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Strings, gravity, and the quest for unification/Cordes, gravitation, et la quête d'unification

What is inside a black hole?

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Abstract

In the conventional picture of a black hole we cannot see the states which account for its entropy, and Hawking radiation is unable to carry out the information of the state of the hole. We outline some computations in string theory which suggest that bound states expand to a size determined by their degeneracy. Thus the state information of a black hole would be distributed throughout the interior of the horizon, and Hawking radiation can 'read' this information. No individual microstate has a horizon, rather the horizon arises upon coarse graining as a boundary of the region where the microstates differ. ***To cite this article: S.D. Mathur, C. R. Physique 6 (2005).***

Résumé

Qui a-t-il dans un trou noir ? Dans la représentation conventionnelle d'un trou noir il est impossible de voir les états qui compose son entropie, et la radiatio d'Hawking est incapable de transporter en dehors du trou noir de l'information sur l'état du trou noir. Nous décrivons des calculs en théorie des cordes qui suggèrent les états liés grandissent juqu'à une taille déterminée par leur dégénérescence. Ainsi l'information à propos d'un trou noir serait distribuée à l'intérieur de l'horizon, et la radiation d'Hawking peut « lire » cette information. Aucun état individuel n'a pas d'horizon ; mais l'horizon apparaît après moyennisation comme la région où les états microscopiques diffèrent. ***Pour citer cet article : S.D. Mathur, C. R. Physique 6 (2005).***

Keywords: String theory; Black hole; Hawking radiation

Mots-clés : Théorie des cordes ; Trou noir ; Radiation d'Hawking

1. Introduction

Quantum mechanics in the presence of black holes generates some sharp paradoxes. Gedanken experiments indicate that we must associate an entropy

$$S_{\mathrm{Bek}} = \frac{A}{4G} \tag{1}$$

with the hole to save the second law of thermodynamics [1]. Statistical mechanics then indicates that the hole must have $e^{S_{\mathrm{Bek}}}$ microstates. However, the geometry of the hole appears to be determined by its mass, charges and angular momenta; i.e., 'black holes have no hair'. Where should we look for the differences between the microstates?

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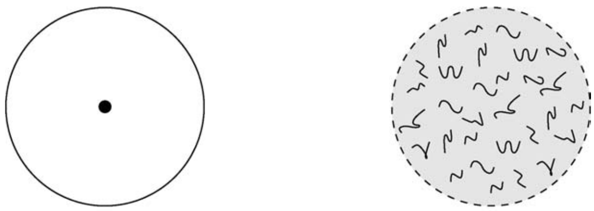

1(a) 1(b)

Fig. 1. (a) The conventional picture of a black hole; (b) the proposed picture—state information is distributed throughout the 'fuzzball'.

Closely related is the 'information puzzle'. Particle–antiparticle pairs are continuously created and annihilated in the vacuum, but in the presence of the tidal forces at the horizon one member of this pair can fall in (lowering the mass of the hole) while the other escapes to infinity as Hawking radiation. But this radiation carries no information about the matter that initially made the hole, so when the hole evaporates away into radiation we find 'information loss', a violation of the basic unitarity of quantum mechanics [2].

The above two problems are closely connected. If we burn a piece of coal and get radiation we do not lose information because the radiation emerges from a region where it can see the state of the coal. By contrast, Hawking radiation appears to emerge from the horizon while the matter making the hole appears to sit at the singularity $r = 0$. The distance between horizon and singularity is macroscopic, and we need to understand how information can be transported across this large distance.

In this article we discuss some computations in string theory which suggest that the picture of the black hole interior might be radically different from the conventional picture given in Fig. 1(a). In the proposed picture (Fig. 1(b)) the state information is distributed throughout the interior of the horizon; different configurations of this 'fuzzball' give different states of the hole. No individual state has a horizon; rather the horizon is a boundary of the region where the states differ significantly from each other. There is no special point to play the role of a central singularity.

