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C. R. Physique 5 (2004) 1011–1020
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String theory and fundamental forces/Théorie des cordes et forces fondamentales

Collisions of cosmic F- and D-strings

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Available online 11 November 2004

Presented by Guy Laval

Abstract

Recent theoretical advances and upcoming experimental measurements make the testing of generic predictions of string theory models of cosmology feasible. Brane anti-brane models of inflation within superstring theory are promising as string theory descriptions of the physics of the early universe. While varied in their construction, these models can have the generic and observable consequence that cosmic strings will be abundant in the early universe. This leads to possible detectable effects in the cosmic microwave background, gravitational wave physics and gravitational lensing. Detailed calculations of cosmic string interactions within string theory are presented, in order to distinguish these cosmic strings from those in more conventional theories; these interaction probabilities can be very different from conventional 4D strings, providing the possibility of experimental tests of string theory. ***To cite this article: N. Jones, C. R. Physique 5 (2004).***

Résumé

Collisions de cordes cosmiques F et D. Les progrès théoriques récents et les mesures expérimentales à venir rendent possibles les tests des prédictions cosmologiques génériques de la théorie des cordes. Les modèles branaires et anti-branaires d'inflations sont des description cordistes prometteuses de la physique des l'univers primordial. Bien que variants dans leurs constructions, ces modèles ont tous la conséquence générique et importante que les cordes cosmiques sont abondantes dans l'univers primordial. Ceci donne des effets détectables dans le rayonnement du fond cosmique, la physique des ondes gravitationnelles et les effets de lentille gravitationnelle. Des calculs détaillés dans la cadre de la théorie des cordes des interactions entre les cordes cosmiques sont présentés, afin de différentier ces cordes cosmiques de celles des théories plus conventionnelles ; la probabilité de ces interactions peut être tres différente des cordes conventionnelles en 4D, fournissant des possibilités de tests expérimentaux de la théorie des cordes. ***Pour citer cet article : N. Jones, C. R. Physique 5 (2004).***

Keywords: String theory models; Brane anti-brane models; Cosmic string interactions

Mots-clés : Théorie des cordes ; Modèles branaires et anti-branaires ; Interactions entre cordes cosmiques

1. Introduction

This article is based on the work [1] with Mark Jackson and Joe Polchinski.

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1631-0705/$ – see front matter
doi:10.1016/j.crhy.2004.10.002

An observation of a cosmic string would be spectacular in itself, but it has long been realised that cosmic strings could provide an understanding of microscopic physics. Long ago, Witten ruled out fundamental strings as long-lived, observable cosmic strings [2] in the phenomenological models of the time; such strings will be unstable, will have tensions too large to be consistent with existing observations, and would be removed by inflation. In [3–5] however, it was noticed that, because of the structure of the brane anti-brane ($D\overline{D}$) tachyon potential, cosmic strings – *and not monopoles or domain walls* – will be produced after inflation that arises from the motion in a $D\overline{D}$ system. Such models include the most well developed embeddings of inflation within string theory, the KKLMMT model [6] (and the subsequent generalisations, [7–13] for instance). The production of fundamental (F-) and Dirichlet (D-) strings in any such $D\overline{D}$ inflation model is generic, depending solely upon causality. Further, [14] showed not only the meta-stability of strings on cosmological timescales in many instances of the KKLMMT model, but also that bound state of F- and D- strings, (p,q)-strings, could be formed and would lead to interesting cosmic physics quite different from that in simple 4D field theory models.

Many of the observationally important properties of cosmic string physics follow from the behaviour of cosmic string networks. The evolution of string networks is governed by the details of string interactions, and much of the resulting late-time physics will depend on these interactions.

Naïvely, averaging over a string network will yield an energy density which scales like a^{-2} (where a is the cosmological scale factor) [15]. This can be easily seen because the energy-density of non-interacting objects of worldvolume n will scale like in 4D spacetime like a^{n-4}: radiation scales like a^{-4}, dust like a^{-3} and dark energy like a^0. Without interactions a^{-2} scaling would be disastrous, since the string network would quickly come to dominate the universe. Adding interactions, however, generally leads to 'scaling solutions', where if sub-dominant, the string network will match onto the scaling behaviour of the dominant energy species.

