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C. R. Physique 4 (2003) 767–774

New frontiers in the Solar System: trans-Neptunian objects/Les nouvelles frontières du système solaire : les objets transneptuniens

A statistical insight into the Edgeworth-Kuiper belt

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Presented by Pierre Encrenaz

Abstract

Ten years after the discovery of the first object entirely outside the orbit of Neptune, the number of detected Edgeworth-Kuiper Belt objects (EKBO) is close to 800: after the discovery of the asteroid 1 Ceres, it took 115 years to discover the same number of asteroids. These large comets dressed as asteroids are very elusive objects, challenging the observers with their faintness. As well as the comets and the asteroids, this group of objects represent a valuable source of information on the physical and chemical environment of the Solar System at the epoch of planet growth. In this paper we summarize the results of the first statistical studies of the bulk physical and chemical properties of EKBO, based on broad band photometry data. ***To cite this article: M. Fulchignoni, A.C. Delsanti, C. R. Physique 4 (2003).***

Résumé

Un aperçu statistique de la ceinture d'Edgeworth-Kuiper. Dix ans après la découverte du premier objet orbitant au-delà de Neptune, le nombre d'objets de la ceinture de Edgeworth-Kuiper détectés avoisine 800 : après la découverte de 1 Ceres, il a fallu en revanche attendre 115 ans pour découvrir un nombre équivalent d'astéroïdes de la ceinture principale. Ces grandes comètes à l'apparence d'astéroïdes sont des objets dont la faible luminosité est un défi pour les observateurs. Ce groupe représente – comme les comètes et les astéroïdes – une source d'information précieuse sur l'environnement physique et chimique du système solaire au moment de la formation des planètes. Dans cet article, nous récapitulons les résultats des premières analyses statistiques des propriétés physiques et chimiques des objets de la ceinture de Kuiper basées sur des données de photométrie à large bande. ***Pour citer cet article : M. Fulchignoni, A.C. Delsanti, C. R. Physique 4 (2003).***

Keywords: Solar system; Small bodies; Trans-Neptunians; Centaurs

Mots-clés : Système solaire ; Petits corps ; Trans-neptuniens ; Centaures

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1. Introduction

With the increasing number of multi-colour studies, it becomes now possible to perform statistical analysis in order to highlight possible correlations between colours and physical parameters. Such results may give some hints about physical and chemical properties of different groups of objects and give indications on the existence of a possible taxonomic scheme, in analogy with the asteroid population.

Most of the data available to date comes from broadband photometry. B, V, R, I data have been obtained for one hundred objects and for fifty or so of them J data are also available. These magnitudes can be also converted into relative reflectivities in

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doi:10.1016/j.crhy.2003.09.013

order to build low resolution reflectance spectra (sampled at the wavelengths of the broadband filters used), that provide a first glance of the shape of the real (primarily featureless in the optical) spectra. Due to the extreme faintness of the objects, spectra are available for less than two dozen of objects (the most in the range 0.36 to 2.2 µm).

EKBOs display a broad and continuous distribution of visible colours from neutral (solar reflected colours) to very red, which is also illustrated in visible spectra by a wide range of spectral slopes. A first hypothesis to explain such a diversity is a different intrinsic composition of the objects. Although it is not very likely (accounting for the small equilibrium temperature gradient across the Kuiper belt), this hypothesis cannot be ruled out yet. A second possibility is that the objects, initially believed to have neutral colours (due to the expected presence of fresh ices), suffer different resurfacing processes during their evolution, leading to a variety of surface colours. The use of different statistical tests that will be described in this paper is an important tool to cast some light on the possible relations between surface colours, properties and physical and orbital parameters. These tests may allow us to isolate families of objects with respect to their surface properties, as well as giving some clues about the importance of the different resurfacing processes that are believed to play a role in EKBOs surface evolution.

Another tricky question that may be solved with the use of statistics is the possible bimodality of the visible colour distributions of EKBOs. A group (Tegler and Romanishin, 1998 [1]) found two families of objects within a sample of 13 objects: one with solar colors, clearly separated from another group of objects with very red colours. With the increasing number of data points, this result has not been confirmed by the subsequent photometric studies; in addition it is difficult to understand in terms of physical processes for the globality of EKBOs classes. However, bimodality in the colour distributions may also happen for individual dynamical subclasses: a recent statistical study from Peixinho et al. [2] show a possible significant bimodality in the Centaurs visible colours. A model by Delsanti et al. [3] gives some clues about physical processes leading to such a bimodality.

