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Ice: from dislocations to icy satellites/La glace : des dislocations aux satellites de glace

# The viscoplastic behaviour of ice in polar ice sheets: experimental results and modelling

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# Abstract

The slow motion of polar ice sheets is governed by the viscous deformation of anisotropic ices. Physical mechanisms controlling the deformation of ice crystal and polycrystal are reviewed. For the low stress conditions prevailing in ice sheets, the stress exponent of the flow law is lower than 2 and the deformation is dominated by the glide of dislocations on the basal plane. The mismatch of slip at grain boundaries induces large strain inhomogeneities partially relieved in ice sheets by grain growth and recrystallisation. The hard X-ray diffraction technique can be used to describe the orientation gradients within grains. The structure of ice along deep ice cores in Antarctica and Greenland exhibits significant changes in the shape, size and orientation of grains. A large variation of ice viscosity with depth is therefore expected. Polycrystal deformation models accounting for the changing rheological properties of polar ice are discussed. These models must predict and take into account the intracrystalline field heterogeneity. *To cite this article: M. Montagnat, P. Duval, C. R. Physique 5 (2004)*.

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# Résumé

**Déformation viscoplastique de la glace polaire : résultats des expériences et modélisation.** L'écoulement des calottes polaires dépend largement de la déformation viscoplastique de glaces anisotropes. Les mécanismes physiques contrôlant la déformation du cristal et du polycristal de glace sont discutés. Dans les conditions de faibles contraintes mises en jeu dans les calottes polaires, le paramètre de sensibilité à la contrainte prend une valeur légèrement inférieure à 2 et la déformation dynamique contribuent à réduire les inhomogénéités de déformation induites par les incompatibilités de déformation entre les grains. L'analyse de la structure de la glace des carottes profondes extraites en Antarctique et au Groenland montre des variations importantes de la forme, de la taille et de l'orientation des grains. De fortes variations de la viscosité avec la profondeur sont donc attendues. Les modèles de déformation des polycristaux rendant compte de l'évolution des propriétés rhéologiques des glaces polaires sont discutés. Ces modèles doivent prédire et prendre en compte les hétérogénéités de contrainte et de vitesse de déformation dans les grains. *Pour citer cet article : M. Montagnat, P. Duval, C. R. Physique 5 (2004).* © 2004 Académie des sciences. Published by Elsevier SAS. All rights reserved.

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# 1. Introduction

Changes in large ice sheets have affected sea levels in the past and may affect them in the future. An understanding of how polar ice sheets interact with the climatic system is of the highest importance in order to predict sea-level changes and assess the effects of a massive discharge of icebergs on ocean circulation [1]. The development of physically-based models describing ice sheet dynamics has recently received special attention [2,3]. The slow motion and changes of large ice masses are governed by the deformation of polycrystalline ice. The constitutive law of polycrystalline ice for ice sheet modelling is that of an incompressible, non-linear viscous fluid. Deviatoric stresses in ice sheets are generally lower than 0.1 MPa and strain rates are typically between  $10^{-10}$  and  $10^{-13}$  s<sup>-1</sup>. It is difficult to obtain valuable information on the ice flow law under such low stress conditions. Laboratory tests take too long to obtain a significant amount of deformation under these conditions and extrapolation from tests performed at higher stresses introduces significant uncertainty. However, much progress has been made with the study of the ice structure in deep ice cores recently retrieved from Antarctica and Greenland. The sensitivity of strain rate to stress in ice sheets is characterised by a stress exponent lower than 2. The rate controlling processes are not totally clear, but basal slip is the dominant deformation mode [4,5].

Finite strain higher than 1 is reached in the large Antarctic and Greenland ice sheets since the age of ice can be more than 100 000 years in the deep ice layers of these two ice sheets. For example, the Vostok ice core (East Antarctica) record extends through four climatic cycles, with ice older than 400 000 years at a depth of 3310 m [6]. The deformation of polar ice induces the development of lattice preferred orientations giving a non-random distribution of the *c*-axe orientations (textures). Given that ice grains are transversely isotropic, *a*-axes textures are not considered. Owing to the very large anisotropy of ice crystal [7] and the preponderance of intracrystalline dislocation glide in polar ice [8], initially isotropic ice formed after transformation of snow and firn near the surface becomes anisotropic as textures develop. Very large variations of strain rates with the applied stress direction have been found using both laboratory and in situ measurements [9–11]. Up to now, models describing the evolution of ice sheets have not accounted for the changing rheological properties of ice with time. A first attempt has been made by Salamatin and Malikova [3] in the framework of the shallow-ice flow-line approximation. The role of recrystallisation processes in the ice sheet dynamics was specially analysed.

