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Wormholes in AdS

Liat Maoz

ITFA, University of Amsterdam, Valckenierstraat 65, Amsterdam 1018XE, The Netherlands

Presented by Guy Laval

Abstract

A few different examples of Euclidean wormholes with AdS asymptotics are constructed. These are geometries which are completely regular, and are solutions of ten- or eleven-dimensional supergravity. We point out that such geometries are puzzling from the AdS/CFT point of view, and try to speculate on possible resolutions of this puzzle. A better understanding of the physics of these geometries could lead to interesting insights into the nature of quantum gravity and to some new interpretations of closed cosmologies. ***To cite this article: L. Maoz, C. R. Physique 6 (2005).***

Résumé

Trous de vers dans AdS. On construit quelques exemples de trous de vers euclidiens avec un espace AdS asymptotique. Ces géométries sont complétement régulières, et sont des solutions des supergravité en dimensions dix et onze. Nous expliquons en quoi ces géométries sont intriquantes du point de vue de la correspondance AdS/CFT, et nous spéculons sur de possibles résolutions de ces questions. Une meilleure compréhension de ces géométries peut amener des progrès intéressants sur la nature de la gravité quantique et à de nouvelles interprétations des cosmologies fermées. ***Pour citer cet article : L. Maoz, C. R. Physique 6 (2005).***

Keywords: Wormhole; AdS/CFT; Holography

Mots-clés : Trous de vers ; AdS/CFT ; Holographie

1. Introduction

This article comes from a talk given at 'Strings '04', based on hep-th/0401024 with J. Maldacena [1].

Wormholes are configurations which have fascinated physicists for a long time. The main interest in wormholes, in the context of quantum gravity, started in the late 1980s, when it was thought that wormholes might cause violations of unitarity [2] and that they are likely to nucleate on large scales, leading to a 'wormhole catastrophe' [3]. Later it was argued that such catastrophes would actually not occur [4–8], and the interest in wormholes in that respect diminished. In the more recent context of the AdS/CFT correspondence, wormholes raise a new and interesting puzzle.

E-mail address: lmaoz@science.uva.nl (L. Maoz).

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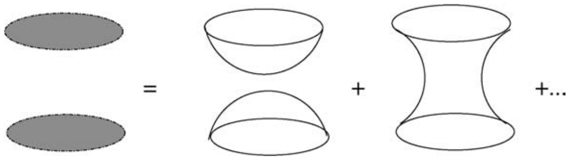

Fig. 1. The CFT expectation value can be approximated as a sum over geometries. The sum includes both connected and disconnected bulk configurations.

The AdS/CFT correspondence [9] basically states that the sum over all geometries with fixed boundary conditions is the same as a (conformal) field theory living on the boundary. More accurately, it states that:

$$\left\langle \exp\left[-\int \varphi_j^{(0)} O_j(x)\mathrm{d}^d x\right]\right\rangle_{\mathrm{CFT}} = Z_{\mathrm{string}}[\varphi_j] \approx Z_{\mathrm{SUGRA}}[\varphi_j] \approx \sum_{\mathrm{cl.sols}} \exp\bigl(-S[\varphi_j]\bigr) \tag{1}$$

where φ_j are some bulk fields with boundary values $\varphi_j^{(0)}$, and on the right-hand side, we first used a small α' approximation and then a saddle point approximation, replacing the path integral by a sum over classical solutions with the given boundary values. One example where the CFT path integral was actually shown to equal a sum over such classical solutions (with appropriate weights), is the case of a CFT with target space $Hilb^k(K3)$ which is dual to IIB string theory on $AdS_3 \times S^3 \times K3$ [10].

A natural question that arises is how such a correspondence could work if the boundary has a few disconnected components.[1] In such a case, the correspondence principle instructs us to sum over all fillings of the boundary. These can be of two kinds—either such where the bulk itself has a few disconnected components, each ending on a different boundary component, or such that the bulk is connected, i.e., such that the spacetime is a wormhole.

Now, on the one hand, the left-hand side of (1) involves a CFT living on the sum of disjoint boundary components and there is no evident way in which they should be coupled. One therefore would expect the CFTs to be completely independent and that correlations between the different boundaries should factorize. On the other hand, the sum on the right-hand side of (1) includes also the wormhole geometries, where the different asymptotic regions are connected (see Fig. 1). Thus at least naively, one should expect to find some correlations between the different boundaries. This puzzle becomes even clearer when the wormhole geometry is not just a saddle point of the path integral, but is actually a local or even global minimum, and therefore is the dominant contribution in the sum over geometries.

