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Fundamental metrology/Métrologie fondamentale

Metrology and general relativity

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Abstract

The accuracy of the measurement of time has led to a reconsideration of metrology in the framework of general relativity. This theory, which up to now has passed successfully all experimental tests, keeps the invariance of physical laws, but restricts it as a local property. However, the main applications of the time standards involve a large extension in space: the whole Earth, the solar system and farther. For example, one of the most striking applications is the development of satellite positioning systems that everyone uses: such systems would not operate without a rigorous relativistic treatment. The role of measurements in extended spatial domains is therefore examined. ***To cite this article: B. Guinot, C. R. Physique 5 (2004).***

Résumé

Métrologie et relativité générale. L'exactitude des mesures de temps a conduit à une réflexion sur la signification de la métrologie dans le cadre de la relativité générale. Cette théorie qui a jusqu'à présent subi avec succès l'épreuve de l'expérience conserve l'invariance des lois de la physique, mais la restreint au domaine local. Or, les principales applications des étalons de temps mettent en jeu une grande étendue spatiale : la Terre, le système solaire et au-delà. Par exemple, une des plus frappantes applications est l'établissement de systèmes satellitaires de positionnement que chacun utilise, parfois sans le savoir. De tels systèmes ne sauraient fonctionner sans un traitement relativiste rigoureux. Le rôle des mesures dans un espace étendu est donc examiné. ***Pour citer cet article : B. Guinot, C. R. Physique 5 (2004).***

Keywords: General relativity; Time standards; Satellite positioning systems; Measurements in extended spatial domains

Mots-clés : Relativité générale ; Étalons de temps ; Systèmes satellitaires de positionnement ; Mesures dans un espace étendu

1. Introduction

General relativity was born in 1916 mainly from Einstein's desire to incorporate special relativity and gravitation in an elegant and simple single theory. Special relativity was widely used by physicists, in particular for evaluation of the Doppler effect and of the energy of a mass moving in the frame of an observer. In contrast, until the 1960s, general relativity was considered as a monument due to the genius of Einstein but *was cut-off from the mainstream of physics* [1]. This situation arose from the lack of confrontation with measurements apart the two well-known classical tests: the explanation of the advance of

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the perihelion of Mercury [2] and the measure of the deflection of light by the Sun [3]. A third classical test was the observation of the gravitational frequency shift predicted by the theory, but this shift could be reliably measured only in 1960 [4]. Early applications to cosmology could not bring a clear conclusion concerning the validity of the theory.

The need to appeal to general relativity in metrology and in some technical applications appeared with the atomic clocks and their rapid progress. The relative frequency inaccuracy of the first caesium clock in 1955 was about 10^{-9}; in 1970 an inaccuracy of 5×10^{-13} was reported. Nowadays the level of 10^{-15} is reached and a further reduction of uncertainties is expected in the near future. These values are to be compared with the relativistic gravitational frequency shift near the ground: 10^{-13} per kilometre in altitude. Let us recall a key experiment: the flights of atomic clocks around the world organised in 1972 in order to check the predictions of general relativity [5]. This experiment may now look naïve, on account of the large uncertainties. It was nevertheless important for psychological reasons: it demonstrated in a spectacular way the need to treat in the framework of general relativity the numerous worldwide synchronizations by air-borne atomic clocks performed in the 1970s. Another striking and much more precise experiment was the launch of hydrogen masers in a rocket in 1976 [6]. The need to appeal to general relativity extended to astronomy, geodesy, space techniques where the most precise observations are based on the time of flight of electromagnetic signals and on the Doppler effect and take full advantage of the frequency accuracy and stability of atomic clocks. In the domain of applications, an example of use of general relativity is provided by the satellite positioning systems such as the Global Positioning System of the USA or the future European GALILEO.

