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Strings, gravity, and the quest for unification/Cordes, gravitation, et la quête d'unification

No-scale supergravity from higher dimensions

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Abstract

We discuss recent results on the interpretation of flux compactifications on certain Type IIB orientifolds in terms of gauged $\mathcal{N}$-extended supergravities of no-scale type. ***To cite this article: R. D'Auria et al., C. R. Physique 6 (2005).***

Résumé

Supergravité sans échelle des dimensions supplémentaires. Nous rapportons des résultats récents sur l'interprétation de-compactifications avec flux sur certains orientifolds de type IIB, entermes de supergravités étendues $\mathcal{N}$ jaugées de type sans échelle. ***Pour citer cet article : R. D'Auria et al., C. R. Physique 6 (2005).***

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1. Introduction

Superstring/M-theory are considered to be the most promising candidates to describe the fundamental theory of gravity. Upon compactification to four dimensions, the effective low-energy dynamics of both bulk and brane degrees of freedom is encoded in a four-dimensional supergravity. Ordinary compactifications typically yield supergravity models which are far from being realistic, since they describe a plethora of massless scalar fields, in part related to the moduli of the internal manifold, which are not observed in nature and whose v.e.v. define a continuum of degenerate vacua. In order to derive phenomenologically viable models from string/M-theory new dynamics should be introduced, which would be described at the level of the low-energy effective theory by a suitable scalar potential V. The effect of this potential should be to lift the vacua degeneracy making the model more predictive and define at the same time vacua with interesting properties like spontaneous supersymmetry breaking, cosmological constant etc… Remarkable progress in this direction has been made in the last four years by considering compactifications in the presence of non-vanishing p-form fluxes across cycles of the internal manifold [1–11]. The presence of fluxes determines indeed a non-trivial scalar potential in the effective low-energy supergravity, which defines in some cases vacua with vanishing cosmological constant (at tree level), in which spontaneous (partial) supersymmetry breaking may occur and (some of) the moduli of the internal manifold are fixed. In fact theories with vanishing cosmological constant

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are generalized no-scale models, which were studied long ago in the pure supergravity context [12,13]. The presence of fluxes gives also rise in the low-energy supergravity to local symmetries gauged by vector fields.[1] Supergravity models with such gauge symmetries (gauged supergravities) have been extensively studied in the literature [14,15], also in connection to flux compactifications or Scherk–Schwarz dimensional reduction [16–26]. Actually in extended supergravities ($\mathcal{N} \geqslant 2$) the *gauging* procedure, which consists in promoting a global symmetry group of the Lagrangian to local invariance, is the only way of introducing a non-trivial scalar potential without explicitly breaking supersymmetry. The global symmetry group of extended supergravities is the isometry group G of the scalar manifold, whose non-linear action on the scalar fields is associated with an electric/magnetic duality action on the n_v vector field strengths and their duals [27]. This duality transformation is required in four dimensions to be symplectic and thus is defined by the embedding of G inside $\mathrm{Sp}(2n_v, \mathbb{R})$. Gauge symmetries deriving from flux compactifications typically are related to non-semisimple Lie groups $\mathscr{G}$ containing Abelian translational isometries acting on axionic fields which originate from ten dimensional R–R forms $C_{(p)}$ ($p = 0, 2, 4$ for Type IIB) or the NS two form $B_{(2)}$. The embedding of $\mathscr{G}$ inside G is defined at the level of the corresponding Lie algebras by the flux tensors themselves, which play the mathematical role of an *embedding matrix* [15].

No-scale models arising from flux compactifications or Scherk–Schwarz dimensional reduction give rise to a semi-positive definite scalar potential which has an interpretation in terms of an $\mathcal{N}$-extended gauged supergravity in four dimensions. Let us recall the general form of such scalar potential $V(\Phi)$ (Φ denoting collectively the scalar fields) [28]:

$$\delta_B^A V(\Phi) = -3S^{AC}S_{BC} + N^{IA}N_{IB}, \tag{1}$$

where $S_{AB} = S_{BA}$, and N^{IA} appear in the gravitino and spin $1/2$ supersymmetry transformations

$$\delta\psi_{A\mu} = \frac{1}{2}S_{AB}\gamma_\mu \varepsilon^B + \cdots, \tag{2}$$

$$\delta\lambda^I = N^{IA}\varepsilon_A + \cdots, \tag{3}$$

and give rise in the supergravity Lagrangian to the following terms:

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \cdots + S_{AB}\bar{\psi}_\mu^A \sigma^{\mu\nu}\psi_\nu^B + \mathrm{i}N^{IA}\bar{\lambda}_I\gamma^\mu\psi_{\mu A} - V(\Phi). \tag{4}$$

Flat space demands that on the extremes $\partial V/\partial\Phi = 0$ the potential vanishes, so

$$3\sum_C S^{AC}S_{CA} = \sum_I N^{IA}N_{IA}, \quad \forall A. \tag{5}$$

The first term in the potential (1) is the square of the gravitino mass matrix. It is Hermitian, so it can be diagonalized by a unitary transformation. Assume that it is already diagonal, then the eigenvalue in the entry (A_0, A_0) is non zero if and only if $N^{IA_0} \neq 0$ for some I. On the other hand, if the gravitino mass matrix vanishes then N^{IA} must be zero.

