String theory and fundamental forces/Théorie des cordes et forces fondamentales

Variations on the warped deformed conifold

Steven S. Gubser [a], Christopher P. Herzog [b], Igor R. Klebanov [a,*]

[a] *Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA*
[b] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*

Abstract

The warped deformed conifold background of type IIB theory is dual to the cascading $SU(M(p + 1)) \times SU(Mp)$ gauge theory. We show that this background realizes the (super-)Goldstone mechanism where the U(1) baryon number symmetry is broken by expectation values of baryonic operators. The resulting massless pseudo-scalar and scalar glueballs are identified in the supergravity spectrum. A D-string is then dual to a global string in the gauge theory. Upon compactification, the Goldstone mechanism turns into the Higgs mechanism, and the global strings turn into ANO strings. *To cite this article: S.S. Gubser et al., C. R. Physique 5 (2004).*

Résumé

Variations sur le conifold voilé déformé. La configuration de fond pour la théorie de type IIB donnée par le conifold voilé déformé est duale à la cascade de théorie de jauge $SU(M(p + 1)) \times SU(Mp)$. Nous montrons que cette configuration donne une réalisation du mécanisme de (super-)Goldstone où la symètrie baryonique U(1) est brisée par la valeur moyenne dans le vide des opérateurs baryoniques. Les boules de glue pseudo-scalaires et scalaires de masse nulle résultantes sont identifiées dans le spectre de supegravité. Une D-corde est alors duale à une corde globale dans la théorie de jauge. Après compactification, le mécanisme de Goldstone devient un mécanisme de Higgs, et une corde globale devient une corde ANO. *Pour citer cet article : S.S. Gubser et al., C. R. Physique 5 (2004).*

1. Introduction

This article, delivered as a talk at Strings '04 by one of us (I.R.K.), is a condensed version of our paper [1].

One of the themes in recent string theory research concerns extensions of the AdS/CFT correspondence [2–4] to confining gauge theories. One such background of type IIB string, the warped deformed conifold, was constructed in [5]. It was argued

---

* Corresponding author.
 E-mail addresses: ssgubser@princeton.edu (S.S. Gubser), herzog@kitp.ucsb.edu (C.P. Herzog), klebanov@princeton.edu (I.R. Klebanov).

to be dual to 4-dimensional $\mathcal{N} = 1$ supersymmetric $SU(M(p + 1)) \times SU(Mp)$ gauge theory [6] whose flow exhibits an RG cascade [5,7]. In each cascade step the integer p decreases by 1 through the Seiberg duality [8].

In this article we show that the warped deformed conifold of [5] incorporates a supergravity dual of the supersymmetric Goldstone mechanism, and identify a pseudo-scalar Goldstone boson and its scalar superpartner. An old puzzle guides our investigation: what is the gauge theory interpretation of D1-branes in the deformed conifold background [5]? The interpretation of the fundamental strings placed in the IR region of the metric is clear: they are dual to confining strings. Like the fundamental strings, the D-strings fall to the bottom of the throat, $\tau = 0$, where they remain tensionful; hence, they cannot be dual to 't Hooft loops which must be screened [5]. We propose instead that in the dual gauge theory they are solitonic strings that create a monodromy of a massless pseudo-scalar Goldstone boson field.[1] For this explanation to make sense, the IR gauge theory must differ from the pure glue $\mathcal{N} = 1$ theory in that it contains a massless pseudo-scalar bound state (glueball). The fact that this massless mode must couple directly to a D-string means that it corresponds to a certain perturbation of the RR 2-form potential, which turns out to mix with the RR 4-form potential. We exhibit the necessary ansatz in Section 3, and indeed find a massless glueball. This mode should be interpreted as the Goldstone boson of spontaneously broken global U(1) baryon number symmetry. Its presence supports the claim made in [5,9] that the cascading gauge theory is on the baryonic branch [10], i.e. certain baryonic operators acquire expectation values. The supersymmetric Goldstone mechanism gives rise also to a massless scalar mode. In Section 4 the supergravity dual of this mode is identified as a massless glueball coming from a mixture of an NS–NS 2-form and a metric deformation. The ansatz for such perturbations was written down some time ago in [11].

Besides being an interesting example of the gauge/gravity duality, the warped deformed conifold background offers interesting possibilities for solving the hierarchy problem along the lines suggested in [12,13]. If the background is embedded into a compact CY space with NS–NS and R–R fluxes, then an exponential hierarchy may be created between the UV compactification scale and the IR scale at the bottom of the throat [5,14]. Models of this type received an additional boost due to a possibility of fixing all moduli proposed in [15], and a subsequent exploration of cosmology in [16]. Recently, a new role was proposed for various (p, q) strings placed in the IR region [17]. Besides being the confining or solitonic strings from the point of view of the gauge theory, they may be realizations of cosmic strings. The exponential warping of the background lowers the tension significantly, and makes them plausible cosmic string candidates. In Section 5 we discuss the Higgs mechanism that occurs upon embedding the warped deformed conifold into a flux compactification, and argue that a D-string placed at the bottom of the throat is dual to an Abrikosov–Nielsen–Olesen string in the gauge theory coupled to supergravity. We conclude in Section 6.