General relativity had thus become an essential tool and was used whenever useful. However, this was often done by 'corrections' to the classical model, without insertion in a comprehensive relativistic framework. Only in 1991, the International Astronomical Union adopted, after much controversy, general relativity as the working theory in the definition of space–time reference systems. The Bureau International des Poids et Mesures gave the first notes on the significance of the second in general relativity in its 1985 edition of the book on the International System of Units (SI) [7]. In 1992, the Comité International des Poids et Mesures decided the creation of a Working Group on the Application of General Relativity to Metrology, whose report [8] has been largely used in the preparation of this article. Part of the reluctance to adopt general relativity explicitly was motivated by some doubts concerning the validity of the theory and by the competition of other relativistic theories. However, in the framework of metrology and applications in physics, astrometry, celestial mechanics, space research, geodesy, where gravitational fields are weak, it can be admitted that general relativity satisfies all the requirements, for the time being. As general relativity is the simplest relativistic theory, it is fully justified to use it as the working theory. General relativity can be seen as a new 'classical' framework for metrology, until tests actively pursued reveal its possible shortcomings.

2. Local physics

The invariance of local physics is a postulate of general relativity, with important restrictions. (i) It is only valid in an infinitesimally small region of space and appears as an approximation as soon as an extent in space is involved. (ii) It also requires the absence of gravitation and rotation. It is then designated as the Einstein's Equivalence Principle (EEP). The form of the EEP given in [8] is reproduced here: "(...) *if the same local, non-gravitational, experiment is performed at two different locations and times, and if the measurement of its evolution is based on identically built local standards, the mathematical description of its evolution is the same. In particular the coupling constants that enter into the local laws of physics, when referred to local standards* (...) *take the same values*". In such a local experiment, the theory of special relativity has to be applied.

Practically (we quote [8] again) "*in the real case of, say, a laboratory at rest on the surface of the Earth, which is accelerated in a space–time sense, the universality of the outcome of the experiment holds only after corrections have been made for the gravitational effects of the external universe as predicted by Einstein's theory itself, treated in a classical manner* (*e.g. the fall of atoms in the resonant cavity of an atomic clock resting on the ground*)." The gravitational effects inside the laboratory, as in the Cavendish's experiment for the measure of the constant of gravitation, can also be treated in a classical manner.

In applications, one has to make sure that the approximation implied by 'local' is compatible with the experiment at hand. Locally the relativistic effects predicted by general relativity do not arise from the external gravitational field of the Earth itself: their cause is the lack of uniformity of this field. For example, this non-uniformity, near the surface of the Earth generates a relativistic blue shift of the frequency of an atomic clock of 10^{-16} per metre of elevation. If a relative uncertainty of 10^{-15} is acceptable, a usual room is local and metrology keeps its familiar form.

The condition of non-rotation, when it matters, is local and is determined by the absence of dynamical rotational effects, such as Coriolis force. In practice, a kinematical definition of non-rotation with respect to the direction of very distant astronomical sources, the quasars, is often sufficient: on the Earth, the difference between the dynamical and kinematical definitions is a rotation of the polar axis at a rate of $2''$ per century. Let us recall that the local invariance of electromagnetism included in the EEP is at the root of special and general relativity.

We shall see later that it is always possible to define mathematically at any location and time (i.e. at any *event*) an ideal *tangent flat space* where it is possible to apply the EEP in an extended domain. A real laboratory in free fall in a gravitational field is an approximation of the tangent flat space in a limited space region.

3. Units

A straightforward consequence of the EEP is that the definition of units in the International System of Units (SI) applies without any additional information in the tangent flat space of the observer, the standards being at rest with respect to him. In their realization in a laboratory submitted to an external gravitational field (or acceleration), as a laboratory on the ground, one has to take account of gravity in the classical manner and to make sure that this laboratory is sufficiently small, so that the non-uniformity of the gravitational potential can be neglected. This is often summarised by saying that units must be seen as *proper units*. Similarly, the measurands and physical constants are *proper quantities* whose values are expressed with proper units.

It has been sometimes suggested to specify the location and time, i.e. the *event*, where the definition of the second (for example) applies. Thus the definition would be free from any theoretical model and a physicist could use the theory which he finds the most appropriate to obtain the unit of time he needs in his laboratory. Such a localized definition might be necessary when a violation of the EEP is demonstrated. At present this would be a useless complication. However the assumptions implied by the underlying postulate on local physics should not be forgotten.