For no-scale models, there is a subset of fields $\lambda^{I'}$ for which

$$3\sum_C S^{AC}S_{CA} = \sum_{I'} N^{I'A}N_{I'A}, \quad \forall A, \tag{6}$$

at any point in the scalar manifold $\mathscr{M}_{\mathrm{scal}}$. This implies that the potential is given by

$$V(\Phi) = \sum_{I\neq I'} N^{IA}N_{IA}, \tag{7}$$

and it is manifestly positive definite. Zero vacuum energy on a point of $\mathscr{M}_{\mathrm{scal}}$ implies that $N^{IA} = 0$, $I \neq I'$ at that point. This happens independently of the number of unbroken supersymmetries, which is controlled by $N^{I'A}$.

In the sequel we shall first discuss in some detail the supergravity description of Type IIB superstring on $K3 \times T^2/\mathbb{Z}_2$ orientifold in the presence of fluxes and D3/D7 branes. Eventually we shall comment on some general properties of the vacua in no-scale supergravities originating from flux compactifications and Scherk–Schwarz dimensional reduction, concluding with a comment on the dynamical generation of a cosmological constant.

[1] In four-dimensional supergravities coupled to linear multiplets, fluxes may give rise to more general couplings.

2. Type IIB on $K3 \times T^2/\mathbb{Z}_2$ orientifold with fluxes and D3/D7 branes

Consider Type IIB superstring theory compactified on $K3 \times T^2/\mathbb{Z}_2$ orientifold [29] to four-dimensions [5]. Let x^μ ($\mu = 0, \ldots, 3$) denote the four-dimensional Minkowski coordinates, x^ℓ ($\ell = 4, \ldots, 7$) the $K3$ coordinates and x^p ($p = 8, 9$) the coordinates of T^2. The low-energy effective theory is a $\mathcal{N} = 2$ supergravity [17,23] which describes the gravitational multiplet coupled to 3 vector multiplets and 20 hypermultiplets. The scalar manifold is the product of a special Kähler manifold spanned by the three complex scalars s, t, u in the vector multiplets and a quaternionic Kähler manifold describing the 20 hyperscalars [17]:

$$\begin{aligned}
&\mathscr{M}_{\text{scal}} = \mathscr{M}_{SK} \times \mathscr{M}_{QK},\\
&\mathscr{M}_{SK} = \left(\frac{\text{SU}(1,1)}{\text{U}(1)}\right)_s \times \left(\frac{\text{SU}(1,1)}{\text{U}(1)}\right)_t \times \left(\frac{\text{SU}(1,1)}{\text{U}(1)}\right)_u,\\
&\mathscr{M}_{QK} = \frac{\text{SO}(4,20)}{\text{SO}(4) \times \text{SO}(20)}
\end{aligned} \tag{8}$$

s, t, u being complex scalars spanning each factor of $\mathscr{M}_{SK}$ are defined as follows:

$$\begin{aligned}
&s = C_{(4)} - \mathrm{i}\,\text{Vol}(K_3),\\
&t = \frac{g_{12}}{g_{22}} - \mathrm{i}\frac{\sqrt{\det g}}{g_{22}},\\
&u = C_{(0)} - \mathrm{i}\,\mathrm{e}^{\varphi},
\end{aligned} \tag{9}$$

where $C_{(4)}$ is the axion originating from the components of the ten-dimensional four-form along the directions of $K3$, $\text{Vol}(K_3)$ is the volume of $K3$ in the ten-dimensional Einstein frame, $C_{(0)}$ and φ are the ten-dimensional axion, dilaton and the matrix g denotes the metric on T^2. The vector fields A_μ^Λ in the bulk sector originate from the components $B_{\mu p}^\alpha$ of the ten-dimensional two forms $\{B_{(2)}^\alpha\} \equiv \{B_{(2)}, C_{(2)}\}$ where $\alpha = 1, 2$ is the doublet index of the ten-dimensional Type IIB duality group $\text{SL}(2,\mathbb{R})_u$, and the index $\Lambda = 0, \ldots, 3$ runs over the **4** of $\text{SL}(2,\mathbb{R})_u \times \text{SL}(2,\mathbb{R})_t$.

