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String theory and fundamental forces/Théorie des cordes et forces fondamentales

Higher spin symmetry (breaking) in $\mathcal{N} = 4$ SYM and holography

Massimo Bianchi

Dipartimento di Fisica and INFN, Università di Roma "Tor Vergata", 00133 Rome, Italy

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Abstract

I concisely review the results of previous articles that shed some light on *"La Grande Bouffe"*, the pantagruelic Higgs mechanism, whereby HS gauge fields eat lower spin Goldstone fields. Mass generation in the AdS bulk is holographically dual to the emergence of anomalous dimensions in the boundary $\mathcal{N} = 4$ SYM theory. ***To cite this article: M. Bianchi, C. R. Physique 5 (2004).***

Résumé

(Brisure de) symétrie de grand spins dans la théorie de SYM $\mathcal{N} = 4$ et holographie. Je passe briévement en revue des résultats précedents qui apportent un certain éclairage sur « *La Grande Bouffe* », le mécanisme de Higgs pantagruélique, où les champs de jauge de HS absorbent les champs de Goldstone de plus bas spin. La génération de masse dans AdS est holographiquement duale à l'apparition de dimensions anormales pour la théorie de SYM $\mathcal{N} = 4$ au bord. ***Pour citer cet article : M. Bianchi, C. R. Physique 5 (2004).***

Keywords: String theory models; HS gauge fields; Pantagruelic Higgs mechanism

Mots-clés : Théorie des cordes ; Champs de jauge de HS ; Mécanisme de Higgs pantagruélique

This article concisely reviews the results of [1–3].

Formulating the dynamics of higher spin (HS) fields is a long standing problem, see e.g. [4–6] and references therein.[1] In the massless bosonic case, Fronsdal wrote down linearized field equations for totally symmetric tensors $\varphi^{(\mu_1\cdots\mu_s)}$ that in $D = 4$ arise from a Lorentz covariant (quadratic) action upon imposing 'double tracelessness' $\eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\varphi_{\mu_1\ldots\mu_s} = 0$. HS gauge invariance correspond to restricted transformations $\delta\varphi_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\epsilon_{\mu_2\ldots\mu_s)}$ with traceless parameters. Fang and Fronsdal then extended the analysis to fermions, while Singh and Hagen formulated equations for massive fields that reduce to Frondal's or Fang–Fronsdal's in the massless limit upon removing certain auxiliary fields. String theory in flat spacetime can be considered as a theory of an infinite number of HS gauge fields of various rank and (mixed) symmetry in a broken phase. At high energies these symmetries should be restored resulting in a new largely unexplored phase. Upon coupling HS fields to (external) gravity, the presence of the Weyl tensor in the variation of the action for $s > 2$, resulting from the Riemann tensor in the commutator of two covariant derivatives, spoils HS gauge invariance even at the linearized level and for on-shell gravitational backgrounds. For

E-mail address: Massimo.Bianchi@roma2.infn.it (M. Bianchi).

[1] For lack of space, I will often need to refer to comprehensive review papers rather than the original literature. I apologize for this inconvenience and warmly invite the interested reader to consult the exhaustive list of reference in [7].

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fields of spin $s \leqslant 2$, where, at most, the Ricci tensor appears. Problems with interactions for HS gauge fields in flat spacetime are to be expected since the Coleman–Mandula theorem and its generalization by Haag–Lopusanski–Sohnius imply a trivial S-matrix whenever the Poincaré group is extended by additional spacetime generators such as HS symmetry currents. Moreover, closure of the HS algebra requires an infinite tower of symmetries as soon as HS fields with $s > 2$ enter the game. A completely new approach to the interactions, if any, is to be expected in order to deal with an infinite number of HS fields and arbitrarily high derivatives [4–6].

According to Fradkin and Vasiliev, the situation improves significantly when the starting point is taken to be a maximally symmetric AdS space[2] with non-vanishing cosmological constant $\Lambda = -(D-2)(D-1)/R^2$ rather than flat spacetime. One can then use the HS analogue of the MacDowell, Mansouri, Stelle, West (MDMSW) $SO(D-1,2)$ formulation of gravity in order to keep HS gauge symmetry manifest and compactly organize the resulting higher derivative interactions and the associated non-locality. Vasiliev has been able to pursue this program till the very end, i.e. at the fully non-linear level, for massless bosons in $D = 4$ [4]. In Vasiliev's equations Λ plays a double role. On the one hand it organizes higher derivative interactions, very much like the string scale $M_s = 1/\sqrt{\alpha'}$ in string theory. On the other hand it allows one to define a generalized $SO(D-1,2)$ curvature that vanishes exactly for AdS. The AdS/CFT correspondence at the HS enhancement point seems exactly what the doctor ordered. At generic radius R, superstring theory should describe HS fields in a broken phase. At some critical radius, Vasiliev's equations or some generalization thereof should govern the dynamics of the exactly massless phase.