In order to resolve this puzzle and try to understand how the AdS/CFT correspondence could work in these cases, one should first obtain some string theory examples of wormhole geometries, which are regular, are asymptotically AdS and have a well defined corresponding field theory. In previous examples of wormhole-like solutions either the two asymptotic regions were not as well understood, or they were not solutions of 10- or 11-dimensional supergravity [6,7,12–17]. In this article we shall describe some new wormhole geometries of the type we are looking for. In Section 2, we first discuss briefly the conditions for the existence of wormhole geometries. Then in Section 3 we will describe some general features of all wormhole solutions we have built. Sections 4–6 describe in more detail three different types of solutions. For each we will see show how it can be built as a reduction of 10- or 11-dimensional supergravity, and try to find perturbative and nonperturbative instabilities, in order to determine which kind of extremum that geometry is. Finally in Section 7 we end by returning to the AdS/CFT related puzzle, discuss the open issues, and try to give some speculation as to how the puzzle might be resolved.

2. The conditions for existence of wormholes

The first question one would like to answer is in which setups wormholes with AdS asymptotics could arise as sensible geometries. In the Lorentzian case, for asymptotically AdS spacetimes, a topological censorship theorem was proven [18], which basically states that under certain physically reasonable conditions, the presence of multiple boundaries forces the bulk to be separated by horizons, in such a way that the different boundary components are not causally connected through the bulk. More precisely, the theorem states that if M' is a globally hyperbolic spacetime with a timelike boundary I satisfying the average null energy condition,[2] and if I_0 is a connected component of I such that either (i) I_0 admits a compact spacelike cut or

[1] See also [11] for a brief survey of some aspects of AdS/CFT related to the role of topology of the boundary in the correspondence.

[2] The average null energy condition states that for each point p in M near I, and any future complete null geodesic $s \to \eta(s)$ in M starting at p with tangent X, $\int_0^\infty R_{ab}X^a X^b\,\mathrm{d}s \geqslant 0$.

(ii) M' satisfies the generic condition,[3] then I_0 cannot communicate with any other component of I, i.e., $J^+(I_0) \cap (I \setminus I_0) = \emptyset$. This theorem implies that the different holographic boundary theories are uncorrelated, and do not interact dynamically. An example where such a situation arises is the extremal AdS Schwarzschild black holes, which have two boundaries separated by horizons. Indeed in that case the dual CFTs are decoupled and the only correlations between them are through an initial entangled state [19].

In the Euclidean case, looking at Einstein manifolds M of negative curvature, the boundary can either have negative or positive curvature. In the first case, it was shown by Seiberg and Witten [20] that the holographic theory living on the negative curvature boundary would be unstable for any boundary dimension $n \geqslant 3$. We will expand on this instability in the next section. In the second case, when one of the boundary components has $R > 0$, a theorem by Witten and Yau basically shows that the boundary must be connected [21]. This theorem was later generalized by Cai and Galloway [22] to cases where the boundary has zero scalar curvature. The precise statement is: Let M^{n+1} be a complete Riemannian manifold which admits a conformal compactification, with conformal boundary N^n, and with the Ricci tensor of M satisfying $Ric \geqslant -ng$ such that $Ric \to -ng$ sufficiently fast on the approach to conformal infinity.[4] If N has a component of nonnegative curvature, then the following holds:

(i) N is connected;
(ii) If M is orientable, then $H_n(\overline{M}, Z) = 0$;
(iii) The map $i_* : \Pi_1(N) \to \Pi_1(\overline{M})$ ($i =$ inclusion) is onto.

This seems to exclude the existence of stable Euclidean wormhole-like solutions. However, as we shall describe in the following sections, one can actually go around these problems. One way to do this is to consider spaces with negative curvature boundaries, but where the field theory is two-dimensional, defined on an arbitrary Riemann surface, and does not suffer from the regular instabilities. Another way is to consider spaces with positive curvature boundaries, but which are not pure Einstein manifolds, but rather solutions of supergravity, where extra fields, apart of the metric, are turned on. These geometries do not obey the constraint on the Ricci tensor, appearing in the Witten–Yau and Cai–Galloway theorems.[5]

3. General features

Before we go into the details of the different solutions, let us describe some common general features of all wormholes we consider. All configurations we construct are completely regular Euclidean geometries, which are asymptotically AdS, and are solutions of 10d or 11d supergravity. The metric of the solutions is of the form

$$\mathrm{d}s_{n+1}^2 = \mathrm{d}\rho^2 + w(\rho)^2\,\mathrm{d}s_{\Sigma_n}^2 \tag{2}$$

where Σ_n—a constant ρ slice—is a compact n-dimensional surface, and $w(\rho) \sim \mathrm{e}^{|\rho|}$ as $\rho \to \pm\infty$. The two disconnected boundaries are at $\rho = \pm\infty$, and the asymptotic behavior of $w(\rho)$ ensures that the geometries are asymptotically AdS. The solutions we construct also have the property that they are reflection symmetric under $\rho \to -\rho$. The Wick rotation of these wormholes geometries into Lorentzian signature, obtained by taking $\rho = \mathrm{i}t$, describes closed universe cosmologies with big bang and big crunch singularities, and compact spatial surfaces Σ_n (see Fig. 2). This gives another important motivation to finding the holographic description of the wormholes, namely finding a field theory interpretation of such closed cosmologies.