A very simple method to estimate the evolutionary behaviour of a string network with interactions is the 'single-scale model' introduced by [16]. Start by assuming that there is a single scale which characterises the string network, $L(t)$; this is for instance the characteristic string length. A single string aligned along the x-axis will have an energy-momentum tensor $T^\mu_\nu = \rho\,\mathrm{diag}(1,1,0,0)$, and averaging over all string orientations, a fluid of strings will therefore have $T^\mu_\nu = \rho\,\mathrm{diag}(1,\frac{1}{3},\frac{1}{3},\frac{1}{3})$, i.e. pressure $p = -\frac{1}{3}\rho$ [15]. ρ can therefore be written in terms of the tension μ as $\rho \sim \mu/L^2$. Assume also there is a definite probability of string self-interaction, $\mathcal{P}$, and the equation of motion for the string fluid becomes

$$\nabla_\nu T^\nu_0 = \{\text{interactions}\} \longrightarrow \dot{\rho} + 2H\rho = -\mathcal{P}\frac{\rho}{L}.$$

This can be solved for $L(t) = t\xi(t)$, which in a 4D universe dominated by objects of worldvolume $n < 2$ has a fixed point $\xi(t) = \mathcal{P}\frac{(4-n)}{2(2-n)}$. The consequences of this are that firstly, with these interactions, $\rho \sim (\mathcal{P}t)^{-2} \sim (\mathcal{P}^2 a^{4-n})^{-1}$ which is the scaling of the dominant energy species. However, simulations reveal that although this picture is qualitatively correct, the model is a little simplistic, and the true scaling of the network is like $\rho \sim (\mathcal{P}t^2)^{-1}$ [17,18]. Physically, the interaction is allowing string loops to break off from long excited strings; the loops then decay into gravitational radiation, and the string network energy density is damped, leading to the scaling behaviour. Also, since the density scales like the inverse of the interaction probability squared, a lower interaction probability leads to enhanced string density, hence the effects shall be augmented.

Therefore, one way in which a cosmic string network which arises in string theory can differ strongly from that in a 4D field theory model is in its interaction probabilities, $\mathcal{P}$. The field theory solitons have essentially $\mathcal{P} \simeq 1$ [19], so to calculate any deviation from this would distinguish string theory cosmic strings. Further, in string theory there is the possibility of producing (p,q) strings which can have 3-string junctions [14]; networks of such string types could also lead to deviations from scaling, so their interactions are important. The detailed calculations of string theory F-, D- and (p,q)- string interactions are presented here with a view to understanding these distinguishing features.

2. Calculating string interactions

In [5] it was recognised that the important string interaction probabilities for string theoretic cosmic strings could be suppressed by the spread of the strings in the extra dimensions. The argument applied to orientifold torus models. In the flux stabilised model [6], this reasoning must be modified because the strings are localised by a potential and see only some effective volume; also different string types can be localised in different throats and the alteration of interaction probabilities is not so clear [14].

The explicit string theory interaction probabilities for macroscopic strings are calculated in [1]. The procedure employed is to first calculate flat-space interaction probabilities and then to embed these into the various brane inflationary models by considering the effective volumes of localisation.

2.1. F–F interaction

The basic calculation of the string reconnection process, if the strings involved are F-strings, is a simple string scattering calculation, using the tricks applied to the calculation for the bosonic string in [20]. In fact the result shall be the same, although the strings are superstrings; this is because the macroscopic limit erases the low-level oscillator information.

The calculation is set-up as follows. Compactifying two of the 4 spacetime dimensions on a torus of radii l, with sides at an angle θ, the incoming macroscopic strings can be represented by closed strings wound on the torus; the final state string is a bent, doubly wound closed string. The transverse dimensions are compactified on a torus of volume $V_\perp$; this volume will be replaced later by an effective volume in the compact manifold within which the strings are localised. The straight wound strings are easily represented by winding vertex operators; the bend string would have an infinite number of oscillator excitations in the macroscopic limit, so the optical theorem is used to calculate the sum of amplitudes to all final states, and the final state vertex operator need not be used. The imaginary part of the forward scattering diagram is the required sum of squared amplitudes to final states. Finally, to further simplify the calculation, the string states are included without any oscillator excitations; this amounts to switching the GSO projection by inserting a factor of $(-1)^{\boldsymbol{F}}$ ($\boldsymbol{F}$ being spacetime fermion number) in the trace over the directions of the torus. Adding a finite number of excitations in the macroscopic, $l \to \infty$, limit cannot change the result.

Following this procedure, the reconnection probability, obtained from the imaginary part of the invariant string scattering amplitude, with the relevant kinematic factors, is [1]:

$$\mathcal{P} = \frac{g_s^2}{V_\perp}(2\pi^2\alpha')^3 \frac{(1-\cos\theta\sqrt{1-v^2})^2}{\sin\theta v\sqrt{1-v^2}}. \tag{1}$$

As is necessary, all factors involving the string length which appear at various stages of the calculation have canceled out to give a finite macroscopic string limit. The important dependence on g_s and extra-dimensional volume could have been guessed: the interaction is suppressed by g_s^2, and can be suppressed by a large spread of the string wavefunction in the extra dimensions, $V_\perp$. As discussed in [1], this result suffers higher order corrections, which cannot be so quickly calculated by the unitarity trick.