We first look at the simplest system that has entropy—the 2-charge extremal D1–D5 system. The microscopic entropy of the D1–D5 bound state is $2\pi\sqrt{2}\sqrt{n_1 n_5}$. We find that the geometries dual to these states behave like Fig. 1(b), and the area of the bounding surface where the geometries start to differ satisfies $A/(4G) \sim \sqrt{n_1 n_5}$. We then move to the 3-charge extremal D1–D5–P system where we have constructed geometries dual to a small subset of the microstates; these geometries also agree with Fig. 1(b) rather than Fig. 1(a).

Why are we getting a change in our picture of the black hole in a region where semiclassical physics appeared to be valid, and suggested 'empty space' as the local description? It is normally assumed that quantum gravity effects must be confined to within a length scale like the Planck length l_p or string length l_s. But in making a large black hole we are dealing with a large number N of quanta, and what we seem to find is that quantum effects extend to a distance $\sim N^{\alpha} l_p$, where the power α is such that the size of the resulting object satisfies a Bekenstein type relation $A/(4G) \sim S$ (e^S is the degeneracy of the quantum state). The key notion seems to be 'fractionation': when we bind together a large number N of quanta in string theory then the excitations of the resulting state come in fractional units $1/N$; thus they are very light for large black holes. A very rough estimate shows that because of this fractionation the 'size' of a 3-charge bound state is of the same order as the naive horizon radius, and this makes it possible for the entire interior of the horizon to be filled with 'quantum fuzz' of fractional brane excitations.

We end with some comments on non-NPS holes, and the implications of this picture for other problems in physics.

2. The 2-charge system

Consider type IIB string theory, compactified to $M_{5,1} \times S^1 \times T^4$. The radius of S^1 is R and the volume of T^4 is $(2\pi)^4 V$. We use coordinates $t = x^0, x^1, \ldots, x^4$ to describe $M_{4,1}$, $y = x^5$ for S^1 and $x^6 \cdots x^9 \equiv z_1 \cdots z_4$ for T^4. Wrap n_1 D1 branes on S^1 and n_5 D5 branes on $S^1 \times T^4$. Make a bound state of these branes, and place it at $r = 0$ in the noncompact space $M_{4,1}$. The geometry usually written down for this configuration (which we shall call the 'naive geometry') is

$$\mathrm{d}s^2_{\text{naive}} = \frac{1}{\sqrt{(1 + Q_1/r^2)(1 + Q_5/r^2)}}\left[-\mathrm{d}t^2 + \mathrm{d}y^2\right] + \sqrt{\left(1 + \frac{Q_1}{r^2}\right)\left(1 + \frac{Q_5}{r^2}\right)}\,\mathrm{d}x_i\,\mathrm{d}x_i + \sqrt{\frac{1 + Q_1/r^2}{1 + Q_5/r^2}}\,\mathrm{d}z_a\,\mathrm{d}z_a. \tag{2}$$

This geometry is flat space at large r, locally $A\,\mathrm{d}S_3 \times S^3 \times T^4$ in the 'throat' and ends in a singularity (due to an accumulation point of identifications $y \sim y + 2\pi R$) at $r = 0$. The horizon coincides with the singularity, and has zero area. We sketch this geometry schematically in Fig. 2(a).

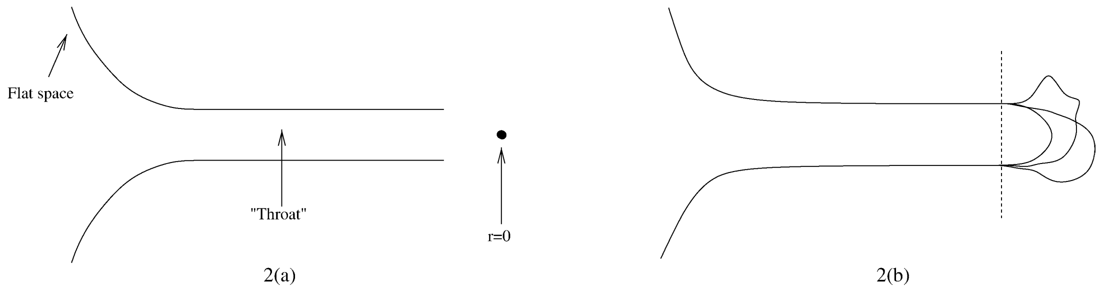

Fig. 2. (a) The naive geometry of extremal D1–D5; (b) the actual geometries; the dashed line gives a 'horizon' whose area gives the entropy.