Finally, another feature of all the configurations we will describe is that they are not supersymmetric. There is a simple argument showing why one should not expect supersymmetric wormhole configurations of this form. Basically the reflection symmetry of the Euclidean solutions (2) implies that the Wick rotated Lorentzian geometry is time-reflection symmetric. However, supersymmetric Lorentzian solutions possess a timelike or null killing vector [24], and if their spatial sections are compact, this implies they cannot have time reflection symmetry.

[3] The generic condition is satisfied iff every timelike or null geodesic with tangent vector X contains a point at which $X^a X^b X_{[c} R_{d]ab[e} X_{f]}$ is nonzero.

[4] I.e., $r^{-2}(Ric + ng) \to 0$ as $r \to 0$ where the bulk metric is expanded in a neighborhood of the boundary as $g = \frac{1}{r^2}(\mathrm{d}r^2 + r_r)$, and the conformal boundary is at $r \to 0$.

[5] More details about the physical interpretation of this feature can be found in [23].

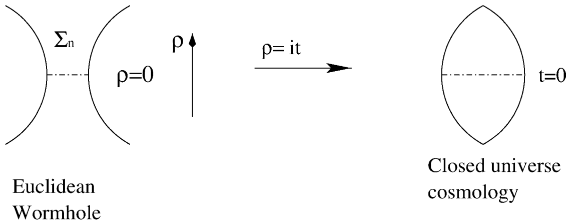

Fig. 2. On the left is a sketch of the Euclidean geometry with two boundaries and reflection symmetry around $\rho = 0$. On the right is the Wick rotated Lorentzian geometry. This is a big bang-big crunch cosmology.

4. Quotients of hyperbolic space

The first family of wormholes we consider, which are probably the simplest wormhole examples is just quotients of hyperbolic space, i.e., of Euclidean AdS. Hyperbolic space is usually written in the following form:

$$\mathrm{d}s^2_{H_{n+1}} = \mathrm{d}y^2 + \sinh^2 y\, \mathrm{d}s^2_{\Omega_n} \tag{3}$$

where y is a radial coordinate $y \in [0, \infty)$ and equal y slices are n-spheres. In particular, the boundary of the space is at $y = \infty$ and is an S^n. However, by a change of coordinates, one can recast this space in the following form:

$$\mathrm{d}s^2_{H_{n+1}} = \mathrm{d}\rho^2 + \cosh^2 \rho\, \mathrm{d}s^2_{H_n} \tag{4}$$

where $\rho \in (-\infty, \infty)$ and equal ρ slices are again hyperbolic spaces. Naively it might seem that this space has two boundary components, at $\rho = \pm\infty$. However, this is, of course, not true. Retracing the coordinate transformation between the two slicings, one can map the sphere at $y = \infty$ back and see that the entire northern hemisphere maps to the $\rho = \infty$ slice, and the southern hemisphere to the $\rho = -\infty$ slice (see Fig. 3). So the boundary is still only one connected component. Equivalently, one can regard these as two boundary disks with 'transparent' boundary conditions.

In order to create a space with two disconnected boundaries out of this, one can perform a quotient of the hyperbolic slices H_n by a discrete subgroup Γ of the hyperbolic symmetry group $SO(1, d)$. We pick Γ such that the resulting slice $\Sigma_n = H_n/\Gamma$ is a compact, smooth and finite volume surface.

To be concrete, let us look at the specific case of $n = 2$, i.e., at quotients of H_3. The relevant group of isometries is $SL(2, C)$, and in order to create a multiple boundary space, one should choose a discrete subgroup $\Gamma \subset SL(2, C)$ to quotient H_3 by. Different choices of Γ lead to different boundary structures. For example, performing a quotient by a fuchsian discrete group[6] results in a space with two boundaries, each a Riemann surface, and both having the same moduli t^α (see Fig. 4).