2.2. F-(p,q) interaction

When one of the interacting strings has D-string charge and the other remains a pure F-string, a tree-level open string diagram can describe the lowest order term in the process. The F-string splits on the D-string to form an open string attached to a D-brane, and the calculation [1] is a generalisation (and supersymmetrisation) of the calculation [21] describing the decay of macroscopic open strings. The D-string can be turned into a (p,q)-string by adding constant electric flux on the disc boundary [22]. As is well known, the constant electric flux on the boundary induces a change in the boundary conditions and hence OPEs for the worldsheet fields, resulting in an altered open string metric with which momenta are contracted [23].

Using the same tricks as in the previous section, the interaction probability is calculated to be [1]:

$$\mathcal{P} = \frac{g_s}{V_\perp}(2\pi^2\alpha')^3 \frac{q^2v^2 + [g_s p - \cos\theta\sqrt{(1-v^2)(g_s^2p^2+q^2)}]^2}{\sin\theta v g_s\sqrt{(1-v^2)(g_s^2p^2+q^2)}}. \tag{2}$$

Note that at zero velocity the numerator vanishes when the angle between the strings is $\tan\theta = q/g_s p$, which is the angle at which the strings are mutually BPS and there is no long-range force between them [24].

The reconnection probability above reduces exactly to (1) when $p \to 0$, $q \to 1$. This may be surprising since the two interaction probabilities are obtained from very different string amplitudes, however in the forward scattering limit, in the t-channel both processes are seen to pick up the supergravity pole; the imaginary part of the full amplitude is then obtained from an average over the s-channel poles in the macroscopic $s \to \infty$ limit. Thus both cases lock onto a result which could be obtained by a supergravity calculation and consequently (2) should reduce to (1).

2.3. D–D reconnection

The calculation of the interaction when both strings have D-string charge is less straightforward. The perturbative method of [1] is to first calculate the probability of string pair production between the moving angled D-strings, then to estimate the number of string pairs that must be produced to 'stick' the D-strings together long enough such that they will recombine, and finally to calculate the probability of that number being produced.

The first part of the calculation was first performed for parallel Dp-branes by Bachas [25] and has been reapplied many times. The procedure is to calculate the one-loop open string vacuum amplitude, the annulus diagram, where either boundary has the boundary conditions of one of the (p,q)-strings. Here only the $(p,q) = (0,1)$ case is presented, but details of the general case appear in [1].

With the spectrum given by the modding above and an impact parameter y, it is straightforward to extend the results of [25] to this case, leading to an amplitude [26,27,22,28],

$$\mathcal{M}(y) = -\frac{\mathrm{i}}{2}\int_0^\infty \frac{\mathrm{d}t}{t}\,\mathrm{e}^{-ty^2/2\pi\alpha'}\left[\eta^6(\mathrm{i}t)\Theta_1\left(\mathrm{i}\frac{\theta t}{\pi}\Big|\mathrm{i}t\right)\Theta_1(\epsilon t|\mathrm{i}t)\right]^{-1}\sum_{k=2}^{4}(-1)^{k-1}\Theta_k(0|\mathrm{i}t)^2\Theta_k\left(\mathrm{i}\frac{\theta t}{\pi}\Big|\mathrm{i}t\right)\Theta_k(\epsilon t|\mathrm{i}t). \tag{3}$$

The imaginary part of this amplitude can be understood as the pair production rate (or directly the probability in this case where the strings are confined to a point) of open strings between the D-strings. Similar calculations in [25,29] can be understood as the production rate of charged particles in an electric field, which is the string-theory version of the field theory calculation of this phenomenon due to Schwinger [30]. As emphasised in [1], the pair production rate will not be the reconnection probability, but it can be deduced from it. The probability of producing *at least one* pair of strings is given by the sum of disconnected annuli diagrams,

$$\mathcal{P}_{\mathrm{pp}}(y) = 1 - \left|\mathrm{e}^{\mathrm{i}\mathcal{M}(y)}\right|^2 = 1 - \left[\frac{\prod_{\mathrm{fermions}\,j}(1-x_j)}{\prod_{\mathrm{bosons}\,i}(1+x_i)}\right], \quad x = \mathrm{e}^{-2\pi\alpha' m^2/\epsilon}. \tag{4}$$

The imaginary part of $\mathcal{M}(y)$ is obtained easily from (3) by summing the contributions from the poles at $t = \frac{n}{\epsilon}$, for positive integer n, and m here is the mass of the given stretched string state at minimum separation.