2. Review of the warped deformed conifold

The conifold may be described by the following equation in four complex variables,

$$\sum_{a=1}^{4} z_a^2 = 0. \tag{1}$$

Since this equation is invariant under an overall real rescaling of the coordinates, this space is a cone and admits the metric [18]:

$$ds_6^2 = dr^2 + r^2 \, ds_{T^{1,1}}^2, \tag{2}$$

where

$$ds_{T^{1,1}}^2 = \frac{1}{9} \left(d\psi + \sum_{i=1}^{2} \cos\theta_i \, d\phi_i \right)^2 + \frac{1}{6} \sum_{i=1}^{2} (d\theta_i^2 + \sin^2\theta_i \, d\phi_i^2) \tag{3}$$

is the metric on $T^{1,1}$. Here ψ is an angular coordinate which ranges from 0 to 4π, while (θ_1, ϕ_1) and (θ_2, ϕ_2) parametrize two $\mathbf{S}^2$s in a standard way. Therefore, this form of the metric shows that $T^{1,1}$ is an $\mathbf{S}^1$ bundle over $\mathbf{S}^2 \times \mathbf{S}^2$. Topologically, $T^{1,1} \sim \mathbf{S}^2 \times \mathbf{S}^3$.

Now placing N D3-branes at the apex of the cone we find the metric

$$ds^2 = \left(1 + \frac{L^4}{r^4} \right)^{-1/2} \left(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) + \left(1 + \frac{L^4}{r^4} \right)^{1/2} (dr^2 + r^2 ds_{T^{1,1}}^2), \tag{4}$$

whose near-horizon ($r \to 0$) limit is $AdS_5 \times T^{1,1}$. The same logic that leads us to the maximally supersymmetric version of the AdS/CFT correspondence now shows that the type IIB string theory on this space should be dual to the infrared limit of the field

___

[1] We are grateful to E. Witten for emphasizing this possibility to us.

theory on N D3-branes placed at the singularity of the conifold. Since Calabi–Yau spaces with these D-branes preserve 1/4 of the original supersymmetries, we have an $\mathcal{N} = 1$ superconformal field theory. This field theory was constructed in [19,20]: it is $SU(N) \times SU(N)$ gauge theory coupled to two chiral superfields, A_i, in the $(\mathbf{N}, \overline{\mathbf{N}})$ representation and two chiral superfields, B_j, in the $(\overline{\mathbf{N}}, \mathbf{N})$ representation.

The continuous symmetries of the gauge theory are $U(1)_R \times U(1)_B \times SO(4)$ where the $SO(4)$ acts on the A's and the B's as $SU(2) \times SU(2)$. The exactly marginal superpotential is fixed uniquely by the symmetries up to overall normalization:

$$W \sim \epsilon^{ij} \epsilon^{kl} \, \mathrm{tr} \, A_i B_k A_j B_l. \tag{5}$$

The $U(1)$ baryon number symmetry acts as $A_k \to e^{i\alpha} A_k$, $B_j \to e^{-i\alpha} B_j$. The massless gauge field in AdS_5 dual to the baryon number current originates from the RR 4-form potential [21,22]:

$$\delta C_4 \sim \omega_3 \wedge A. \tag{6}$$

Also important for our discussion is the $\mathbf{Z}_2$ symmetry generated by the interchange of A_1, A_2 with B_1, B_2 accompanied by charge conjugation, i.e. the interchange of the fundamental and the antifundamental representations, in both $SU(N)$ gauge groups [19,20]. We will call this interchange symmetry the $\mathcal{I}$ symmetry. The corresponding transformation in the IIB string theory on $AdS_5 \times T^{1,1}$ is the interchange of (θ_1, ϕ_1) with (θ_2, ϕ_2) (i.e., of the two $\mathbf{S}^2$'s) accompanied by the $-I$ of the $SL(2, \mathbf{Z})$ S-duality symmetry [19,20]. The action of the $-I$ of the $SL(2, \mathbf{Z})$ reverses the sign of the NS–NS and R–R 2-form potentials, B_2 and C_2.