We review below the different statistical tools that has been applied to date to cast some light on the physical properties and possible taxonomical classes of EKBOs.

2. Uni- and bivariate analyses

2.1. The MBOSS colour database

In the perspective of statistical studies, Hainaut and Delsanti [4] compiled EKBOs measured magnitudes available in the literature in a database maintained up-to-date in an on-line version (referred later in the paper at the MBOSS, Minor Bodies of the Outer Solar System colour database, http://www.sc.eso.org/~ohainaut/MBOSS/). Colour indices were computed with great care, i.e., using only simultaneous magnitudes from a same epoch (defined as a few hours) by the same observer. Published colour indexes computed from magnitudes of different epochs were *not* entered in the database. Then, for a given object, the obtained colours from the different observers were combined in a weighted average (the weight of a measurement was set to $1/\sigma$, therefore trusting the published error bars). In this way, a measurement with a large error have a small contribution to the final average value. For each object, an averaged absolute magnitude was also computed, as well as a spectral gradient over the VRI range (which is the slope in %/100 nm of the reflectivity spectrum reconstructed from the broadband photometry, see Delsanti et al. [5] and references therein). The dispersion of the measurements before averaging is similar to the error bars, suggesting that no dramatic systematic effects affect the different teams measurements.

Various statistical tests were performed on this dataset (that encompasses to date colors for $\sim$150 objects); the more relevant results are summarized below.

2.2. Correlations: colours versus physical and orbital parameters

From a dynamical point of view, EKBOs are currently classified as trans-Neptunian objects (classical objects with quasi-circular orbits with small inclinations with respect to the ecliptic plane, resonant objects trapped in mean motion resonances with Neptune, scattered disk objects on very eccentric orbits) and the Centaurs (orbiting beyond Jupiter and inside the orbit of Neptune) which are trans-Neptunians injected in the inner part of the solar system, considered as the source of the Jupiter family comets. We refer the reader to the paper by Davies [6] for a more detailed definition of the EKBOs classes.

Pearson's correlation was applied [4] to the MBOSS database described above to search for correlations between all colour indexes (and spectral gradient) versus orbital parameters (semi-major axis a, eccentricity e, inclination i, as well as orbit excitation (defined as $\varepsilon = \sqrt{e^2 + \sin^2 i}$) and size (quantified by the absolute magnitude). If we consider the several classes of EKBOs all together, the colours and gradient are not correlated with any of the orbital parameters a, e, i, ε. For only classical objects, there is a systematic anti-correlation between the visible colours (and spectral gradient) with the orbit inclination and orbit energy ε: objects with higher inclination and orbital energy are bluer, the effect is the strongest for B–R.

Doressoundiram et al. [7], checked for relations between visible colours and rms excitation defined by $V_{\text{rms}} = V_k(e^2 + i^2)^{1/2}$, where V_k is the Keplerian velocity of the object. Spearman's rank correlation statistics gives $r_{\text{corr}} = 0.72$ for 22 classical

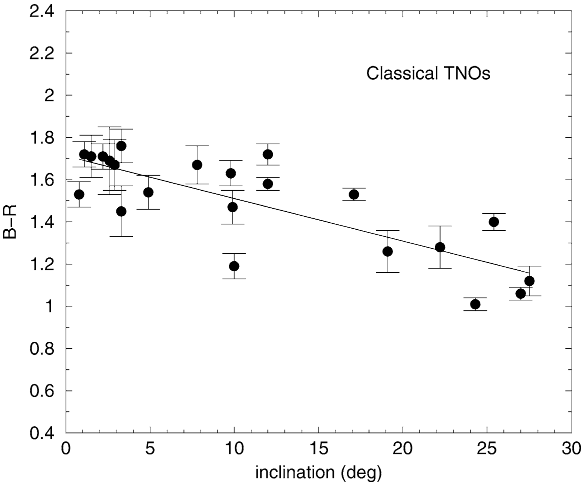

Fig. 1. Plot of inclination versus B–R colour for classical objects. A correlation ($r_{\text{corr}} = 0.72$, 3.8σ significance) exists only for the classical objects. This correlation points out that dynamically excited classical objects tend to have less red surfaces. The least-squares line has been superposed to the observed data (from Doressoundiram et al. [7] published with permission of the author, and of the American Astronomical Society).

objects, which corresponds for Gaussian statistics to a 3.8σ significance level. This correlation, illustrated in Fig. 1, points out that dynamically excited classical objects tend to have less red surfaces.