One could also perform a quotient of H_3 by a quasi-fuchsian subgroup. This would result again in a space with two boundaries, which are Riemann surfaces, but this time the two Riemann surfaces can have different moduli (see Fig. 5). In fact a theorem by Bers [25] ('the Bers simultaneous uniformization theorem') states that there is a 1–1 correspondence be-

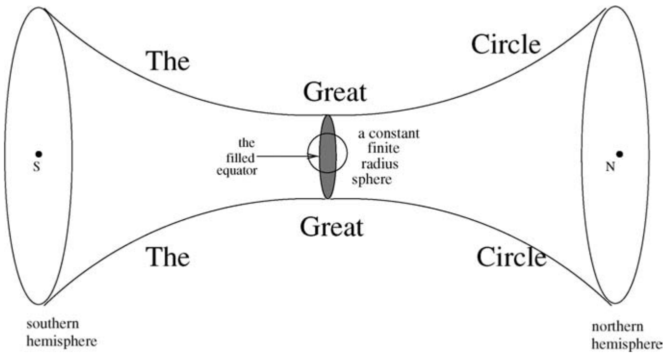

Fig. 3. A sketch of the hyperbolic slicing of hyperbolic space. ρ increases from $-\infty$ on the left to $+\infty$ on the right. The mapping of the spherical slicing into this one is indicated.

[6] This is a group which is actually a subgroup of $SL(2, R)$. Any Riemann surface can be represented as a quotient of H_2 by an appropriate such group.

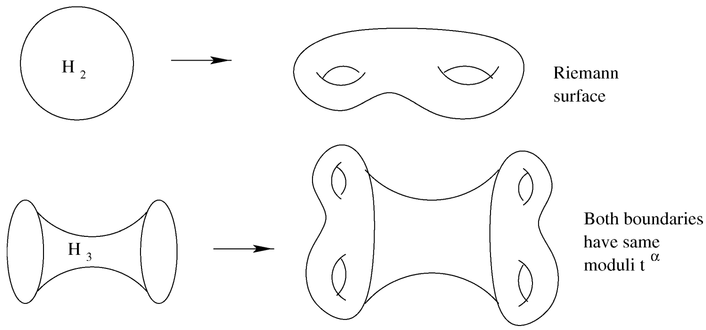

Fig. 4. The quotient of the disk H_2 by a fuchsian subgroup is a Riemann surface. The quotient of H_3 by a fuchsian subgroup is a space with two boundaries, both Riemann surfaces with the same moduli.

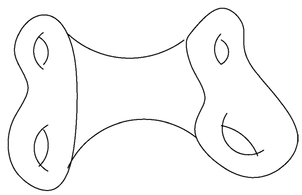

Fig. 5. Performing a quotient of H_3 by a quasi-fuchsian subgroup results in a space with two boundaries which are Riemann surfaces, of different moduli.

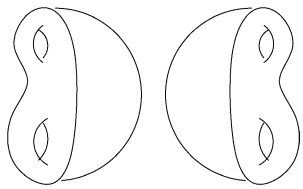

Fig. 6. Two disconnected three-dimensional manifolds, each ending on a different boundary. Each of the manifolds results from the quotient of H_3 by a Schottky group.

tween the choice of such groups and the choice of two points in Teichmuler space, i.e., that $QF(S_{g,n})$ is homeomorphic to $Teich(S_{g,n}) \times \overline{Teich(S_{g,n})}$.

Additionally, one could quotient $SL(2,C)$ by a Schottky subgroup. This quotient results in a space with only one boundary, which is a Riemann surface (see Fig. 6).[7] Taking two such spaces, we can recover the same boundary structure as in the previous cases, but this time the bulk itself is also disconnected.

As hyperbolic spaces come up very frequently as solutions of 10d and 11d supergravity, by performing such quotients, one can easily create wormhole geometries in string theory.

The next question that naturally arises is whether such solutions are stable and thus consist of local or even global minima of the path integral over geometries, so as to dominate it.

Looking first for perturbative instabilities, we check for negative modes of a scalar field in the bulk. Consider the metric:

$$ds_{n+1}^2 = d\rho^2 + \cosh^2\rho \, ds_{\Sigma_n}^2 \tag{5}$$

where $\Sigma_n = H_n/\Gamma$ is a constant negative curvature compact manifold. Solving the scalar field equation of motion in this background $[-\nabla^2 + m^2]\phi = \lambda\phi$ for regular normalizable fields, gives the following modes:

$$\lambda_k = \Delta(\Delta - n) + k(n-k) \tag{6}$$

where $k \in Z$, $0 < k \leqslant n/2$ and $\Delta = n/2 + \sqrt{(n/2)^2 + m^2}$. Thus negative modes appear when $\Delta < n$, i.e., when the corresponding operator in the CFT is relevant.