From a philosophical point of view, it is somewhat disturbing that metrology, the science of rigour, only holds as an approximation as soon as a space extent is involved; but we have to accept the world as it is. In extended domains, the key to model phenomena is the use of coordinates as we shall see later. Comments on some units are now given.

3.1. The second

Let us recall first that general relativity only uses one space–time unit. In theoretical work, one does not distinguish units for time and length, and the velocity of light c is taken equal to unity (dimensionless). Although for astronomical distances the light year is a familiar unit, it is convenient for practical measurements to define two base units, the second and the metre. With the definition of the metre adopted in 1983, which introduces a conventional relationship between these units ($c = 299792458$ m/s exactly), the definitions of the second and the metre are in conformity with the theory. The choice of the second as sole independent unit is justified by the accuracy which can be attained in the realization.

The definition of the second is valid on the world line of the caesium-133 atom. To a high degree of accuracy, the frequency of the defining transition does not depend on the direct effect of gravity (if the atom is not in free fall). The only relativistic effect which has to be considered when operating an atomic time/frequency standard was, until recently, the second order Doppler shift of special relativity due to the velocity of the atom with respect to the observer (which can be defined as a specified connector which delivers the frequency of the atomic transition, or a known sub-multiple of it). However, time has come where the variation of the gravitational potential within the structure of an atomic clock is not negligible; the gravitational frequency shift being 10^{-16} in relative value for an elevation of one metre, close to the surface of the Earth. The theoretical modelling of a clock in the framework of general relativity is becoming a necessity [9].

3.2. The metre

The velocity of light c being only a conversion factor, the definition of the metre should not raise any other problems than those of the definition of the second. However, formally, the definition of the metre has to be considered as valid only in the flat tangent space of the observer. This is also true for the wavelengths provided for the practical realization of the metre. In contrast with proper time, a finite proper length, strictly speaking, has no physical existence in presence of a gravitational field. In practice, the relativistic ambiguity of the proper metre does not matter over length of, say, a few metres, because it is much smaller than the uncertainties of measurement. Over large distances, such as those which are encountered in geodesy and astronomy, one deals with proper time measurements: flight time of photons reflected by a mirror (laser and radar telemetry), Doppler effect, time of arrival of signals emitted by satellites (satellite positioning systems). This is a domain where relativistic coordinates are needed; the use of metres instead of seconds is merely a matter of habit.

3.3. The kilogram

The uncertainties on all SI units other than the second and the metre are such as the whole Earth appears as a sufficiently ‘small’ laboratory. Nevertheless a comment on the kilogram may be useful. The concept of mass in special relativity is a source of misunderstandings as pointed out by Okun [10], because some authors consider that mass increases with the velocity. In special relativity and in general relativity, the mass of a body is an attribute of this body, a constant, independent of the frame of reference. In these theories, as in Newtonian mechanics, inertial mass and gravitational mass are identical. Strictly speaking, the mass of a body includes the equivalent of energy accumulated in this body, such as temperature and self-gravitation, and remains constant if the body does not exchange radiation with the external universe. The possible variation due to such exchanges is totally negligible in practice, for macroscopic systems, in laboratory experiments.

4. Physics in extended domains

Local metrology keeps its familiar form. However, metrology includes comparisons between distant standards, located on the Earth, and also in space laboratories. On the other hand, it is often necessary to establish a theoretical model for an experiment extended in space and time, on the basis of local measurements performed at different locations. An example is the determination of the orbit of an artificial satellite of the Earth, based on laser telemetry from several ground stations. The solution of these problems requires the definition of space–time reference systems, the use of relativistic coordinates and of a conventional, but rigorous, definition of synchronisation.

In the theory of general relativity, the basic mathematical object is the *metric tensor* **G**, which depends on the distribution of mass and energy through the Einstein field equations. This tensor contains all the information needed to represent gravitational effects.

