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Steady flows in networks of microfluidic channels: building on the analogy with electrical circuits

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Abstract

This paper gathers comments and elaborations on the classical representation of networks of microfluidic channels in terms of equivalent electrical circuits. Basics on pressure-driven and electro-osmotic flows are first recalled. A unified formalism for hydrodynamic and electro-kinetic effects in networks of arbitary topology is then presented. Eventually, comments on the representation of pumps by generators are proposed. ***To cite this article: A. Ajdari, C. R. Physique 5 (2004).***

Résumé

Écoulements stationnaires dans des réseaux de microcanaux : élaborations sur l'analogie avec les circuits électriques. Différents aspects de la représentation de réseaux de canaux microfluidiques par des circuits électriques équivalents sont discutés. Après un rappel sur les écoulements par gradient de pression et par électro-osmose, un formalisme général est proposé pour les effets hydrodynamiques et électrocinétiques dans des réseaux de topologies variées. Suivent quelques remarques sur la description de pompes par des générateurs équivalents. ***Pour citer cet article : A. Ajdari, C. R. Physique 5 (2004).***

Keywords: Microfluidics; Electro-kinetic effects; Linear response; Equivalent circuits

Mots-clés : Microfluidique ; effets électrocinétiques ; Réponse linéaire ; Circuits équivalents

1. Introduction

As emphasized by the common abreviation µTAS (Micro Total Analysis System), one of the aims of microfluidics is the development of microsystems to perform more or less autonomously various analytical tasks. This requires the integration of various elements with different functionalities in networks with often many inlets and outlets (for buffer, sample, reagents, waste removal, etc.). Up to now a large part of microfluidic studies have focused on the design of elements for specific tasks (pumps, mixers, sorters, on-chip columns, etc.) [1,2]. The actual realization of systems integrating many elements lags somewhat behind, although there has been recently a developing activity on that side, with a few massive, parallel realizations [3]. It is therefore likely that the field will require, at some point, design strategies and methods, as is already well-practiced in microelectronics.

In this paper, we explore very non-exhaustively the virtues and limitations of the analogy between simple microfluidic circuits and electrical circuits, which is a powerful engineering approach that many people have used to model micro-systems. *We limit ourselves to steady monophasic flows*. We first recall basics about hydrodynamics and electro-kinetic effects in a single micro-channel (Section 2). In Section 3 we present the basics of the representation of a network of such microchannels in terms of an equivalent circuit, which is simple and well-documented for hydrodynamics, but less so for electro-kinetic effects.

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Section 4 sets a more general formalism to describe junctions or multiply connected zones in such networks. Eventually the description of micropumps in terms of equivalent generators is briefly commented upon (Section 5).

2. Single channels as linear elements

2.1. Hydrodynamics

For low Reynolds number hydrodynamics [4] the equations relating fluid velocities to applied forces are linear, so that for a channel the flow rate Q (volume/time) is linear in the pressure difference Δp between its exit and its entrance:

$$Q = M(-\Delta p) \quad \text{or} \quad (-\Delta p) = RQ, \tag{1}$$

where M is the (positive) hydrodynamic conductance and R the hydrodynamic resistance of the channel. This formula is in exact similarity with Ohm's law for a resistive element such as a conducting wire, with Q the equivalent of the current and $-\Delta p$ the equivalent of the voltage drop.

Note, however, that the scaling of the transport coefficient with dimensions is different in the two realms, as the hydrodynamic flow is not homogeneous in the channel (due to friction on the wall), Fig. 1 top. For example for a channel of constant rectangular section (length L, width w, height h), the resistance scales as $R \simeq \eta L/(h^3 w)$ where η is the viscosity of the fluid. Compared to the electrical resistance of a conducting wire of the same dimensions, there is an additional factor of h^{-2}, which make these resistances grow enormously with miniaturization.

2.2. Electro-osmosis

This difficulty has prompted the use of electro-osmosis as a means for moving efficiently electrolyte solutions. This phenomenon, largely studied in the contexts of the electrophoresis of colloids [5] and fluid transport in porous media, is interfacial in origin, but has bulk manifestations.