Since textures develop with strain, models were recently developed to account for the anisotropic behaviour of ice and the evolution of its strain-induced anisotropy. Most of them are based on micro–macro approaches that derive the macroscopic behaviour of a polycrystal with a given texture from the assumed known behaviour of the ice grain using a homogenisation procedure [12–16]. The reliability of these models is limited by the assumptions made on the behaviour of ice crystal and the mechanical conditions at the grain boundaries. Due to the very large anisotropy of ice crystal, grains that are well oriented for basal slip (soft grains) deform much more than hard grains [14]. Stress equilibrium and strain compatibility between grains are maintained at the expense of the build-up of deformation inhomogeneities within grains [17,18]. Prediction of the mechanical fields inside grains is therefore required to correctly describe the behaviour of the polycrystal. Strong lattice distortion has been observed in ice grains from several deep ice cores [5]. The observed lattice curvature supports basal slip as the dominant deformation mode in ice sheets [5]. The large grain anisotropy of ice leading to high heterogeneity contrast has recently attracted the interest of theoreticians to test the accuracy of various self-consistent approaches used to predict the mechanical response of viscoplastic polycrystalline materials [19,20].

This work focuses on the viscoplastic behaviour of ice crystal and the deformation modes of polycrystalline ice for conditions prevailing in ice sheets. We will show that intracrystalline dislocation glide is compatible with in situ deformation measurements and the development of textures. Models used to describe the behaviour of the polycrystalline polar ice will be discussed. Emphasis is placed on the influence of heterogeneities and the necessity of describing stress and strain rate fluctuations within grains.

#### 2. Deformation of ice single-crystal

The main feature of the plasticity of ice crystal is its outstanding anisotropy. For shear stresses corresponding to those found in glaciers and ice sheets, ice deforms by basal slip. Basal slip is caused by the glide motion of basal dislocations with the  $\langle 11\bar{2}0 \rangle$  Burgers vector. Creep curves show that strain rate increases continuously up to more than 10% strain [21]. Variations of steady state strain rates with stress for basal creep are shown in Fig. 1. Almost all authors report a stress exponent of about 2 and activation energy close to 63 kJ mol<sup>-1</sup>. Slip lines parallel to the basal plane can be seen by shadow photography [22,23] or by X-ray diffraction [24]. The formation of slip lines is probably due to the correlated movement of many dislocations [25,26]. The increase of strain rate during creep tests is likely related to the increasing rate of formation of basal slip bands [24]. The value of the stress exponent n = 2 can be explained by the linear variation of both slip dislocation velocity and dislocation density with stress, considering dislocation interactions within slip bands. The outer surface of single crystals is considered as the main source of dislocations when single crystals are deformed at relatively high temperature (> -20 °C).



Fig. 1. Strain rate as a function of stress for the basal slip of ice crystal and for isotropic polycrystalline ice [72–78].

When ice crystals are loaded such that there is no resolved shear stress on the basal plane, creep rates are so small that measurements are uncertain. A slight misorientation of the *c*-axis with respect to the direction of applied axial stress induces basal slip. According to Duval et al. [7], under the same prescribed stress, a crystal sheared parallel to its basal plane exhibits a strain rate that can be up to three orders of magnitude greater than that measured when ice crystal is compressed along or perpendicular to the *c*-axis. The rapid glide of short edge dislocations on non-basal planes has been observed by X-ray topography [27,28]. The fact that basal slip is dominant, in spite of faster movement of such non-basal dislocations, is attributed by these authors to the large difference in dislocation densities. According to Hondoh et al. [29], the long basal screw dislocations should be dissociated on the basal plane thereby impeding the glide of these dislocations on non-basal planes.