In the much studied case of $\mathcal{N} = 4$ SYM theory in $d = 4$ with $SU(N)$ gauge group, the holographic correspondence with type IIB superstring theory on $AdS_5 \times S^5$ with N units of RR 5-form flux has been a fruitful source of insights in the duality between (superconformal) gauge theories and AdS gravity [8–11]. At vanishing coupling, $\mathcal{N} = 4$ SYM exposes HS symmetry enhancement [12–14]. Conformal invariance indeed implies that a spin s current, such as $J_{(\mu_1\mu_2\cdots\mu_s)|} = \mathrm{Tr}(\varphi_i D_{(\mu_1} \cdots D_{\mu_s)|}\varphi^i) + \cdots$ saturating the unitary bound $\Delta = 2 + s$ be conserved. $\mathcal{N} = 4$ superconformal symmetry $(P)SU(2,2|4)$ implies that twist two operators are either conserved currents or superpartners thereof [15–17]. Altogether they form the *doubleton* representation of $HS(2,2|4)$, the HS extension of $(P)SU(2,2|4)$. The weak coupling regime on the boundary should be holographically dual to a highly stringy regime in the bulk, where the curvature radius R is small in string units $R \approx \sqrt{\alpha'}$ and the string is nearly tensionless. Although quantizing the superstring in $AdS_5 \times S^5$ is a difficult and not yet accomplished task [18], Sezgin and Sundell have been able to write down linearized field equations for the 'massless' $HS(2,2|4)$ doubleton [19–21]. As in Vasiliev's case the field content can be assembled into a master connection A and a master scalar (curvature) Φ. The former transform in the adjoint representation of $HS(2,2|4)$ and contains physical gauge fields with $s \geqslant 1$ and charge $B = 0, \pm 1$. The latter transform in the twisted adjoint representation and contributes physical fields with spin $s \leqslant 1/2$ or $s \geqslant 1$ and charge $|B| \geqslant 3/2$ such as self-dual two-form potentials. The field strengths $F_A = \mathrm{d}A + A \wedge *A$ and $D_A\Phi = \mathrm{d}\Phi + A * \Phi - \Phi * \tilde{A}$ transform covariantly $\delta F_A = [F_A, \epsilon]_*$, $\delta D_A\Phi = D_A\Phi * \tilde{\epsilon} - \epsilon * D_A\Phi$ under HS gauge transformations $\delta A = \mathrm{d}\epsilon + [A, \epsilon]_*$, whereby $\delta\Phi = \Phi * \tilde{\epsilon} - \epsilon * \Phi$. The linearized constraints and integrability conditions lead after some tedious algebra to the correct linearized field equations for the 'matter' fields with $s \leqslant 1/2$, for the HS gauge fields and for the antisymmetric tensors with generalized self-duality [19–21].

Possibly because of the presence of these generalized self dual tensors, an interacting $HS(2,2|4)$ gauge theory has not yet been formulated. In some sense, however, non-linear equations of Vasiliev's type encode combinatorial interactions which are present even in a free field theory, where HS symmetry is unbroken, or couplings to multi-particle states at finite N.[3] Although truncation to the HS massless multiplet (doubleton) should be consistent at the point of HS enhancement this should no more be the case for generic R. When interactions are turned on, i.e. at $\lambda \neq 0$, only a handful of HS fields remain massless. The vast majority participates in a pantagruelic Higgs mechanism, termed "*La Grande Bouffe*"[4] in [1–3], whereby HS gauge fields eat lower spin Goldstone fields. In the dual $\mathcal{N} = 4$ SYM description only the 1/2 BPS short multiplets with $2^8 p^2(p^2-1)/12$ components, corresponding to $\mathcal{N} = 8$ gauged supergravity and its Kaluza–Klein (KK) recurrences, are protected against quantum corrections to their dimensions. Except for the $\mathcal{N} = 4$ supercurrent multiplet, the infinite tower of conserved doubleton multiplets acquire anomalous dimensions which violate the conservation of the HS currents at the quantum level. At one-loop, one has [23]

$$\gamma_{1\text{-loop}}(2n) = \frac{g^2_{\mathrm{YM}} N}{2\pi^2} h(2n), \quad h(j) = \sum_{k=1}^{j} \frac{1}{k}, \tag{1}$$

[2] Results for dS space can be formally obtained by analytic continuation.

[3] Precisely for this reason they are relevant in the $d = 3$ $\mathrm{O}(N)$ model on the boundary of AdS_4 [22].

[4] Several people asked me the origin of this terminology. It is the title of a movie directed by Marco Ferreri, interpreted, among others, by Marcello Mastroianni and Ugo Tognazzi and presented in 1973 at *Festival du Cinema* in Cannes where it received the International Critics Award.

This elegant 'number theoretic' formula gives a clue on how to compute generic anomalous dimensions at first order in perturbation theory relying on HS symmetry breaking considerations and leads to integrability of the super spin chain Hamiltonian that represents the action of the (one-loop) dilatation operator [23]. For the purpose of describing the breaking of $HS(2,2|4)$ to $PSU(2,2|4)$ one should identify the Goldstone multiplets that provide the 'longitudinal' lower spin modes to the massless HS doubleton and hopefully determine their couplings by imposing (linearized) HS symmetry. At large N, this problem might turn out to be easier to solve than constructing a fully non-linear massless $HS(2,2|4)$ theory because it should only require a little bit more than the knowledge of the linearized field equations. For the (long) $\mathcal{N}=4$ Konishi multiplet [24,25], with 2^{16} components, dual to the first massive level of string excitations, for instance one expects a decomposition like

$$\mathcal{K}_{\text{long}} \leftrightarrow \mathcal{K}_{\text{short}} + \mathcal{K}_{1/4} + \mathcal{K}_{1/8} + \mathcal{K}^{*}_{1/8}, \tag{2}$$

i.e. the HS semishort multiplet ($\mathcal{K}_{\text{short}}$) eats the lower spin Goldstone multiplets $\mathcal{K}_{1/8}$, $\mathcal{K}^{*}_{1/8}$ and $\mathcal{K}_{1/4}$, belonging to the 'massive' HS multiplets associated to the totally antisymmetric 'tripleton' and the window-like 'tetrapleton'.