Student's t-test and Fisher's F-test have been also applied to the EKBOs data [4]. The t-test checks whether the means of two continuous, uni-dimensional distributions (for instance V–R colours of two groups of EKBOs) are significantly different. The direct interest of each estimator in itself is limited, a much more interesting value is derived from the estimator: the probability (*Prob*) that the statistical estimator is as large as measured by chance. Small values of *Prob* indicates that the chances of getting the observed estimator by chance while extracting the two samples from the same distributions are small, or, in other words, that the two samples are not statistically compatible. F-test evaluates whether two distributions have significantly different variances.

The Student's t-test applied to classical objects reveals a strong, highly significant trend: objects with higher ε and i are significantly bluer (*Prob* in the 10^{-3} range). This effect is systematic in all colours and spectral gradient. This support the result presented by Tegler and Romanishin [8] 2000, Doressoundiram et al. 2001 [9] and quantified by Trujillo et al. 2001 [10]. They interpreted this as evidence for low ε objects being less affected by (collisional) re-surfacing than objects with a high excitation (because the collisions are on average less violent, and/or less numerous). This trend vanishes for Centaurs, resonant and scattered disk objects.

The F-test also points out that classical objects with higher orbital excitation, eccentricity and inclination have significantly broader visible colour distributions than the others. The t-test also indicates that resonant objects with a fainter absolute magnitude (i.e., the smallest) are slightly bluer (the significance is quantified by a 10^{-2} to 10^{-1} *Prob* for all colour indexes and gradient).

2.3. Population comparisons

Mean colours of the different classes of objects (e.g., classical, resonant, scattered disk objects, Centaurs) were computed. The colour of an object is function of the nature of its surface as well as the current stage of resurfacing. For a given population, the mean colour will therefore gives information on the stage of competition between the aging (reddening by high energy particle bombardment) and the different rejuvenating processes (such as for instance non-disruptive collisions between EKBOs, dust re-deposition or gas re-condensation after a cometary activity event).

Hainaut and Delsanti [4] checked for incompatibilities between colours of the different classes of objects using the Student's t-test (cf. paragraph above). It appeared that V–R, R–I and spectral gradient of classical objects are incompatible (with a high significance of 10^{-3}), as well as, to a lesser extent, V–I. Infrared colours cannot be tested properly as the available data sample is too small. Also B–V, and less significantly B–R colours of classical and scattered disk objects are not compatible. Apart from these particular cases, tests show that – to date – there is no significant incompatibilities between colours of classical, resonant objects and Centaurs.

Obviously, the whole information from a distribution is not contained in its two first moments (mean and variance). A more complete comparison of the colour distributions can be done for instance with the Kolmogorov–Smirnoff (KS) test in which two samples are compared through their complete Cumulative Probability Function (*CPF*). A quantity d is computed which is the maximum (vertical) distance between the two *CPF*s of the distributions compared. The associated probability Prob gives the probability to get a d larger than the observed one while dealing with data sets extracted from the same distribution. Small values of *Prob* indicate that the distributions compared are incompatible. The KS test confirms to a significant level ($\sim 10^{-3}$) the incompatibility of B–R and B–V of classical and scattered disk objects. This result is also present, less significantly for V–R and R–I index, which tends to indicate that visible colours of classical and scattered EKBOs may not be compatible.

2.4. Distribution bimodality

B–V, V–R, and V–I data are available to date for a hundred EKBOs and for fifty or so of them J data are also available, while only forty or so asteroids have U–B and B–V colours determined. Despite this small number of asteroid colours (∼2% of the population known at that time) Wood and Kuiper [11] suggested, on the basis of the observed data, the existence of two different compositional classes clustering around the colours indices of the Moon and of the Sun. These groups were the ancestors of the S and C classes known today. Tegler and Romanishin [1] noted two groups of objects by analysing the B–V and V–R s of 13 EKBOs (∼20% of the known EKBOs population at that time) one very red and the other quite neutral with respect to the Sun.

The question of bimodality of EKBOs colour distributions have been claimed again by Tegler and Romanishin [12]. On the other hand, various multi-colour photometric surveys [3,7,13–15] (see the above quoted on-line MBOSS database for a fairly complete listing of the other references) found a broad, continuous distribution of colours from neutral to very red. Tegler and Romanishin argued that their result comes from more accurate photometric measurements. However, most of the photometric studies are now performed on 8 m-class telescopes with excellent S/N ratios, and the dispersion of the measurements of the various groups in the optical is rather small.