Let us recall some properties of the $K3$ cohomology. The second order cohomology group $H^{(2)}(K3,\mathbb{Z})$ is isomorphic to the lattice $\Gamma^{3,19}$ in which the following inner product between harmonic two-forms is defined: $(\alpha,\beta) = \int_{K3} \alpha \wedge \beta$. Let us denote by ω_I, $I = 1, \ldots, 22$, a basis of $H^{(2)}(K3,\mathbb{Z})$, and let $m = 1, 2, 3$ and $a = 1, \ldots, 19$ be the indices running over the positive and negative signature directions respectively. The manifold $\mathscr{M}_{QK}$ can be written in the form:

$$\mathscr{M}_{QK} = \left[\frac{\text{SO}(3,19)}{\text{SO}(3) \times \text{SO}(19)} \times \text{O}(1,1)\right] \ltimes \{\mathbf{22}_+\} \tag{10}$$

where $\{\mathbf{22}_+\}$ denote a subspace generated by 22 Abelian isometries Z_I (with positive grading with respect to the $\text{O}(1,1)$ generator). These are parametrized by the axions C^I originating from the components of the four form with two indices along $K3$ and two indices along T^2. The $\text{O}(1,1)$ factor is parametrized by the volume of T^2: $\sqrt{\det(g)} = \mathrm{e}^{\phi}$. Finally the 40 complex structure moduli and the 17 Kähler moduli (except the volume) of $K3$ are described by a 3×19 matrix e_a^m which span the $\text{SO}(3,19)/\text{SO}(3) \times \text{SO}(19)$ submanifold. These scalars are arranged in the 20 hyperscalars as follows: $\{C^m, \phi\}$, $\{C^a, e_a^m\}$.

Let us now add to the microscopic setting a stack of n_3 space-filling D3 branes and one of n_7 space-filling D7 branes wrapped around $K3$. The low-energy brane dynamics is described by a SYM theory on their world volume. We shall consider the SYM theories on the D3/D7 branes to be in the Coulomb phase (namely the branes to be separated from each other), so that the gauge group and the massless bosonic modes on the world volume theories are:

$$\begin{aligned}
&\text{D3:}\quad \text{gauge group} = \text{U}(1)^{n_3};\quad \text{bosonic 0-modes: } A_\mu^r,\ y^r = y^{8,r} + t y^{9,r}\ (r = 1, \ldots, n_3),\\
&\text{D7:}\quad \text{gauge group} = \text{U}(1)^{n_7};\quad \text{bosonic 0-modes: } A_\mu^k,\ x^k = x^{8,k} + t x^{9,k}\ (k = 1, \ldots, n_7),
\end{aligned}$$

where y^r and x^k are complex scalars describing the position of each D3, D7-brane along T^2 respectively. The massless brane degrees of freedom will enter the low-energy theory as $n_3 + n_7$ additional vector multiplets, causing the special Kähler manifold to enlarge to a homogeneous non-symmetric $(3 + n_3 + n_7)$-dimensional space denoted by $L(0, n_3, n_7)$ [30]. The metric of this manifold was computed in terms of the bulk/brane fields, using the solvable Lie algebra parametrization, in [25].

2.1. Geometry of $\mathscr{M}_{SK}$

Let us briefly recall the main formulae of special Kähler geometry. The geometry of the manifold is encoded in the holomorphic section $\Omega = (X^\Lambda, F_\Sigma)$ which, in the *special coordinate* symplectic frame, is expressed in terms of a prepotential $\mathscr{F}(s,t,u,x^k,y^r) = F(X^\Lambda)/(X^0)^2 = \mathscr{F}(X^\Lambda/X^0)$, as follows:

$$\Omega = \big(X^\Lambda, F_\Lambda = \partial F/\partial X^\Lambda\big). \tag{11}$$

In our case $\mathscr{F}$ is given by

$$\mathscr{F}(s,t,u,x^k,y^r) = stu - \frac{1}{2}sx^kx^k - \frac{1}{2}uy^ry^r. \tag{12}$$

The Kähler potential K is given by the symplectic invariant expression:

$$K = -\log\big[\mathrm{i}\big(\overline{X}^\Lambda F_\Lambda - \overline{F}_\Lambda X^\Lambda\big)\big]. \tag{13}$$