The addition of M fractional 3-branes (wrapped D5-branes) at the singular point of the conifold changes the gauge group to $SU(N + M) \times SU(N)$ [6]. The M units of magnetic 3-form flux cause the conifold to make a 'geometric transition' to the deformed conifold

$$\sum_{a=1}^{4} z_a^2 = \epsilon^2, \tag{7}$$

in which the singularity of the conifold is removed through the blowing-up of the $\mathbf{S}^3$ of $T^{1,1}$. Therefore, the dual of the cascading $SU(M(p+1)) \times SU(Mp)$ gauge theory is the warped deformed conifold [5]. Below we collect some necessary formulae about this background (for reviews see [23]).

The ten-dimensional metric is

$$ds_{10}^2 = h(\tau)^{-1/2} \left(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) + h(\tau)^{1/2} ds_6^2, \tag{8}$$

where

$$ds_6^2 = \frac{\epsilon^{4/3} K(\tau)}{2} \left[\frac{1}{3K^3} \left(d\tau^2 + (g^5)^2 \right) + \cosh^2 \left(\frac{\tau}{2} \right) \left((g^3)^2 + (g^4)^2 \right) + \sinh^2 \left(\frac{\tau}{2} \right) \left((g^1)^2 + (g^2)^2 \right) \right] \tag{9}$$

is the usual Calabi–Yau metric on the deformed conifold. The one forms are given in terms of angular coordinates as

$$g^1 = \frac{e^1 - e^3}{\sqrt{2}}, \quad g^2 = \frac{e^2 - e^4}{\sqrt{2}}, \quad g^3 = \frac{e^1 + e^3}{\sqrt{2}}, \quad g^4 = \frac{e^2 + e^4}{\sqrt{2}}, \quad g^5 = e^5, \tag{10}$$

where

$$\begin{aligned} e^1 &\equiv -\sin\theta_1 \, d\phi_1, \qquad e^2 \equiv d\theta_1, \qquad e^3 \equiv \cos\psi \sin\theta_2 \, d\phi_2 - \sin\psi \, d\theta_2, \\ e^4 &\equiv \sin\psi \sin\theta_2 \, d\phi_2 + \cos\psi \, d\theta_2, \qquad e^5 \equiv d\psi + \cos\theta_1 \, d\phi_1 + \cos\theta_2 \, d\phi_2. \end{aligned} \tag{11}$$

Note that

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh\tau}. \tag{12}$$

The warp factor is

$$h(\tau) = (g_s M \alpha')^2 2^{2/3} \epsilon^{-8/3} I(\tau), \tag{13}$$

where

$$I(\tau) \equiv \int_{\tau}^{\infty} dx \, \frac{x \coth x - 1}{\sinh^2 x} \left(\sinh(2x) - 2x \right)^{1/3}. \tag{14}$$

Since $h(\tau)$ decreases monotonically from a finite value at $\tau = 0$, the tension of the fundamental string is minimized at $\tau = 0$, where it is found to be $1/(2\pi\alpha'\sqrt{h(0)})$. This means that this background is dual to a confining gauge theory.

The NS–NS two form field is

$$B_2 = \frac{g_s M \alpha'}{2}\left[f(\tau)g^1 \wedge g^2 + k(\tau)g^3 \wedge g^4\right], \tag{15}$$

while the RR three form field strength is

$$F_3 = \frac{M\alpha'}{2}\left\{g^5 \wedge g^3 \wedge g^4 + d\left[F(\tau)(g^1 \wedge g^3 + g^2 \wedge g^4)\right]\right\}. \tag{16}$$

The auxiliary functions in these forms are

$$F(\tau) = \frac{\sinh \tau - \tau}{2\sinh \tau}, \qquad f(\tau) = \frac{\tau \coth \tau - 1}{2\sinh \tau}(\cosh \tau - 1), \qquad k(\tau) = \frac{\tau \coth \tau - 1}{2\sinh \tau}(\cosh \tau + 1). \tag{17}$$

In the warped deformed conifold the SO(4) and the $\mathcal{I}$ global symmetries are preserved, but the $U(1)_R$ symmetry is broken in the UV by the chiral anomaly down to $\mathbf{Z}_{2M}$ [24]. Further spontaneous breaking of this discrete symmetry to $\mathbf{Z}_2$, which acts as $z_i \to -z_i$, does not lead to appearance of a Goldstone mode. The $U(1)_B$ symmetry is not anomalous, and its spontaneous breaking does produce a Goldstone mode, which we exhibit in Section 3.

3. The Goldstone mode

To begin, consider a D1-brane extended in two of the four dimensions in $\mathbf{R}^{3,1}$. Because the D1-brane carries electric charge under the R–R three-form field strength F_3, it is natural to think that a pseudo-scalar a in four dimensions, defined so that $*_4 da = \delta F_3$,[2] experiences monodromy as one loops around the D1-brane world-volume.