Consider the simplest example, a straight homogeneous rectangular channel filled with salty water. Typically the walls in contact with water acquire a surface charge and release counter-ions in the solution. The net result of the competition between thermal agitation and electrostatic effects is the build-up of a thin 'screening' charged layer in the vicinity of the wall (thickness λ_D a few nanometers to a few tens of nanometers depending on the ionic strength). If an electric field runs through the electrolyte channel, then a net shear stress is exerted on the fluid only where it is charged, i.e., in these thin layers. The result in a channel with homogeneous surface charge is a 'quasi plug-like' flow (see Fig. 1) with a velocity $v = \mu_{eo} E$, proportional to the applied field E through an 'electro-osmotic mobility' μ_{eo} that depends on the surface charge, and on the fluid viscosity and ionic strength. This corresponds to a net flow rate $Q_{eo} \simeq (hw)\mu_{eo} E$ in the channel, proportional to the electric potential drop $\Delta\phi = -LE$ applied to its ends.

In a linear picture, Δp and $\Delta\phi$ can be used in an additive way to generate flows,

$$Q = M(-\Delta p) + M_{eo}(-\Delta\phi). \tag{2}$$

A virtue of electro-osmotic flows compared to pressure-driven flows is that the typical velocity does not decrease if the system is miniaturized (as long as h remains larger than the thickness of the charged Debye layer λ_D). Forgetting again numerical

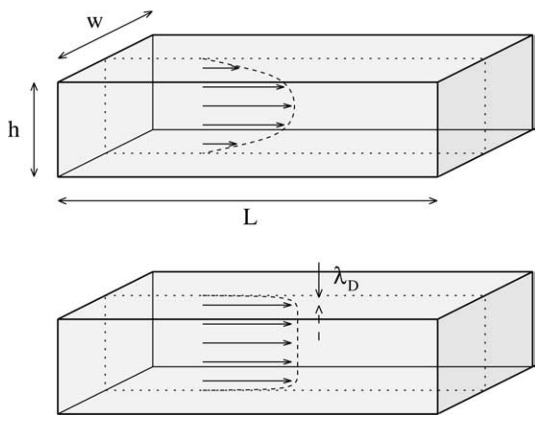

Fig. 1. Schematic flow profiles in a microchannel. Top: pressure driven parabolic flow. Lower: electro-osmotic flow induced by an electrical field in a channel with homogeneous negatively charged walls; the flow is generated by shear in the thin (λ_D-thick) regions, and almost plug-like.

prefactors, $M/M_{eo} \simeq (h^2/\eta\mu_{eo})$ so that electro-osmosis always dominates for thin channels. For aqueous solutions, and for pressure drops of about a bar and potential drops in the kiloVolt range, electro-osmosis dominates for h in the micron range or less (for water $\eta = 10^{-3}$ Pa.s. and typically $\mu_{eo} \simeq 10^{-8}\ \mathrm{m^2\,V^{-1}\,s^{-1}}$). For pressures in the millibar range, the crossover occurs for thicker channels $h \sim 50$ µm.

2.3. Unified description including electro-kinetics for a single channel

Electro-osmosis, where an electric field generates a flow, has its counterpart (streaming effects), where a flow generates an electric current, mostly by convecting the charged layers in the vicinity of the walls. So one should in general describe in a combined way the hydrodynamic and electrical effects [5–7].

The flow rate of liquid Q and the electrical current I_{el} through a micro-channel (see Fig. 3 left) are given by a *generalized conductance matrix*:

$$\begin{bmatrix} Q \\ I_{el} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} -\Delta p \\ -\Delta\phi \end{bmatrix}. \tag{3}$$

General principles require that this matrix be symmetric $M_{12} = M_{21}$ [6]. Its coefficients describe various phenomenologies [7]: M_{11} characterizes hydraulic permeability, M_{12} characterizes electro-osmosis, M_{21} quantifies streaming effects, and M_{22} is an electric conductance. These coefficients scale differently with the dimensions of the channel. For a homogeneous channel of rectangular cross-section (with h larger than the charged layer thickness), the electro-kinetic coefficients typically scale as $M_{12} = M_{21} \simeq (hw)\mu_{eo}/L$, the hydrodynamic permeability $M_{11} \simeq h^3 w/(\eta L)$, and the electrical conductance $M_{22} \simeq hw\sigma/L$, where σ is the conductivity of the liquid. Of course one can equivalently describe the channel by a resistance matrix $\mathbf{R} = \mathbf{M}^{-1}$.