In terms of K the metric has the form $g_{i\bar{j}} = \partial_i\partial_{\bar{j}}K$. The matrices $U^{\Lambda\Sigma}$ and $\overline{\mathscr{N}}_{\Lambda\Sigma}$ are respectively given by:

$$\begin{aligned}
&U^{\Lambda\Sigma} = \mathrm{e}^K \mathscr{D}_i X^\Lambda \mathscr{D}_{\bar{j}}\overline{X}^\Sigma g^{i\bar{j}} = -\frac{1}{2}\mathrm{Im}(\mathscr{N})^{-1} - \mathrm{e}^K\overline{X}^\Lambda X^\Sigma,\\
&\overline{\mathscr{N}}_{\Lambda\Sigma} = \hat{h}_{\Lambda|I}\circ\big(\hat{f}^{-1}\big)^I_\Sigma, \quad \text{where } \hat{f}^\Lambda_I = \begin{pmatrix}\mathscr{D}_i X^\Lambda\\ \overline{X}^\Lambda\end{pmatrix};\ \hat{h}_{\Lambda|I} = \begin{pmatrix}\mathscr{D}_{\bar{i}}F_\Lambda\\ \overline{F}_\Lambda\end{pmatrix}.
\end{aligned} \tag{14}$$

For our choice of $\mathscr{F}$, K has the following form:

$$K = -\log\bigg[-8\bigg(\mathrm{Im}(s)\mathrm{Im}(t)\mathrm{Im}(u) - \frac{1}{2}\mathrm{Im}(s)\big(\mathrm{Im}(x)^k\big)^2 - \frac{1}{2}\mathrm{Im}(u)\big(\mathrm{Im}(y)^r\big)^2\bigg)\bigg], \tag{15}$$

with $\mathrm{Im}(s), \mathrm{Im}(t), \mathrm{Im}(u) < 0$ at $x^k = y^r = 0$. The components X^Λ, F_Σ of the symplectic section which correctly describe our problem, are chosen by performing a constant symplectic change of basis from the one in (11) given in terms of the prepotential in Eq. (12). The rotated symplectic sections then become [23]

$$\begin{aligned}
&X^0 = \frac{1}{\sqrt{2}}\bigg(1 - tu + \frac{(x^k)^2}{2}\bigg), \qquad X^1 = -\frac{t+u}{\sqrt{2}},\\
&X^2 = -\frac{1}{\sqrt{2}}\bigg(1 + tu - \frac{(x^k)^2}{2}\bigg), \qquad X^3 = \frac{t-u}{\sqrt{2}},\\
&X^k = x^k, \qquad X^r = y^r,\\
&F_0 = \frac{s(2 - 2tu + (x^k)^2) + u(y^r)^2}{2\sqrt{2}}, \qquad F_1 = \frac{-2s(t+u) + (y^r)^2}{2\sqrt{2}},\\
&F_2 = \frac{s(2 + 2tu - (x^k)^2) - u(y^r)^2}{2\sqrt{2}}, \qquad F_3 = \frac{2s(-t+u) + (y^r)^2}{2\sqrt{2}},\\
&F_i = -sx^k, \qquad F_r = -uy^r.
\end{aligned} \tag{16}$$

Note that, since $\partial X^\Lambda/\partial s = 0$, the new sections do not admit a prepotential, and the no-go theorem on partial supersymmetry breaking [31] does not apply in this case. As in [17], we limit ourselves to gauge shift-symmetries of the quaternionic manifold of the $K3$ moduli-space. Other gaugings which include the gauge group on the branes will be considered elsewhere.

2.2. Fluxes

Let us consider the effect of switching on fluxes of the three-form field strengths across cycles of the internal manifold. The only components of $F^\alpha_{(3)} = dB^\alpha_{(2)}$ which survive the orientifold projection are: $F^\alpha_{(3)} = F^{\alpha I}_p\omega_I \wedge dx^p$. We can describe these flux components in terms of four integer vectors f^I_Λ, $\Lambda = 0,\ldots,3$:

$$F^{\alpha I}_p \equiv F^I_\Lambda = \frac{4\pi^2}{R^3}\alpha' f^I_\Lambda; \quad f^I_\Lambda = \{f^m_\Lambda, h^a_\Lambda\} \in \Gamma^{3,19}, \tag{17}$$

where R is the linear size of the internal manifold and last property follows from the flux quantization condition.

The presence of these fluxes imply local invariance in the low-energy supergravity. A way to see this is to consider the dimensional reduction of the kinetic term for $C_{(4)}$:

$$D = 10 \to D = 4,$$
$$F_{(5)} \wedge^* F_{(5)} \to \big(\partial C^I - f^I_\Lambda A^\Lambda_\mu\big)^2, \tag{18}$$