The small velocity limit of this expression is just given by the expansion in the open string channel in which the lightest modes dominate - the tachyon and 4 massless fermions; higher modes are suppressed by $\exp(-2\pi\alpha' m^2/\epsilon)$. The lightest state is the boson with $m^2 = y^2/(2\pi\alpha')^2 - \theta/2\pi\alpha'$ [26]. As $\epsilon \to 0$, for impact parameters smaller than the critical separation $y^2 < 2\pi\alpha'\theta$ when there is a tachyon in the spectrum, the tachyon completely dominates (4) and $\mathcal{P}_{\mathrm{pp}} \to 1$. The rolling of the tachyon when the stationary branes are at a non-zero angle describes the reconnection of the branes [31,32]. In cosmic string networks, the velocities are moderately relativistic, so that a typical string collision will have $v \sim 0.7$ or $\epsilon \sim 0.3$ [17,33]. This is not so different from the small velocity limit, in that only the lightest open strings are produced.

As previously stated, this probability of pair generation is not the probability of D-string recombination; basically because at weak string coupling F-strings are much lighter than D-strings, a single F-string pair will not strongly perturb the motion of the D-strings. Suppose only a small number (compared to g_s^{-1}) of F-string pairs are created in the collision; then the D-strings will pass through each other and continue on their trajectories, stretching the F-strings as they do so. The F-strings pairs are localised about the point of intersection, and they will quickly find each other and annihilate, leaving excited D-strings, with the excitation energy provided by a net slowing of the strings. If, however, a number of F-strings sufficient to glue the D-strings together is produced, the tachyonic mode will rapidly roll and the strings will reconnect.

This reasoning appears in [1], where a precise condition is formulated to determine the reconnection probability. Firstly, for N F-string pairs, balance of forces implies that the angle between the F-strings and D-strings in the rest frame of the junction is $\frac{1}{2}\pi + \phi$, where $N = \sin\phi/g_s$. The effect of the collision can only travel with the speed of light, so on each string there are two kinks traveling away from the point of the collision. If the centre-of-mass frame speed, u, satisfies $u > \tan\phi$ then the system contains sufficient energy to stretch the F-string pairs and separate the D-string junctions. For lower speeds, the junctions do not separate and the D-strings remain in contact, and the tachyon will be given time to roll down its potential, thus allowing the strings to combine. Thus the condition for recombination is

$$N > \frac{1}{g_s}\sinh\left(\frac{\pi}{2}\epsilon\right), \quad \epsilon \equiv \frac{2}{\pi}\tanh^{-1}(u). \tag{5}$$

The annulus calculation of the Schwinger pair production result only dealt with the production of at least one pair. For the case for which the string impact parameter vanishes, in the low velocity limit, it is simple to calculate the probability of producing N F-string pairs. Being localised to the intersection region, the tachyon pairs are produced in a squeezed state. For a squeezed state of a single oscillator, if the probability of producing at least one pair is p, then the probability of producing at least k pairs is just p^k. Here, $p \simeq 1 - \mathrm{e}^{-\theta/\epsilon}$ and so $p^k \simeq \exp(-k\mathrm{e}^{-\theta/\epsilon})$. Subtracting the four massless fermionic pairs which will necessarily be produced, the probability that the reconnection condition (5) is satisfied is

$$\mathcal{P} = \exp\left(\left[4 - \frac{1}{g_s}\sinh\left(\frac{\pi}{2}\epsilon\right)\right]\mathrm{e}^{-\theta/\epsilon}\right). \tag{6}$$

$\mathcal{P}$ decreases as $g_s \to 0$, because the reconnection condition (5) becomes more stringent while the probability of producing a given number of pairs is constant. In fact, it falls as $\mathrm{e}^{-\mathrm{O}(1/g_s)}$ and so is non-perturbative, even though it was deduced from a perturbative calculation.

For realistic choices of parameters, there is a range of small angles where D-strings will sometimes pass through one another, but this will have likely have a small effect on the network behavior. This discussion extends directly to the general case. The general conclusion is that reconnection almost always occurs unless g_s is very small.

3. Warped compactification effects

3.1. General results

The string reconnection probabilities, Eqs. (1), (2) and (6) all depend on the compactification volume as $1/V_\perp$, reflecting the fact that the strings have to come roughly within a string radius in order to interact [34,5,18]. It is therefore essential to determine the effective value of $V_\perp$.