The huge enhancement of symmetry allows us to not only determine the HS content of the free $\mathcal{N}=4$ SYM spectrum at large N [3] but also match it with the superstring spectrum extrapolated to the point of HS symmetry enhancement [1,2]. Even at finite coupling, nearly BPS operators with large R-charge J and dimension $\Delta \approx J$ can be built by successively inserting *impurities* inside Chiral Primary Operators (CPOs) of the form $\mathrm{Tr}(Z^J)$. Berenstein, Maldacena and Nastase (BMN) argued that the sector with $J \approx \sqrt{N}$ is described by type IIB superstring on the maximally supersymmetric pp-wave emerging from a Penrose limit of $AdS_5 \times S^5$ [26]. Despite the presence of a null RR 5-form flux $F_{+1234} = F_{+5678} = \mu$, superstring fluctuations can be quantized in the light-cone gauge, where $p^+ = J/\mu\alpha'$. The spectrum of the light-cone Hamiltonian

$$H_{\text{LC}} = p^- = \mu(\Delta - J) = \mu \sum_n N_n \omega_n, \quad \omega_n = \sqrt{1 + \frac{n^2 \lambda}{J^2}}, \tag{3}$$

with the level matching condition $\sum_n n N_n = 0$, represents a prediction for the spectrum of anomalous dimensions of the so-called BMN operators, that form $(P)SU(2,2|4)$ multiplets at large but finite J. For our purposes it is crucial that any single-trace operator in $\mathcal{N}=4$ SYM be identified with some BMN operator with an arbitrary but finite number of impurities.

In flat spacetime, the single-particle type IIB superstring spectrum results from combining left- and right-moving modes with the same chirality projection on the vacuum and imposing level-matching $\ell = \sum n N^L_n = \sum_n n N^R_n$. In the light-cone, where only $SO(8) \subset SO(9,1)$ is manifest, the chiral groundstates $|\mathcal{Q}\rangle_{L/R}$ consists of $\mathbf{8}_V$ bosons and $\mathbf{8}_S$ fermions. At $\ell = 0$ one finds the 'transverse' modes of type IIB $\mathcal{N}=(2,0)$ supergravity $(\mathbf{8}_V - \mathbf{8}_S) \times (\mathbf{8}_V - \mathbf{8}_S)$. At higher levels, $\ell \geqslant 1$, the (chiral) spectrum assembles into full representations of the massive transverse Lorentz group $SO(9)$. For instance at $\ell = 1$, one finds $\mathbf{44} + \mathbf{84} - \mathbf{128}$ of $SO(9)$, corresponding to a symmetric tensor ('spin 2'), a 3-index totally antisymmetric tensor and a spin-vector ('spin 3/2'). At higher levels the situation is similar. The spectrum actually can be organized into $\mathcal{N}=(2,0)$ supermultiplets, whose groundstates are annihilated by half of the 32 supercharges. For $\ell = 1$ the groundstate cannot be other than an $SO(9)$ singlet $V^{L/R}_{\ell=1} = \mathbf{1}$, i.e. a scalar, since $2^8 \times 2^8 = 2^{16}$ equals the number of d.o.f. at this level. At higher levels the situation is not so straightforward, but one can eventually deduce a recurrence relation that yields $V^{L/R}_{\ell=1} = \mathbf{1}$, $V^{L/R}_{\ell=2} = \mathbf{9}$, $V^{L/R}_{\ell=3} = \mathbf{44} - \mathbf{16}, \ldots$ for the first few levels. In summary, the Hilbert space of type IIB superstring excitations in flat space can be written as

$$\mathcal{H}_{\text{flat}} = \mathcal{H}_{\text{sugra}} + \mathcal{T}_{\text{susy}} \sum_{\ell} V^L_{\ell} \times V^R_{\ell} \tag{4}$$

where $\mathcal{T}_{\text{susy}}$ represents the action of the 16 'raising' supercharges. States with maximum spin $s_{\text{Max}} = 2\ell + 2$ at level ℓ belong to the first Regge trajectory which is generated by oscillators with lowest non-trivial mode number. Moreover, the partial sums $\sum_{\ell}^{1,K} V^L_{\ell} \times V^R_{\ell}$ form $SO(10)$ multiplets. This is related to the possibility of 'covariantizing' the massive spectrum of type IIB, which is identical to the one of type IIA, to $SO(10)$, by lifting it to $D = 11$ [27], or to $SO(9,1)$, by introducing worldsheet (super)ghosts [18].

In order to extrapolate the massive string spectrum from flat space to $AdS_5 \times S^5$ at the HS symmetry enhancement point one should first decompose $SO(9)$ into $SO(4) \times SO(5)$, the relevant stability group of a massive particle. This straightforwardly determines two of the quantum numbers of the $(P)SU(2,2|4)$ superisometry group, namely the two spins (j_L, j_R) of $SO(4) \subset SO(4,2)$. The set of allowed representations of the S^5 isometry group $SO(6) \approx SU(4)$ are those that contain a given representation of $SO(5)$ under the decomposition $SO(6) \to SO(5)$. Denoting irreps by their Dynkin labels, $[m,n]$ for $SO(5)$ and $[k,p,q]$ for $SO(6)$, group theory yields the KK towers

$$\mathrm{KK}_{[m,n]} = \sum_{r=0}^{m} \sum_{s=0}^{n} \sum_{p=m-r}^{\infty} [r+s, p, r+n-s] + \sum_{r=0}^{m-1} \sum_{s=0}^{n-1} \sum_{p=m-r-1}^{\infty} [r+s+1, p, r+n-s]. \tag{5}$$

Any ambiguity in the lift, say, of the (pseudo-real) spinor **4** of $SO(5) \approx Sp(4)$ to the complex **4** of $SO(6) \approx SU(4)$ or to its complex conjugate $\mathbf{4}^*$ is resolved by the infinite sum over KK recurrences. Once the $SO(4) \times SO(6)$ quantum numbers are determined, the perturbative superstring spectrum turns out to be encoded in

$$\mathcal{H}_{\rm AdS} = \mathcal{H}_{\rm sugra} + \mathcal{T}_{\rm KK}\mathcal{T}_{\rm susy} \sum_{\ell} V_\ell^L \times V_\ell^R \tag{6}$$

where $\mathcal{T}_{\rm KK} = \sum_p [0, p, 0]$ represents KK towers that boil down to sums over scalar spherical harmonics, i.e. p-fold symmetric traceless tensors of $SO(6)$. $\mathcal{T}_{\rm susy}$ represents the action of the 16 'raising' Poincaré supercharges Q and $\overline{Q}$. $V_\ell^{L/R}$, defined in flat space, are to be decomposed under $SO(4) \times SO(5)$ and lifted to $SO(4) \times SO(6)$. Formula (6) looks deceivingly simple, almost trivial, since the most interesting information, the scaling dimension Δ, denoted by Δ_0 at the HS enhancement point, is still missing.