In conclusion, the deformation of ice crystal is dominated by the glide of dislocations on the basal plane. The glide of short edge dislocations on non-basal planes makes little contribution to the deformation of ice crystal [30]. This mechanism could however contribute to the production of basal screw dislocations [28,31]. Widely extended partial dislocations limit the occurrence of basal dislocation cross slip or climb [32].

## 3. Deformation of isotropic polycrystalline ice

## 3.1. Viscous behaviour at high stresses

Creep data for isotropic polycrystalline ice are given in Fig. 1. They concern the minimum creep rate reached after a strain of about 1%. The stress exponent is close to 3 for stresses between 0.1 and 2 MPa. During primary creep, strain rate decreases by more than 2 orders of magnitude (Fig. 2). This behaviour is directly related to the strong anisotropy of ice crystal [7]. On initial loading, stress state within the polycrystalline ice sample is almost uniform. Deformation is essentially produced by basal slip. Because grain boundaries act as obstacles to dislocation slip, resolved stress on the basal plane is relaxed through load transfer to harder slip systems. As a result, an increasingly non-uniform state of internal stress develops with peaks in amplitude near grain boundaries. Initial creep rate is largely determined by basal slip systems, but the steady strain rate is determined by an appropriate average over the resistance of all slip systems [33]. According to Hutchinson [34], extensive plasticity of polycrystalline ice is possible with four independent slip systems. Basal slip provides two independent systems. Slip or climb of dislocations on non-basal planes, as discussed above, would give additional independent systems.

## 3.2. Viscous behaviour of ice at low stresses; application to polar ice sheets

#### 3.2.1. Deformation modes

For conditions prevailing in ice sheets, (equivalent stress lower than 0.2 MPa), stress exponent is slightly lower than 2, a value close to that found in isolated single crystals (Fig. 1). This result is supported by densification measurements of bubbly ice at Vostok [35]. The high difference in strain rate cannot be explained by a geometric effect related to the random orientation of crystals. As at high stresses, basal slip is the dominant deformation mode and other deformation modes are needed to



Fig. 2. Strain rate plotted against strain for isotropic polycrystalline ice using reduced variables (from Ashby and Duval [33]).

assure the compatibility of deformation at grain boundaries. However, the internal stress field induced by the incompatibility of deformation between grains is reduced under the low stress conditions of ice sheets by grain boundary migration. Grain growth driven by the free energy of grain boundaries is observed in the first hundred meters [36–41]. Dynamic recrystallisation associated with the progressive misorientation of sub-boundaries is observed below the zone where normal grain growth occurs [40]. By sweeping away dislocations located in front of moving grain boundaries, grain boundary migration associated with grain growth and recrystallisation prevents kinematic hardening caused by the incompatibility of deformation between grains. Accommodation of slip by grain boundary migration in polar ice must be taken into account to explain the low stress exponent observed at low stresses.

A physical deformation model, considering dislocation density within an average grain as an internal variable, was developed by Montagnat and Duval [4]. The increase of dislocation density by work hardening is balanced by grain boundary migration and by the formation of new grain boundaries. This model accounts for the transition between grain growth and recrystallisation in several locations in the Greenland and Antarctic ice sheets. However, the low stress exponent of the flow law cannot be deduced from such a model. It is also difficult to assume that the same value of stress exponent for single crystals and polycrystalline ice at low stresses (Fig. 1) implies the same rate controlling processes.

Grain boundary sliding is also suggested to accommodate basal slip, but the occurrence of such process in the flow conditions of ice sheets has not been proven. This mechanism was proposed as the predominant deformation mode of polar ice by Goldsby and Kohlstedt [42,43]. Polar ice is, therefore, considered by these authors as a superplastic material. Grain boundary sliding accommodated by dislocation climb is, indeed, generally considered as the deformation mode of fine-grained superplastic materials when n = 2 [44]. The microstructure and the development of preferential orientations of ice crystals in ice sheets are clearly not compatible with grain boundary sliding as the dominant deformation mode in polar ice sheets [45].