As mentioned above, it is known that the 2-charge D1–D5 bound state has degeneracy $e^{2\pi\sqrt{2}\sqrt{n_1 n_5}}$, and we might wonder what we have to do to see the differences between these states in the gravity description. Classically we can get many supergravity solutions by breaking the D1–D5 bound state into two or more pieces which we can then separate. But we are interested in describing the *bound* states of the D1, D5 branes, since it is these bound states which have the stated degeneracy and which give the analogue of 'one black hole' rather then 'many holes'.

While it is not easy to see in the D1–D5 language which geometries describe bound states, we can solve the problem by performing a set of dualities

$$\begin{aligned} &\mathrm{D1\ D5(IIB)} \to S \to \mathrm{NS1\ NS5(IIB)} \to T_5 \to \mathrm{P\ NS5(IIA)} \to T_6 \to \mathrm{P\ NS5(IIB)} \\ &\quad \to S \to \mathrm{P\ D5(IIB)} \to T_{6789} \to \mathrm{P\ D1(IIB)} \to S \to \mathrm{P\ NS1(IIB)}. \end{aligned} \tag{3}$$

This gives the fundamental string (NS1) wrapped n_5 times on S^1, carrying n_1 units of momentum (P) along S^1. A *bound* state of these charges needs to have all strands of the NS1 string joined up into one 'multiwound' string, and all the momentum must be carried as traveling waves on this string.

We now come to a crucial step: the fundamental string has no longitudinal vibration modes, so all the momentum must be carried by transverse oscillations. Thus the string must bend away from its central axis at $r = 0$, and this transverse bending will increase with the amount of momentum the string carries. It is convenient to go to a covering space where we open up the n_5 windings of the string and get just a single strand, identified after a distance $\Delta y = 2\pi R n_5$. The vibration profile is given by a transverse displacement $\mathbf{F}(v)$, where $v = t - y$. For the moment consider the vibrations where $\mathbf{F}$ lies in the four noncompact directions.[1] In the physical space we find that the n_5 strands of the string spread themselves over a certain region of the transverse R^4. The metric of a single strand carrying momentum is known, and the metric of multiple stands can be obtained by superposing the harmonic functions appearing in the metric of each strand. In the classical limit the number of strands goes to infinity, so we take them to form a continuous 'ribbon'. Finally, we reverse the dualities (3) to go from this P–NS1 geometry to the D1–D5 geometry. We find [3]

$$\mathrm{d}s^2 = \sqrt{\frac{H}{1+K}}\left[-(\mathrm{d}t - A_i\,\mathrm{d}x^i)^2 + (\mathrm{d}y + B_i\,\mathrm{d}x^i)^2\right] + \sqrt{\frac{1+K}{H}}\,\mathrm{d}x_i\,\mathrm{d}x_i + \sqrt{H(1+K)}\,\mathrm{d}z_a\,\mathrm{d}z_a, \tag{4}$$

$$H^{-1} = 1 + \frac{Q}{L}\int_0^L \frac{\mathrm{d}v}{|\mathbf{x} - \mathbf{F}(v)|^2}, \quad K = \frac{Q}{L}\int_0^L \frac{\mathrm{d}v(\dot{F}(v))^2}{|\mathbf{x} - \mathbf{F}(v)|^2}, \quad A_i = -\frac{Q}{L}\int_0^L \frac{\mathrm{d}v\dot{F}_i(v)}{|\mathbf{x} - \mathbf{F}(v)|^2}. \tag{5}$$

