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Collisions, accretion, and erosion in the Kuiper Belt

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Abstract

Collisional modeling has been a fertile area of Kuiper Belt research for almost a decade. Such modeling has yielded important results concerning expected KBO surface properties, the KBO size distribution, the origin of KBOs and the properties of the primoridal Kuiper Belt, and most recently, the formation of KBO satellites. In what follows we briefly review some isolated aspects of these research results. A far more comprehensive, but older review of this topic was provided by Farinella et al. (in: Protostars and Planets IV, Mannings et al. (Eds.), University of Arizona Press, 2001). *To cite this article: S.A. Stern, S.J. Kenyon, C. R. Physique 4 (2003).*

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Résumé

Collisions, accrétion, et érosion dans la ceinture de Kuiper. La modélisation des collisions constitue depuis presque dix ans un domaine fertile de la recherche sur la ceinture de Kuiper. Cette modélisation a fourni des résultats importants concernant les propriétés de surface des objets de la ceinture de Kuiper (KBO), la distribution en taille des KBO, leur origine, les propriétés de la ceinture de Kuiper primitive et, tout récement, la formation des satellites de KBO. Dans cet article, nous passons brièvement en revue quelques aspects particuliers de ces résultats. Farinella et al. (dans : Protostars and Planets IV, Mannings et al. (Eds.), University of Arizona Press, 2001) ont donné un traitement bien plus complet, mais antérieur, de ce sujet. *Pour citer cet article : S.A. Stern, S.J. Kenyon, C. R. Physique 4 (2003)*.

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1. KBO collision rates and outcomes in the Kuiper Belt today

The Kuiper Belt (KB) represents a major population of small bodies whose evolution has been primarily shaped by collisions [1-4]. Studies such as these have yielded numerous insights into the history of the Kuiper Belt.

Although typical, present-day intrinsic collision rates (i.e., the number of collisions per square kilometer per year averaged over all KBO orbits) are lower by a factor of ~ 1000 in the KB than the main asteroid belt, the population of objects in the KB is ~ 1000 times greater. As a result, the overall collisional frequency collisional processing of individual objects is of similar scale to that in the main belt.

Among the important findings in the literature cited above regarding collision rates and collisional outcomes in the presentday Kuiper Belt are:

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- (1) The current collision timescale in the KB for 1 km radius objects impacting 100 km radius objects is $\sim 6 \times 10^7 4 \times 10^8$ yr. Over the 3.5 to 4 Gyr since the KB must have completed its accretional phase, this amounts to $\sim 8-60$ such impacts onto a single 100 km target. Given the estimated population of such objects in the present KB, there should be one such impact somewhere in the 30-50 AU region every $\sim 1-9 \times 10^4$ yr. Impacts of 4 m radius projectiles onto 1 km radius objects occur on $2-5 \times 10^7$ yr time scales, so over the entire population of $\sim 2 \times 10^9$ objects in the KB, there should be one such collision every few days.
- (2) The cumulative fraction of the surface area of 1 to 100 km radius objects cratered by projectiles with r > 4 m ranges from a few to a few tens of percent over 3.5 to 4 Gyr, depending upon the assumptions made regarding the detailed small body (r < 1 km) population size-frequency histogram.
- (3) Adopting typical relative encounter speeds of $\sim 1.1-1.4 \text{ km s}^{-1}$ between objects in the 30–50 AU region, and using impact strengths from published scaling laws, it has been found that < 1% of the 100 km-scale radius KBOs have been catastrophically disrupted over time. In contrast, the catastrophic disruption time scales for 1 km radius objects are 100 to 1000 times shorter, indicating that virtually all 'cometary' sized bodies in the KB are both young and have been heavily damaged by collisions.
- (4) Time-dependent simulations have shown that even starting purely from a population of large (i.e., R > 50 km) KBOs 4 Gyr ago, sufficient collisions have taken place to completely populate a collisional equilibrium, small-body tail to the size distribution, thereby providing a population of small objects (comets) to feed the so-called, Jupiter Family of short period (nearly ecliptic) comets.
- (5) There is evidence for a statistical correlation between the colors of KBOs and the mean random impact speed that these objects experience [5]. This lends evidence to suggestions that a competition between collisional resurfacing and radiation reddening may contribute to the surface appearance of KBOs. As such, collisions may play a role in creating time-variable albedo- and color-variegation effects (e.g., [6]).