3. Microfluidic networks as analogs of electrical circuits

Microfluidic 'on-chip' devices often consist in two-dimensional networks of channels, so that the natural following step is to extend the analogy to circuits.

3.1. Pure hydrodynamics

Consider for example the network of Fig. 2, where four 'ports' connect the channel network to the outer 'macro-world'. At these points pressure or flow rate can be imposed by pumps or valves. From a purely hydrodynamical point of view, the system is exactly analogous to the electrical circuit on the right of Fig. 2 with each channel being replaced by a resistor, with either potential or electrical current imposed at the ports $P_1, \ldots, P_4$. To compute the response of the system, one then needs to solve for the pressure (potential in the equivalent circuit) at the nodes N_1, N_2, writing for each of them that the sum of incoming flow rates (electrical currents in the equivalent circuit) is zero (Kirchoff's law).

3.2. Unified description including electro-kinetics for a network of channels

To include electro-kinetic effects, a similar but intrinsically more complex scheme must be followed (Fig. 3):

– each channel in the circuit is described by a matrix as in Eq. (3), and should be represented as a 2-way linear element as on the left of Fig. 3;

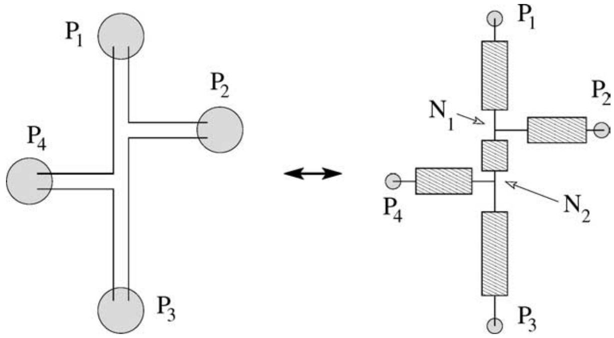

Fig. 2. A network of microfluidic channels connected to the outer world by four ports (left), and its equivalent electrical circuit (right).

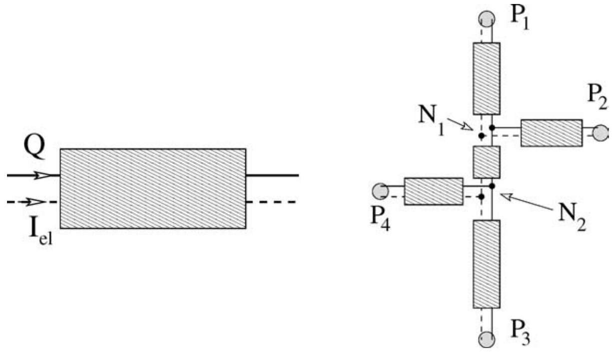

Fig. 3. Modifications of the electrical circuit analogy to take into account electro-kinetic effects. Left: a single microchannel is now equivalent to a linear element with two ways, a fluidic one (full line) and an electric one (dashed line), that are coupled within the element so that the response is described by a matrix (see Eq. (3)). Right: in a simple picture, a microfluidic network should thus be described as a circuit of such elements with fluidic and electric lines meeting independently at each node.

– at each port a mechanical quantity (flow rate or pressure) and an electrical quantity (potential or current) are imposed;
– at each node u (N_1, N_2 in Fig. 3), one must determine the local pressure p_u and potential ϕ_u by writing that the sum of the incoming flows and the sum of the incoming electrical currents are zero.

In many cases this complexity is by-passed by neglecting streaming effects, i.e., neglecting the feed-back of the flow on the electrical problem. The procedure is then the following (see, e.g., the theoretical analysis in [10] of the pump presented in [9]). First, compute the electrical potential everywhere using Ohm's law (this is a pure electrical-circuit problem), and deduce the electric field in each channel. Second, replace the resistor picture for each channel by an equation combining electro-osmosis and pressure-driven flow as in (2) to solve the hydrodynamic problem and deduce the flow rate distribution.

4. More complex nodes and multiport elements

In some cases, the previous picture for circuits (Figs. 2 and 3), has to be extended so as to include multiport elements in addition to the 2-port elements described above. This is true for extended zones of the network with multiple inlets (e.g., microstructured separation areas), but also in principle for nodes.