Here $L = 2\pi n_5 R$ is the total length of the NS1 string, and $\mathrm{d}B = -*_4\,\mathrm{d}A$ ($*_4$ is the duality operation in the 4-d transverse space $x_1 \cdots x_4$ using the flat metric $\mathrm{d}x_i\,\mathrm{d}x_i$). The dualities only involve the compact directions, and thus the transverse spread of the NS1–P state implies a corresponding transverse spread of the D1–D5 bound state. The naive geometry (2) has been resolved into a family of different geometries labeled by the function $\mathbf{F}(v)$. One may be concerned about a singularity at the points where $\mathbf{x} = \mathbf{F}(v)$, but it was shown in [4] that the singularity here is just a coordinate one, similar to that at the center of a Kaluza–Klein monopole. Each individual geometry (determined by a choice of $\mathbf{F}(v)$) has no singularity and no horizon; it is flat space at infinity, it is approximately locally $AdS_3 \times S^3 \times T^4$ in the 'throat', then there is a smooth 'cap', with the shape of the cap depending on $\mathbf{F}(v)$. We sketch this schematically in Fig. 2(b).

[1] In [4] the computations were extended to cover the case of vibrations in the T^4 directions as well.

Classically we have a continuous family of geometries, but semiclassical quantization of this moduli space should give us a finite set of BPS states. The simplest way to count states is to look at the weak coupling description of states of the NS1 string carrying momentum. The total length of the string is $L_T = 2\pi R n_5$, and the momentum is

$$P = \frac{n_1}{R} = \frac{2\pi n_1 n_5}{L_T}. \tag{6}$$

The left movers are thus excited to a level $N = n_1 n_5$, and we get a large degeneracy from the number of ways the momentum can be partitioned among harmonics. Noting that there are 8 transverse directions of oscillation, and 8 corresponding fermions, we have $c_{\text{eff}} = 8 + 4 = 12$, and we get

$$\#states \sim \mathrm{e}^{2\pi\sqrt{c_{\text{eff}} N/6}} = \mathrm{e}^{2\pi\sqrt{2}\sqrt{n_1 n_5}}. \tag{7}$$

2.1. The 'horizon'

Let us draw a surface in the throat where the typical geometries start to deviate from each other by order unity (Fig. 2(b)). We find that the area A of this surface satisfies [5]

$$\frac{A}{4G} \sim \sqrt{n_1 n_5} \sim S \tag{8}$$

so we get a Bekenstein type relation for the size of the generic state.

It retrospect it is appealing that individual geometries do not have horizons, since if they did, we would have to associate an entropy with that horizon, and this makes no sense since we are discussing a given microstate. The bounding surface that we have drawn as a 'horizon' does a kind of 'coarse-graining'; it keeps the part of the geometry where all microstates agree and cuts off the region where the microstates differ. Thus the area entropy (8) arises upon coarse-graining, which is in line with how we get entropy in all other physical systems.

It is worth noting that the bounding surface which we have drawn acts like a 'horizon' in the sense that if a quantum goes past this surface in the typical geometry then due to the complexity of the 'cap' created by a generic $\mathbf{F}(v)$ it stays trapped for a time $\Delta t \sim$ (radius of S^3) $\times \sqrt{S}$ ($S \sim \sqrt{n_1 n_5}$ is the entropy) [5]. In the classical limit of large charges this time diverges when measured in units of the radius of the S^3 so the quantum does not return on the classical time scale.

Note that the dilaton in the D1–D5 geometries is given by $\mathrm{e}^{2\Phi} = H(1+K)$ and so is bounded above and below everywhere. Thus Planck length l_p and string length l_s are of the same order. By contrast, the radius of the 'fuzzball' created by the spreading of the D1–D5 geometry is

$$r_{\text{D1–D5}} \sim \left(\frac{g^2 \alpha'^4}{VR}\right)^{1/3} (n_1 n_5)^{1/6} \gg l_p, l_s. \tag{9}$$

This illustrates the basic idea we are seeking—bound states spread over a radius much bigger than the naively expected scales l_p, l_s in the problem, and the size of the bound state reflects its entropy through a Bekenstein type relation like (8).

3. The D1–D5 CFT

To have a better idea of the geometries that we have constructed, we identify the corresponding states in the dual CFT. The low energy dynamics of a large number of D1, D5 branes is conjectured to be given by a $(1+1)$-dimensional CFT which arises as a sigma model with target space $(T^4)^N/S_N$—the symmetric product of $N = n_1 n_5$ copies of T^4. Each copy of T^4 gives a $c = 6$ CFT with 4 free bosons X_i, 4 free fermions left moving fermions and 4 free right moving fermions. The orbifolding gives rise to twist sectors, created by twist operators σ_n which link together n copies of the $c = 6$ CFT (living on circles of length $2\pi R$) to give a $c = 6$ CFT living on a circle of length $2\pi n R$. We call each such set of linked copies a 'component string'.