In any space–time coordinate system x^α, $\alpha = 0, 1, 2, 3$ (assuming continuity conditions not discussed here), the *metric* is written with the component $g_{\alpha\beta}$ of **G** as

$$\mathrm{d}s^2 = g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta, \tag{1}$$

with the summation convention on repeated indices, and with $g_{\alpha\beta} = g_{\beta\alpha}$.

The scalar $\mathrm{d}s^2$ (invariant under coordinate change) is the squared interval between two infinitesimally close events described by their coordinates. It is assumed to be measurable with rods and clocks.

The field equations do not lead to any particular coordinates. Any coordinate change of the form $\underline{x}^\mu \to \underline{x}^\mu(x^0, x^1, x^2, x^3)$, with $\mu = 0, 1, 2, 3$, is possible (with mathematical restrictions). The choice of coordinates is a matter of convenience. In practice, different coordinate systems, which are approximate solutions of the field equations are used in limited regions of space. For example, metrology on the Earth, and the motion of artificial satellites, are treated in geocentric coordinate systems, but the motion of planets in the solar system is treated with coordinates centred at the barycentre of this system.

In classical physics, a simplicity criterion leads to conceive coordinates which are directly measurable everywhere: Euclidian space coordinates graduated by transport of a rod, absolute time measured by a clock which can be located anywhere. These fine properties do not exist in general relativity: coordinates are only implicitly defined by the expression of the metric. They can receive values, but they are not directly measurable with physical standards. To stress this aspect, some theoreticians consider them as dimensionless parameters used to label events, some sort of telephone numbers [11]. In practice, usual relativistic coordinates have clearly the dimensions of time and length, and are expressed in seconds and metres, as we shall see later. This may be a cause of confusion: it is essential to keep in mind the distinction between *proper quantities*, locally measurable with standards in the usual manner, and *coordinate quantities*, constructed with coordinates arbitrarily defined.

Mathematically, the symmetry of **G** and its signature allow, at any event, a coordinate change of major importance leading to

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}\tau^2 + \mathrm{d}l^2, \tag{2}$$

with $\mathrm{d}\tau = \mathrm{d}\underline{x}^0/c$, where c is the velocity of light, and $\mathrm{d}l^2 = (\mathrm{d}\underline{x}^1)^2 + (\mathrm{d}\underline{x}^2)^2 + (\mathrm{d}\underline{x}^3)^2$. One recognizes here the Minkowski metric of the flat space–time of special relativity. Physically, this means that in general relativity, it is always possible to find locally an infinitesimally small laboratory where special relativity holds: this should not be a surprise after the success of this theory. Another way to express this property is to say that there always exists a *tangent flat space*, an expression we have used previously. In Eq. (2), τ and l are *proper time* and *proper length*. These quantities are assumed to be measurable, with ideal standards, in the classical sense of metrology. Let us assume that a clock can be moved (case of a *timelike* separation) from event A to event B along a trajectory C, described, for example, by the expression of the four space–time coordinates as functions of a parameter λ. At any point of C Eq. (2) holds with $\mathrm{d}l = 0$. Thus the increase of proper time $\Delta\tau$ is

$$\Delta\tau = c^{-1}\int_C (-\mathrm{d}s^2)^{1/2}. \tag{3}$$

In case of *spacelike* separation ($\mathrm{d}s^2 > 0$), the increase of proper length, Δl, is obtained using a similar integral as for $\Delta\tau$.

Among the possible world lines joining A and B, one is especially important. This is the one which gives to $\Delta\tau$ or Δl an extremal value. This world line is called a *geodesic*. General relativity postulates that a free falling particle follows a *timelike geodesic* and that along the trajectory of photons, $\mathrm{d}s^2$ is constantly zero.