4.1. Pure hydrodynamics

Consider first a purely hydrodynamic picture in which electro-kinetic effects are neglected.

The reason why nodes are described in Fig. 2 as simple points is that usually they display hydrodynamic cross-sections comparable to those of the channels (say hw), and that their extent (length) is much shorter than those of the channels (say of order w instead of L), so that overall their resistance is negligible. However, if the nodes have fine internal feature that increase their hydraulic resistance they have to be described as elements on their own.[1] The proper description is then a matrix representation as the effects are still linear.

Generally, a system with 4 inlets (see Fig. 4 left) is characterized by a *symmetric* matrix:

$$\begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} G_{00} & G_{01} & G_{02} & G_{03} \\ G_{10} & G_{11} & . & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix} \cdot \begin{bmatrix} -p_0 \\ -p_1 \\ -p_2 \\ -p_3 \end{bmatrix} \tag{4}$$

that relates the exiting flow through the four branches to the four pressures at each inlet. Actually, taking into account flow conservation, one can explicit that flows are generated by pressure *differences*, which leads to another matrix, which is symmetric and positive:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \cdot \begin{bmatrix} -\Delta p_1 \\ -\Delta p_2 \\ -\Delta p_3 \end{bmatrix} \tag{5}$$

[1] A similar consideration for the drainage of foams which occurs through the network delimited by the soap films, and where the contribution of the nodes between Plateau borders is sometimes important. See, e.g., [11].

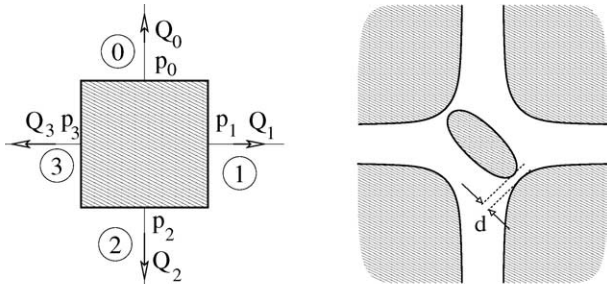

Fig. 4. Left: a generic 4-branch linear hydrodynamic element that can represent either an extended area, a subset of the microfluidic circuit, or a simple node. Right: a node with thin gaps and low symmetry. In an equivalent circuit approach such a node should be represented as on the left side (and quantitatively characterized by a matrix such as Eq. (4) or (5)), rather than by a single point as in the simplistic representation of Fig. 2 right. Due to the node symmetry, a vertical pressure difference also generates a horizontal flow and conversely (transverse hydrodynamic effects).

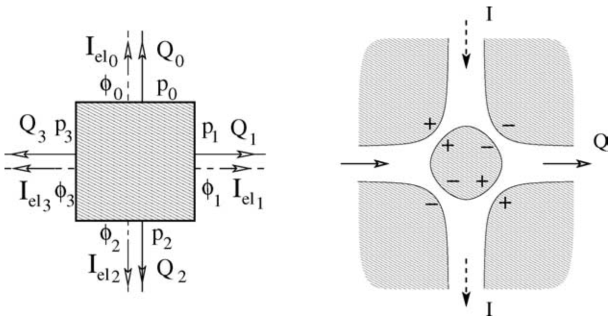

Fig. 5. Left: a generic 4-branch linear electro-hydrodynamic element that can represent either an extended area, a subset of the microfluidic circuit, or a simple node. Right: a node with thin gaps and a surface charge pattern of low symmetry that in an equivalent circuit (such as Fig. 3 right) should be described as on the left (i.e., by a matrix such as Eq. (6) or (7)). Due to the symmetry a vertical electrical current generates an electro-osmotic flow only in the horizontal direction (transverse electro-kinetic effects).

with $\Delta p_i = p_i - p_0$. Q_0 is given by $Q_0 = -(Q_1 + Q_2 + Q_3)$. Eq. (5) coincides with Eq. (1) for a 2-inlet system.

The above conductance matrices reflect the symmetry of the 4-inlet element, which can be designed through fabrication. For example, it is obvious that for the 4-branch node on the right of Fig. 4, the left-right symmetry is broken so that a pressure difference applied from top to bottom leads to a flow from left to right. Of course, whether or not such effects are detectable has to do with the internal resistance of this element, i.e., in this example how thin d is. Actually, in the limit $d \to 0$, the 4-inlet element becomes a set of two disconnected 2-inlet turns, and the matrix **G** splits into two 2×2 blocks describing 0,1 and 2,3 separately.