Our CFT on the D1–D5 branes is in the Ramond sector, since the periodicity of fermions on the branes is induced from the periodicity of supergravity fermions in the bulk and the fermions in the bulk are periodic around the compact circles. On each component string we have 4 left and 4 right fermion zero modes, which give 8 bosonic and 8 fermionic ground states for the component string. The number of ways to partition $n_1 n_5$ copies of the CFT into component strings, together with this zero mode degeneracy, reproduces the count (7) of states in the D1–D5 language.

By looking at the Poincare polynomials one can establish a direct link between the NS1–P and D1–D5 states. Consider a state of the NS1 string carrying left oscillators $\alpha^{i_1}_{-n_1} \cdots \alpha^{i_k}_{-n_k}$. In the D1–D5 CFT this maps to a Ramond ground state with component strings of lengths $n_1, \ldots, n_k$. The polarizations $i_1, \ldots, i_n$ give the fermion zero modes on the component strings. Using this map we can find the D1–D5 geometry that we want to associate (in the classical limit) with a Ramond ground state of the D1–D5 CFT. From the twist orders n_m and polarizations i_m we find the oscillator excitations of the NS1 string in the NS1–P dual. These oscillators give, in the classical limit, a profile function $\mathbf{F}(v)$. But for a given $\mathbf{F}(v)$ we know the NS1–P geometry, and by duality the D1–D5 geometry (5).

3.1. Comparing the CFT and gravity pictures

Consider the following special subset of states. Take $\mathbf{F}(v)$ of the form

$$\mathbf{F}(v) = a\big[\sin(kv)\hat{x}_1 + \cos(kv)\hat{x}_2\big] \tag{10}$$

so that the NS1 string makes a uniform helix of k turns when opened up to the covering space. The corresponding CFT state turns out be to be

$$\big|\Psi^{--}(k,0)\big\rangle = \big[(\sigma_k^{--})^{N/k}|0\rangle\big]_{NS\to R}. \tag{11}$$

Here the superscripts $(--)$ on σ_k indicate the fermion zero modes, $|0\rangle$ is the NS vacuum of the CFT and $NS \to R$ indicates that after constructing the state in the NS sector we spectral flow to the R sector.

In the CFT the twist operators σ_k create effective string components with length $2\pi Rk$. The excitation levels of such components come in units of

$$\Delta E = \frac{1}{Rk} + \frac{1}{Rk} = \frac{2}{Rk}. \tag{12}$$

(We have one left and one right mover so that the change in momentum is zero.)

Turning to the gravity description of the same states, we find that the depth of the throat depends on k—geometries dual to states with larger k have deeper throats. The energy levels for excitations in the throat can be computed by solving the corresponding supergravity wave equation for small perturbations, and the result (12) is exactly reproduced [6] by the gravity duals. The same comparison can also be recast in terms of the time taken for left and right movers to travel around the component string of the CFT; in the gravity description this maps to the time taken for a quantum to travel down the throat and back up [3]. In the latter description the backreaction of the quantum on the geometry was also checked and found to be small for the generic state, so we can indeed distinguish between different microstates in the gravity description by performing scattering experiments.

4. The 3-charge system

If we take the D1–D5 system above and add n_p units of momentum along S^1 then we get the D1–D5–P system. The microscopic entropy is $S_{\text{micro}} = 2\pi\sqrt{n_1 n_5 n_p}$. The naive geometry this time is flat space at infinity and a locally $AdS_3 \times S^3 \times T^4$ 'throat', but this time there is a horizon at $r = 0$, and if we continue the geometry past this horizon we find the singularity expected of an extremal Reissner–Nordstrom hole. The Bekenstein entropy computed from the area of the horizon agrees with the microscopic entropy exactly [8].