2. KBO formation

We now turn to the subject of KBO formation, which is believed to have occurred \sim 4 billion years ago.

Theoretical models for KBO formation begin with the planetesimal hypothesis. In this picture, small 1 m to 1 km objects, called planetesimals, lie in a disk-shaped protosolar nebula orbiting the Sun. As planetesimals collide, they merge into larger objects if the collisions are sufficiently gentle. Because the requisite velocities for accretion among 1 km and smaller bodies are much smaller than the mean random orbit crossing velocities in the current KB, the ancient KB must have been a far less excited orbital environment than today.

As planetesimals grow, dynamical friction circularizes the orbits of the largest bodies, but the orbits of the smallest bodies become more and more eccentric. This process leads to a period of runaway growth, where the largest objects grow rapidly relative to the smaller bodies. As the largest objects reach radii of 100 km and larger, viscous stirring rapidly increases the eccentricities, e, and inclinations, i, of the smallest objects. Eventually, the relative collision velocities become large enough to shatter small objects. This shattering leads to a collisional cascade, where the leftover 1 m to 1 km planetesimals are ground to dust, halting accretion. The largest bodies sweep up some of this dust; radiation pressure and Poynting–Robertson (i.e., radiation) drag remove the rest.

Numerical simulations of planet growth require a statistical calculation. A minimum mass solar nebula (the minimum amount of solid material needed to explain the masses of the planets in our solar system) contains roughly 10^{20} to 10^{21} , 1 m to 1 km planetesimals, equivalent to ≈ 40 Earth-masses of solids of planetesimals in orbit at 40–50 AU. Modern *n*-body codes can only follow the orbits of order 10^9 objects. To bridge this gap, [7] developed the particle-in-a-box method, which follows the evolution of distributions of the masses and orbital parameters of planetesimals in a set of discrete circular annuli surrounding a star. As in kinetic theory or nuclear reactions inside stars, the collision rate is $n\sigma v$, the product of the number density, the collision cross-section, and the relative velocity. For small objects, σ is the geometric cross-section. For large objects in nearly circular orbits, gravity enhances σ by factors of 10^2-10^4 . This 'gravitational focusing' produces runaway growth of the largest planetesimals in an annulus [8].

As they collide and merge, several processes modify the orbits of planetesimals. Collisions that produce debris tend to increase the eccentricities of smaller bodies relative to larger bodies. Gravitational scattering is also important. Dynamical friction transfers orbital energy from large bodies to small bodies and tends to drive a system of planetesimals to energy equipartition. Viscous stirring transfers angular momentum between bodies and increases e and i for all planetesimals. Finally gas drag and Poynting–Robertson drag damp the velocities of the smallest objects, pulling these bodies towards the Sun. Most numerical simulations treat these interactions with a Fokker–Planck integrator, which averages interactions over many orbits [9]. Hybrid simulations treat gravitational interactions between individual large bodies with an n-body code, while treating the small bodies statistically, as described above.



Fig. 1. Cumulative number distributions N_C and velocity distributions for Kuiper Belt models at 40–47 AU. The legend lists evolution times in Myr for each curve.

Numerical simulations by different investigators produce a consistent picture for KBO formation [1,2,10–13]. Fig. 5 shows a typical result [14]. Modern simulations begin with small objects (1 m up to 1–10 km) in roughly circular orbits ($e \sim 10^{-5}$ to 10^{-2}) around the Sun. Planetesimals with low *e* merge and grow on short timescales. An ensemble of 1 m to 1 km planetesimals thus produces 10 km objects in ~1 Myr and 100 km objects in 6 Myr. During runaway growth, the orbits of the smallest objects become more and more eccentric (Fig. 1, right panel). Gravitational focusing factors decrease. Because growth then proceeds more slowly, it takes ~75 Myr for 100 km objects to merge and grow into a few 1000 km objects.