In space–time measurements, time has a privileged role. We can hardly imagine how proper length could be precisely measured, since the standard would involve an extent in space. On the contrary, a clock can be small enough to provide a good approximation to the theoretical proper time. Clearly, such a clock should be insensitive to gravity, temperature and other environmental effects. We *believe* that atomic clocks, which fulfil these conditions with a high degree of accuracy, are good realizations of the ideal clock of the theory. Is that true? All we can say is that up to now the use of atomic clocks has not led to discrepancies between the predictions of general relativity and the observations, within the limits of experimental uncertainties.

5. An example of the role of coordinates

The success of Newtonian mechanics suggests that simplifications may result from the use of space–time coordinates which are close, but not equivalent, to the Newtonian absolute time and Cartesian coordinates. Such coordinate systems are implicitly defined by the expressions of the barycentric (centred at the centre of masses of the solar system) and geocentric metrics adopted by the International Astronomical Union (IAU) in 2000, with all terms up to c^{-4} for g_{00}, c^{-3} for g_{0i}, c^{-2} for g_{ij} $(i, j = 1, 2, 3)$ [12]. These expressions are too complex to be reproduced here, but they are needed for modelling the motion of natural and artificial bodies in the solar system. Only an approximation of the IAU 2000 metric for geocentric coordinates is considered here, under the following form:

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}\tau^2 = -(1-2U/c^2)(\mathrm{d}x^0)^2 + (1+2U/c^2)\big[(\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2\big]. \tag{4}$$

The potential U is the sum of the Newtonian gravitational potential U_E generated by the Earth and of the Newtonian tidal potential U_T of external bodies, with the conventions that the potentials are positive, that U_E vanishes at infinity and that U_T vanishes at the geocentre. Near the ground $U_\mathrm{E}/c^2 = 0.70 \times 10^{-9}$ and decreases by 1.1×10^{-16} per metre of altitude. The value of U_T/c^2 increases with the distance from the geocentre and is of order 10^{-15} at the altitude of geostationary satellites.

Usually one defines a *coordinate time* t by $t = x^0/c$. In the IAU 2000 metric and in (4), the coordinate time t, corresponding to the use of the SI second for τ, and after the choice of an origin, received the name Geocentric Coordinate Time (TCG).

In order to illustrate the role of coordinates, we shall take the example of the orbit of a satellite of the Earth, considered as a point-mass. Neglecting here perturbing forces such as atmospheric friction, radiation pressure, the satellite follows a *timelike geodesic.* The equation of the geodesic takes the form of a system of four differential equations parameterized with an affine parameter which may be the time of a clock carried by the satellite. These equations contain the four coordinates, and derivatives of the components of the metric.

Satellite tracking stations on the ground measure proper time with their clocks such as duration between the emission and reception of photons reflected by the satellite, time of reception of a satellite signal, measurement of a frequency emitted by the satellite. The measurements are converted in TCG and are used together with the geodesic equations to fit by statistical methods a model of the orbit, establishing relations between space and time coordinates. With a convenient organisation of the observations, one may obtain, for example, (i) an ephemeris which gives the space coordinates of the satellite, using as argument TCG, in the reference system implied by (4); (ii) geodetic coordinates of the observing stations in a system rotating with the Earth, where they are almost constant; (iii) the proper frequency of a clock carried by the satellite, referred to TCG.

In most cases, the direct use of space coordinates satisfies the needs of users. One of the outputs of space geodesy is the realization of an International Terrestrial Reference Frame which consists in the coordinates of several hundreds of sites with uncertainties at centimetre or millimetre level. These coordinates are used directly by the users of positioning systems.

In some applications, very accurate coordinates are employed to derive local ‘distances’. Theoretically the term distance has no rigorous meaning when the mode of measurement is not specified. As an example, let us suppose that an observer in A measures the distance to a point B by laser telemetry, both A and B being on the ground, say about one kilometre apart. The return flight of photons is measured in proper time of the observer, $\Delta\tau$, and a distance D_1 is $D_1 = \frac{1}{2}c\Delta\tau$. If the coordinates of A and B are known, one can also compute a *coordinate distance* D_2 from their differences Δx^i as

$$D_2 = (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2. \tag{5}$$

The difference between D_1 and D_2 is of order of 10^{-9} in relative value, i.e. in that case 10^{-6} metre. In practice this ambiguity can be usually ignored. A concrete example is the observation of the variation of mean sea level by satellite altimetry, where an accuracy of the millimetre per year measured over a few years is a reasonable goal. The use of coordinates is quite sufficient, but it is important to make sure that no inconsistencies arise in the relativistic treatment by different centres, at different epochs.