Several teams probed the significance of the claimed bimodality of B–V and V–R EKBOs distributions through different methodologies and statistical tests. Jewitt et al. [16] 2001 applied three tests to the data samples published in 1998 and 2000 by Tegler and Romanishin [1,8] such as the bin test, the dip test and the interval distribution test. They projected all the points of the B–V versus V–R space along a line of principal variation and tested the null hypothesis that the colours are distributed randomly along this line. They determined three equally sized bins and used the bin test, which quantify the probability that objects of the central colour bin are extracted from a uniform distribution. Tegler and Romanishin's 1998 sample had a 3% likelihood (corresponding to a 2.2σ confidence level) to be drawn by chance from a uniform distribution. It goes up to 13% for the 2000 and $1998 + 2000$ samples: the significance of the result increases with the increasing number of measurements, which points out a lack of evidence for a bimodal distribution of visible colours. The dip test [17] was applied to the same samples. The dip test computes the maximum difference between the observed distribution and the unimodal distribution that minimizes that maximum difference. In other terms, the larger the dip, the larger the incompatibility. Jewitt et al. reported that the dip test does not support any bimodality for the merged $1998 + 2000$ dataset, although the (smallest) 1998 sample alone displays a non-significant (less than 3σ) indication for bimodality. Lastly they performed the interval distribution test. In a real bimodal distribution, colour intervals between modes are larger than intervals between members of the mode, whereas for a continuous distribution all intervals are expected to be roughly equal. Jewitt et al. generated Monte Carlo distributions in which they picked colours from a uniform distribution to assess the likelihood that the large interval observed in Tegler and Romanishin's colours might happen by chance from random uncorrelated data. At the 3σ confidence level, the results of the interval distribution test are consistent with random sampling of a uniform colour distribution. The absence of bimodality in the colour distribution is interpreted as a hint for collisional resurfacing of the EKBOs.

Davies et al. [14] 2000 also checked for the optical-infrared V–J colours distributions and found not hint for bimodality.

In Hainaut and Delsanti [4] 2002, all observed colour distributions (e.g., not only B–V and V–R) were compared with 1D and 2D simple models of continuous and bimodal distributions whose parameters were fitted to maximize the Kolmogorov–Smirnoff probability (e.g., to maximize the compatibility of the compared real and model distributions). These tests indicate that in no case is there enough data to rule out the validity of simple, continuous distributions. This does not mean that the colour distributions are continuous, but one has to be extremely careful if saying that they are not.

Until recently, the number of measured colours of Centaurs did not allow one to perform any statistical tests. By visually checking the colours distributions, it appears that the Centaurs tend to have rather bimodal distributions [4,5,17,18]. Peixinho et al. [2] considered a sample of 20 Centaurs merged from their own photometric data and the MBOSS database. They are the first to quantify the significance of the Centaurs bimodality, and the dip test gives a confidence level above 3σ. This result deserves to be followed and confirmed by further studies implying a larger number of Centaurs. A model of EKBOs surface evolution by Delsanti et al. [3] provides some hints to explain such a bimodality.