These considerations are not formal speculations for theoreticians: for example, nodes with a design very similar to Fig. 4 right (a diagonal porous membrane in the center of a 4-branch node) have recently been fabricated for dialysis purposes [12].

4.2. Unified description with multiport elements

If we now move to describing hydrodynamic and electro-kinetic effects on the same footing, nodes and multi-inlets structures are characterized by matrices relating *all applied potentials and pressures* to *all the flows and electrical currents*.

For example for a 4-inlet structure (Fig. 5) one needs now a 8×8 matrix:

$$\begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \\ I_{\text{el}0} \\ I_{\text{el}1} \\ I_{\text{el}2} \\ I_{\text{el}3} \end{bmatrix} = \begin{bmatrix} G_{00} & G_{01} & G_{02} & . & . & . & . & . \\ G_{10} & G_{11} & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \end{bmatrix} \cdot \begin{bmatrix} -p_0 \\ -p_1 \\ -p_2 \\ -p_3 \\ -\phi_0 \\ -\phi_1 \\ -\phi_2 \\ -\phi_3 \end{bmatrix} \tag{6}$$

or again taking port 0 as the reference and using the conservation of flow $\Sigma_{i=0}^{3} Q_i = 0$ and electrical current $\Sigma_{i=0}^{3} I_{\mathrm{el}i} = 0$:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ I_{\mathrm{el}1} \\ I_{\mathrm{el}2} \\ I_{\mathrm{el}3} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & . & . & . & . \\ M_{21} & M_{22} & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{bmatrix} \cdot \begin{bmatrix} -\Delta p_1 \\ -\Delta p_2 \\ -\Delta p_3 \\ -\Delta\phi_1 \\ -\Delta\phi_2 \\ -\Delta\phi_3 \end{bmatrix} \tag{7}$$

with $\Delta p_i = p_i - p_0$ and $\Delta\phi_i = \phi_i - \phi_0$. This definition coincides with Eq. (3) for a 2-inlet system.

These symmetric matrices now describe many couplings, e.g., that a hydrodynamic driving on a branch generates an electric effect on another branch. For example in the 4-branch node described in Fig. 5 right, the electro-kinetic effects are purely transverse: an electrical current running vertically generates a left to right electro-osmotic flow, but none vertically (as is easily seen by studying the electro-osmosis in each of the four 'sub-channels' of the node, and the symmetries of the structure). Conversely, a flow along one direction generates a streaming potential in the transverse one.

A more thorough discussion of such effects is given in [13], where an explicit proof of the symmetry of the above matrices is given. This article also points out the fact neglected here, that a complete description should actually take into account all the chemical species (ions) dissolved in the electrolyte, so that even larger matrices relate the fluxes of all species (including the solvent) to the drops of chemical potentials. In the absence of concentration inbalance between the ports of the system one can simplify the description to the format adopted here.

To sum up, a general modelization of a microfluidic circuit would be that of Fig. 3, with however the nodes and possible multi-inlet structures described as in Fig. 5 left, and characterized by matrices coupling hydro-dynamic and electro-kinetic effects (such as Eqs. (6) and (7)).

5. Pumps as active elements

In microfluidic systems, the circulation of the fluid in microchannels can be generated from the outside (applied potential or pressure drops), or/and generated by micropumps within the channel. In the latter case it is appealing to describe the 'internal' pumps in a way similar to that used for generators in electrical circuits. In particular, one can use two possible pictures to describe a pump (see Fig. 6): (i) a 'pressure generator', i.e., a perfect pressure generator of strength Δp_0 in series with an internal resistance R_0; or (ii) a 'flow generator', i.e., a perfect flow (or current) generator of strength Q_0 in parallel with an internal resistance R_0. For linear pumps, these three parameters (Q_0,R_0,Δp_0) are (by definition) constants and the two pictures are equivalent with $\Delta p_0 = R_0 Q_0$, but yet, as for electrical circuits, they convey different intuitions.