The perturbation ansatz we therefore adopt is

$$\begin{aligned}
\delta F_3 &= *_4 da + f_2(\tau)\, da \wedge dg^5 + f_2'\, da \wedge d\tau \wedge g^5, \\
\delta F_5 &= (1 + *)\delta F_3 \wedge B_2 = \left(*_4\, da - \frac{\epsilon^{4/3}}{6K^2(\tau)}h(\tau)\, da \wedge d\tau \wedge g^5\right) \wedge B_2,
\end{aligned} \tag{18}$$

where $f_2' = df_2/d\tau$, $h(\tau)$ is given by (13), and $K(\tau)$ by (12). The variations of all other fields, including the metric and the dilaton, vanish. The last two terms in δF_3 sum to the exact form $-d(f_2\, da \wedge g^5)$. As shown in [1], all linearized SUGRA equations are satisfied if $a(x^0, x^1, x^2, x^3)$ is a harmonic function, i.e. $d *_4 da = 0$, and $f_2(\tau)$ satisfies

$$-\frac{d}{d\tau}[K^4 \sinh^2 \tau\, f_2'] + \frac{8}{9K^2}f_2 = \frac{(g_s M\alpha')^2}{3\epsilon^{4/3}}(\tau \coth \tau - 1)\left(\coth \tau - \frac{\tau}{\sinh^2 \tau}\right). \tag{19}$$

The normalizable solution of (19) that is regular both for small and for large τ is

$$f_2(\tau) = -\frac{2c}{K^2 \sinh^2 \tau}\int_0^\tau dx\, h(x)\sinh^2 x, \tag{20}$$

where $c \sim \epsilon^{4/3}$. We find that $f_2 \sim \tau$ for small τ, and $f_2 \sim \tau\, e^{-2\tau/3}$ for large τ.

The zero-mass glueball we are finding is due to the spontaneously broken global $U(1)$ baryon number symmetry [9]. The form of the δF_5 in (18) makes the connection between our zero-mode and $U(1)_B$ evident. Asymptotically, at large τ, there is a component $\sim \omega_3 \wedge da \wedge d\tau$ in δF_5. Thus from (6), we have $A \sim da$. For the 4-d effective Lagrangian, there should be a coupling between the baryon number current J^μ and a:

$$\frac{1}{f_a}\int d^4x\, J^\mu \partial_\mu a = -\frac{1}{f_a}\int d^4x\, a(x)(\partial_\mu J^\mu), \tag{21}$$

where the pseudo-scalar a enters as the parameter of the baryon number transformation. It is important that this transformation does not leave the vacuum invariant!

As discussed in [5,9] the field theory is on the baryonic branch: 'the last step' of the cascade takes place through giving expectation values to baryonic operators in the $SU(2M) \times SU(M)$ gauge theory coupled to bifundamental fields A_i, B_j,

---

[2] The 4-dimensional Hodge dual $*_4$ is calculated with the Minkowski metric, $\mathrm{vol}_4 = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$.

$i, j = 1, 2$. In addition to mesonic operators $(N_{ij})^\alpha_\beta \sim (A_i B_j)^\alpha_\beta$, the gauge theory has baryonic operators invariant under the $\mathrm{SU}(2M) \times \mathrm{SU}(M)$ gauge symmetry:

$$\mathcal{B} \sim \epsilon_{\alpha_1 \alpha_2 \cdots \alpha_{2M}} (A_1)^{\alpha_1}_1 (A_1)^{\alpha_2}_2 \cdots (A_1)^{\alpha_M}_M (A_2)^{\alpha_{M+1}}_1 (A_2)^{\alpha_{M+2}}_2 \cdots (A_2)^{\alpha_{2M}}_M,$$
$$\overline{\mathcal{B}} \sim \epsilon^{\alpha_1 \alpha_2 \cdots \alpha_{2M}} (B_1)^1_{\alpha_1} (B_1)^2_{\alpha_2} \cdots (B_1)^M_{\alpha_M} (B_2)^1_{\alpha_{M+1}} (B_2)^2_{\alpha_{M+2}} \cdots (B_2)^M_{\alpha_{2M}}.$$

(22)

The baryonic operators are invariant under the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ global symmetry rotating A_i, B_j. These operators contribute an additional term to the usual mesonic superpotential:

$$W = \lambda (N_{ij})^\alpha_\beta (N_{k\ell})^\beta_\alpha \epsilon^{ik} \epsilon^{j\ell} + X\big(\det\big[(N_{ij})^\alpha_\beta\big] - \mathcal{B}\overline{\mathcal{B}} - \Lambda^{4M}_{2M}\big),$$

(23)

where X can be understood as a Lagrange multiplier.