Naïvely it would seem that one could obtain very small values of $\mathcal{P}$ in models with large compact dimensions. However, from the point of view of the world-sheet field theory, the position of the string in the compact dimensions is a scalar field, which is not protected by any symmetry. One therefore expects that at some scale this modulus will be fixed, like the compactification moduli. That is, there is an effective potential which localises the string. The effective potential for (p,q)-string fluctuations in compact directions Y^i, when the general warped metric takes the form

$$\mathrm{d}s^2 = H^{-1/2}(Y)\eta_{\mu\nu}\,\mathrm{d}X^\mu\,\mathrm{d}X^\nu + H^{1/2}(Y)g_{ij}(Y)\,\mathrm{d}Y^i\,\mathrm{d}Y^j,$$

is obtained by the small Y expansion of the (p,q)-string action, leading to [1]

$$V(Y) = \frac{\nu(Y)}{2\pi\alpha' H^{1/2}(Y)}, \tag{7}$$

in a static worldsheet gauge. The string tension is proportional to $\nu = (p^2 + q^2\,\mathrm{e}^{-2\Phi(Y)})^{1/2}$, and general dilaton dependence on the compact coordinates $\Phi(Y)$ gives a position-dependent string tension. If the dilaton is nontrivial then the position of the minimum will depend on p and q. For the strongly warped geometries [6], the variation of the dilaton is negligible. However, for F-theory compactifications (see for example [35]) it should be noted that this effect has the possibility in principle to localise the different (p,q) strings far enough apart that they will evolve as essentially independent networks. They must be separated by more than a string length for this to happen.

The behavior of scalar fields in 1+1 dimensions implies that the effective volume over which the string wavefunction spreads depends only logarithmically on the mass scale of the moduli, as the cube of the logarithm, to be precise [14]. As a result, $\mathcal{P}$ can be suppressed somewhat, but not by many orders of magnitude. A UV momentum cut-off is necessary on the 2D worldsheet, and for the present purpose, the cutoff is the string scale as seen by a ten-dimensional observer, but in four-dimensional units this is red-shifted to $\Lambda^2 \sim 1/\alpha' H^{1/2}(0)$. Rotating coordinates to make $\partial_i\partial_j V(0)$ diagonal gives

$$\langle Y^i Y^i\rangle = \frac{\alpha'}{2\nu(0)}\omega_i,\ \omega_i = \ln\left[1 + \frac{\nu(0)}{2\pi\alpha'^2 H^{1/2}(0)V_{,ii}(0)}\right] \quad (\text{no sum on } i). \tag{8}$$

The fluctuations of the string in the transverse dimensions scale only logarithmically as the scale of the potential that localises the string is lowered; the linear scale of the fluctuations goes as the square root of the logarithm, and the volume goes as the cube of the logarithm. The fluctuation (8) is proportional to $1/\nu(0)$, so for strings with D-brane charge it vanishes to leading order in perturbation theory.

This calculation is meaningful only when $V_{,ii}$ is small in string units; i.e. the geometry varies slowly on the string scale.[1] In this case it is expected to be able to combine the flat spacetime calculation with an effective wavefunction for the string thus calculated. When $V_{,ii}$ is of order one in string units, so that there is no separation of scales, there is no way to use the flat spacetime calculation. However for the present purposes, all physical compactifications have small radius of curvature in string units.

From the information above, the overlaps of the string wavefunctions in the compact directions are easily calculated in [1] for the different string types:

$$\left(\frac{1}{V_\perp}\right)_{\mathrm{FF}} = \frac{1}{(2\pi\alpha')^3\prod_i \omega_i^{1/2}}, \tag{9}$$

$$\left(\frac{1}{V_\perp}\right)_{\mathrm{FD}} = \frac{1}{(\pi\alpha')^3\prod_i \omega_i^{1/2}}\exp\left[-\sum_i \frac{Y_{\mathrm{D}}^i Y_{\mathrm{D}}^i}{\alpha'\omega_i}\right]. \tag{10}$$

The minimum of the F-string potentials are always coincident, however in dilatonic theories, F- and D- strings can be localised about different points (separated by distance $Y = Y_{\mathrm{D}}$, say), leading to the expected large suppression when the separation of the minima is larger than the string scale.

[1] Also in this limit the effective quartic coupling is small, so the free worldsheet calculation that has been done is valid.

For the general D-D collision the effect of a non-zero impact parameter y is to replace

$$x \to x\,\mathrm{e}^{-y^2/2\pi\alpha'\epsilon}$$

in the general interaction probability (4). Applying this in the case that only the bosonic tachyon and the massless fermions are important, as holds over most of the parameter space,

$$1 - \mathcal{P}_{\mathrm{pp}}(y) \simeq \left[\frac{(1-\mathrm{e}^{-y^2/2\pi\alpha'\epsilon})^4}{1+\mathrm{e}^{\theta/\epsilon}\,\mathrm{e}^{-y^2/2\pi\alpha'\epsilon}}\right]. \tag{11}$$

Again for strings in different minima, the interaction probability falls rapidly for separations large compared to the string scale. For strings in the same minima, the quantum fluctuations (8) imply that the result (11) must be averaged over a Gaussian wavefunction of this width.