However, one may wonder if the 3-charge microstate 'swells up' in a manner similar to (9), in which case the naive geometry inside the horizon would be inaccurate and we might get 'caps' to the throat region as in Fig. 2(b). In the 3-charge case the diameter of the throat stabilizes to a constant as we go down towards $r = 0$, so if we draw a boundary where the 'caps' begin to differ (Fig. 2(b)) then we will get the same area A from this boundary as we get from the horizon of the naive geometry. So for the 3-charge case all we need to check is that the geometries get 'capped'; if they do, the boundary area of the 'fuzzball' will satisfy the desired Bekenstein type relation anyway.

It was argued in [16] that the 3-charge state does indeed swell up to a size of order the horizon radius, and this estimate (which we discuss in more detail below) spurred us to start with the simpler 2-charge system and look carefully at the geometries corresponding to microstates. We cannot as yet construct all the 3-charge geometries, but we have constructed some families of axisymmetric ones and identified the corresponding microstates, and these geometries turn out to look like Fig. 2(b) rather than Fig. 2(a). In the CFT description we get a D1–D5–P microstate by adding left moving excitations to a D1–D5 ground state. As an example, for the states

$$\Big(\prod\big[J^-_{-2n}\cdots J^-_{-4/k}J^-_{-2/k}\big]\Big)\big|\Psi^{--}(k,0)\big\rangle \tag{13}$$

(the product runs over the N/k connected components of the CFT created by the twists) we find the geometry [10]

$$
\begin{aligned}
\mathrm{d}s^2 = {} & -\frac{1}{h}(\mathrm{d}t^2-\mathrm{d}y^2)+\frac{Q_p}{hf}(\mathrm{d}t-\mathrm{d}y)^2+hf\left(\frac{\mathrm{d}r^2}{r^2+(\tilde\gamma_1+\tilde\gamma_2)^2\eta}+\mathrm{d}\theta^2\right)\\
& + h\left(r^2+\tilde\gamma_1(\tilde\gamma_1+\tilde\gamma_2)\eta-\frac{Q_1Q_5(\tilde\gamma_1^2-\tilde\gamma_2^2)\eta\cos^2\theta}{h^2f^2}\right)\cos^2\theta\,\mathrm{d}\psi^2\\
& + h\left(r^2+\tilde\gamma_2(\tilde\gamma_1+\tilde\gamma_2)\eta+\frac{Q_1Q_5(\tilde\gamma_1^2-\tilde\gamma_2^2)\eta\sin^2\theta}{h^2f^2}\right)\sin^2\theta\,\mathrm{d}\phi^2+\frac{Q_p(\tilde\gamma_1+\tilde\gamma_2)^2\,\eta^2}{hf}(\cos^2\theta\,\mathrm{d}\psi+\sin^2\theta\,\mathrm{d}\phi)^2\\
& - \frac{2\sqrt{Q_1Q_5}}{hf}(\tilde\gamma_1\cos^2\theta\,\mathrm{d}\psi+\tilde\gamma_2\sin^2\theta\,\mathrm{d}\phi)(\mathrm{d}t-\mathrm{d}y)\\
& - \frac{2\sqrt{Q_1Q_5}(\tilde\gamma_1+\tilde\gamma_2)\eta}{hf}(\cos^2\theta\,\mathrm{d}\psi+\sin^2\theta\,\mathrm{d}\phi)\,\mathrm{d}y+\sqrt{\frac{H_1}{H_5}}\,\mathrm{d}z_a\,\mathrm{d}z_a
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
&\eta=\frac{Q_1Q_5}{Q_1Q_5+Q_1Q_p+Q_5Q_p},\\
&f=r^2+(\tilde\gamma_1+\tilde\gamma_2)\eta(\tilde\gamma_1\sin^2\theta+\tilde\gamma_2\cos^2\theta),\\
&H_1=1+\frac{Q_1}{f},\quad H_5=1+\frac{Q_5}{f},\quad h=\sqrt{H_1H_5},
\end{aligned}
\tag{15}
$$

$$
\tilde\gamma_1=-\frac{\sqrt{Q_1Q_5}}{R}n,\quad \tilde\gamma_2=\frac{\sqrt{Q_1Q_5}}{R}\left(n+\frac{1}{k}\right),\quad Q_p=\frac{Q_1Q_5}{R^2}n\left(n+\frac{1}{k}\right).
\tag{16}
$$

It was shown in [9,10] that this geometry has no horizon or closed timelike curves, and the only possible singularities are orbifold singularities which can be understood as arising as a limiting case of a family of smooth manifolds. (Similar conical defects arise in the 2-charge D1–D5 family of geometries [11,12].)