6. Metrology of time

The distinction between proper and coordinate quantities is especially important for time, because, in addition to the realization of a unit for duration, which is a proper quantity needed in laboratory work as we have seen previously, one needs time scales for dating events and synchronizing activities in extended domains. These time scales must be coordinate times.

It appears now evident that time scales should be coherent with the definition of the SI second for proper time adopted in 1967, based on an atomic frequency. However, at the beginning of the era of caesium clocks (1955), the construction of atomic time scales by integration over their frequency was leading to a cumulated error in time which was judged unacceptable. The International Atomic Time was officially adopted only in 1971 [13].

Before the advent of caesium atomic clocks, the best measurement of time was deduced from the motion of planets and of the Moon, treated in the Newtonian model of mechanics. The primary output was a time scale (the Ephemeris Time), from which the unit of time was derived. Presently, the observations of the motion of planets are treated in the framework of general relativity and the method would lead to the realization of a coordinate time, the Barycentric Coordinate Time TCB. The adopted metrics and relativistic coordinate transformations can provide the unit of proper time for a terrestrial user. On theoretical grounds this process is sound. It has been abandoned simply because it cannot compete in accuracy with atomic clocks.

The development of the metrology of time based on atomic frequency standards and clocks confers an essential role to time comparisons.

(i) The measurement of the frequency difference of atomic frequency standards is, as for all metrological standards, a requirement. Usually these standards cannot be brought side by side. Their frequency difference is based on comparison at distance of their proper times.
(ii) Time scales for use in extended domains, especially for terrestrial use, have to be realized. These time scales, which are representations of relativistic coordinate times, must be based on the proper time of numerous atomic clocks in order to eliminate the risk of interruption and to ensure the best quality by a statistical treatment. Here time comparisons are needed quasi permanently. Time comparisons are not only important, they are also critical because their uncertainties are often too large for an optimum use of the standards. Let us recall that one nanosecond over ten days corresponds to a relative frequency difference of 10^{-15}, which is the uncertainty of the best standards.
(iii) The dissemination of a time scale consists in time comparisons between this scale and a user's clock. This is especially important for systems based on synchronization (satellite positioning systems, communications, ...).

The definition and the realization of time comparisons require the use of a metric. It is often useful to define a coordinate system rotating with the Earth by a simple rotation of the space coordinates of (4) around the polar axis. This transformation keeps unchanged the coordinate time TCG. In polar coordinates, the metric takes the form

$$\mathrm{d}s^2 = -(1-2\tilde{U}/c^2)\,\mathrm{d}t^2c^2 + (1+2U/c^2)\big[\mathrm{d}r^2 + r^2\,\mathrm{d}\phi^2 + r^2\cos^2\phi\,(2\omega\,\mathrm{d}L\,\mathrm{d}t + \mathrm{d}L^2)\big]. \tag{6}$$

In this expression

$$\widetilde{U} = U + \tfrac{1}{2}r^2\omega^2\cos^2\phi + \mathrm{O}(c^{-2})$$

is the sum of the gravitational potential of the Earth and of the potential of the centrifugal force, $r = [(x^1)^2 + (x^2)^2 + (x^3)^2]^{1/2}$, ϕ is the geocentric latitude and L is the longitude (reckoned positive toward East), ω is the velocity of the Earth's rotation. The surface $\tilde{U} =$ constant which coincides approximately with the mean sea level is the *geoid*.