#### 3.2.2. Lattice distortion and dislocation density in polar ice

The analysis of the microstructure of single crystals from the Vostok ice core (East Antarctica) by hard X-ray diffraction [5] has revealed a significant distortion of the lattice accommodated by geometrically necessary dislocations, as defined by Ashby [46]. Fig. 3 shows the diffraction pattern of an ice sample from 3516 m in depth. Diffraction peaks from the basal, prismatic and pyramidal planes are observed. The inclination of the (100) diffraction line is explained by the torsion of the lattice around the *c*-axis whereas the inclination of the basal (002) line is due to the bending of the basal plane. Dislocation density associated with this lattice distortion is of the order  $10^9 \text{ m}^{-2}$  [5]. By comparison, the dislocation density of a 'good' ice crystal grown in the laboratory can be less than  $10^8 \text{ m}^{-2}$ . The dislocation density associated with statistically stored dislocations, as opposed to geometrically necessary ones, produces a homogeneous enlargement of the diffraction lines. It is observed to be generally low in Vostok samples [5]. The total dislocation density would be lower than  $10^9 \text{ m}^{-2}$  for the 3516 m sample. It



Fig. 3. Diffraction patterns of a single crystal extracted from Vostok ice core at a depth of 3516 m. Diffraction lines on the (002) basal and (100) prismatic planes are shown The other lines represent diffractions on several pyramidal planes [5].

is significant to point out that sub-boundaries and new grains should form as soon as dislocation density exceeds a value of about  $10^{10}$  m<sup>-2</sup>. A dislocation density higher than  $10^{11}$  m<sup>-2</sup> is expected in the depth range of 500 to 1500 m at Vostok where sub-boundaries can be seen with the naked eye [40]. The relatively low dislocation density at 3516 m is due to the high in situ temperature at this depth (> -5 °C) and the low horizontal shear stress caused by the presence of the lake below a depth of 3700 m [47]. Subgrains with misorientations of about 20' were observed at a depth of 2755 m by X-ray diffraction. The dislocation density, deduced from X-ray diffraction measurements, must be considered as a lower bound since dislocations associated with pile-up at grain boundaries could not be seen since only isolated ice crystals extracted from ice cores were studied. Note that grain boundary migration, by sweeping away dislocations located in front of grain boundaries, should impede the formation of dislocation pile-ups in ice sheets.

Most dislocations in deformed ice samples appear to be associated with strain gradients [48]. Thus, dislocations accommodating the lattice distortion shown in Fig. 3 and those contained in dislocation pile-ups are geometrically necessary.

## 3.2.3. Rate-controlling processes in the creep of polar ice

The deformation of polar ice is essentially produced by dislocation slip on basal planes. The mismatch of slip at grain boundaries induces lattice distortion and strain gradients. Grain boundary migration associated with grain growth and recrystallisation reduces the dislocation density and the internal stress field. As a consequence, strain rate is higher than that extrapolated from high stress conditions with a stress exponent n = 3. The large difference between the basal slip of single crystals and the creep behaviour of polycrystalline ice (Fig. 1) shows that the incompatibility of deformation between grains is significant even at low stresses. However, the amount of non-basal slip or climb of dislocations required for compatibility reasons should be reduced in the flow conditions of ice sheets. With regard to non-basal slip, the bending of basal planes observed by hard X-ray diffraction must be taken into account when considering deformation along the *c*-axis. Mechanisms, which give the value of the stress exponent n = 2 at low stresses, cannot be determined on the basis of this analysis alone. The slowest deformation mechanism could control strain rate if it occurs in series with basal slip. It is probably the case at high stresses where non-basal slip or climb of dislocations should be the rate-controlling process. At low stresses, the amount of these hard deformation systems is reduced by several processes such as the nucleation of grains and grain boundary migration. An analysis of dislocation microstructure by X-ray diffraction made during the deformation of ice samples will provide useful information on the level of internal stress field under low stress conditions.

# 4. The mechanical behaviour of anisotropic ices in ice sheets

Significant variations of textures with depth are observed along deep ice cores [36,38,39,49–51]. A progressive clustering of crystal *c*-axes toward the vertical is generally observed. These textures are interpreted by the rotation of the lattice by dislocation slip under the effect of vertical compression in the first thousand meters and horizontal shear in the deepest layers. Textures along the Vostok core are characterised by a clustering of *c*-axes within a vertical plane [52]. They result from uniaxial tension along a horizontal direction. Textures with a strong vertical single maximum are preferentially found in fine-grained glacial ice. Grain size variations with climate are related to dust content which is relatively high during the glacial periods [6]. From Weiss et al. [41], the pinning of grain boundaries by micro particles is at the origin of grain size variations with climate. Grain boundary migration appears to be strongly hindered by this pinning force in glacial ice.