5. Fractionation

We have argued for a change in the conventional picture of the black hole not just within Planck distance of the singularity but in the entire interior of the classical sized horizon. How can quantum gravity effects act across such large distances? The key notion appears to be 'fractionation', which we now discuss.

Consider a string wrapped on a circle of length L. The lowest excitation of this string (with no net momentum charge) is given by one left mover and one right mover, with total energy $\Delta E=2\pi/L+2\pi/L=4\pi/L$. Now consider the bound state of N such strings, which is just one 'multiwound string' of length $L_T=NL$. The excitations now come in units of $\Delta E=2\pi/L_T$, with the constraint that the total momentum be of the form $P=2\pi m/L$. The lowest excitation is now $\Delta E=2\pi/L_T+2\pi/L_T=4\pi/(NL)$. We see that if we make a bound state of a large number N of strings then excitations are very light, due to a 'fractionation' by a factor N [7].

This is of course a very simple effect, not peculiar to string theory. It becomes important though when we note that dualities in string theory map momentum modes into branes, and thus we get very light branes of fractional tension. Such branes can stretch far, and can give effects at macroscopic length scales, as we now argue.

Let us recall the 'correspondence principle' proposed by Horowitz and Polchinski [13]. Start with an excited string at weak coupling, and increase g till it just makes a black hole. At this critical value g_c it is found that the entropy of the string state matches the Bekenstein entropy of the hole, suggesting a continuous transition between string states and black holes.

However, a closer look reveals that while the entropy may appear continuous, the dynamical degrees of freedom change radically at the string/black hole transition. To see this note that we can apply the correspondence principle also to the case where the string has winding and momentum charges, so we might as well adapt it to the case we have been discussing: $M_{9,1}$ compactified on $S^1\times T^4$ and the NS1 string wrapped with winding n_1 and momentum n_p along S^1.

One may think that the lowest excitation for this string is given by a fractional P–$\overline{\mathrm{P}}$ pair (momentum modes running up and down the string) with threshold

$$
\Delta E=\frac{4\pi}{n_1L}.
\tag{17}
$$

However, recall that in the low energy model of the 3-charge hole we start with D1 and D5 branes, and the excitation were given by fractional P–$\overline{\mathrm{P}}$ pairs with threshold $\Delta E=4\pi/(n_1n_5L)$ [14,15]. The charges D1–D5–P permute under dualities, so if we started with D1–P then the excitations should come as fractional D5–$\overline{\text{D5}}$ pairs, or by an S duality, if we start with NS1–P then the excitations should come as fractional NS5–$\overline{\text{NS5}}$ pairs, with threshold

$$\Delta E = \frac{2}{n_1 n_w} \frac{VL}{2\pi g^2 \alpha'^3}. \tag{18}$$

Is the correct threshold (17) or (18)? For very small g (18) is very large, and it is entropically unfavorable to excite the heavy NS5 brane pairs. But for somewhat larger g the 'double fractionation' in (18) will win over the 'single fractionation' in (17). It turns out that the crossover to (18) happens when $g \sim g_c$, the value at the correspondence point! [16]. Thus we see that when studying black holes we will encounter fractional brane excitations, something that we might not have intuitively considered when considering quantum effects.[2]