Once Pluto-sized ($R \sim 1000$ km) objects form, viscous stirring increases the eccentricities of the smallest objects to $e \sim 10^{-2}$. Collisions between the leftover smaller planetesimals then produce debris instead of mergers. The debris experiences a cascade of collisions which produces copious amounts of dust. This collisional cascade removes 90% or more of the initial solid material in the disk (e.g., [11]). Radiation pressure ejects most of this debris into the outer solar system. Poynting–Robertson drag and subsequent dynamical interactions with Neptune remove the rest.

After 5 Gyr, the results of most simulations depend on remarkably few of the starting conditions. The timescale to produce a Pluto is inversely proportional to the initial mass of solid material M_0 , with $t_P \approx 40-50$ Myr $(M_0/M_{MMSN})^{-1}$ for models that start with 1 m to 1 km planetesimals, where M_{MMSN} is the mass of a minimum mass solar nebula. Calculations that start with more material in 1–10 km objects take 2–3 times longer to produce a Pluto. Planetesimals with large initial *e* damp rapidly and lead to similar results as small *e* simulations; models with larger initial *e* take longer to make a Pluto than small *e* simulations. The tensile strength S_0 of planetesimals sets the amount of debris. Planetesimals with a tensile strength comparable to terrestrial snow, $S_0 \sim 10^6$ erg g⁻¹, produce less debris than weak planetesimals with $S_0 \sim 1$ erg g⁻¹. Most models yield a roughly power-law size distribution, where the cumulative number of objects is $N_C \propto r_i^{-q}$ and $q \approx 2.5-3.5$. Calculations with weak planetesimals tend to produce steeper power laws.

Stochastic processes are important in KBO formation. Occasionally, a single large body grows rapidly to a radius of 1000 km or larger in \sim 10 Myr, before other objects can grow to sizes of 100 km. Based on multiple simulations with identical initial conditions, single rapid runaways occur \sim 20% of the time. Smaller fluctuations in the growth rate produce factor of 2 variations in the timescale to reach Pluto-sized objects.

Comparisons of model predictions with observations are encouraging. The calculations yield a slope for the power-law size distribution of 1 km and larger objects, $q \approx 2.5$ –3.5, that agrees with the slope derived from observations [15,16]. Once the collisional cascade removes most of the leftover planetesimals, the average surface density of 1 km and larger KBOs is close to the observed value. For $S_0 \leq 10^3$ erg g⁻¹, the predicted surface density is a factor of 2–3 larger than observed. Models with larger S_0 leave factors of 2–10 more material in large KBOs. Because dynamical interactions with Neptune remove 50% to 70% of KBOs over 1–5 Gyr [17–20], formation models with $S_0 \approx 10^2$ to 10^4 erg g⁻¹ that include dynamical interactions between KBOs and Neptune should yield better agreement between theory and observations.

Calculations of KBO formation provide an interesting link between the outer solar system and possible planet formation in debris disks around other stars. The dusty disks surrounding α Lyr, β Pic, and other $1-3M_{\odot}$ stars may be remnants of planetbuilding [21]. Although there is as yet no direct detections of planetary mass objects, the luminosity and surface brightness of dust in these disks provide an indirect measure of the outcomes of planet formation, [23] describe planetesimal calculations in disks surrounding $3M_{\odot}$ stars and show that the collisional cascade produced by the formation of Pluto-sized or larger objects leads to an observable amount of dust. Dusty rings of debris similar to those observed in HR 4796A are often produced in these calculations. Planet formation in the KB also produces observable amounts of dust. For the model in Fig. 5, the ratio of the dust luminosity L_d to the stellar luminosity L_{\star} reaches $L_d/L_{\star} \approx 3-5 \times 10^{-4}$ during the first 1 Gyr of evolution and then fades to $L_d/L_{\star} \leqslant 10^{-5}$ after 5 Gyr. Thus, an extraterrestrial observer with our current technology could have observed the KB as a luminous dusty ring surrounding the Sun some 4–5 Gyr ago.