where the four form field strength is defined as: $F_{(5)} = dC_{(4)} + \frac{1}{2}\varepsilon_{\alpha\beta} B^\alpha_{(2)} \wedge F^\beta_{(3)}$. The Stueckelberg-like kinetic terms for C^I in four dimensions are clearly invariant under the local translations $C^I \to C^I + f^I_\Lambda \xi^\Lambda$, ξ^Λ being four local parameters, provided the bulk vectors are subject to the gauge transformation $A^\Lambda_\mu \to A^\Lambda_\mu + \partial_\mu \xi^\Lambda$. Thus from general arguments we expect that in the presence of three form fluxes, the low-energy supergravity should be invariant under a four-dimensional Abelian gauge group $\mathcal{G}$, subgroup of G whose generators $X_\Lambda = f^I_\Lambda Z_I$ are gauged by the bulk vectors. The $\mathcal{N} = 2$ supergravity originated from the flux compactification is obtained therefore from the ungauged theory through the gauging procedure which consists in promoting the subgroup $\mathcal{G}$ of the isometry group of $\mathscr{M}_{QK}$ to local invariance of the Lagrangian. Supersymmetry then requires the introduction of additional terms (fermion shifts) in the fermion supersymmetry transformation rules, fermion mass terms, and a scalar potential $V(\Phi)$ whose expression is constrained to be a well defined bilinear in the fermion shifts [32]. In the sequel we shall denote by $\mathscr{P}^x_\Lambda$ and k^I_Λ the momentum maps and the Killing vectors of the gauged isometries X_Λ:

$$k^I_\Lambda = f^I_\Lambda; \qquad \mathscr{P}^x_\Lambda = \sqrt{2}\,\mathrm{e}^\phi \big(\big[\big(1 + \mathrm{e}\mathrm{e}^t\big)^{1/2}\big]^x_m f^m_\Lambda + e^x_a h^a_\Lambda\big). \tag{19}$$

In terms of these quantities the scalar potential can be written as follows:

$$\begin{aligned} V = {} & 4\mathrm{e}^{2\phi}\big(f^m_\Lambda f^m_\Sigma + 2e^a_m e^a_n f^m_\Lambda f^n_\Sigma + h^a_\Lambda h^a_\Sigma\big)\bar{L}^\Lambda L^\Sigma \\ & + 2\mathrm{e}^{2\phi}\big(U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma\big)\big(f^m_\Lambda f^m_\Sigma + e^a_m e^a_n f^m_\Lambda f^n_\Sigma + 2\big[\big(1 + \mathbf{e}\mathbf{e}^T\big)^{1/2}\big]^n_m e^n_a f^m_{(\Lambda} h^a_{\Sigma)} + e^n_a e^n_b h^a_\Lambda h^b_\Sigma\big). \end{aligned} \tag{20}$$

Once the potential is known then we can study the vacua of the theory, that is bosonic backgrounds which extremize $V(\Phi)$. If we are interested in supersymmetric vacua we need to look for bosonic backgrounds Φ_0 which admit a Killing spinor ε, namely directions in the supersymmetry parameter space along which:

$$\delta_\varepsilon(\text{fermions})|_{\Phi_0} = 0. \tag{21}$$

If a background admits a Killing spinor, it can be shown that it is also a vacuum of the theory. The spinors of the theory consist of the gravitini ψ^A_μ, the gaugini $\lambda^{i,A}$ $(i = 1, \ldots, n_v)$ and the hyperini $\zeta^{1,A}$, $\zeta^{a,A}$. From the Killing spinor equation $\delta_\varepsilon \zeta^{a,A} = 0$ we derive the following conditions which should hold for any supersymmetric vacua:

$$e^a_m f^m_\Lambda = e^m_a h^a_\Lambda = 0, \tag{22}$$
$$h^a_\Lambda X^\Lambda = 0. \tag{23}$$

Conditions (22) will fix $K3$ complex structure moduli, while Eq. (23) will fix the T^2 complex structure t and the axion/dilaton u. The Killing spinor equations $\delta_\varepsilon \zeta^{1,A} = 0$ and $\delta_\varepsilon \psi^A_\mu = 0$ turn out to be equivalent for this gauging and, together with the equations $\delta_\varepsilon \lambda^{i,A} = 0$, will impose restrictions on the fluxes.

2.2.1. $\mathcal{N} = 2$ vacua

From the gravitino Killing spinor equation we derive $\mathscr{P}^x_\Lambda \equiv 0$, which, upon implementation of Eqs. (22) implies

$$f^m_\Lambda = 0, \tag{24}$$

which can be restated as the requirement that no flux vector among the f_Λ in $\Gamma^{3,19}$ have positive norm, consistently with the results by Tripathy and Trivedi [5]. Let us, for the sake of simplicity, choose as the only non-vanishing components of the flux

$$h^{a=1}_2 = g_2; \qquad h^{a=2}_2 = g_3. \tag{25}$$

Condition (23) then imply:

$$X^2 = X^3 = 0 \quad \Leftrightarrow \quad t = u,\ 1 + t^2 = \frac{(x^k)^2}{2}, \tag{26}$$

so that t, u are fixed, while s and the brane coordinates x^k, y^r remain moduli. Finally conditions (22) imply $e^m_{a=1,2} = 0$. Since the two axions $C^{a=1,2}$ are Goldstone bosons which provide mass to A^2_μ, A^3_μ, the whole two hypermultiplets $a = 1, 2$ will not appear in the low-energy effective theory. This theory will be no-scale since the potential at the minimum vanishes identically in the moduli.