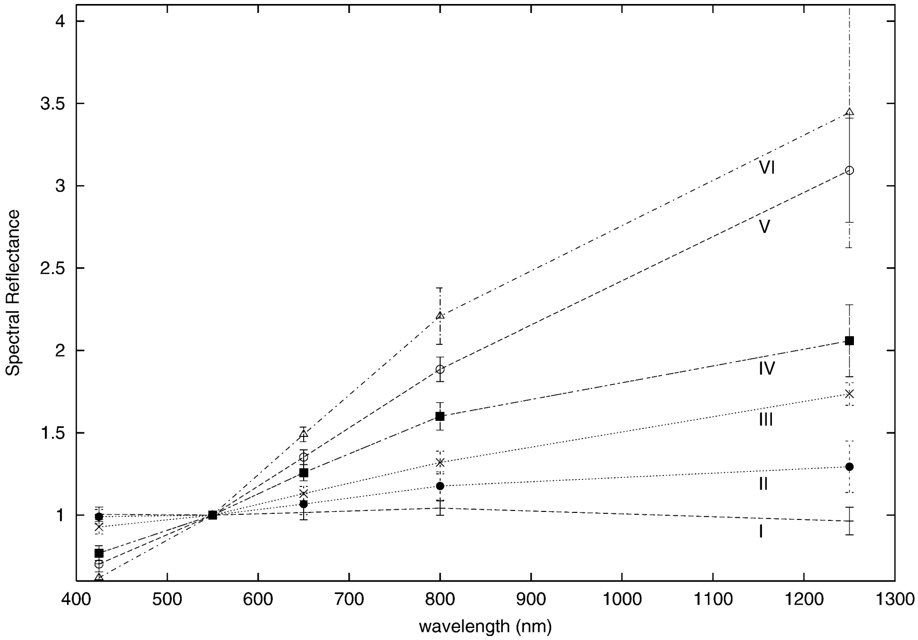

Fig. 2. The average broad band reflectance spectra (normalized to the V band) for the groups found by the G-mode analysis. The error bar is the standard deviation of the reflectance mean value within each group. The group spectra spread from the neutral (group I) to the very red one (group VI).

3. Multivariate analyses

In the early 1980s a few thousand asteroids were known, a large survey [19] was devoted to measure in eight filters (range 0.35–1.0 µm) the sunlight reflected by several hundred asteroids (∼10% of the known objects). Then the IRAS data [20] provided the albedo data for most of these objects. Tholen [21] and Barucci et al. [22] used these data bases in classifying the asteroids by means of independent multivariate statistical techniques (the principal components analysis based on the eight colours data, and the G-mode analysis based on the eight colours and IRAS albedo data, respectively). Both the methods provide a similar classification scheme which constitute the bulk of the current asteroid taxonomy, separating the asteroid population in a dozen compositionally homogeneous groups. Barucci et al. [23] and Fulchignoni et al. [24] applied the same techniques to two samples of 22 and 34 EKBOs respectively (∼5% of the EKBOs population known when the analyses have been done) characterized by 4 colours (B–V, V–R, V–I and V–J) obtained by the same observers, during the same run, or inter-calibrated through the V measurements. The results indicate a clear compositional trend within the examined sample and suggest the possible existence of some homogeneous groups.

3.1. The G-mode analysis

The same sample has been analysed with the G-mode multivariate statistics [25,26], evidencing the existence of a finer structure of the sample. The G-mode statistics analyse our sample of 34 objects (described by 4 variables, the colours B–V, V–R, V–I and V–J). The total number of degrees of freedom (136) allow us to use this kind of statistical analysis. The goal of the analysis is to find groups of objects having homogeneous behaviour in terms of their variables. The method provides a quantitative estimation of the weight of each variable in separating the groups.

The G-mode analysis has been carried out transforming the colour data in reflectance values by $R_{c_\lambda} = 10^{\pm 0.4(c_\lambda - c_{\lambda\,\mathrm{sun}})}$ where c_λ and $c_{\lambda\,\mathrm{sun}}$ are the λ–V colours of the object and of the Sun, respectively. The method separated the 34 objects in six groups at a significance level of 99%. In Fig. 2 the average broadband reflectance spectra (normalized to the V bands) for the groups found by the G-mode analysis are reported.

Group I (formed by 2 classical objects and 1 Centaur – Chiron – the only one for which cometary activity has been observed) contains the objects having reflectance spectra neutral with respect the Sun. The objects of group II (one out of each EKBOs class) have a higher value of V–J, which distinguish this group from the previous one. Group III (1 scattered disk, 3 Centaurs and an "unusual" – Halley comet type?) and group IV (5 classic, 3 resonant, 1 scattered disk and 1 Centaur) clearly distinguished each other by the V–I colour while V–J separate these two groups from all the other groups their spectra are redder than those

Table 1
Eigenvectors, eigenvalues and percentage of total variance contributed by each eigenvalue

Variable	Eigenvector 1	Eigenvector 2	Eigenvector 3	Eigenvector 4
B–V	0.3029	0.4733	0.8208	−0.1025
V–R	0.2171	0.2266	−0.3223	−0.8931
V–I	0.4469	0.6236	−0.4701	0.4365
V–J	0.8133	−0.5794	0.0386	0.0367
Eigenvalues	0.2828	0.0185	0.0023	0.0007
% Total variance	92.93	6.08	0.75	0.24

of group I and group II. Group V (3 classical, 5 resonant, 1 Centaur and 1 scattered disk) contains objects still redder and group VI is formed by the reddest objects (2 Centaurs) of the solar system. V–R and V–I separate the two last groups, which have the same dramatic reddening, completely different from the trend of all the other spectra.