3. Some implications of Kuiper Belt evolution for the terrestrial planets

The planetesimal hypothesis predicts that the timescale to form planets in the protosolar nebula is a sensitive function of a, the radial distance from the central star. For a nebula with a standard surface mass density $\Sigma \propto a^{-3/2}$, the timescale is $t \propto P/\Sigma \propto a^3$ where P(a) is the orbital period. The strong sensitivity of the planet formation timescale with distance suggests that planets at small a are relatively unaffected by planet formation at very large a. However, [22] has shown that icy planetesimals leftover from the formation of Uranus and Neptune can be scattered into the inner solar system and swept up by terrestrial planets, potentially providing a substantial fraction of the available volatiles on these worlds. It is interesting to consider whether material from KBOs outside the orbits of Uranus and Neptune might also contribute material to other planets.

There are several main mechanisms to transport KB material into the inner solar system: One is collisions, which can inject material into unstable resonances or Neptune crossing orbits. Others are gas drag and Poynting–Robertson (PR) drag, which pull small objects towards the Sun. Although drag forces are effective in the inner solar system, gas and Poynting–Robertson drag probably removes less than a lunar mass from the KB because radiation pressure ejects most of the dust: collisions grind objects to 1 micron sizes before PR drag can remove somewhat larger objects. Gas drag is ineffective because the scale height of the disk is large and the gas density is small. Neptune probably sweeps up most of this material.

Dynamical interactions with Neptune are a promising way to transport material from the KB to the inner solar system. After Neptune is fully formed, its gravity eventually ejects objects that cross Neptune's orbit (or nearly do so) from the KB [17,18]. The inner KB can lose 50% to 75% of its initial mass in these interactions. Roughly half of the lost material is injected into the inner solar system as short-period comets and smaller debris; the rest is ejected out into the scattered KB or the Oort cloud. It has been known for decades now that short-period comets impact all the planets (e.g., [22]), so it is now clear that short-period comets with origins in the KB must impact all the planets, thereby providing a mechanism for transporting volatiles, organics, and other KB materials to Earth and other worlds.

Without detailed calculations, there are many uncertainties in estimating the amount of KB material that might have impacted the terrestrial planets 3–4 Gyr ago. [22] show that the Moon might sweep up a fraction $f \sim 10^{-8}$ of material leftover from the formation of Uranus and Neptune. With $f \sim 10^{-7}$, Mars might sweep up more material. For a standard initial mass of $\sim 6 \times 10^{29}$ g in the Uranus–Neptune region, these estimates imply a total mass in impacts of $\sim 1 \times 10^{24}$ g for the Earth, $\sim 6 \times 10^{21}$ g for the Moon, and $\sim 6 \times 10^{22}$ g for Mars. If these fractions hold for KBO ejections, then KBOs might contribute as much as $\sim 1 \times 10^{23}$ g to the Earth, $\sim 6 \times 10^{20}$ g to the Moon, and $\sim 6 \times 10^{20}$ g to the Moon, and $\sim 6 \times 10^{21}$ g to Mars.

The relative formation times of Neptune and large KBOs are crucial unknowns in these estimates. If Neptune forms well before large KBOs, dynamical interactions will initiate a collisional cascade that robs the KB of most of its icy material. Because dynamical interactions might deplete the KB on shorter timescales, $\sim 10-30$ Myr, than the collisional cascade $\sim 30-100$ Myr, Uranus and Neptune might scatter significant material from the KB into the terrestrial zone. If Neptune forms after large KBOs, however, the collisional cascade will deplete the KB rapidly and prevent incorporation of KBO material into terrestrial planets. Resolution of this uncertainty requires a better understanding of Uranus and Neptune formation, including calculations which incorporate both collisional processes and orbital dynamics.

4. Regarding KBO satellite formation

Just as recently as 2001, the first satellite of a KBO was detected [24]. Since then, another 7 KBOs have been identified to have satellites [25]. These discoveries have made it clear that at least a few percent of all large KBOs have satellites, though the actual number may be higher due to obvious selection effects associated with the small size and great distance of KBOs and their satellites.