[7] See [26] for further discussion of this.

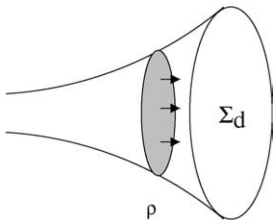

Fig. 7. A codimension 1 BPS brane in AdS, sitting at some finite ρ coordinate, would be unstable towards moving to the boundary.

Let us look at a few examples. For $n = 2$, we consider type IIB string theory on $AdS_3 \times S^3 \times K3$. In this case there are no negative modes, but there is a zero mode for an operator of $\Delta = 1$. This operator transforms as $(1/2, 1/2)$ under the $SU(2)_L \times SU(2)_R$ parameterizing the S^3 rotations. It can be projected out of the theory, if one performs a quotient of it by a $Z_N \subset U(1)_L \subset SU(2)_L$. This quotient has no fixed points, and results on the CFT side in a $(4, 0)$ supersymmetric theory, which was studied in [27]. Another example is when $n = 4$. We consider type IIB string theory on $AdS_5 \times S^5$ and see that in this case one obtains negative modes for $\Delta = 2$. It can be shown that differently from the previous example, here there is no subgroup $\Gamma \subset SO(6)$, such that $AdS_5 \times (S^5/\Gamma)$ has no fixed points and projects out all the $\Delta = 2$ operators (which are in the 20 of $SO(6)$).

Let us now turn to examine possible non-perturbative instabilities of such quotients. These are the ones mentioned in Section 2, arising from the fact that the boundary has negative curvature.

As was shown by Seiberg and Witten [20], if one looks at a codimension 1 BPS brane at a constant large ρ (see Fig. 7), its action is roughly given by:

$$S \approx T \cdot (area) - n \cdot q \cdot (volume) \approx -\frac{2n}{2^n(n-2)} e^{(n-2)\rho} + \cdots. \tag{7}$$

As the leading nonvanishing contribution is negative, and becomes more negative as ρ increases, one finds an instability towards the nucleation of a brane–antibrane pair in the bulk and moving one of them towards the boundary at $\rho \to \pm\infty$. This instability is non-perturbative as one needs to overcome a barrier $\sim 1/g_s$ in order to create the branes. On the field theory side, this instability translates into the existence of a scalar field with negative mass squared, coming from its conformal coupling to the negative curvature surface: $m^2_{\mathrm{conf}} = \frac{n-2}{4(n-1)} R < 0$.

Such instabilities could in principle be removed in two different ways. One can try and add some mass terms to the field theory Lagrangian, which would basically overwhelm the negative conformal mass. This would be something in the spirit of the deformations described in [28,29], only while they were discussing a field theory on flat space, here one needs to consider negatively curved spaces.[8] Another way to try and eliminate the instabilities is to look at the H_3 case again. If one goes to a generic point in the moduli space of the CFT and turns on some RR fields on the $K3$, the branes are not BPS anymore, and their tension is always greater than their charge. We denote $\varepsilon \equiv T/q - 1$. Repeating the calculation of the brane action as in (7), one finds that in this case the action behaves as:

$$S \approx \frac{q}{4}[\varepsilon e^{2\rho} - 4\rho + 2]. \tag{8}$$

Here, for large ρ, the leading term, which is proportional to the non-BPSness parameter ε, is positive and grows with ρ. Therefore for large ρ there is no instability, and the asymptotic boundary conditions are stable. However, for large Q_1, Q_5 there exist branes with $\varepsilon \ll 1$, for which it is still favorable to nucleate a pair of brane–antibrane and move one of them to some finite position ρ_0. We conclude that there still is some nonperturbative instability, which we expect to lead to a new stable configuration with the same asymptotics.

Finally, let us try and compute the correlation function of boundary operators using gravity. Before performing the quotient, one can simply compute the bulk to boundary propagator in hyperbolic coordinates, and find:

$$G(r, \rho; r_0) \sim \frac{1}{[\cosh\rho]^{\Delta}[\cosh s - \tanh\rho]^{\Delta}} \tag{9}$$

where (r, ρ) are the coordinates of the bulk point, and r_0 labels a boundary point. s is the distance between the points r and r_0 with the boundary hyperbolic metric.

[8] In fact, in a recent paper by Buchel [30] a similar thing was attempted, but after adding the mass terms in that case, it was found that the geometries developed naked singularities.