Let us apply this lesson to estimate the 'size' of the D1–D5–P extremal bound state. Let us bring a test quantum to a distance d from the bound state. Do we expect that fractional brane–antibrane pairs will reach out to distance d and absorb the quantum? We have taken an extremal bound state, so the only energy we have to make pairs is from the quantum itself, which is at least $\sim 1/d$. This is very small for macroscopic d, but we expect fractionation to help us. We do not really know how to picture branes stretching out to absorb the quantum, so what we do instead is to put the bound state on a transverse circle of length d and ask if brane–antibrane pairs can wrap around this circle using the energy $1/d$. With this extra circle $\widetilde{S}^1$ compactified (in addition to $S^1 \times T^4$) we have the compactification needed to make a black hole in $3+1$ noncompact dimensions, and we know that this time there are four kinds of charges involved: D1–D5–P as before and in addition KK monopoles which have $\widetilde{S}^1$ as the nontrivially fibred circle [17]. If we start with the charges D1–D5–KK then the excitations are P–$\overline{\text{P}}$ pairs with energy threshold $4\pi/(n_1 n_5 n_K L)$. Dualities permute the four charges, so if we have D1–D5–P charges (as we do in our problem) then the excitations are given by fractional KK–$\overline{\text{KK}}$ pairs with threshold

$$\Delta E = \frac{2}{n_1 n_5 n_k} \frac{RVd^2}{g^2 (2\pi)^2 \alpha'^4}. \tag{19}$$

Let us also require that converting the available energy $1/d$ into these pairs increases the entropy by at least $\Delta S = 1$. Thus there is e times more phase space if we make these pairs as compared to the case where we do not. With all this we have branes that stretch far enough to overlap with the test quantum (which makes a transition from test quantum to brane–antibrane pairs *possible*), and also phase space volumes which make the transition *likely*. A short computation shows that the critical d where $\Delta S = 1$ is

$$d \sim \left(\frac{g^2 \alpha'^4}{VR}\right)^{1/3} (n_1 n_5 n_p)^{1/6} \sim R_S \tag{20}$$

where R_S is the horizon radius of the D1–D5–P hole! Note that the entropy is a power of $N = n_1 n_5 n_p$, and so the horizon radius is a power of N too, but fractionation involves powers of this same N, and generates correspondingly long objects which can invalidate naive expectations about the range of quantum effects [16].

6. Non-BPS holes

It was shown in [18] that if we take black holes in $4+1$ dimensions with any value of mass M and charges n_1, n_5, n_p then the Bekenstein entropy of the hole is *exactly* reproduced if we take a collection of branes and antibranes that reproduce the total mass and charges and optimises the following direct extension of the extremal expression for the entropy

$$S = 2\pi \left(\sqrt{n_1} + \sqrt{\bar{n}_1}\right)\left(\sqrt{n_5} + \sqrt{\bar{n}_5}\right)\left(\sqrt{n_p} + \sqrt{\bar{n}_p}\right). \tag{21}$$

But if we can understand neutral holes in terms of branes and antibranes this way, then our above crude estimates for the 'size' of the brane bound state extend to neutral holes as well, and we conclude that the inside of a Schwarzschild hole is filled with fractional brane–antibrane pairs.

A natural question to ask is: what do we see if we fall into a black hole? We have looked at states of the black hole, and not at dynamics, so we cannot directly address this question with what we know. But note that there are two very different time scales associated to the hole: the crossing time for light across the diameter of the hole, and the much longer Hawking evaporation time. A collapsing shell may well seem to have made a conventional hole when it passes through its horizon and shrinks towards $r = 0$, but this is a very special initial condition with low entropy, and interaction over a longer time may cause

[2] The fact that in the black hole phase we have the excitations (18) rather than (17) also resolves some contradictions encountered in the study of greybody factors; investigation of these contradictions led to the arguments quoted above [16].

it to 'swell up' into the generic state which is a horizon sized 'fuzzball'. Inside the fuzzball all the modes of string theory are excited, not just the graviton, so classical spacetime ceases to hold and we cannot say that classical light cones will forbid such a 'swelling up'.

If we have such large distance quantum effects in black holes, then perhaps we can have similar effects in any situation where we have a large number of quanta close together, as in the early Universe. If we get macroscopic distance correlations from such effects then we might be able to bypass the horizon problem without requiring inflation. Also the cosmological constant Λ receives contributions from the zero modes of all quantum fields, but if high energy modes are dominated by large sized fractional brane–antibranes pairs then we might need to revisit our understanding of the regularizations needed to understand Λ.

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