The known ensemble of KBO satellites, though small in number, has already revealed several important attributes: Satellites have been found around KBOs with a significant range of heliocentric semi-major axes (from 39 to 48 AU). Satellites have been found around both classical (i.e., non-resonant) and resonant KBOs. Satellite-bearing KBOs have been found with widely ranging heliocentric orbital eccentricities (from ~ 0.00 to at least 0.37) and inclinations (ranging from 3 to 17 degrees). Large

satellite radii (80–170 km) have been inferred from photometric data after assuming low (4%) albedos characteristic of cometary nuclei, a standard assumption for KBOs.

Prior to these discoveries, of course, Pluto's satellite Charon had been detected in 1978 [26]. Like many of the KBO satellites that have been found, Charon's diameter is a large fraction of the dimater of its primary, Pluto. Unlike most (but not all) KBO satellites, however, Charon orbits quite close to Pluto $(17R_{pl})$, indicating there has been strong tidal evolution. The fact that tidal evolution has occurred is supported by the spin–spin–orbit synchonicity of the Pluto–Charon system.

Owing to both the inferred, relatively low KBO-to-satellite diameter ratios, and their predominently large separation distances from their primaries, dynamicists have hoped for capture (rather than co-accretion or impact) mechanisms to explain the formation of KBO-satellite pairs. Several such mechanisms have been identified, typically relying on dissipative multi-body [27,28] effects.

Nonetheless, collisions may also be an important formation mode of KBO satellites, as illustrated by the large (circumstantial) body of evidence that the Pluto–Charon system was formed via a physical collision (e.g., see the reviews [29,30]).

One of us (see [31]) examined the energetics of KBO satellite formation via collisions. It was found that collisional formation requires a dynamically excited Kuiper Belt such as we see today, and cannot have operated successfully in the dynamically $(e \sim 10^{-4.5})$ environment expected to have been extant during most of the KBO accretion era. (Thus only high collision velocities can eject sufficient material into orbit about the primary.)

Furthermore, even under energetically optimistic collisional formation assumptions, collisional processes cannot make sufficiently many KBO satellites (by a factor of order 40) to explain observations, unless the standard assumptions about the Kuiper Belt and/or the KBOs themselves is seriously in error. In it was found that if either the KBO primary and/or KBO satellite surface albedos have been significantly underestimated (by making the canonical assumption of 4% KBO surface albedos), then satellite formation by collisions could occur (though the issue of dynamical evolution to distant orbits would for many KBO satellites remain problematic). In particular, it was found that if KBO-primary (or KBO-satellite) albedos are 15% or higher, then the number of KBO-satellite pairs could be created in 4 Gyr to match the observed satistics. Although no known KBO has a measured albedo as low as 4%, and the albedos of some Centaurs, Charon, and even a few KBOs have been shown by thermal IR techniques to be in the range 15–40%, one awaits the much larger sample of KBO albedo determinations expected to be obtained by the SIRTF infrared observatory mission after its launch later in 2003 before this hypothesis can be further tested.

5. Concluding remarks

In the coming years we expect collisional, accretional, and erosional simulations of the KB to yield additional, valuable insights into the nature and history of the Kuiper Belt. As in the past, however, observations are expected to feed progress by guiding modelers. Among the important observational and modeling advances we eagerly anticipate in the next two decades are:

- (1) The detection of far-IR KB emission to directly constrain the current KB debris population and collision rate.
- (2) In situ dust density measurements and surface crater counts by spacecraft like the planned NASA New Horizons Pluto-Kuiper Belt mission.
- (3) Crater counts on Pluto, Charon, and KBOs by spacecraft like New Horizons to anchor the small body debris tail of both the ancient and the current-day KB.
- (4) Deep imaging observations by 20 m and larger telescopes like OWL, by ACS on HST, and with JWST, to detect km-class KBOs.
- (5) Size, albedo, and mass determinations of specific KBOs (the former by IR techniques, the latter obtainable by spacecraft flybys or by application of Kepler's Laws to KBO satellite orbits).
- (6) Accurate, SIRTF-derived albedos and sizes of KBOs.
- (7) Detections of more binaries will begin to set limits on formation mechanisms, in much the same way that the size distribution of single KBOs constrains formation models.

Additionally, we look forward to the application of sophisticated smoothed-particle hydrodynamics (SPH) codes to the study of KBO (and Pluto–Charon) formation.

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