Applied on a purely 'resistive' channel of resistance R, a pressure generator generates a pressure drop $\Delta p = \Delta p_0/(1 + R_0/R)$, and a flow generator generates a flow $Q = Q_0/(1 + R/R_0)$. A good pressure generator should be able to apply a given (strong) pressure independent of R. This requires a high value of Δp_0, and a low R_0. Conversely a good current generator requires a high Q_0 and a high R_0! There is no contradiction here given the fact that the three quantities are related.

Note that for arbitrary pumps, both representations above are linearizations of the flow/pressure drop function of the pump around a given operation point $(Q^*, \Delta p^*)$: $Q \simeq Q^* + (\mathrm{d}Q/\mathrm{d}\Delta p)^*(\Delta p - \Delta p^*) + \cdots$. This can be rewritten for example $Q \simeq Q_0 - \Delta p/R_0$ with $(\mathrm{d}Q/\mathrm{d}\Delta p)^* = -1/R_0$ and $Q_0 = Q^* - (\mathrm{d}Q/\mathrm{d}\Delta p)^*\Delta p^*$. The linear representations are therefore usually efficient only within a certain window of drivings, that can, however, be extended by allowing some variation of the parameters, e.g., $Q_0 = Q_0(\Delta p)$ and $R_0 = R_0(\Delta p)$.

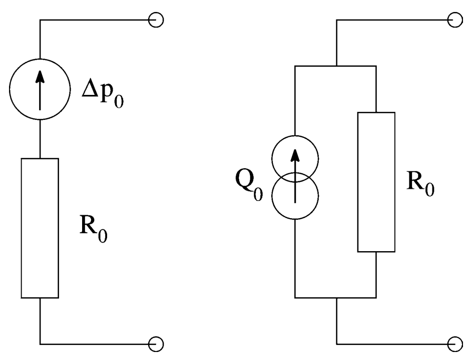

Fig. 6. Schematic representation of a pressure generator (left) and a flow generator (right).

One should look at the physical mechanism that generates the pumping to decide which description (flow or pressure generator) is better suited or more intrinsic, e.g., to get an understanding of how the performance of the pump changes upon miniaturization. For example, there are a few methods using electro-osmotic effects to generate pumping: usual DC electro-osmosis (see IIB), transverse DC electro-osmosis [14], and AC electro-osmosis [15–18]. In all of these, a typical slip velocity v_0 is generated and thus typically a net flow $Q_0 \simeq (hw)v_0$ independent of the length L_0 of the pumping section. This together with $R_0 \simeq \eta L_0/(h^3 w)$ provides all that is needed to model the performance of the pump, once the mechanism that generates v_0 is specified: for usual electro-osmosis, $v_0 \simeq \mu_{\mathrm{eo}}(-\Delta\phi/L_0)$ (this actually leads back to equation (2), with Q_0 the second term on the right-hand side); for a transverse electro-osmotic pump with lateral electrodes in the channel $v_0 \simeq \mu_{\mathrm{eo}}(-\Delta\phi/w)$ [14]; and for AC electro-osmostic pumping with an embedded asymmetric array of interdigitated electrodes of period λ [15–18], $v_0 \simeq \mu_{\mathrm{eo}}(-\Delta\phi/\lambda)$. This kind of analysis also applies to situations in which back-flow is prevented by the use of porous sections of large hydrodynamic resistance, $R_0 \simeq \eta L_0/(\xi^2 hw)$ with ξ the small pore-size (see, e.g., [8,9]), situations where the representation in terms of equivalent elements is a powerful tool (see, e.g., [10]).

6. Concluding remarks

Section 4-2 provides a general formalism to describe networks of *passive linear* elements including both fluidics and electro-kinetic effects (as long as osmotic effects due to gradients of concentrations of solutes are negligible, see [13]). Whether or not such an elaborate procedure is required in a given practical situation depends on what can be safely omitted. In many cases, a simple inspection is enough to assert that some of the ingredients are negligible (e.g., weak node resistance, minute streaming effects in a fully pressure-driven system, etc.). One can then turn to simpler formalisms such as those described in Section 4.1 (for negligible electro-kinetic effects), or in Sections 3.1 and 3.2.