Taking $\rho \to \pm\infty$ and renormalizing by $e^{\Delta\rho}$ should roughly give the boundary to boundary propagators.[9] For operators on the same boundary we find:

$$\langle O(r,\theta^i)_1 O(r',\theta'^i)_1\rangle \sim \frac{1}{[\sinh s/2]^{2\Delta}} \tag{10}$$

and for operators on different boundaries:

$$\langle O(r,\theta^i)_1 O(r',\theta'^i)_2\rangle \sim \frac{1}{[\cosh s/2]^{2\Delta}}. \tag{11}$$

Now we need to implement the quotient H_n/Γ, and this can be done on the gravity side by summing over all images. Performing this sum, gives a result which is generically nonzero and coordinate dependent. Thus it seems that from the gravity point of view, there are correlations between the two boundaries.[10]

5. Meron wormholes with AdS_4 asymptotics

Let us now describe a different kind of wormhole having two boundaries of positive curvature. This is a 4-dimensional space with boundaries which are 3-spheres, and asymptotics at both boundaries of AdS_4. This wormhole solution, which was also described in [17,32,33], can actually be embedded in 11-dimensional supergravity. Starting with 11-dimensional supergravity, it is possible to make the reduction to the following 4-dimensional action, containing no scalars or Chern–Simons terms

$$S \sim N^{1/2} \int d^4x \sqrt{g}\left[-(R+6) + F^a_{\mu\nu} F^{a\mu\nu}\right] \tag{12}$$

where $F^a_{\mu\nu}$ is an $SU(2)$ gauge field. This reduction can be done in two different ways. One way is to look at the $SO(8)$ gauged supergravity obtained by the S^7 compactification of 11-dimensional supergravity [34]. Then one can take the 8_c spinor representation of $SO(8)$, map it under triality to the $SO(8)_v$ and truncate keeping only fields which are invariant under the $SO(5)$ in the subgroup $SO(3) \times SO(5) \subset SO(8)$. This projects out the scalars of the $SO(8)$ gauged supergravity. Another way is to take the $SO(4)$ 4-dimensional truncation described in [35] and truncate it to the diagonal $SU(2)$ in the decomposition $SO(4) = SU(2)_L \times SU(2)_R$. One can show that the 8_v of $SO(8)$ transforms as a pair of 2s of $SU(2)$. In any case, one is left with the action (12) with $A = i\frac{\sigma^a}{2}A^a$ and $F = dA + A^2$.

In this setting we are able to exactly realize the scenario mentioned in the introduction and depicted in Fig. 1: we have a CFT defining some boundary conditions, and we will manage to construct both solutions which are wormhole like, and also solutions which have two disconnected bulk components, having the same boundary conditions.

Having at hand the string theory description in terms of M2 branes, enables one to determine the field theory dual. It is just the field theory on a stack of M2-branes, coupled to a fixed background $SU(2)$ connection. It is not known how to write down the Lagrangian for a system of multiple M2-branes. However, it can be shown that this theory breaks supersymmetry.

The 4-dimensional wormhole solution, with the given asymptotics is the following:

$$ds_4^2 = d\rho^2 + \frac{1}{2}(\sqrt{5}\cosh 2\rho - 1)\, ds^2_{S^3}; \quad A^a = \frac{1}{2}w^a \tag{13}$$

where w^a are $SU(2)$ left-invariant spin-connections, and the S^3 metric is given by $ds^2_{S^3} = \frac{1}{4}w^a w^a$.

The metric is written in an S^3 slicing, and the gauge connection is in a meron configuration, i.e., it is proportional to the $SU(2)$ left-invariant spin connections on the S^3.

This wormhole solution is definitely an extremum of the path integral in (1). In order to check if it is actually a minimum dominating the path-integral, or merely a saddle point, one needs to check for perturbative instabilities. A natural deformation of (13) is by a mode of the form:

$$A^a = \left[\frac{1}{2} + \varepsilon(\rho)\right] w^a \quad \text{with} \lim_{\rho\to\pm\infty} \varepsilon(\rho) = 0. \tag{14}$$

[9] The correct procedure is a bit more involved, but this gives the correct result up to a Δ-dependent factor [31].

[10] It might be that one needs to average these correlators over the isometry group of the hyperbolic space, as from the boundary theory point of view the result should be invariant under these. However, from the gravity point of view, it is not clear why such an averaging should be done. We thank E. Witten for his comment in this matter.

Indeed, when one inserts this back into the Euclidean action, and expands it to second order in the perturbation, one sees that this is a negative mode. This means that the wormhole solution is perturbatively unstable, and might decay to a different solution with the same boundary conditions.