If the deformation of ice is only due to the thinning of annual layers, a constant strain rate along the major part of the ice core would be expected [53]. The development of textures should therefore be uniform along the ice core as observed at GRIP

[38] and Dome Summit (East Antarctica) [50]. Under uniaxial compression and tension, ice becomes harder with strain since resolved shear stress in the basal plane decreases with strain [54]. This negative feedback mechanism could also hinder changes of directional viscosity with climate.

Assuming that ice viscosity decreases with grain size [55,56] glacial ice could deform more readily than interglacial ice. Horizontal shear stress, which is proportional to ice thickness and surface slope, can become dominant in deep ice and far from a dome. Under this imposed stress, any variation of viscosity between two layers will be strengthened since textures with a clustering of c-axes toward the vertical direction that develop in simple shear [36] make ice softer for horizontal shear. Variations of textures and ice viscosity with climate are consistent with this explanation [57]. Any variation of viscosity between two layers will be strengthened since stress is imposed. Such positive feedback is also amplified since textures with a clustering of c-axes toward the vertical direction make ice softer for horizontal shear. Variations of textures and ice viscosity with climate are consistent with this explanation. Shear zones observed in deep polar ice should result from these mechanisms.

A detailed analysis of the viscosity of textured ices from deep ice cores was made by Castelnau et al., [58]. A GRIP ice core sample exhibiting a strong concentration of c-axes along the vertical direction was deformed under uniaxial compression along and at 45° from this direction. For the same level of compression stress, strain rate varies by a factor higher than 100 for the two configurations. This variation of strain rate is directly related to the level of resolved shear stress in the basal plane. This result gives clear evidence of the high viscoplastic anisotropy of textured polycrystals in ice sheets. Models accounting for the anisotropy of polar ice must be used to improve the simulation of the flow of ice sheets.

## 5. Modelisation of the viscoplastic behaviour of polycrystalline ice

Simulations of glacier flow are commonly based on the assumption that ice has an isotropic viscosity. The anisotropy of polar ice was initially incorporated in flow models by introducing an enhancement factor [59,60]. Models accounting for the anisotropic behaviour of polar ice and the evolution of its strain-induced anisotropy were recently used to simulate the flow of glaciers in simple situations [61,62]. Significant effects of ice anisotropy were obtained. For example, flow is less sensitive to bedrock topography variations; a partial stagnation of ice in the holes of the bedrock is observed. Strong textures could be at the origin of flow disturbances and facilitate folding [63].

Several polycrystal models are used to simulate the development of textures and the corresponding evolution of the mechanical behaviour in simple stress (or strain) configurations. Most of them are based on micro-macro approaches that derive the macroscopic behaviour of a polycrystal with a given texture from the assumed known behaviour of ice crystal using a homogenisation technique [12-16,64]. Ice crystal is generally assumed to deform by slip on the basal plane only and stress within the polycrystal is assumed to be uniform, i.e. the stress on each grain is equal to the macroscopic stress applied to the polycrystal (Reuss bound). These assumptions are justified when such models are introduced in large-scale ice sheet flow models. Castelnau et al. [14] first applied the Visco-Plastic Self-Consistent (VPSC) approach, which does not assume uniform stress within the polycrystal and requires slip on non-basal planes. This VPSC approach was first applied to anisotropic materials by Lebensohn and Tomé [65]. Each grain represents one crystalline orientation and is considered as an ellipsoidal inclusion in a Homogeneous Equivalent Medium (HEM) with a behaviour representing that of the polycrystal (1-site approach). This model allows stress and strain rate to be different in each grain. The behaviour of the HEM is linearised following a tangent or a secant approach. In its tangent formulation, it well reproduces all experimental macroscopic behaviours of isotropic and anisotropic ices at the expense of a non-negligible amount of pyramidal slip, which is not observed experimentally [58], but it fails to simulate texture development at higher deformation. Furthermore, this formulation tends towards the Reuss or lower bound (Fig. 4) when the anisotropy between slip systems increases. It is not consistent with the exact solution for highly anisotropic ice polycrystals [19].