The coordinate time TCG runs faster than the proper time on the surface of the Earth by about 7.0×10^{-10}, i.e. 22 ms per year. It has been found convenient, in 1991, to define a Terrestrial Time TT by applying to TCG a frequency offset such that the second of TT coincides with the proper SI second on the geoid. However, in 2000, the IAU, in order to avoid difficulties inherent in the definition and realization of the geoid, recommended to define TT by

$$\mathrm{d}TT/\mathrm{d}TCG = 1 - L_\mathrm{G},$$

where $L_\mathrm{G} = 6.969290134\times10^{-10}$ is a defining constant, chosen so that the rate of proper time and of TT are in very close agreement on the geoid.

In the following, we will use TT, designated by $\underline{t}$, in place of TCG.

6.1. Simultaneity and time comparisons

Simultaneity has no absolute meaning in general relativity. The convention of *coordinate simultaneity* actually applied is that two events at different location in space are said *simultaneous* when they have the same coordinate time t in some specified coordinate system. For applications on the Earth, or in its vicinity, t is conveniently taken equal to TCG. Let us remark that a change of coordinates of the general form $\underline{t} = \underline{t}(t)$, $\underline{x}^i = \underline{x}^i(t, x^1, x^2, x^3)$ keeps the coordinate simultaneity unchanged. Thus, the application of this convention to the non-rotating and rotating systems implied by (4) and (6) and to TT leads to the same results.

Comparisons of distant standards, for any quantity, should be *coordinate simultaneous.* Of course, the strict application of this convention does not matter for standards providing nearly constant quantities. But the convention is essential for clocks. Consider clocks A and B, their readings (proper times) at date $\underline{t}$(coordinate time) are denoted, according a common practice, $A(\underline{t})$ and $B(\underline{t})$. The time comparison at date $\underline{t}$ is expressed by $A(\underline{t}) - B(\underline{t})$, often written as

$$A - B = x \text{ seconds at } \underline{t} = y, \quad y \text{ being a date.}$$

In most cases, $A - B$ varies slowly, so that the date of the comparison in $\underline{t}$ can be specified loosely. The condition of *coordinate synchronisation* of A and B during an interval $(\underline{t}_1, \underline{t}_2)$ is defined by $x = 0$ during this interval. The same definition of time comparisons can be applied to coordinate times, but in general, the location must be also mentioned (i.e. the coordinates of the *event* of the time comparison).

Presently, when no material link is available, all precise time comparisons, even over short distances, say of a few kilometres, involve the propagation of an electromagnetic signal between a ground station and a satellite. Different configurations are developed in order to minimize some perturbations. For example in the two-way satellite time and frequency transfer (TWSTFT), signals relayed by a satellite are sent from ground stations A to B and B to A, nearly simultaneously, in order to eliminate the uncertainties on the refraction and on the position of A, B and of the satellite. From a relativistic point of view, all methods lead us to consider the basic problem of time comparison between clocks on the ground and on a satellite, the positions being known. This problem is treated, for example, in [8]. Let us only recall that it requires to express the propagation path, which is a null geodesic, as a function of the selected coordinate time $\underline{t}$, and to solve $\mathrm{d}s^2 = 0$ in $\mathrm{d}\underline{t}$. The integration along the propagation path provides the propagation time $\Delta\underline{t}$. This is a complicated process. Fortunately, in the non-rotating system, the gravitational bending of the world line of the photons has still negligible effects and one obtains as a sufficient approximation

$$\Delta\underline{t} = \rho/c + (\Delta\underline{t})_{\mathrm{G}}, \tag{7}$$

where ρ is the coordinate distance between a ground station and the satellite defined as in (5). Quantity $(\Delta\underline{t})_{\mathrm{G}}$ depends on the coordinates; it is small (56 ps from a ground station to a geostationary satellite vertically above), but may have to be evaluated. Let us mention some more trivial, non-relativistic, problems. The propagation time $\Delta\underline{t}$ being a priori unknown, the position of the satellite at the time of reception of the ground signal, is not precisely known: this is solved by successive approximations. On the other hand, one has to use the rate $\mathrm{d}\tau/\mathrm{d}\underline{t}$ of the satellite clock in order to extrapolate its reading at the time of emission from the ground station; on account of the short duration involved, this is not critical.