(32 929) 1995QY9 (scattered disk) and (35 671) 1998SN165 (classic), characterized by the lower PC2 scores, remain isolated forming two '*single object groups*' in a similar way (4) Vesta, (1862) Apollo and (349) Dembowska formed the V, Q and R classes in the Tholen asteroid taxonomy [21] (dozens of small asteroids now populate the V class and some new objects have been added to the R and Q classes today [27,28]).

The relative weights of the variables in structuring the EKBO's sample in these groups are 38% for V–J, 30% V–I, 17% V–R and 15% B–V. The V–J colour discriminate the groups each other at a high significance level ($>3\sigma$), V–I plays the same role at a slightly lower level. Minor contributions are provided by V–R and B–V.

Summarizing, G-mode analysis has allowed us to distinguish six groups of homogeneous objects and two single class objects, as far as the colour behaviour is concerned.

The significance level (99%) of this grouping is larger than the one (93%) obtained by Barucci et al. [22] in classifying a sample of 438 asteroids with the same statistical technique. This constitutes a strong indication that colours reveal real differences in the surface nature of EKBOs, probably originating in their physico-chemical evolution.

In the Barucci et al. preliminary analysis [23] the authors found a similar result analysing a smaller sample of 22 EKBOs: recognizing four groups of objects. The 50% increase of the sample size refines the description of the EKBOs population, concerning particularly the intermediate groups. In fact, the two extreme groups (neutral and reddest spectra) have the same average spectra both in the Barucci et al. and in the present analysis, while each of the two Barucci's intermediate groups splits in two more homogeneous groups.

3.2. The Principal Component Analysis (PCA)

Barucci et al. [23] analysed a set of 22 EKBOs using Principal Component Analysis (PCA) [29]. Fulchignoni et al. [24] extended the analysis to 34 objects. We refer hereafter to the results described in the latter work.

In the quoted paper the principal components are linear combinations of the original variables (B–V, V–R, V–I and V–J) whose coefficients reflect the relative importance of each variable (colour) within each principal component. These coefficients are the eigenvectors of the variance–covariance matrix of the colours. The sum of the eigenvalues of this matrix (which is equal to its trace) accounts for the total variance of the sample. Each eigenvalue reflects the percentage of the total variance contributed by each principal component. The eigenvectors, the percentages of total variance contributed by each eigenvector and the eigenvalues of the variance–covariance matrix of the sample are reported in Table 1.

The first eigenvector accounts for most of the sample variance (92.93%) and the larger contribution comes from V–J (46%). The second eigenvector adds only 6.08% of the total variance and is weighted in quite equal measure by V–I (33%) and V–J (30%). The first and the second principal component account for more than 99% of the total variance, therefore the PC1 versus PC2 plane contains practically all the information on the variance of the variables characterizing the considered sample. It is possible to infer from this result that the degree of reddening is the main distinctive character of the EKBOs population. In Fig. 3 the 34 EKBOs sample is plotted in the PC1–PC2 plane. The predominance of PC1 (i.e., of V–J colour) in characterizing the EKBOs behaviour is shown by the PC1 scores, which span three times more than the PC2 ones. The objects having a neutral colour with respect to the Sun have the lower values of the PC1 scores and fall in the left part of the plot, for larger PC1 scores the objects are redder. The main result of the PC analysis is the clear evidence of the existence of a quasi continuous trend from neutral to very red spectra.

The six groups found by the G-mode constitute a finer structure overlapping the general trend from neutral to very red spectra resulting from the PC analysis, as shown in Fig. 3.