The supersymmetry-preserving vacua include the baryonic branch:

$$X = 0; \quad N = 0; \quad \mathcal{B}\overline{\mathcal{B}} = -\Lambda^{4M}_{2M},$$

(24)

where the $\mathrm{SO}(4)$ global symmetry rotating A_i, B_j is unbroken. Since the supergravity background of [5] also has this symmetry, it is natural to identify the dual of this background with the baryonic branch of the cascading theory. The expectation values of the baryonic operators spontaneously break the $\mathrm{U}(1)$ baryon number symmetry $A_k \to e^{i\alpha} A_k$, $B_j \to e^{-i\alpha} B_j$. The deformed conifold as described in [5] corresponds to a vacuum where $|\mathcal{B}| = |\overline{\mathcal{B}}| = \Lambda^{2M}_{2M}$, which is invariant under the exchange of the As with the Bs accompanied by charge conjugation in both gauge groups. As noted in [9], the baryonic branch has complex dimension 1, and it can be parametrized by ξ where

$$\mathcal{B} = i\xi \Lambda^{2M}_{2M}, \qquad \overline{\mathcal{B}} = \frac{i}{\xi} \Lambda^{2M}_{2M}.$$

(25)

The pseudo-scalar Goldstone mode must correspond to changing ξ by a phase, since this is precisely what a $\mathrm{U}(1)_B$ symmetry transformation does. As usual, the gradient of the pseudo-scalar Goldstone mode $f_a \partial_\mu a$ is created from the vacuum by the action of the axial baryon number current, J_μ (we expect that the scale of the dimensionful 'decay constant' f_a is determined by the baryon expectation values).

Thus, the breaking of the $\mathrm{U}(1)$ baryon number symmetry necessitates the presence of a massless pseudo-scalar glueball, which we have found. By supersymmetry, this field falls into a massless $\mathcal{N} = 1$ chiral multiplet. Hence, there will also be a massless scalar mode and corresponding Weyl fermion. The scalar must correspond to changing ξ by a positive real factor.

4. The scalar zero-mode

The presence of the pseudo-scalar zero mode found in Section 3, and the $\mathcal{N} = 1$ supersymmetry, require the existence of a scalar zero-mode. In this section we argue that this zero-mode comes from a metric perturbation that mixes with the NS–NS 2-form potential.

The warped deformed conifold of [5] preserves the $\mathbf{Z}_2$ interchange symmetry which we called the $\mathcal{I}$ symmetry in Section 2: see (6). However, the pseudo-scalar mode we found breaks this symmetry: from the form of the perturbations (18) we see that δF_3 is even under the interchange of (θ_1, ϕ_1) with (θ_2, ϕ_2), while F_3 is odd; δF_5 is odd while F_5 is even. Similarly, the scalar mode must also break the $\mathcal{I}$ symmetry because in the field theory it breaks the symmetry between expectation values of $|\mathcal{B}|$ and of $|\overline{\mathcal{B}}|$. We expect that turning on the zero-momentum scalar modifies the geometry because the scalar changes the absolute value of $|\mathcal{B}|$ and $|\overline{\mathcal{B}}|$ while the pseudo-scalar affects only the phase. The necessary perturbation that preserves the $\mathrm{SO}(4)$ but breaks the $\mathcal{I}$ symmetry is a mixture of the NS–NS 2-form and the metric:

$$\delta B_2 = \chi(\tau)\, \mathrm{d}g^5, \qquad \delta G_{13} = \delta G_{24} = m(\tau),$$

(26)

where, for example, $\delta G_{13} = m(\tau)$ means to add $2m(\tau)g^{(1}g^{3)}$ to $\mathrm{d}s^2_{10}$. To see that these components of the metric break the $\mathcal{I}$ symmetry, we note that

$$(e^1)^2 + (e^2)^2 - (e^3)^2 - (e^4)^2 = g^1 g^3 + g^3 g^1 + g^2 g^4 + g^4 g^2.$$

(27)

We find it convenient to define $m(\tau) = h^{1/2} K \sinh(\tau) z(\tau)$.

In [1] it was shown that all the linearized SUGRA equations are satisfied provided that

$$\frac{((K\sinh(\tau))^2 z')'}{(K\sinh(\tau))^2} = \left(\frac{2}{\sinh(\tau)^2} + \frac{8}{9} \frac{1}{K^6 \sinh(\tau)^2} - \frac{4}{3} \frac{\cosh(\tau)}{K^3 \sinh(\tau)^2} \right) z$$

(28)

and

$$\chi' = \frac{1}{2}g_s M z(\tau)\frac{\sinh(2\tau) - 2\tau}{\sinh^2\tau}. \tag{29}$$

The solution of (28) for the zero-mode is remarkably simple:

$$z(\tau) = s\frac{(\tau\coth(\tau) - 1)}{[\sinh(2\tau) - 2\tau]^{1/3}}, \tag{30}$$

with s a constant. Like the pseudo-scalar perturbation, the large τ asymptotic is again $z \sim \tau\,e^{-2\tau/3}$. We note that the metric perturbation also has the simple form $\delta G_{13} \sim h^{1/2}[\tau\coth(\tau) - 1]$. Note that the perturbed metric $d\tilde{s}_2^6$ differs from the metric of the deformed conifold, Eq. (9), by

$$\sim (\tau\coth\tau - 1)(g^1 g^3 + g^3 g^1 + g^2 g^4 + g^4 g^2), \tag{31}$$

which grows as $\ln r$ in the asymptotic radial variable r.