When treated in the rotating coordinate system, which is especially convenient for geostationary satellites, a complementary term appears in (7), which represents the Sagnac effect. This term may exceed 250 ns for a geostationary satellite.

6.2. Frequency comparison of distant standards

Accurate frequency comparisons of distant standards are usually based on time comparisons at two dates, $\underline{t}_1$ and $\underline{t}_2$. During this interval, the above methods provide, for two clocks A and B, $\Delta\tau_{\mathrm{A}}/\Delta\underline{t}$ and $\Delta\tau_{\mathrm{B}}/\Delta\underline{t}$. The theoretical quantity $\mathrm{d}\tau/\mathrm{d}\underline{t}$ which depends on the world lines of each clock has a central role in the frequency comparison. We will not enter into the details, we only note that here again the defects in the realization of $\underline{t}$ have no critical role, the limiting factor in accuracy being the uncertainty of time comparisons at $\underline{t}_1$ and $\underline{t}_2$.

For two standards A and B fixed on the ground, $\mathrm{d}\tau/\mathrm{d}\underline{t}$ depends only on the total potential $\widetilde{U}$. It is thus easy, in principle, to compute $\mathrm{d}\tau_{\mathrm{A}}/\mathrm{d}\tau_{\mathrm{B}}$ the standards being assumed at the same location and thus the ratio of relative proper frequencies $f_{\mathrm{A}}/f_{\mathrm{B}} = \mathrm{d}\tau_{\mathrm{B}}/\mathrm{d}\tau_{\mathrm{A}}$. In practice, the main difficulty is the evaluation of the potential which is not straightforward and which, at present, limits the accuracy to about 3×10^{-17} [14]. A better accuracy may be obtained if the clocks are in space because we benefit of a better representation of the gravitational potential U. In the future, frequency comparisons between standards in space and on the ground may become a geophysical tool for the study of the geoid [15].

6.3. Definition of times and realization of time scales

We use here the terms *time* for theoretical times and *time scales* for realizations, affected by uncertainties.

The International Atomic Time is a time scale which is a realization of TT, but with a time offset (for historical reasons), such as

$$TT = TAI + 32.184 \text{ s (exactly)} \quad \text{on } TAI = 1977 \text{ January 1, 0h 0m 0s.} \tag{8}$$

Indeed, expression (8) is a definition of the origin of TT. It is puzzling that a theoretical time has its origin experimentally determined. An origin of a time must be fixed by dating an event. In all coordinate times needed in astronomy, this event, specified by its four coordinates, is conventionally the geocentre at date in TAI given in (8). The memory of this event is kept by TAI (or any other time scale). Concretely, that means that a realization of a coordinate time on the basis of TAI, for example, is available at any instant, the integration constant being fixed by the above convention.

The time scale TAI is obtained by averaging the data of numerous atomic clocks (about 250) and of frequency standards providing the best realizations of the SI (proper) second. Before averaging, proper times and proper frequencies are converted into coordinate quantities in the system where TT is the coordinate time. An essential tool in this enterprise is the comparison of distant clocks and frequency standards. This is treated with the convention of coordinate simultaneity, as explained previously.

The departure of TAI with respect to $TT - 32.184$ s, null in 1977 by definition of TT, results from the integration of the errors in frequency of caesium clocks. It may increase indefinitely. The cumulated error from 1977 to the present might be of about 20 µs. It is generated mostly by frequency inaccuracy and long term frequency instabilities. It might not be negligible in some astronomical studies, for example in studies of the rotation of millisecond pulsars whose phase can be recorded with uncertainties of order of 1 µs [16]. Fortunately the rapid improvement of atomic clocks ensures that the error will grow at a much smaller rate.

7. Final remarks

Although this article deals with metrology and general relativity in general terms, this topic is especially important for time. More details can be found in [8] and in [17]. In these documents the developments are limited at the c^{-2} order. However, the progress of time and frequency standards calls for a better approximation. Developments at the c^{-3} order can be found in [18] and [19].

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