2.2.2. $\mathcal{N}=1,0$ *vacua*

Let us look for $\mathcal{N}=1$ vacua by requiring the component ε_2 to be the Killing spinor. Upon implementation of (22), we obtain the following conditions:

$$\begin{array}{l}\delta_\varepsilon\psi_\mu^A=0\\ \delta_\varepsilon\lambda^{i,A}=0\end{array}\quad\Rightarrow\quad\begin{cases}(f_\Lambda^{x=1}+\mathrm{i}f_\Lambda^{x=2})X^\Lambda=0,\\ (f_\Lambda^{x=1}+\mathrm{i}f_\Lambda^{x=2})\partial_i X^\Lambda=0,\\ f_\Lambda^{x=3}=0.\end{cases}\tag{27}$$

Condition $f_\Lambda^{x=3}=0$ in particular can be rephrased as the statement that the flux should be defined by at most two positive norm vectors in $\Gamma^{3,19}$, consistently with the *primitivity* condition on the complexified 3-form field strength $G_{(3)}$ as found by Tripathy and Trivedi [5].

Suppose, for the sake of simplicity, that the only non-vanishing flux components are the following

$$f_0^{m=1}=g_0;\qquad f_1^{m=2}=g_1;\qquad h_2^{a=1}=g_2;\qquad h_2^{a=2}=g_3,\tag{28}$$

then from the vanishing of the D7-brane gaugini variations in (27) we have the condition $x^k=0$, namely that the D7 branes be stuck at the origin of T^2. Condition (23) then implies:

$$X^2=X^3=0\quad\Leftrightarrow\quad t=u=-\mathrm{i}.\tag{29}$$

The four axions $C^{m=1,2}$, $C^{a=1,2}$ are Goldstone bosons which provide mass to all the bulk vectors. Finally conditions (22) will fix the 40 complex structure moduli of $K3$:

$$e_{a=1,2}^x=0;\qquad e_{a>2}^{x=1,2}=0\tag{30}$$

leaving the 17 Kähler moduli $e_{a>2}^{x=3}$ unfixed. The unfixed moduli will enter chiral multiplets in the effective $\mathcal{N}=1$ theory as the following complex scalars:

$$s,\ y^r,\ C^{m=3}+\mathrm{i}\,\mathrm{e}^{\phi},\ C^{a>2}+\mathrm{i}\,e_{a>2}^{m=3},\tag{31}$$

which span the scalar manifold:

$$\mathscr{M}_{\mathrm{scal}}=\frac{\mathrm{U}(1,1+n_3)}{\mathrm{U}(1)\times\mathrm{U}(1+n_3)}\times\frac{\mathrm{SO}(2,18)}{\mathrm{SO}(2)\times\mathrm{SO}(18)},\tag{32}$$

the former factor being parametrized by s, y^r. We have not dealt with all conditions (27) yet. In particular in the effective $\mathcal{N}=1$ we can construct a superpotential using $\mathcal{N}=2$ quantities:

$$W=\left[\mathrm{e}^{-\phi}\left(\mathscr{P}_\Lambda^{x=1}+\mathrm{i}\mathscr{P}_\Lambda^{x=2}\right)X^\Lambda\right]\big|_{\Phi_0}\propto g_0-g_1\ \text{moduli independent}.\tag{33}$$

On the other hand the expressions in (27) $(f_\Lambda^{x=1}+\mathrm{i}f_\Lambda^{x=2})X^\Lambda$ and $(f_\Lambda^{x=1}+\mathrm{i}f_\Lambda^{x=2})\partial_i X^\Lambda$ turn all out to be proportional to g_0-g_1. Therefore if $W=0$ we have $\mathcal{N}=1$ otherwise the vacuum will break all supersymmetry. In both cases the potential at the minimum vanishes identically in the moduli so that the effective supergravity is no-scale.