A typical value of the correction factor for D-strings is

$$\mathrm{e}^{-y^2/2\pi\alpha'\epsilon} \sim \exp\left[-\frac{\sum_i \omega_i}{2\pi\nu(0)\epsilon}\right], \tag{12}$$

assuming strings of the same type, thus summing fluctuations as in (9) which contributes a factor of 2 to the exponent. For D-strings, $\nu^{-1}(0) = g_s$, so this is formally higher order in perturbation theory. However, the fluctuations in the different directions add to give an effective factor of 6, and so for the typical $\epsilon \sim 0.3$ the exponent can be of order one if the logarithm ω_i is large. This would lift the suppression due to the fermion zero modes. The probability to produce tachyon string modes remains large until the suppression factor (12) approaches $\mathrm{e}^{-\theta/\epsilon}$, but if the scale of $V_{,ii}$ is low this can be the case, at least for some range of angles. Thus there is the possibility that D-strings can pass through one another without reconnecting.

3.2. KKLMMT model parameters

In the KKLMMT model [6], inflation takes place in a highly warped throat whose local geometry is given by the Klebanov–Strassler solution [36]. The warp factor also produces a potential well for the transverse coordinates of the string. The geometry near the base of this solution is locally $\mathrm{R}^3 \times \mathrm{S}^3$, with the radius of the S^3 fixed by M units of RR 3-form flux, $R_3^2 \sim g_s M\alpha'$. The energy scale of inflation in this model is of order 10^{-4} in Planck units, so the warp factor is of the order of 10^{-4}. The dilaton in this solution is constant, and the three and five-form fluxes do not affect the string action. In order for the supergravity approximations used in constructing the solution to be valid, $g_s M \gg 1$.

This solution has the special property that the warp factor has its minimum not at a point but on the entire three-sphere at the tip of the throat. The effective $V_\perp$ is given by combining the volume of the S^3 with the quantum fluctuations on the R^3. However, the fact that the potential is constant on the S^3 reflects an $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry of the Klebanov–Strassler solution. This local geometry is part of a larger Calabi–Yau solution, which does not possess this isometry; hence this symmetry will be broken on some scale, generating a potential for the strings along the S^3. The warp factor is a measure of the size of the tip of the Klebanov–Strassler throat in terms of the underlying Calabi–Yau geometry [35]. It therefore governs the extent to which the throat feels the curvature of the geometry, and so the size of the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ breaking and the size of the potential. Assuming that the curvature of the Calabi–Yau manifold will have an effect on the throat geometry of order the warp factor squared, relative to the other scales in the throat gives $\omega_i \sim \ln H^{1/2}(0)$ in the S^3 directions. Obviously, the spread of the string wavefunctions in the extra dimension can only grow as much as the size of the S^3, so once $H^{1/2}(0)$ becomes sufficiently large, the volume of localisation is just the volume of the S^3.

For D–D collisions, the relevant quantity is

$$\frac{y^2}{2\pi\alpha'\epsilon} \sim \mathrm{Min}\left[\frac{g_s M}{2\pi\epsilon}, \frac{3g_s \ln H(0)}{8\pi\epsilon}\right], \tag{13}$$

depending on whether the quantum fluctuations fill out the S^3. Again, the fermion zero modes are lifted to the extent that this is non-zero, and the tachyonic modes are not excited for collisions at angles less than $\epsilon \times (13)$.

4. Final interaction probabilities

Having assembled all of the relevant calculations, it is interesting now to insert some typical parameter values and obtain estimates for $\mathcal{P}$. Estimations of the self-interaction probabilities, as already explained, will yield useful estimates of the enhancement of the string network density relative to standard cosmic string models, given that $\mathcal{P}$ describes the efficiency of the lossy interaction responsible for string loops breaking off and radiating energy away from the network. The probabilities for the

Table 1
Final interaction probabilities for the different string types in the various brane inflation models. In the KKLMMT models, $g_s M \gg 1$ to ensure that the supergravity approximations made are valid, and that the curvature at the tip of the inflationary throat is small compared to the string scale. Individually, g_s can be between its G.U.T. value, 1/20 and 1 (in some F-theory compactifications); M must be at least 12, perhaps somewhat larger [37]. The KKLMMT probabilities are generally somewhat less than 1, but shown is the range from best to worst case of the model parameters