So far we have tacitly assumed that there are no non-perturbative states that can appear in the single-particle spectrum as a result of strings or branes wrapping nontrivial cycles. Indeed there are no such states with finite mass at small g_s, i.e. large N, since the only nontrivial cycles of S^5 are a 0-cycle (a point) or a 5-cycle (the full space). Although there can be ambiguities in extrapolating the perturbative spectrum from large radius, where KK technology is reliable, to small radius, where HS symmetry is restored, we expect 'level repulsion' at large N [14]. This guarantees that any state identified at large radius (strong 't Hooft coupling) can be smoothly if not explicitly followed to small radius (weak coupling) since trajectories of different fields/operators with the same quantum numbers never intersects.

One can thus start with identifying the string excitations that are expected to become massless at the point of enhanced HS symmetry [19–21]. In particular, the totally symmetric and traceless tensors of rank $2\ell - 2$ at level $\ell > 1$ appearing in the product of the groundstates $V_\ell^L \times V_\ell^R$ become massless and thus correspond to the sought for conserved HS currents on the boundary if one assigns them $\Delta_0 = 2\ell$, that works fine for $\ell = 1$, too. The states with $PSU(2,2|4)$ quantum numbers $\{2\ell; (\ell - 1, \ell - 1); [0,0,0]\}$ are HWS's of semishort multiplets [15–17,25]. Moreover the KK recurrences of these states at floor p arising from the action of $\mathcal{T}_{\rm KK}$ are naturally assigned [1]

$$\Delta_0 = 2\ell + p \tag{7}$$

which represents the $PSU(2,2|4)$ unitary bound for a spin $s = 2\ell - 2$ current in the $SO(6)$ irrep with Dynkin labels $[0, p, 0]$. It is remarkable how simply assuming HS symmetry enhancement fixes the AdS masses, i.e. scaling dimensions, of a significant fraction of the spectrum. Nevertheless, even at this particularly symmetric point there should be operators/states well above the unitary bounds. Surprisingly, (7) turns out to be correct for all primary states with mass/dimension $\Delta_0 \leqslant 4$. Notice that 'commensurability' of the two contributions – spin $s \approx \ell$ and KK 'angular momentum' $J \approx p$ – suggests that $R = \sqrt{\alpha'}$, for what this might mean. In order to find a mass formula that could extend and generalize (7), it is convenient to take the BMN formula (3) as a hint. Although derived under the assumptions of large λ and J [26], there seems to be no serious problem in extrapolating it to finite J at vanishing λ, where $\omega_n = 1$ for all n. Indeed, Niklas Beisert has shown that (two-impurity) BMN operators form $PSU(2,2|4)$ multiplets at finite J and are thus amenable to the extrapolation [23]. The resulting formula can be written [2]:

$$\Delta_0 = J + \nu \tag{8}$$

where $\nu = \sum_n N_n$ is the number of oscillators applied to the 'vacumm' $|J = \mu\alpha' p^+\rangle$ and J is the $U(1)$ charge in the decomposition of $SO(10)$ into $SO(8) \times U(1)_J$, where $SO(8)$ is the massless little group. In turn, $SO(10)$ arises from 'covariantizing' $SO(9)$. Although cumbersome, the procedure is straightforward and can be easily implemented on a computer. Given the $SO(10)$ content of the flat space string spectrum, Eq. (8) uniquely determines the dimensions Δ_0 of the superstring excitations around $AdS_5 \times S^5$ at the HS point. The case $\ell = 1$ is almost trivial, as an illustration, let us thus consider the string levels $\ell = 2, 3$:

$$\begin{aligned} V_2 &= [1,0,0,0,0]^2 - [0,0,0,0,0]^3 \\ &\overset{SO(8)\times SO(2)}{\rightarrow} [1,0,0,0]^2_0 + [0,0,0,0]^2_1 + [0,0,0,0]^2_{-1} - [0,0,0,0]^3_0 \end{aligned} \tag{9}$$

$$\overset{(8)}{\rightarrow} [1,0,0,0]_2 + [0,0,0,0]_1, \tag{10}$$

$$\begin{aligned} V_3 &= [2,0,0,0,0]^3 - [1,0,0,0,0]^4 - [0,0,0,0,1]^{5/2} \\ &\overset{SO(8)\times SO(2)}{\rightarrow} [2,0,0,0]^3_0 + [1,0,0,0]^3_1 + [1,0,0,0]^3_{-1} + [0,0,0,0]^3_0 + [0,0,0,0]^3_2 \\ &\quad + [0,0,0,0]^3_{-2} - [1,0,0,0]^4_0 - [0,0,0,0]^4_1 - [0,0,0,0]^4_{-1} - [0,0,0,1]^{5/2}_{1/2} - [0,0,1,0]^{5/2}_{-1/2} \\ &\overset{(8)}{\rightarrow} [2,0,0,0]_3 + [1,0,0,0]_2 + [0,0,0,0]_1 - [0,0,1,0]_3 - [0,0,0,1]_2. \end{aligned} \tag{11}$$

With the above assignments of Δ_0, negative multiplicities are harmless since they cancel in the sum over KK recurrences after decomposing $SO(10)$ w.r.t. $SO(4) \times SO(6)$. For these low massive levels, the conformal dimensions determined by (8) all saturate $SO(10)$ unitary bounds of the form $\Delta_\pm = 1 + k + 2l + 3m + 2(p+q) \pm (p-q)/2$. At higher levels, starting from the $\Delta_0 = 3$ singlet at level $\ell = 5$, this bound is still satisfied but no longer saturated: the correct conformal dimensions are rather obtained from (8). The results for string levels $\ell = 4$ and $\ell = 5$ are displayed in the following tables and organized under $SO(10) \times SO(2)_{\Delta_0}$, with Dynkin labels $[k,l,m,p,q]$ and $[k,l,m,p,q]^* \equiv [k,l,m,p,q] - [k-1,l,m,p,q]$.