2.3. More general $\mathcal{N}=1$ *vacua*

We may generalize the above choice of fluxes so as to have vacua for more general (complex) values for t, u (in the positivity domain of the Lagrangian), namely:

$$t=a_t-\mathrm{i}\,\mathrm{e}^{2\lambda_t};\qquad u=a_u-\mathrm{i}\,\mathrm{e}^{2\lambda_u},\tag{34}$$

$a_t, a_u, \lambda_t, \lambda_u$ being generic real numbers. To this end we use the property of the gauged Lagrangian to be still duality invariant, *provided* we transform under duality symmetry the fluxes as well. The isometry transformation in $\mathrm{SU}(1,1)_t\times\mathrm{SU}(1,1)_u$ which maps the values $t=u=-\mathrm{i}$ into those in (34) is represented by the following symplectic matrix:

$$\mathscr{O}=\mathscr{O}_t\mathscr{O}_u;\quad \mathscr{O}_t=\begin{pmatrix}A_t^{-1T} & 0\\ 0 & A_t\end{pmatrix};\ \mathscr{O}_u=\begin{pmatrix}A_u^{-1T} & 0\\ C_u & A_u\end{pmatrix}.\tag{35}$$

One can verify indeed that

$$\begin{aligned}&\mathscr{O}\Omega(s,t,u,x,y)=\mathrm{e}^{-\lambda_t-\lambda_u}\Omega(s,t',u',x',y'),\\ &t'=a_t+\mathrm{e}^{2\lambda_t}t;\quad u'=a_u+\mathrm{e}^{2\lambda_u}u;\quad y'^r=\mathrm{e}^{\lambda_t}y^r;\quad x'^k=\mathrm{e}^{\lambda_t+\lambda_u}x^k.\end{aligned}\tag{36}$$

The flux vectors f_Λ^I are electric since they fill the lower part of a symplectic vector. Due to the perturbative form of $\mathscr{O}$, its action on the flux vectors will not produce magnetic charges but will transform them as follows:

$$f'^m_\Lambda = A^\Sigma_\Lambda f^m_\Sigma; \qquad h'^a_\Lambda = A^\Sigma_\Lambda h^a_\Sigma, \tag{37}$$

which, in components, read:

$$\begin{aligned}
f'^1_0 &= \frac{1}{2}\mathrm{e}^{-\lambda_t-\lambda_u}\big(1+\mathrm{e}^{2(\lambda_t+\lambda_u)}-a_t a_u\big)g_0; \qquad f'^1_1 = -\frac{1}{2}\,\mathrm{e}^{-\lambda_t-\lambda_u}(a_t+a_u)g_0,\\
f'^1_2 &= \frac{1}{2}\,\mathrm{e}^{-\lambda_t-\lambda_u}\big(1-\mathrm{e}^{2(\lambda_t+\lambda_u)}+a_t a_u\big)g_0; \qquad f'^1_3 = \frac{1}{2}\,\mathrm{e}^{-\lambda_t-\lambda_u}(-a_t+a_u)g_0,\\
f'^2_0 &= \left(\frac{\mathrm{e}^{-\lambda_t+\lambda_u}a_t}{2}+\frac{\mathrm{e}^{\lambda_t-\lambda_u}a_u}{2}\right)g_1; \qquad f'^2_1 = \left(\frac{\mathrm{e}^{\lambda_t-\lambda_u}}{2}+\frac{\mathrm{e}^{-\lambda_t+\lambda_u}}{2}\right)g_1,\\
f'^2_2 &= \left(-\frac{\mathrm{e}^{-\lambda_t+\lambda_u}a_t}{2}-\frac{\mathrm{e}^{\lambda_t-\lambda_u}a_u}{2}\right)g_1; \qquad f'^2_3 = \left(\frac{\mathrm{e}^{\lambda_t-\lambda_u}}{2}-\frac{\mathrm{e}^{-\lambda_t+\lambda_u}}{2}\right)g_1,\\
h'^1_0 &= -\frac{1}{2}\mathrm{e}^{-\lambda_t-\lambda_u}\big(-1+\mathrm{e}^{2(\lambda_t+\lambda_u)}+a_t a_u\big)g_2; \qquad h'^1_1 = -\frac{1}{2}\mathrm{e}^{-\lambda_t-\lambda_u}(a_t+a_u)g_2,\\
h'^1_2 &= \frac{1}{2}\mathrm{e}^{-\lambda_t-\lambda_u}\big(1+\mathrm{e}^{2(\lambda_t+\lambda_u)}+a_t a_u\big)g_2; \qquad h'^1_3 = \frac{1}{2}\mathrm{e}^{-\lambda_t-\lambda_u}(-a_t+a_u)g_2,\\
h'^2_0 &= \left(-\frac{\mathrm{e}^{-\lambda_t+\lambda_u}a_t}{2}+\frac{\mathrm{e}^{\lambda_t-\lambda_u}a_u}{2}\right)g_3; \qquad h'^2_1 = \left(\frac{\mathrm{e}^{\lambda_t-\lambda_u}}{2}-\frac{\mathrm{e}^{-\lambda_t+\lambda_u}}{2}\right)g_3,\\
h'^2_2 &= \left(\frac{\mathrm{e}^{-\lambda_t+\lambda_u}a_t}{2}-\frac{\mathrm{e}^{\lambda_t-\lambda_u}a_u}{2}\right)g_3; \qquad h'^2_3 = \left(\frac{\mathrm{e}^{\lambda_t-\lambda_u}}{2}+\frac{\mathrm{e}^{-\lambda_t+\lambda_u}}{2}\right)g_3.
\end{aligned} \tag{38}$$

One can verify that with this choice of fluxes $\mathcal{N}=1$ residual supersymmetry imply $g_0=g_1$, $x^k=0$ and t,u fixed at the values in (34).