The existence of the scalar zero-mode makes it likely that there is a one-parameter family of supersymmetric solutions which break the $\mathcal{I}$ symmetry but preserve the SO(4) (an ansatz with these properties was found in [11], and its linearization agrees with (26)). We will call these conjectured backgrounds *resolved warped deformed conifolds*. We add the word *resolved* because both the resolution of the conifold, which is a Kaehler deformation, and these resolved warped deformed conifolds break the $\mathcal{I}$ symmetry. As we explained in Section 3, in the dual gauge theory turning on the $\mathcal{I}$ breaking corresponds to the transformation $\mathcal{B} \to (1 + s)\mathcal{B}$, $\overline{\mathcal{B}} \to (1 + s)^{-1}\overline{\mathcal{B}}$ on the baryonic branch. Therefore, s is dual to the $\mathcal{I}$ breaking parameter of the resolved warped deformed conifold.

One might ask whether the resolved warped deformed conifolds are still of the form $h^{-1/2}\,dx_\parallel^2 + h^{1/2}\,d\tilde{s}_6^2$ where $d\tilde{s}_6^2$ is Ricci flat. At linear order in our perturbation, our conifold metric $d\tilde{s}_6^2$ is indeed Ricci flat: the first order corrections vanish if (28) is satisfied. We also showed [1] that the complex 3-form field strength $G_3 = F_3 - \frac{i}{g_s}H_3$ remains imaginary self-dual at linear order, i.e. $*_6 G_3 = iG_3$. It will be interesting to see if these properties continue to hold for the exact solution.

5. Compactification and Higgs mechanism

As we argued above, the non-compact warped deformed conifold exhibits a supergravity dual of the Goldstone mechanism. It was crucial for our arguments that the U(1)$_B$ symmetry is not gauged in the field theory, and the appearance of the Goldstone boson in the supergravity dual confirms that the symmetry is global.

If the warped deformed conifold is embedded into a flux compactification of type IIB string on a 6-dimensional CY manifold, then we expect the global U(1)$_B$ symmetry to become gauged, because the square of the gauge coupling becomes finite. In the compact case we may write $\delta C_4 \sim \omega_3 \wedge A$, where ω_3 is harmonic in the full compact case and A is the 4-d gauge field. If we ignore subtleties with the self-duality of the 5-form field strength, then the kinetic terms for it is

$$\frac{1}{2g_s^2}\int d^{10}x\sqrt{-g}\,F_5^2. \tag{32}$$

Substituting $F_5 = F_2 \wedge \omega_3$ and reducing to 4 dimensions, we find the U(1) kinetic term

$$\frac{1}{2g^2}\int d^4x\,F_2^2, \tag{33}$$

where

$$\frac{1}{g^2} \sim \frac{1}{g_s^2}\tau_m, \tag{34}$$

where we assumed that the effect of compactification is to introduce a cut-off at $\tau_m \gg 1$.

The finiteness of the gauge coupling in the compact case means that the Goldstone mechanism should turn into a Higgs mechanism. The Goldstone boson a enters as a gauge parameter of A and gets absorbed by the U(1) gauge field to make a massive vector field. As usual in the supersymmetric Higgs mechanism, the scalar acquires the same mass which originates from the D-term potential. In $\mathcal{N} = 1$ notation, gauge invariance means we have to introduce factors of $e^{\pm gV}$ into the D-terms for $\mathcal{B}$ and $\overline{\mathcal{B}}$:[3]

$$\mathcal{B}^*\,e^{gV}\mathcal{B} + \overline{\mathcal{B}}^*\,e^{-gV}\overline{\mathcal{B}}. \tag{35}$$

---

[3] $\mathcal{B}$ and $\overline{\mathcal{B}}$ have charge of order M, a charge which we have for simplicity neglected to include in the coupling to V.

Expanding these D-terms to second order in g, we find

$$g^2 \big(|\mathcal{B}|^2 + |\overline{\mathcal{B}}|^2 \big) V^2. \tag{36}$$

As a result, we find an $\mathcal{N} = 1$ massive vector supermultiplet containing a massive vector, a scalar (the Higgs boson), and their fermion superpartners.