$\ell = 4$:

Δ_0	$\mathcal{R}$
4	$[3,0,0,0,0]^*$
7/2	$[1,0,0,0,1]^*$
3	$[0,1,0,0,0]$

$\ell = 5$:

Δ_0	$\mathcal{R}$
5	$[4,0,0,0,0]^*$
9/2	$[2,0,0,0,1]^*$
4	$[0,0,1,0,0] + [1,1,0,0,0]^*$
7/2	$[1,0,0,0,1]$
3	$[0,0,0,0,0]$

In order to test the above prediction for the single-particle superstring spectrum on $AdS_5 \times S^5$ at the HS point against the spectrum of free $\mathcal{N} = 4$ SYM theory at large N, one has to devise an efficient way of computing gauge-invariant single trace operators [13,14,1]. For an $SU(N)$ gauge group this means taking care of the ciclicity of the trace in order to avoid multiple counting. Moreover one should discard operators which would vanish because of the field equations and deal with the statistics of the elementary fields. The mathematical tool one has to resort to is Polya theory that allows one to count 'words' $A, B, \ldots$ of a given 'length' n composed of 'letters' chosen from a given 'alphabet' $\{a_i\}$, modulo some symmetry operation: $A \approx B$ if $A = gB$ for $g \in \mathcal{G}$. In order to compute Polya cycle index it is convenient to decompose the discrete group $\mathcal{G} \subset \mathcal{S}_n$ into conjugacy classes whose representatives $[g] = (1)^{b_1(g)}(2)^{b_2(g)} \ldots (n)^{b_n(g)}$ are characterized by the numbers $b_k(g)$ of cycles of length k. For cyclic groups, $\mathcal{G} = Z_n$, conjugacy classes are labelled by divisors d of n, $[g]_d = (d)^{n/d}$, and the cycle index is simply expressed

$$\mathcal{P}_{Z_n}\big(\{a_i\}\big) = \frac{1}{n} \sum_{d|n} \mathcal{E}(d) \bigg(\sum_i a_i^d \bigg)^{n/d} \tag{12}$$

where $\mathcal{E}(d)$ is Euler's totient function which counts the number of elements in the conjugacy class $[g]_d$. $\mathcal{E}(d)$ equals the number of integers relatively prime to and smaller than d, with the understanding that $\mathcal{E}(1) = 1$, and satisfies $\sum_{d|n} \mathcal{E}(d) = n$.

For $\mathcal{N} = 4$ SYM, the alphabet is given by the elementary fields and their derivatives (modulo the field equations) $\{\partial^k \varphi, \partial^k \lambda, \partial^k F\}$ that transform in the *singleton* representation of $PSU(2,2|4)$. As a first step, one computes the on-shell single letter partition function or rather the Witten index $\mathcal{Z}_1(q) = \mathrm{Tr}(-)^F q^{\Delta_0}$, in order to take statistics into account. For a single (Abelian) $\mathcal{N} = 4$ vector multiplet, one has

$$\mathcal{Z}_1(q) = 2q \frac{(3 + \sqrt{q})}{(1 + \sqrt{q})^3}. \tag{13}$$

Plugging (13) into (12) one finds the single-trace partition function [13,14,1]

$$\mathcal{Z}_{\mathcal{N}=4}(q) = \sum_{n=2}^{\infty} \sum_{n|d} \frac{\mathcal{E}(d)}{n} \bigg[\frac{2q(3 + q^{d/2})}{(1 + q^{d/2})^3} \bigg]^{n/d} \tag{14}$$

$$\begin{aligned} &= 21q^2 - 96q^{5/2} + 376q^3 - 1344q^{7/2} + 4605q^4 - 15456q^{9/2} + 52152q^5 - 177600q^{11/2} \\ &\quad + 608365q^6 - 2095584q^{13/2} + 7262256q^7 - 25299744q^{15/2} + 88521741q^8 - 310927104q^{17/2} \\ &\quad + 1095923200q^9 - 3874803840q^{19/2} + 13737944493q^{10} + \mathcal{O}(q^{21/2}) \end{aligned} \tag{15}$$

for $SU(N)$ at large N, where mixing with multi-trace operators is suppressed.

In order to identify superconformal primaries, one can pass $\mathcal{Z}_{\mathcal{N}=4}(q)$ through an Eratosthenes super-sieve, that removes superdescendants. This task can be accomplished by first subtracting 1/2 BPS multiplets

$$\mathcal{Z}_{BPS}(q) = \frac{q^2(20 + 80q^{1/2} + 146q + 144q^{3/2} + 81q^2 + 24q^{5/2} + 3q^3)}{(1-q)(1+q^{1/2})^8}, \tag{16}$$

from (14) and then dividing by

$$\mathcal{T}_{SO(10,2)}(q) = (1-q^2)\frac{(1-q^{1/2})^{16}}{(1-q)^{10}} = \mathcal{T}_{\text{susy}}(q)\mathcal{T}_{\text{KK}}(q) \tag{17}$$

that not only removes superconformal descendants generated by $\mathcal{T}_{\text{susy}}(q) = (1-q^{1/2})^{16}/(1-q)^4$ but also the operators dual to the KK recurrences generated by $\mathcal{T}_{\text{KK}}(q) = (1-q^2)/(1-q)^6$, where the numerator implements the $SO(6)$ tracelessness condition. For $\mathcal{Z}_{SO(10,2)}(q) = [\mathcal{Z}_{\mathcal{N}=4}(q) - \mathcal{Z}_{BPS}(q)]/\mathcal{T}_{SO(10,2)}(q)$ one eventually finds the expansion

$$\begin{aligned}\mathcal{Z}_{SO(10,2)}(q) = {}& q^2 + 100q^4 + 236q^5 - 1728q^{11/2} + 4943q^6 - 12928q^{13/2} + 60428q^7 - 201792q^{15/2} + 707426q^8 \\ & - 2550208q^{17/2} + 9101288q^9 - 32568832q^{19/2} + 116831861q^{10} + \mathcal{O}(q^{21/2})\end{aligned}$$

that can be reorganized in the form

$$\begin{aligned}\mathcal{Z}_{SO(10,2)}(q) = {}& (q^1)^2 + (10q^2 - q^3)^2 + (-16q^{5/2} + 54q^3 - 10q^4)^2 \\ & + (45q^3 - 144q^{7/2} + 210q^4 + 16q^{9/2} - 54q^5)^2 + \cdots.\end{aligned} \tag{18}$$