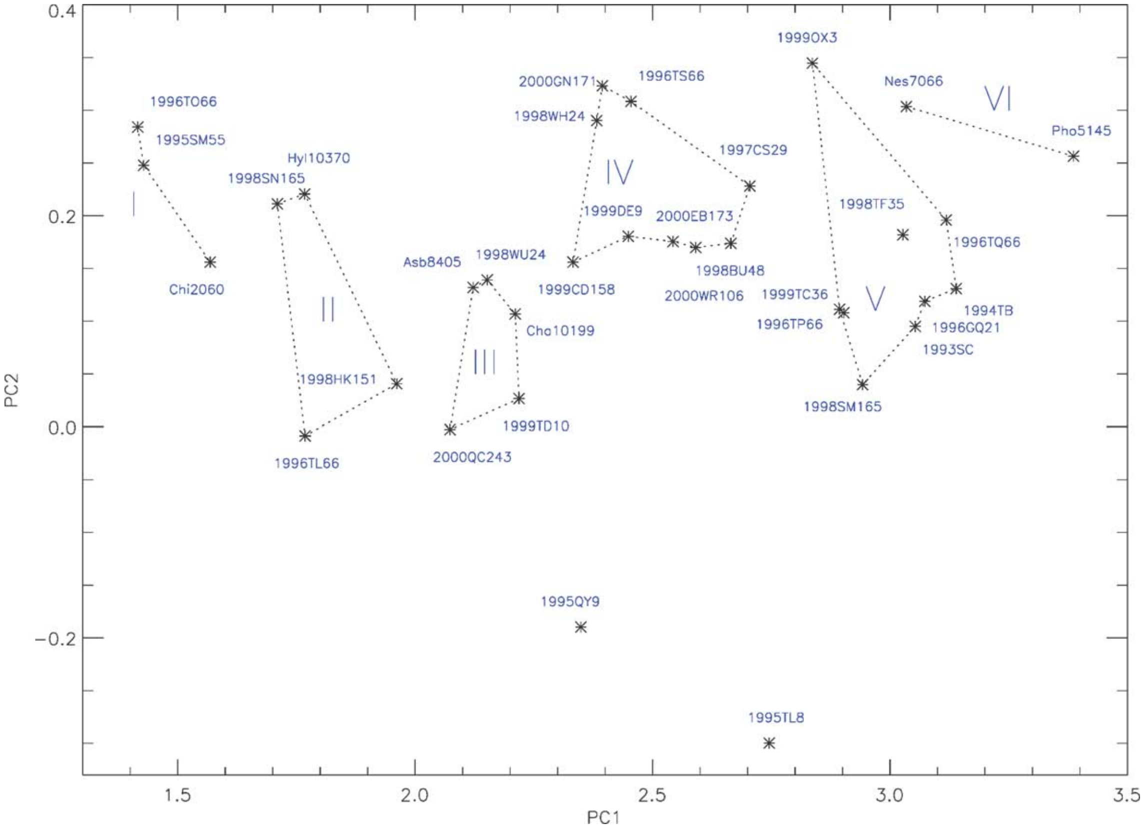

Fig. 3. EKOs and Centaurs designation for each object in the PC1–PC2 plot are reported. The dotted lines indicates the groups found by the G-mode analysis. The error bars have been obtained propagating the errors of the colours.

4. Conclusions

More data are needed to confirm the preliminary results of the different statistical analyses described above and to allow us to provide a better description of the meaning of the EKBOs colour differences.

It is clear from Fig. 2 that the bimodality found by some authors analysing the behaviour of EKBOs in the B–V, V–R plane is a consequence of the scarce significance of these colours in discriminating the object surface characteristic described by the colours. In fact, the B–V average values of groups I, II and III cluster together and are a little larger than those of the cluster of groups IV, V, VI, this trend is reversed for the V–R index which exhibits a quasi continuous increasing average value from group I to group VI. Then the groups are more and more separated with the IR colours. This implies that in the B–V versus V–R plot the EKBOs will cluster preferentially around two points slightly different in B–V, which is marginal in discriminating the homogeneous groups of EKBOs. It follows that the processes responsible for a strong reddening of EKBO spectra have only a minor affect on their reflectance properties in the B band.

The multivariate analysis of broadband spectro-photometric data of EKBOs provides strong indications for differences in the surface nature of these objects. The principal component analysis highlights a quasi continuous trend, which is probably a witness for the evolution sequence of each object, consequence of the alteration processes undergone by their surfaces while the different groups, obtained with the G-mode, indicate the present physico-chemical state of the analysed objects.

Fig. 3 provides a possible scenario (mostly qualitative) of the EKBOs evolution. Starting from a given initial state (original or consequence of a resetting event), the position of each object along the trend from the neutral to the reddest EKBOs give an indication on the length of its exposure to the space weathering (disruptive collisions, micro and macro cratering, energetic particles bombardment…). The groups of objects having an homogeneous behaviour in the four considered colours indices would represent successive homologous stages in the evolution of the population and the relative number density of the objects in each group might account for how long that stage is lasting. Finally, the presence of single objects groups might imply the existence of other evolutionary paths.

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