As for electrical circuits, many functionalities can be accessed if the toolbox contains intrinsically *non-linear* elements. In the present microfluidic context, this non-linearity can for example be achieved using the elasticity of the channels (see, e.g., [19]), or by embedding mobile parts in the channel (see, e.g., [20]), or using non-Newtonian fluids where the non-linearity of the hydraulic response of a channel stems from the non-linear flow curve of the fluid. A set of non-linear elements has recently been demonstrated using polymer solutions in [21] (see also the introduction of that paper for the non-applicability of inertia-based devices valid for macro-fluidics).

For *active* elements, where an external command acts on the system, one needs additional ad-hoc modelization for each kind of elements. Section V provides elements for the description of micropumps. An elegant tunable element is the Flow-FET (Field effect Transistor) of [22], a channel the surface charge of which is controlled by an independent electrical connection.

Obviously, the picture also needs to be extended if the geometry is responsive to chemical concentrations or reactions within the flow, or if the flow is biphasic.

Eventually we have focused only on global fluxes within the network (current, flow), whereas the laminar structure of the flow permits that many streamline structures run in parallel within a single channel geometry without loosing their identity (i.e., in terms of solute concentration, temperature, etc.) if the transverse diffusion of these quantities is slow enough. In such situations, the above modelling in terms of 'elements' is inappropriate, and a case by case local study is required.

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References

[1] H.A. Stone, S. Kim, AIChE J. 47 (2001) 1250–1254.
[2] G. Whitesides, A.D. Stroock, Phys. Today 54 (2001) 42–48.
[3] T. Thorsen, S.J. Maerkl, S.R. Quake, Science 298 (2002) 580–584.
[4] J. Happel, H. Brenner, Low Reynolds Number Hydrodynamics, Martinus Nijhoff, The Hague, 1983.
[5] R.J. Hunter, Foundations of Colloid Science, Oxford University Press, New York, 1991.
[6] P. Mazur, J.T.G. Overbeek, Recl. Trav. Chim. 70 (1951) 83.
[7] P.B. Lorenz, J. Phys. Chem. 56 (1952) 775–778.
[8] P.H. Paul, D.W. Arnold, D.W. Neyer, K.B. Smith, in: A. van den Berg, et al. (Eds.), Micro Total Analysis 2000, Kluwer Academic, Dordrecht, 2000, pp. 583–590, and references therein.
[9] Y. Takamura, et al., in: J.M. Ramsey, A. van den Berg (Eds.), Micro Total Analysis 2001, Kluwer Academic, Dordrecht, 2001, p. 230.

[10] A. Brask, G. Goranovic, H. Bruus, Sensors and Actuators B 92 (2003) 127–132.
[11] S.A. Koehler, S. Hilgenfeldt, H.A. Stone, Phys. Rev. Lett. 82 (1999) 4232–4235.
[12] B. Kirby, A.K. Singh, in: Y. Baba, et al. (Eds.), Micro Total Analysis 2002, Kluwer Academic, Dordrecht, 2002, p. 742.
[13] E. Brunet, A. Ajdari, Phys. Rev. E., in press.
[14] I. Gitlin, A.D. Stroock, A. Ajdari, G. Whitesides, Appl. Phys. Lett. 83 (2003) 1486–1488.
[15] A. Ajdari, Phys. Rev. E 61 (2000) R45.
[16] A.B.D. Brown, C.G. Smith, A.R. Rennie, Phys. Rev. E 63 (2001) 016305.
[17] V. Studer, A. Pépin, Y. Chen, A. Ajdari, Microelectron. Engrg. 61-62 (2002) 915–920.
[18] A. Ramos, A. Gonzàlez, A. Castellanos, N.G. Green, H. Morgan, Phys. Rev. E 67 (2003) 056302.
[19] K.R. King, M. Toner, in: A. Northrup, et al. (Eds.), Proceedings of Micro Total Analysis 2003, Transducers Research Foundation catalog number 03TRF-0001, ISBN 0-9743611-0-0, 2003, p. 49.
[20] E.F. Hasselbrink, T.J. Shepodd, J.E. Rehm, Anal. Chem. 74 (2003) 4913–4918.
[21] A. Groisman, M. Enzelberger, S.R. Quake, Science 300 (2003) 955–958.
[22] R.B.M. Schasfoort, S. Schlautmann, L. Hendrikse, A. van den Berg, Science 286 (1999) 942–945.