One candidate for such a solution is the one we mentioned earlier, made up of two disconnected bulk components, each ending on a different boundary component. Each one of these components has just the AdS_4 metric, but contains a nonvanishing gauge connection, which is (anti)self dual and asymptotes the appropriate constant valued gauge connection on the boundary. The self dual solution is given by:

$$\begin{aligned} &\mathrm{d}s^2 = \mathrm{d}y^2 + \sinh^2 y\, \mathrm{d}s^2_{S^3}, \\ &A^a = f(y) w^a; \quad f^{-1}(y) = 1 + \frac{1}{\tanh^2 y/2}. \end{aligned} \tag{15}$$

Note that $\lim_{y\to\infty} f^{-1}(y) = (1/2)^{-1}$, so that on the boundary the gauge connection does not vanish, and in all, this is 'half an instanton' in AdS_4. It can be verified that the instanton breaks the supersymmetries of AdS_4. Evaluating the action of this one boundary solution, and comparing it to that of the wormhole solution, we get:

$$2S_{1\mathrm{bdy}} < S_{\mathrm{wormhole}}.$$

This confirms that the solution where the bulk is disconnected is more stable than the wormhole one, and that the instability we found for the wormhole solution might deform it into such a disconnected configuration.

6. Instanton–antiinstanton wormholes with AdS_5 asymptotics

In this section we describe another example of wormholes, rather similar to the previous one. Here, too, the boundaries are of positive curvature, and are, in fact, again spheres. However, this example seems to be more stable than the one described in the last section. It is a 5-dimensional wormhole, where only the metric and a gauge field are excited, and is a solution of a consistent 5-dimensional reduction of IIB string theory on an S^5. This reduction was described in [36] and keeps an $SO(6)$ gauge field and some scalars. The 5-dimensional reduced action is:

$$\begin{aligned} L_5 = {} & R * 1 - \frac{1}{4} T^{-1}_{IJ} * DT_{JK} \wedge T^{-1}_{KL} DT_{LI} - \frac{1}{4} T^{-1}_{IK} T^{-1}_{JL} * F^{IJ} \wedge F^{KL} - V * 1 \\ & - \frac{1}{48} \varepsilon_{I_1\cdots I_6} \Big(F^{I_1I_2} F^{I_3I_4} A^{I_5I_6} - g F^{I_1I_2} A^{I_3I_4} A^{I_5J} A^{JI_6} + \frac{2}{5} g^2 A^{I_1I_2} A^{I_3J} A^{JI_4} A^{I_5K} A^{KI_6} \Big) \end{aligned} \tag{16}$$

where A^{IJ} are $SO(6)$ gauge fields, T^{IJ} a 6×6 symmetric unimodular matrix of scalars, and the potential V is given by $V = \frac{1}{2} g^2 (2T_{IJ}T_{IJ} - (T_{II})^2)$.

We are interested in finding solutions, similar to the one we had in 4 dimensions, where the gauge field is turned on, making an instanton configuration on the S^4 boundaries. However, due to the Chern–Simons terms and the scalar couplings in (16), turning on only an $SU(2)$ instanton is not a consistent solution. Instead, it is possible to separate the $SO(6)$ gauge group to $SO(3) \times \widetilde{SO(3)} \subset SO(6)$ that rotate the first three coordinates and last three coordinates of R^6. We denote by $L^{a,IJ}$, $I, J = 1,2,3$; $a = 1,2,3$, the generators of $SO(3)$ and by $\tilde{L}^{a,IJ}$, $I, J = 4,5,6$; $a = 1,2,3$, the generators of $\widetilde{SO(3)}$. Then we build a gauge field configuration, consisting of an instanton on S^4 for the $SO(3)$ and an antiinstanton for the $\widetilde{SO(3)}$. The instantons are $SO(5)$ symmetric under rotations of the S^4. These boundary conditions determine the field theory: it is just $N = 4$ SYM with an external gauge field coupled to the $SO(6)$ currents. This field theory seems well defined and stable.

The wormhole solution with this boundary behaviour is given by the following set of fields:

$$\begin{aligned} &\mathrm{d}s_5^2 = \mathrm{d}\rho^2 + \mathrm{e}^{2w(\rho)} \mathrm{d}s^2_{S^4} = \mathrm{d}\rho^2 + \mathrm{e}^{2w(\rho)} \left[\mathrm{d}\theta^2 + \sin^2\theta \, \frac{1}{4} w^a w^a \right]; \quad \mathrm{e}^{2w(\rho)} = \frac{1}{2}(\sqrt{5}\cosh 2\rho - 1), \\ &A^{IJ}_\mu = i A^a_\mu L^{a,IJ} + \mathrm{i} \tilde{A}^a_\mu \tilde{L}^{a,IJ}; \quad A^a = \cos^2\theta \left(\frac{\theta}{2}\right) w^a; \quad \tilde{A}^a = \sin^2\left(\frac{\theta}{2}\right) w^a, \\ &T^{IJ} = \delta^{IJ}. \end{aligned} \tag{17}$$

This configuration actually has zero total $SO(6)$ instanton number, and thus the equations of motion, including the Chern–Simons terms are obeyed.