Quantity	KKLMMT		General Brane Inflation
	S^3 localised	S^3 filled	
$\mathcal{P}_{\mathrm{FF}}$	$\frac{1.5g_s^2}{\ln^{3/2}(1+g_s M)}$ $10^{-2} \leftrightarrow 1$	$\frac{100g_s^{1/2}}{M^{3/2}\ln^{3/2}(1+g_s M)}$	$0.5g_s^2\left[\frac{\pi n}{\ln(g_s/16\pi G\mu_{\mathrm{D}}\delta)}\right]^{n/2}$ $10^{-3} \leftrightarrow 1$
$\frac{y^2}{2\pi\alpha'\epsilon}$	$15g_s$	$0.5g_s M$	$g_s\{\mathcal{O}(10 \to 25) + \ln(g_s/\delta)\}$
$\mathcal{P}_{\mathrm{DD}}$	$10^{-1} \leftrightarrow 1$		$10^{-2} \leftrightarrow 1$

interactions of strings of different types will be necessary for detailed simulations of (p,q)-string networks, but shall not be applicable to the simple estimates of this section. The range of interaction probabilities indicates the extent to which it might be able to probe stringy physics by measuring the various intercommutation probabilities.

Consider first F–F reconnection. The reconnection probability is the earlier result (1) with the appropriate value of $1/V_\perp$, depending on whether the fluctuations fill out the S^3 or not. The kinematic part of (1), depending on v and θ is roughly 0.5 when averaged over angles and velocities. The results are summarised in Table 1, which also has the final numerical probabilities for appropriate values of the model parameters. Table 1 also includes results calculated in [1] for the general brane inflation models (which are defined here as those models in which there is no highly warped region), using the fact that the scale $G\mu_{\mathrm{D}}$ is likely to lie between 10^{-6} and 10^{-12} [3,5].

For the D–D reconnection probability, the result (13) is combined with appropriate values of the model parameters in Table 1. The multiplicative contribution of each would-be fermion zero mode to (11) is $(1 - \mathrm{e}^{-y^2/2\pi\alpha'\epsilon})$, so the fermion zero modes are largely lifted in all cases, and for larger values of g_s (but still less than 1) the production of fermionic open strings is negligible. The suppression of producing tachyonic strings is angle dependent, and the effective value of the impact parameter determines the range of angles for which recombination is suppressed.

For (p,q)–(p',q') collisions the result depends primarily on the values of q and q'. Making one or both of these larger than 1 enhances reconnection in two ways: the production of strings is enhanced by the Chan–Paton degeneracy, and the fluctuations are decreased due to the greater tension of the (p,q) string; however, the great tension also requires that more F-string pairs are produced between the (p,q)-strings in order to 'glue' the (p,q)-strings together long enough to allow the tachyon to roll.

5. From string theory to astrophysics

It has been argued that cosmic strings are a general consequence of $\mathrm{D\overline{D}}$ inflation, and string theory calculations of the basic properties of these cosmic strings were presented. At this juncture the interface of these basic properties with the large-scale astrophysical characteristics of the string networks needs to be understood. While some rudimentary estimates of the connection have been made, understanding here is incomplete mainly because detailed simulations of string network behaviour and evolution are required with these new 'stringy' properties.

5.1. *Current bounds and detection at $\mathcal{P} = 1$*

Since most of the work on cosmic strings to date considers 'standard' 4D cosmic strings, the bounds and detection sensitivities are formulated for $\mathcal{P} = 1$. As outlined in e.g. [17,33], cosmic string networks have a variety of astrophysical signatures, the most significant being:

– *Cosmic Microwave Background:* Cosmic strings were originally considered an alternative to inflation because they seed density perturbations and structure formation. This fact and the WMAP data [38] supporting inflation place a constraint on the contribution of a string network to the temperature fluctuations, and so on the string tension. The current bound at $\mathcal{P} = 1$ is $G\mu < 7 \times 10^{-7}$ [18], which is well satisfied by most string models: strings in KKLMMT have $10^{-10} \lesssim G\mu \lesssim 10^{-9}$ [6] and more general brane inflation models have $10^{-12} \lesssim G\mu \lesssim 10^{-6}$ [5].

Also a cosmic string network leads to significant B-mode polarisations in the CMB above that expected from gravitational waves in slow-roll inflationary models [18]. For the most optimistic tensions, these could be detectable by future ground and space-based polarisation measurements.

– *Gravitational Wave Bursts:* As described by Vilenkin and Damour [39], dynamical excitations – cusps and kinks – on cosmic strings, which are caused by their interactions, give rise to gravitational wave bursts. Each individual gravitational wave burst depends on the string tension, but there will be $\mathcal{P}$-dependence on the net signal, since $\mathcal{P}$ affects the total string network energy density. The $\mathcal{P}=1$ estimates for the sensitivity to such gravitational wave bursts are:
  - LIGO II, in its frequency band, can detect $G\mu$ as low as 10^{-11} [39].
  - LISA would be able to detect bursts in its frequency band from cusps on strings of tension $G\mu > 10^{-13}$ [39].