In the preceding, we ignored the linear term in g:

$$g \big(|\mathcal{B}|^2 - |\overline{\mathcal{B}}|^2 + \zeta \big) V, \tag{37}$$

where we have included a Fayet–Iliopoulos parameter ζ. Depending on details of the compactification, it may be that ζ is nonzero. Then the potential for the scalar has its minimum for $|\xi| \neq 1$, and the baryon VEVs in (25) are unequal in magnitude. In other words, the Fayet–Iliopoulos parameter breaks the $\mathcal{I}$ symmetry. Thus, further study of the one-parameter family of the supersymmetric backgrounds dual to the baryonic branch is of interest to the understanding of flux compactifications.

While in the non-compact case D-strings are global strings, in the compact case they should be interpreted as Abrikosov–Nielsen–Olesen vortices of an Abelian–Higgs model, where the charged chiral superfields breaking the gauge symmetry are the baryon operators $\mathcal{B}$ and $\overline{\mathcal{B}}$.[4] Since there is a finite number K of NS–NS flux units through a cycle dual to the 3-sphere [14], the D-string charge takes values in $\mathbf{Z}_K$. Indeed, K D-strings can break on a wrapped D3-brane [17]. Correspondingly, we do not expect the ANO vortex duals to be BPS saturated.

6. Discussion

Our work sheds new light on the physics of the cascading $SU(M(p+1)) \times SU(Mp)$ gauge theory, whose supergravity dual is the warped deformed conifold [5]. In the infrared the theory is not in the same universality class as the pure glue $\mathcal{N} = 1$ supersymmetric $SU(M)$ theory: the cascading theory contains massless glueballs, as well as solitonic strings dual to the D-strings placed at $\tau = 0$ in the supergravity background.

As suggested in [5,9] and reviewed in Section 3 above, the infrared field theory is better thought of as $SU(2M) \times SU(M)$ on the baryonic branch, i.e. with baryon operators (22) having expectation values. Since the global baryon number symmetry, $U(1)_B$, is broken by these expectation values, the spectrum must contain a Goldstone bosons which we find explicitly. We also construct at linear order a Lorentz-invariant deformation of the background which we argue is a zero-momentum state of the scalar superpartner of the Goldstone mode. Our calculations confirm the validity of the baryonic branch interpretation of the gauge theory. This also resolves a puzzle about the dual of the D-strings at $\tau = 0$: they are the solitonic strings that couple to these massless glueballs. We further argue that, upon embedding this theory in a warped compactification, the global $U(1)_B$ symmetry becomes gauged; then the gauge symmetry is broken by the baryon expectation values through a supersymmetric version of the Higgs mechanism. Thus, in a flux compactification, we expect the D-string to be dual to an Abrikosov–Nielsen–Olesen vortex.

In [5] it was argued that there is a limit, $g_s M \to 0$,[5] where the physics of the cascading gauge theory should approach that of the pure glue $\mathcal{N} = 1$ supersymmetric $SU(M)$ gauge theory. How can this statement be consistent with the presence of the Goldstone bosons? We believe that it can. Returning to the $SU(2M) \times SU(M)$ gauge theory discussed in Section 4, we expect that in the limit $g_s M \to 0$ the scale Λ_{2M} of the $SU(2M)$, i.e. that of the baryon condensates, is much higher than the scale Λ_M of the $SU(M)$. Hence, the decay constant f_a should be much greater than the confinement scale Λ_M. Since the Goldstone boson interactions at the confinement scale are suppressed by powers of Λ_M / f_a, they appear to decouple from the massive glueballs containing the physics of the pure glue supersymmetric $SU(M)$ gauge theory. Obviously, this heuristic argument needs to be subjected to various checks.

Our work opens new directions for future research. Turning on finite scalar perturbations is expected to give rise to a new class of Lorentz invariant supersymmetric backgrounds, *the resolved warped deformed conifolds*, which preserve the SO(4) global symmetry but break the discrete $\mathcal{I}$ symmetry of the warped deformed conifold. The ansatz for such backgrounds was proposed in [11]. We have argued that these conjectured backgrounds are dual to the cascading gauge theory on the baryonic branch. It would be desirable to find them explicitly, and to confirm their supersymmetry.

A more explicit construction of the solitonic string in the gauge theory is desirable. It is also interesting to explore the consequences of our results for cosmological modeling.

---

[4] Representation of D-strings by ANO vortices in low-energy supergravity was recently advocated in a different context [25] (see also earlier work, [26]).

[5] No string theoretic description of this limit is yet available, because it is the opposite of the limit of large $g_s M$ where the supergravity description is valid.