Having this solution at hand, one would like to check whether it is a stable solution. One natural deformation to consider, is deforming the $SO(6)$ gauge field. As can be checked, the gauge field on S^4 is topologically trivial as an $SO(6)$ gauge field, and therefore can be continuously deformed into a pure gauge configuration. In fact considering the $SO(6)$ gauge theory on the

S^4, such a deformation indeed produces a negative mode. However, requiring also normalizability of the modes at $\rho \to \pm\infty$ removes the negative modes from the full 5-dimensional geometry. Another candidate for negative modes is the fluctuations of the scalar fields. However, an analysis shows that such negative modes too do not exist. In principle one should check all other fields of the 10-dimensional supergravity theory to determine if there are any negative modes leading to instability of this solution. It would also be interesting to find a single boundary solution, with the same boundary behaviour, and check whether it is more or less stable than the wormhole solution.

7. Summary and discussion

In the previous section we have described the construction of a variety of Euclidean geometries, of the wormhole type, i.e., which connect two boundaries. They all have AdS asymptotics on both boundaries, and are all completely regular consistent solutions of string theory. The first family of such geometries is quotients of hyperbolic space. These have boundaries of negative curvature and although generically unstable, we were able to construct the example of $AdS_3/\Gamma \times S^3/Z_N \times K3$ which seems rather stable. The second type of solutions are wormholes with AdS_4 asymptotics, and an $SU(2)$ gauge field in a meron configuration. They have two boundaries of positive curvature. Yet we have found some perturbative instabilities in this case, and another configuration of lower action and same boundary conditions, where the bulk itself is disconnected. The last example is of a wormhole with AdS_5 asymptotics and again with boundaries of positive curvature. In this example the $SO(6)$ gauge fields are in an instanton–antiinstanton configuration, and as far as we have checked, we could not find any instabilities, or any lower action solution with the same asymptotics.

The configurations we described seem to correspond to perfectly well-defined field theories, and therefore provide concrete examples to the puzzle mentioned in the introduction, related to the AdS/CFT correspondence. To recapitulate, on the field theory side, it seems the correlation functions across the two boundaries should factorize, while from the gravity point of view, there seem to be correlations between the boundaries. In the examples where the wormhole geometry seemed to be stable, this puzzle is most evident, as the wormhole dominates the sum over geometries.

What could be the possible resolutions to this puzzle?

One resolution could be that maybe after summing over all geometries, including the wormhole geometry, the gravity correlators would factorize. It is hard to see how such a magical cancellation could happen, but maybe one can get some inspiration from the works of Coleman from the 1980s [8,4]. Coleman analyzed a somewhat different problem, namely, the nonlocal interactions in axionic wormholes in Lorentzian spacetimes. It was believed that these nonlocal interactions, and the existence of disconnected baby universes would lead to violations of unitarity [2]. However, Coleman suggested that contrary to these concerns, in fact the sum over all possible wormholes has a different effect. It leads to the appearance of superselection sectors characterized by some parameters α_i. The physics remain unitary, but the parameters α_i cannot be computed by the observers, and have to be measured experimentally, and regarded as arbitrary constant of nature. Thus loss of unitarity is replaced by loss of predictive power (or in Coleman's words: "*The predictive power of the theory of everything has gone down the wormhole*" [5]). It is possible that something similar to Coleman's factorization would occur in our context as well. Then it would seem that the field theory on the boundary would determine the α parameters and thus also the wavefunction for the associated closed universes.

A different resolution of the puzzle could be that in the end there is some subtle correlation between the two field theories, and the partition function would be of the form: $Z = \sum_i Z_i^1 Z_i^2$ where $1, 2$ indicate the two field theories on the two boundaries, and i runs over some 'sector' of the field theory. Such a situation may arise if the partition functions of the field theories are not well defined. An example for this is a chiral boson in 2 dimensions, for which no modular invariant partition function exists. Only if one considers both left and right moving sectors, there is a modular invariant partition function, and it is of the coupled form above, where $1, 2$ are left and right, and i runs over spin structures on the 2-dimensional surface.[11]

Either way, a resolution of this puzzle, and a better understanding of the physics of wormholes and their holographic description, would be very interesting, and would shed some new light on the nature of quantum gravity.

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[11] It is hard to see why such an effect would arise in the meron and instanton–antiinstanton examples we have described, as they seem well defined in that sense, and are left–right symmetric.

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