presence of the nearby continuum that are encountered in the neighborhood of the neutron drip line.

The current discussion of neutron halo structure has especially focused attention on the approximate decoupling of the halo particles from the core of the nuclear system. This results from the extreme difference in the radial motion of weakly bound  $\ell = 0$  and  $\ell = 1$  neutrons as compared with the radial distribution of the core particles. The effect of this decoupling on pairing, deformation, and collective rotation poses new problems that need to be addressed and which provide special interest to the expanding investigations of the drip line nuclei.

Fig. 1, showing the variation of the energy eigenvalues of single-particle orbits as a function of the parameter  $(R/r_0)^3$  of the Woods–Saxon potential, where  $R$  expresses the radius and  $r_0 = 1.27$  fm, exhibits patterns that are important for the description of weakly-bound neutron systems [3]. Energy eigenvalues of the orbits with  $\ell \gtrsim 2$  approach zero binding as a smooth continuation of those of deeply bound orbits. But  $\ell = 0$  neutrons may have an appreciable probability of being outside the region of the potential as the binding energy becomes small. (Indeed as  $\varepsilon \rightarrow 0$ , the probability of  $\ell = 0$  neutrons being outside,  $P_{\text{out}}(\ell = 0)$ , approaches unity.) This radial structure implies that the dependence of the eigenvalue of weakly bound orbits on the strength (radius) of the potential is much weaker for  $\ell = 0$  than for the  $\ell \gtrsim 3$  orbits. The  $\ell = 1$  orbit exhibits a similar but smaller effect in the weak binding limit ( $P_{\text{out}}(\ell = 1) = 2/3$  for  $\varepsilon = 0$  for a finite square-well potential and considerably larger

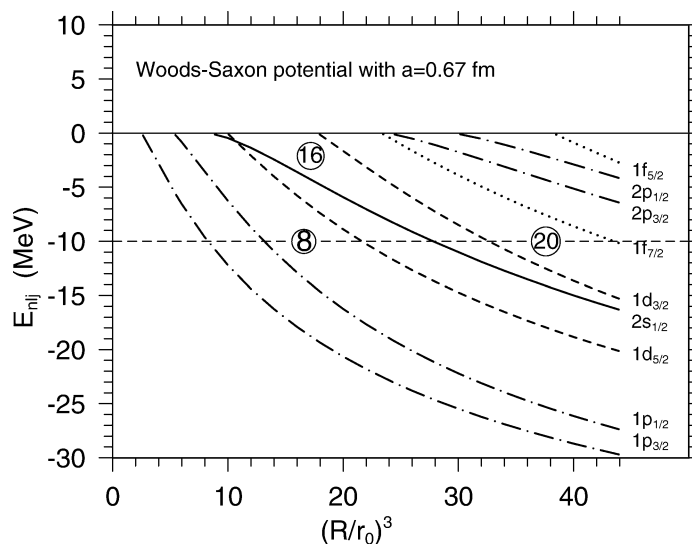


Fig. 1. Energies of neutron orbits for the Woods–Saxon potential with standard parameters in  $\beta$ -stable nuclei [4], as a function of  $(R/r_0)^3$  where  $R$  expresses the radius and  $r_0 = 1.27$  fm. The potential parameters for  $N = Z$  are used. The neutron number obtained by occupying the levels from the bottom is indicated for  $N = 8, 16$  and  $20$  with a circle.

than  $2/3$  for diffuse potentials). These  $\ell$ -dependent variations of the single-particle energies imply significant changes in the shell structure, namely in the ordering of one-particle levels in the weakly-bound neutron systems [3]. In a harmonic oscillator potential the single particle eigenvalues depend only on the combination  $2n + \ell$  of the radial and angular quantum numbers. For deeply bound orbits in potentials with well defined surface these degeneracies are lifted with the eigen energy decreasing as  $\ell$  increases. For a finite square-well potential the level order persists even to the limit of zero binding energy. However, for a potential with diffuse surface the weakly bound low- $\ell$  orbits are better able to exploit the gradual decrease of the attractive potential at the surface and thus the sequence of the former harmonic oscillator degeneracy can become reversed as compared with the sharp surface potential, with the low- $\ell$  orbits having now the lowest energy (see Fig. 1). One may expect that in neutron drip line nuclei the effective diffuseness of the potential can be larger than  $a = 0.67$  fm which is used in Fig. 1. Examining Fig. 1 and Fig. 2–30 of [4], it is seen that the neutron number  $N = 8, 20, 28$  and  $50$  may no longer exhibit closed shell effects in neutron drip line nuclei, as a result of the rearrangement of the sequence of single neutron orbits in the region of very small binding.

Since a role in pairing and deformation is confined to orbits within a few MeV of the Fermi energy, the difference between such an orbit as a bound state or being in the continuum is rather minor provided the orbital angular momentum is appreciable. For Woods–Saxon potentials with standard parameters we have, for example, for  $A \sim 100$  a neutron with  $\ell = 4$  (3) and energy  $2$  MeV in the continuum has a width  $\Gamma \sim 0.04$  ( $\sim 0.6$ ) MeV corresponding to a centrifugal barrier of  $10$  (6) MeV. However for  $\ell = 0$  and  $1$  a significant difference is found for orbits in the low energy continuum. The  $\ell = 0$  single particle resonances have decay widths equal to or larger than their energy as soon as the energy becomes of order  $100$  keV while for  $\ell = 1$  the resonance may still play a role in pairing and deformation up to the energies of a few hundreds keV but with a much reduced weight. In deformed nuclei the consequence of the decay of unbound  $\ell = 0$  orbits is considerably extended by the fact that all  $\Omega\pi = 1/2+$  intrinsic states acquire  $\ell = 0$  components induced by the deformation, and thus these orbits will also decay at relatively low energies. Similar comments apply to the decay of  $\ell = 1$  orbits that affects the  $\Omega\pi = 1/2-$  and  $3/2-$  deformed states.