It is not difficult to recognize that

$$\mathcal{Z}^{(\mathcal{N}=4)}_{SO(10,2)}(q) = \sum_\ell V^L_\ell(q) \times V^R_\ell(q) \tag{19}$$

where q keeps track of the dimensions assigned via $\Delta_0 = J + \nu$ after lifting $SO(9)$ to $SO(10)$. The origin of the $SO(10,2)$ spectrum symmetry calls for deeper understanding possibly in connection with Bars's two-time formulations of the superstring [27].

In order to set the stage for interactions that lead to HS symmetry breaking, one has to decompose the spectrum of single-trace operators in free $\mathcal{N} = 4$ SYM at large N or, equivalently, of type IIB superstring on $AdS_5 \times S^5$ extrapolated to the point of HS symmetry into HS multiplets [3]. To this end, following [4,19] we need to recall some basic properties of the infinite dimensional HS (super)algebra $\mathfrak{hs}(2,2|4)$, that extends the $\mathcal{N} = 4$ superconformal algebra $\mathfrak{psu}(2,2|4)$. The latter can be realized in terms of (super-)oscillators $\zeta_\Lambda = (y_a, \theta_A)$ with $[y_a, \bar{y}^b] = \delta_a{}^b$ and $\{\theta_A, \bar{\theta}^B\} = \delta^B{}_A$. y_a, $\bar{y}^b$ are bosonic oscillators with $a, b = 1, \ldots, 4$, a Weyl spinor index of $\mathfrak{so}(4,2) \sim \mathfrak{su}(2,2)$, while $\theta_A, \bar{\theta}^B$ are fermionic oscillators with $A, B = 1, \ldots, 4$ a Weyl spinor index of $\mathfrak{so}(6) \sim \mathfrak{su}(4)$.

Generators of $\mathfrak{psu}(2,2|4)$ are 'traceless' bilinears of superoscillators: $J^a{}_b = \bar{y}^a y_b - \frac{1}{4}K\delta^a{}_b$ with $K = \bar{y}^a y_a$, $T^A{}_B = \bar{\theta}^A\theta_B - \frac{1}{4}B\delta^A_B$ with $B = \bar{\theta}^A\theta_A$, $\mathcal{Q}^A_a = \bar{\theta}^A y_a$, and $\bar{\mathcal{Q}}^a_A = \bar{y}^a\theta_A$. The central element $C \equiv K + B = \bar{\zeta}^\Lambda\zeta_\Lambda$ generates an Abelian ideal that can be modded out e.g. by consistently assigning $C = 0$ to the elementary SYM fields and their (perturbative) composites. The hypercharge B acts as an external automorphism of $\mathfrak{psu}(2,2|4)$.

The HS extension $\mathfrak{hs}(2,2|4)$ is roughly speaking generated by odd powers of the above generators i.e.

$$\mathfrak{hs}(2,2|4) = \bigoplus_\ell \mathcal{A}_{2\ell+1} = \sum_{\ell=0}^{\infty}\{\mathcal{J}_{2\ell+1} = P^{\Lambda_1\ldots\Lambda_{2\ell+1}}_{\Sigma_1\ldots\Sigma_{2\ell+1}}\bar{\zeta}^{\Sigma_1}\cdots\bar{\zeta}^{\Sigma_{2\ell+1}}\zeta_{\Lambda_1}\cdots\zeta_{\Lambda_{2\ell+1}}\}, \tag{20}$$

with elements $\mathcal{J}_{2\ell+1}$ in $\mathcal{A}_{2\ell+1}$ at level ℓ parameterized by (graded) traceless rank $(2\ell+1)$ symmetric tensors $P^{\Lambda_1\ldots\Lambda_{2\ell+1}}_{\Sigma_1\ldots\Sigma_{2\ell+1}}$. More precisely, one first considers the enveloping algebra of $\mathfrak{psu}(2,2|4)$, which is an associative algebra and consists of all powers of the generators, then restricts it to the odd part which closes as a Lie algebra modulo the central charge C, and finally quotients the ideal generated by C. It is easy to show that B is never generated in commutators (but C is!) and thus remains an external automorphism of $\mathfrak{hs}(2,2|4)$ [19].