Acknowledgements

References

[1] S.S. Gubser, C.P. Herzog, I.R. Klebanov, Symmetry breaking and axionic strings in the warped deformed conifold, hep-th/0405282.
[2] J.M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231, Int. J. Theor. Phys. 38 (1999) 1113, hep-th/9711200.
[3] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B 428 (1998) 105, hep-th/9802109.
[4] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.
[5] I.R. Klebanov, M.J. Strassler, Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities, JHEP 0008 (2000) 052, hep-th/0007191.
[6] S.S. Gubser, I.R. Klebanov, Baryons and domain walls in an $N = 1$ superconformal gauge theory, Phys. Rev. D 58 (1998) 125025, hep-th/9808075;
I.R. Klebanov, N.A. Nekrasov, Gravity duals of fractional branes and logarithmic RG flow, Nucl. Phys. B 574 (2000) 263, hep-th/9911096.
[7] I.R. Klebanov, A.A. Tseytlin, Gravity duals of supersymmetric $SU(N) \times SU(N + M)$ gauge theories, Nucl. Phys. B 578 (2000) 123, hep-th/0002159.
[8] N. Seiberg, Electric-magnetic duality in supersymmetric non-Abelian gauge theories, Nucl. Phys. B 435 (1995) 129, hep-th/9411149.
[9] O. Aharony, A note on the holographic interpretation of string theory backgrounds with varying flux, JHEP 0103 (2001) 012, hep-th/0101013.
[10] P.C. Argyres, M.R. Plesser, N. Seiberg, The moduli space of $N = 2$ SUSY QCD and duality in $N = 1$ SUSY QCD, Nucl. Phys. B 471 (1996) 159, hep-th/9603042.
[11] G. Papadopoulos, A.A. Tseytlin, Complex geometry of conifolds and 5-brane wrapped on 2-sphere, Class. Quant. Grav. 18 (2001) 1333, hep-th/0012034.
[12] L. Randall, R. Sundrum, A large mass hierarchy from a small extra dimension, Phys. Rev. Lett. 83 (1999) 3370, hep-ph/9905221.
[13] H. Verlinde, Holography and compactification, Nucl. Phys. B 580 (2000) 264, hep-th/9906182.
[14] S.B. Giddings, S. Kachru, J. Polchinski, Hierarchies from fluxes in string compactifications, Phys. Rev. D 66 (2002) 106006, hep-th/0105097.
[15] S. Kachru, R. Kallosh, A. Linde, S.P. Trivedi, De Sitter vacua in string theory, Phys. Rev. D 68 (2003) 046005, hep-th/0301240.
[16] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister, S.P. Trivedi, Towards inflation in string theory, JCAP 0310 (2003) 013, hep-th/0308055.
[17] E.J. Copeland, R.C. Myers, J. Polchinski, Cosmic F- and D-strings, hep-th/0312067;
M.G. Jackson, N.T. Jones, J. Polchinski, Collisions of cosmic F- and D-strings, hep-th/0405229.
[18] P. Candelas, X.C. de la Ossa, Comments on conifolds, Nucl. Phys. B 342 (1990) 246.
[19] I.R. Klebanov, E. Witten, Superconformal field theory on threebranes at a Calabi–Yau singularity, Nucl. Phys. B 536 (1998) 199, hep-th/9807080.
[20] D.R. Morrison, M.R. Plesser, Non-spherical horizons. I, Adv. Theor. Math. Phys. 3 (1999) 1, hep-th/9810201.
[21] I.R. Klebanov, E. Witten, AdS/CFT correspondence and symmetry breaking, Nucl. Phys. B 556 (1999) 89, hep-th/9905104.
[22] A. Ceresole, G. Dall'Agata, R. D'Auria, S. Ferrara, Spectrum of type IIB supergravity on AdS(5) × T(11): Predictions on $N = 1$ SCFT's, Phys. Rev. D 61 (2000) 066001, hep-th/9905226.
[23] C.P. Herzog, I.R. Klebanov, P. Ouyang, Remarks on the warped deformed conifold, hep-th/0108101;
C.P. Herzog, I.R. Klebanov, P. Ouyang, D-branes on the conifold and $N = 1$ gauge/gravity dualities, hep-th/0205100.
[24] I.R. Klebanov, P. Ouyang, E. Witten, A gravity dual of the chiral anomaly, Phys. Rev. D 65 (2002) 105007, hep-th/0202056.
[25] G. Dvali, R. Kallosh, A. Van Proeyen, D-term strings, JHEP 0401 (2004) 035, hep-th/0312005;
P. Binetruy, G. Dvali, R. Kallosh, A. Van Proeyen, Fayet–Iliopoulos terms in supergravity and cosmology, hep-th/0402046.
[26] J.D. Edelstein, C. Nunez, F.A. Schaposnik, Supergravity and a Bogomolny bound in three-dimensions, Nucl. Phys. B 458 (1996) 165, hep-th/9506147.