Though there is no centrifugal potential barrier for  $\ell = 0$  neutrons, low energy  $\ell = 0$  neutrons may exhibit sharp resonances if the spectroscopic factor is sufficiently small (as observed for neutrons with eV energies incident on heavy nuclei). In  $\beta$ -stable nuclei with neutron separation energy  $6$ – $8$  MeV, the level density in the relevant excitation energy region is very high. Nevertheless, the clear  $\ell = 0$  ‘resonance’ structure with both the width and the resonance energy of order electron volts, which is about  $10^{-6}$  times the single-particle estimate, is observed in almost all nuclei. In neutron drip line nuclei there may be a chance to observe the ‘resonance’ phenomena of  $\ell = 0$  neutrons coming from the same mechanism not with  $10^{-6}$  times the single-particle unit but with the strength of the order of single-particle estimate. Unfortunately, the low-energy neutron scattering on neutron drip line nuclei (or the one using the inverse kinematics) is not at the present time an accessible experimental technique. However, examining the neutron shell-structure around zero binding, possible candidates, in which  $\ell = 0$  neutron orbits may lie slightly above zero binding, are, for example, around  ${}^{12}_4\text{Be}_8$  for the  $2s_{1/2}$  orbit and  ${}^{60}_{20}\text{Ca}_{40}$  for the  $3s_{1/2}$  orbit.

Among the correlation effects in neutron rich nuclei it is useful to distinguish the two-neutron halo in drip line configurations from the larger group of weakly bound ( $S_n \lesssim 4$  MeV) neutron excess systems. The former structures are special in the sense that the partial decoupling of the last bound neutron pair may provide useful simplification for the interpretation of the low energy structure. The latter configurations provide systems in which the neutron structure involves many body issues that go beyond the solution of a two body problem. When the last two neutrons are effectively decoupled from the core the dynamics of these two neutrons can be approximately treated in terms of a problem involving an effective two body interaction and a truncation of the basis states to a small energy interval in the neighborhood of the Fermi surface. A useful scheme for analyzing this problem has been formulated in the pioneer work by Bertsch and Esbensen [5] and applied to the low energy spectrum of nuclei in the neighborhood of  ${}^{11}\text{Li}$ . When the conditions for separation of the pair from the core are satisfied this scheme should provide a valuable tool for analyzing the low energy spectrum of the halo nuclei. However, it should be emphasized that in most drip line configurations it may be expected that more than two neutrons will be involved in the low energy dynamics and thus more general issues in particular the competition between pairing and deformation must be taken into account.

The element of competition between pairing and shape deformation cannot be defined for two-particle configurations. Thus the effect of weak binding on this competition requires the examination of more general configurations involving the interactions of more than two valence nucleons. The competition between pairing and deformation involves partly the effective interactions that are acting and partly the available single-particle configurations provided by the mean field. It must be expected that for neutrons moving in the very neutron-rich environment near the nuclear surface, the effective interactions may be significantly different from those that apply to particles immersed in the more usual nuclear medium where the neutron and proton densities are comparable. This expectation is based on the fact that the strongest attractive terms are provided by the neutron–proton interactions. Thus the mean field will be significantly less attractive in the neutron skin than in more normal nuclear matter. The pairing interaction is related to the  ${}^1\text{S}$  phase shift of the colliding particles and since this phase shift is a rapidly decreasing function of the relative velocity of the particles, this interaction will be stronger for the slowly moving neutrons in the neutron skin than for the more energetic collisions in the nuclear interior. The large- $\ell$  orbits which have much smaller extension into the region outside the nucleus will be less able to exploit this effect even when the binding energy of the orbit is small. For

small  $\ell$  and small binding the neutrons spend ample time in the region outside the nucleus but now their contribution to the many-body pair correlation is reduced because their radial motion fails to overlap with the pair field created by the rest of the neutrons. These effects have to be evaluated with careful consideration of the effect of the nearby continuum on pair correlation in the exotic system [6], together with competing deformation which is associated with the long-range part of the effective interactions. The net result of these different and conflicting effects on pairing/deformation competition for weakly bound neutrons is a challenging and presently poorly understood problem.

## References

- [1] S.G. Nilsson, *Mat. Fys. Medd. Dan. Vid. Selsk.* 29 (16) (1955).
- [2] J. Bardeen, L.N. Cooper, J.R. Schrieffer, *Phys. Rev.* 106 (1957) 162;  
J. Bardeen, L.N. Cooper, J.R. Schrieffer, *Phys. Rev.* 108 (1957) 1175.
- [3] I. Hamamoto, S.V. Lukyanov, X.Z. Zhang, *Nucl. Phys. A* 683 (2001) 255.
- [4] A. Bohr, B.R. Mottelson, in: *Nuclear Structure, Vol. I*, Benjamin, Reading, MA, 1969.
- [5] G.F. Bertsch, H. Esbensen, *Ann. Phys. (NY)* 209 (1991) 327.
- [6] M. Grasso, N. Sandulescu, Nguyen Van Giai, R.J. Liotta, *Phys. Rev. C* 64 (2001) 064321.