To each element in $\mathcal{A}_{2\ell+1}$ with spins (j_L, j_R) is associated an HS currents and a dual HS gauge field in the AdS bulk with spins $(j_L + \frac{1}{2}, j_R + \frac{1}{2})$. The $\mathfrak{psu}(2,2|4)$ quantum numbers can be read off from (20) by expanding the polynomials in powers of θ's up to 4, since $\theta^5 = 0$. There is a single superconformal multiplet $\mathcal{V}_{2\ell}$ at each level $\ell \geqslant 2$. The lowest spin cases $\ell = 0, 1$, i.e. $\widehat{\mathcal{V}}_{0,2}$, are special. They differ from the content of doubleton multiplets $\mathcal{V}_{0,2}$ by spin $s = 0, 1/2$ states [19]. The fundamental representation of $\mathfrak{hs}(2,2|4)$ turns out to coincide with the singleton $\mathcal{V}^{1,0}_{(0,0)[0,1,0]}$ of $\mathfrak{psu}(2,2|4)$. Its HWS $|Z\rangle$ or simply Z is one of the complex scalars of $\mathcal{N} = 4$ SYM. Any state A in this representation can be found by acting on the vacuum Z, or any other state B, with a sequence of superconformal generators. Looking at the singleton as an irrep of $\mathfrak{hs}(2,2|4)$ the sequence of superconformal generators connecting B to A is replaced by a single HS generator $\mathcal{J}_{A\bar{B}}$. This property is crucial in proving the irreducibility of YT-pletons with respect to the HS algebra. Indeed the tensor product of $L \geqslant 1$ singletons is generically reducible not only under $\mathfrak{psu}(2,2|4)$ but also under $\mathfrak{hs}(2,2|4)$ since the HS generators $\mathcal{J}_{2\ell+1} \equiv \sum_{s=1}^{L}\mathcal{J}^{(s)}_{2\ell+1}$, being completely symmetric, commute with (anti)symmetrizations of the indices. The tensor product thus decomposes into a sum of representations characterized by Young tableaux YT with L boxes. To prove irreducibility of L-pletons associated to a specific YT's under $\mathfrak{hs}(2,2|4)$, it is enough to show that any state in the L-pleton under consideration can be found by acting on the relevant HWS with HS generators. This is easy for the totally symmetric YT. More effort is needed to extend the argument to generic YT's [3].

Only a subset of YT's, those compatible with cyclicity of $SU(N)$ traces, enters the generating function of single-trace operators in $\mathcal{N}=4$ SYM theory, i.e. of cyclic words of length $L=n$.

$$\mathcal{Z}(q)=\sum_{n\geqslant 2}\mathcal{Z}_n(q)=\sum_{n\geqslant 2,d|n}\frac{\mathcal{E}(d)}{n}\mathcal{Z}_{\square}(q^d)^{n/d}. \tag{21}$$

Observe that $\mathcal{Z}_{\square}(q^d)$ can be rewritten as the alternating sum over length-d YT's of hook type:

$$\mathcal{Z}_{\square}(q^d)=\mathcal{Z}_{\square\square\square\square\square\cdots\square\square}(q)-\mathcal{Z}_{\substack{\square\square\square\square\cdots\square\square\\ \square}}(q)+\mathcal{Z}_{\substack{\square\square\square\cdots\square\square\\ \square\\ \square}}(q)-\mathcal{Z}_{\substack{\square\square\cdots\square\square\\ \square\\ \square\\ \square}}(q)+\cdots. \tag{22}$$

Plugging this expansion into (21), we find for the first few cases:

$$\begin{aligned}
&\mathcal{Z}_2=\mathcal{Z}_{\square\square},\\
&\mathcal{Z}_3=\mathcal{Z}_{\square\square\square}+\mathcal{Z}_{\substack{\square\\ \square\\ \square}},\\
&\mathcal{Z}_4=\mathcal{Z}_{\square\square\square\square}+\mathcal{Z}_{\substack{\square\square\\ \square\\ \square}}+\mathcal{Z}_{\substack{\square\square\\ \square\square}},\\
&\mathcal{Z}_5=\mathcal{Z}_{\square\square\square\square\square}+\mathcal{Z}_{\substack{\square\square\square\\ \square\square}}+2\mathcal{Z}_{\substack{\square\square\square\\ \square\\ \square}}+\mathcal{Z}_{\substack{\square\square\\ \square\square\\ \square}}+\mathcal{Z}_{\substack{\square\\ \square\\ \square\\ \square\\ \square}},\quad \text{etc.}
\end{aligned} \tag{23}$$

As anticipated, HS multiplets associated to the tableaux $\substack{\square\\ \square}$, $\substack{\square\square\\ \square}$, and two out of the three of type $\substack{\square\square\\ \square\\ \square}$ are projected out. Under the superconformal group $\mathfrak{psu}(2,2|4)$, the HS multiplet $\mathcal{Z}_{YT}$, associated to a given Young tableau YT with L boxes, decomposes into an infinite sum of multiplets. The HWS's can be found by computing $\mathcal{Z}_{YT}$ and eliminating the superconformal descendants by passing $\mathcal{Z}_{YT}$ through a sort of Eratosthenes (super)sieve [1]. In the $\mathfrak{psu}(2,2|4)$ notation $\mathcal{V}^{\Delta,B}_{(j_L,j_R)[q_1,p,q_2]}$ one finds for $L=2,3$

$$\mathcal{Z}_{\square\square}=\sum_{n=0}^{\infty}\mathcal{V}^{2n,0}_{(-1+n^*,-1+n^*)[0,0,0]}, \tag{24}$$

$$\begin{aligned}
\mathcal{Z}_{\square\square\square}&=\sum_{n=0}^{\infty}c_n\Big[\mathcal{V}^{1+n,0}_{(-1+\frac{1}{2}n^*,-1+\frac{1}{2}n^*)[0,1,0]}+\Big(\mathcal{V}^{\frac{11}{2}+n,\frac{1}{2}}_{(\frac{3}{2}+\frac{1}{2}n^*,1+\frac{1}{2}n^*)[0,0,1]}+\text{h.c.}\Big)\Big]\\
&\quad+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}c_n\Big[\mathcal{V}^{4+4m+n,1}_{(1+2m+\frac{1}{2}n^*,\frac{1}{2}n)[0,0,0]}+\mathcal{V}^{9+4m+n,1}_{(\frac{7}{2}+2m+\frac{1}{2}n^*,\frac{3}{2}+\frac{1}{2}n)[0,0,0]}+\text{h.c.}\Big],
\end{aligned} \tag{25}$$

$$\begin{aligned}
\mathcal{Z}_{\substack{\square\\ \square\\ \square}}&=\sum_{n=0}^{\infty}c_n\Big[\mathcal{V}^{4+n,0}_{(\frac{1}{2}+\frac{1}{2}n^*,\frac{1}{2}+\frac{1}{2}n^*)[0,1,0]}+\Big(\mathcal{V}^{\frac{5}{2}+n,\frac{1}{2}}_{(\frac{1}{2}n^*,-\frac{1}{2}+\frac{1}{2}n^*)[0,0,1]}+\text{h.c.}\Big)\Big]\\
&\quad+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}c_n\Big[\mathcal{V}^{6+4m+n,1}_{(2+2m+\frac{1}{2}n^*,\frac{1}{2}n)[0,0,0]}+\mathcal{V}^{7+4m+n,1}_{(\frac{5}{2}+2m+\frac{1}{2}n^*,\frac{3}{2}+\frac{1}{2}n)[0,0,0]}+\text{h.c.}\Big].
\end{aligned} \tag{26}$$