– *Gravitational Lensing:* It is well-known that an isolated line-like source of stress-energy, as a realistic 4D cosmic string will approximate on some scale, generates a conical spacetime. The deficit angle of the cone is proportional to $G\mu$. Such a spacetime provides a means of lensing a distant object, which importantly shall lead to an even number of images. Given that point-like or spherical lensing objects lead only to odd numbers of images, the string lensing signature is therefore distinctive. String tension could be measured from the image separation.

  While there are recent suggestions of such lensing events being detected ([40,41] for instance), there is of course no definitive evidence yet. A particularly promising proposal for the detection of lensing events is the Square Kilometre Array radio-telescope, which in principle would have milli arc-second resolution and such a wide field of view that an extensive search for such lensing events would be possible.[2] Importantly, this resolution equates to an ability to detect string of tensions down to $G\mu \gtrsim 10^{-10}$, and so all of the strings of KKLMMT models, and most of the more general high-scale supersymmetric models.

5.2. *Avenues to detecting $\mathcal{P} < 1$*

In order to ascertain whether astrophysical cosmic strings have 'stringy' characteristics, the above section needs to be revised for $\mathcal{P} < 1$. What follows is largely qualitative, since most of the important supporting calculations are yet to be done.

F-string reconnection probabilities were found to be in the range $10^{-3} \lesssim \mathcal{P} \lesssim 1$, and those for D-strings $0.1 \lesssim \mathcal{P} \lesssim 1$. As described, the network energy density scales with string tension and loop-formation probability like

$$\rho \sim \frac{\mu}{\mathcal{P}t^2}.$$

This behaviour for $\mathcal{P} < 1$ can permeate many of the conclusions of the previous section. The pertinent question is, given that 4D gauge theory strings have interconnection probability $\mathcal{P} = 1$ exactly, whether there are any experimental measures which can determine $\mathcal{P}$ and μ independently. Fortunately of the various proposed observable effects of cosmic strings on astrophysical phenomena, each has a different sensitivity to $\mathcal{P}$ and μ; not all effects are sensitive only to the energy density of the network. It is important to reiterate that any detection of $\mathcal{P} < 1$, while not a definitive test of $\mathrm{D}\overline{\mathrm{D}}$ inflation and string theory, would be convincing evidence.

The string network influences the CMB by the combination $\mu^2/\mathcal{P}$ [18]. Individual gravitational wave bursts are sensitive to μ only, whereas the burst frequency (per month, say) depends on the network energy density and hence, $\mathcal{P}$, although the precise dependence on $\mathcal{P}$ and μ is not yet known. For these astrophysical effects, because μ and $\mathcal{P}$ cannot be individually determined from any one measurement, at least two different experiments would be needed to disentangle μ and $\mathcal{P}$ dependence, exposing any 'stringy' properties.

Finally and perhaps most promising are the lensing experiments (particularly the proposed Square Kilometre Array). These can detect the tension of individual strings. The number of such lensing events would give a measure of the string network density and therefore of $\mathcal{P}$. Further a survey of double images in the sky would be able to map a spectrum of string tensions, and ascertain if heavier strings were present, whether they were (p,q) or Kaluza–Klein excitations, giving an clear window into the microscopic origins of the universe.

6. Conclusions

As proposed in [3–5], cosmic strings can be a new and exciting window into the microscopic physics of string theory. This talk builds on the promising result of [3–5] that cosmics strings, and not monopoles nor domain walls, will be copiously produced following $\mathrm{D}\overline{\mathrm{D}}$ inflation. This prediction of such models, independent of many of the details of the construction, leads to a conceivable way of testing string theory. The lessened string interaction probability is one clear signal of – at least the

[2] Ira Wasserman brought this possibility to the attention of the author.

presence of extra-dimensions, and more optimistically – string theory itself. The possibility of a spectrum of string tensions from either Kaluza–Klein like excitations of a basic string, or from (p, q)-strings is a signature that could give a more detailed illustration of the physics of hidden dimensions and the string scale.

These results render such phenomena extremely worthy of attention, particularly in that so much of both the underlying string construction and the astrophysical properties of these modified strings are yet to be understood. With progress in these directions, the fascinating possibility of seeing strings in the sky may not be too distant.

Acknowledgements

I would like to thank Henry Tye for many thoughtful questions and discussions, and Steve Drasco, Louis Leblond, Sash Sarangi, Ira Wasserman and Mark Wyman for useful comments on an early version of the talk. I also wish to heartily thank the organisers of Strings 2004 for the invitation and their hospitality. This material is based on work supported by the National Science Foundation under Grant number PHY-0098631.

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