Tomé [66] suggested an alternative formulation of the VPSC model which allows variations in the matrix-inclusion interaction strength as a function of the ability of each grain (orientation) to accommodate the deformation imposed externally. The reasoning is that a grain 'harder' than the matrix is supposed to deform less than the average, while a grain 'softer' than the matrix will follow the latter's deformation. Therefore, interaction is biased towards the secant for 'soft' grains and towards the tangent for 'hard' grains. We applied this 'variable' formulation to ice [67] as it appeared better suited to dealing with the strain heterogeneity between orientations, together with softening by recrystallisation processes. This formulation is in agreement with the 'variational' approach of Gilormini et al. [19] and does not tend toward a Reuss bound for highly anisotropic material (Fig. 4). It offers better stability for texture development simulation than the VPSC tangent scheme, and allows a fairly good representation of texture evolution when compared with ice core observations. However, a stronger non-basal activity (30% in average) is associated with a more homogeneous deformation between grains. Together with the results obtained on ice by Gilormini et al. [19], our results point out the inability of '1-site' homogenisation schemes to reproduce the impact of observed strain gradients and recrystallisation processes on deformation mechanisms of polar ice polycrystals without introducing a *strong non-basal activity*. The intracrystalline field heterogeneity, which is of the highest importance in strongly anisotropic



Fig. 4. Overall reference stress for isotropic ice as a function of the anisotropy parameter  $\tau_{\rm B}/\tau_{\rm A}$  (ratio of critical shear stress on the hard glide plane to the one of the easy system) [19,67].

materials, can be predicted only in an average fashion, by computing the second order statistical moments of the corresponding distributions inside each grain [68].

Lebensohn [69] adapted the FFT formulation of Moulinec and Suquet [70] to viscoplastic materials, and provide a '*n*-sites' micro–macro model well suited to the modelling of ice polycrystal, a material with a random distribution of heterogeneities [71]. The '*n*-sites' scheme takes into account the interactions between neighbouring grains and does not require homogenisation for the calculation of grain behaviour within the polycrystal. The viscoplastic FFT formulation is based on solving the governing equations inside a unit cell with periodic boundary conditions and consists in finding a strain-rate field that minimises the average of local strain energies, under the constraint imposed by the strain compatibility conditions. The unit cell is discretised into a 3D Fourier grid, such that each grain is represented by a large number of Fourier points. Stress and velocity gradients are obtained for each point, thus allowing the computation of heterogeneities and misorientations within each grain.

Results for 2D columnar ice [71] have provided a good representation of strain and stress heterogeneities within grains regarding the interaction with neighbouring grains, together with a basal activity greater than 90%. Analysis of lattice curvature with regard to strain gradients and recrystallisation processes should provide valuable information and is in progress for 3D granular ice as found in polar ice sheet.

# 6. Conclusions

The deformation of ice crystal is dominated by the glide of dislocations on the basal plane. Slip or climb of dislocations on non-basal planes cannot be completely dismissed with regard to dislocation multiplication. The large anisotropy of ice crystal lies at the origin of the development of a non-uniform state of internal stresses during the transient creep of polycrystalline ice. Large strain inhomogeneities within grains are induced by the incompatibility of deformation between grains. For the low stress conditions prevailing in polar ice sheets, grain boundary migration and dynamic recrystallisation are efficient recovery processes. Orientation gradients, observed by X-ray diffraction, accommodate basal slip and contribute to sub-boundaries formation. The amount of non-basal slip or climb of basal dislocations required for compatibility reasons should therefore be small in polar ice. The low stress exponent corresponding to the deformation of ice at low stresses is in accordance with the primordial role of recrystallisation in the accommodation of basal slip.

*N*-site polycrystal models, based on micro–macro approaches, appear to be better suited to simulate the mechanical behaviour of polar ice than '1-site' homogenisation schemes. The intracrystalline field heterogeneity, which is of the highest importance in anisotropic materials, must be predicted. Ice appears to be a good model material to test polycrystal models simulating mechanical behaviour. Models accounting for the anisotropy of polar ice must be used to simulate the flow of ice sheets and to improve ice cores dating.

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