The multiplicities $c_n\equiv 1+[n/6]-\delta_{n,1\ \mathrm{mod}\ 6}$ with $[m]$ the integral part of m, of $\mathfrak{psu}(2,2|4)$ multiplets inside $\mathfrak{hs}(2,2|4)$ count the number of ways one can distribute derivatives (HS descendants) among the boxes in the tableaux.

In addition to the $\frac{1}{2}$-BPS with $n=0$, the symmetric doubleton $\mathcal{Z}_{\square\square}$, corresponding to the quadratic Casimir δ_{ab}, contains the multiplets of conserved HS currents $\mathcal{V}_{2n}$. The antisymmetric doubleton $\mathcal{Z}_{\substack{\square\\ \square}}$ is ruled out by cyclicity of the trace, cf. (23). The 'symmetric tripleton' $\mathcal{Z}_{\square\square\square}$, corresponding to the cubic Casimir d_{abc}, contains the first KK recurrences of twist 2 semi-short multiplets, the semishort-semishort series $\mathcal{V}_{\pm 1,n}$ starting with fermionic primaries and long-semishort multiplets. The antisymmetric tripleton $\mathcal{Z}_{\substack{\square\\ \square\\ \square}}$, corresponding to the structure constants f_{abc}, on the other hand contains the Goldstone multiplets that merge with twist 2 multiplets to form long multiplets when the HS symmetry is broken, in particular, fermionic semishort-semishort multiplets and long-semishort multiplets [3].

The holographic formulation of *La Grande Bouffe* we have in mind is of the Stückelberg type [28,29]. Let us illustrate it for a broken singlet vector current. The Lagrangian describing the bulk Higgs mechanism à la Stückelberg should be (schematically) of the form

$$L=-\frac{1}{4}F(V)^2+\frac{1}{2}(\partial\alpha-MV)^2 \tag{27}$$

where F is the field-strength of the bulk vector field V dual to the current J and α is the bulk (pseudo)scalar dual to the 'anomaly' $A = \partial_\mu J^\mu$. Gauge invariance under

$$\delta V_m = \partial_m \vartheta, \qquad \delta\alpha = M\vartheta \tag{28}$$

is manifest for constant M. For $M = 0$, V and α decouple. For $M \neq 0$, V eats α and becomes massive. In practice M should depend on the dilaton and other massless scalars. Since we want to preserve superconformal invariance, M can at most acquire a constant vev and the above analysis seems valid, at least for a vector current. Although little is known about massless HS bosonic and fermionic fields with mixed symmetry [4–6], whenever they are part of the HS doubleton multiplet, supersymmetry should be enough to determine their equations from the more familiar equations for symmetric tensors. By the same token, the various manifestations of the symmetry breaking mechanism should be related by (extended) supersymmetry. For instance, for the $\mathcal{N} = 4$ Konishi multiplet the axial anomaly is part of an on-shell anomaly supermultiplet [10,24,17]

$$\overline{D}^A \overline{D}^B \mathcal{K}_{\rm long} = g_{ym}\,\mathrm{Tr}\big(\mathcal{W}_{EF}[\mathcal{W}^{AE}\mathcal{W}^{BF}]\big) + \frac{g_{ym}^2}{8\pi^2} D_E D_F\,\mathrm{Tr}\big(\mathcal{W}^{AE}\mathcal{W}^{BF}\big). \tag{29}$$

For symmetric tensors of rank s in any dimension D, the set of Stückelberg fields that participate in the spontaneous breaking of HS symmetry can be elegantly derived performing a formal KK reduction of the (quadratic) HS Lagrangian from $D+1$ dimensions [6]. The a priori complex Fourier modes $\psi^M_{(s-t)}(x)\exp(\mathrm{i}My)$ with $t = 0,\ldots,s$ exactly account for the correct number of d.o.f.'s $\nu_{M\neq 0}(D,s)$. Indeed it is easy to check that $\nu_{M\neq 0}(D,s) = \nu_{M=0}(D,s) + \nu_{M\neq 0}(D,s-1)$ and, by iteration, that $\nu_{M\neq 0}(D,s) = \sum_{t=0}^{s} \nu_{M=0}(D,t)$. After reduction, i.e. integration over y, one can take real combinations $\phi_{(s-t)}$ of $\psi_{(s-t)}$'s. More explicitly, from a massless doubly traceless spin s field $\Phi_{(s)}$ in $D+1$ dimensions one gets 'massless' fields $\phi_{(s-t)}$ with $t = 0,\ldots,s$ satisfying certain trace conditions in D dimensions. The resulting HS field equations are invariant by construction under gauge transformations $\delta\phi_{(s-t)} = \partial_{(1)}\epsilon_{(s-t-1)} + tM\epsilon_{(s-t)}$ with $t = 0,\ldots,s$, resulting from $\delta\Phi_{(s)} = \partial_{(1)}\mathcal{E}_{(s-1)}$ with restricted (traceless) parameters that expose the role of the lower spin fields in the Higgsing of the HS symmetry.

Although the way to go is still long, we wish the reader could at least glimpse *La Grande Bouffe*.

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