The possibility of fixing the effective string coupling constant to small values as important implications. For instance it makes it possible to apply the model to the construction of inflationary models [33–38], in which the *slow-roll* of the inflaton (one of the y^r moduli) is realized once perturbative corrections to the potential are taken into account.[2]

2.3.1. D7 brane world volume fluxes

Within this framework we can consider the effect of switching on fluxes of the D7 gauge field strengths $\mathcal{F}^k_{\mu\nu}$ across two cycles of $K3$. This corresponds for instance to gauging additional Z^a isometries by means of D7 brane vectors A^k_μ [23]. The constant Killing vectors are $k^u_\Lambda = g^k_4$, $\Lambda = 3+k, k=1,\dots,n_7$, along the direction $q^u = C^{a=3,\dots,2+n_7}$ (recall that the isometries $Z_{a=1,2}$ have already been gauged by the vectors $A^{2,3}_\mu$).

As far as supersymmetric vacua are concerned, from inspection of the fermion shifts it is straightforward to verify that the existence of a constant Killing spinor always requires $X^2, X^3, X^{3+k}=0$ which implies $x^k=0$ and $t=u=-\mathrm{i}$ even in the $\mathcal{N}=2$ case (still corresponding to the choice $g_0=g_1=0$). As before we have $\mathcal{N}=1$ if $g_0=g_1\neq 0$ and $\mathcal{N}=0$ otherwise.

3. No-scale supergravity from Scherk–Schwarz generalized dimensional reduction

As pointed out in the introduction, spontaneously broken supergravity can also be obtained through a Scherk–Schwarz dimensional-reduction from $D+1$ to D dimensions [39,16,15,24]. In order for the theory to admit a stable vacuum the Scherk–Schwarz phases should be taken to be in the Cartan subalgebra of the maximal compact subgroup of the isometry group G of the theory in $(D+1)$-dimensions. The scalar potential is obtained from the non-linear σ-model describing the $(D+1)$-dimensional scalar fields:

$$\sqrt{-\det(g)}\,g^{\mu\nu}P^I_\mu P^I_\nu, \tag{39}$$

where P^I_μ are the pull-back on space-time of the vielbeins P^I_i of the $(D+1)$-dimensional scalar manifold. By taking $\mu=\nu=D+1$ we have the following potential in D dimensions:

[2] We are grateful to R. Kallosh for explaining this point to us.

$$V = \mathrm{e}^{-2\frac{D-1}{D-2}\sigma} P^I_{D+1} P^I_{D+1} \geqslant 0, \tag{40}$$

where σ is the modulus associated to the radius of the internal dimension and $P^I_{D+1} = P^I_i M^i_j \phi^j$, M^j_i being the Scherk–Schwarz phases. The potential has an absolute minimum (at the origin of the scalar manifold) only if M^j_i besides being a global symmetry of the $D+1$ theory is also compact, so that there exist a point in the moduli space in which $P^I_{D+1} = 0$. All the scalars are fixed at this minimum except σ and all the $(D+1)$-dimensional scalars ϕ^i for which $M^j_i \phi^j = 0$. Finally the gravitino mass matrix is provided by Q_{D+1} which is the pull-back on the direction $D+1$ of the R-symmetry connection Q_i on the scalar manifold in $D+1$ dimensions.

4. Type IIB on $T^6/\mathbb{Z}_2$ orientifold with fluxes and D3 branes

As a final example of let us briefly mention the gauged supergravity which describes the (classical) low-energy limit of Type IIB on $T_6/\mathbb{Z}_2$ orientifold in the presence of space-filling D3 branes and three form NS and RR fluxes [3,19,20,10]. It is an $\mathcal{N} = 4$ model with an Abelian gauge symmetry generated by twelve independent combinations of the fifteen translational isometries acting on the axions which originate from the internal components of the ten dimensional 4-form $C_{(4)}$. This model exhibits vacua with vanishing cosmological constant at tree level and a hierarchical supersymmetry breaking $\mathcal{N} = 4 \to 3 \to 2 \to 1 \to 0$ in which the masses of the gravitini are provided by four independent flux parameters m_i, $i = 1, \ldots, 4$, expressed in units of $\alpha'/\mathrm{Vol}(T^6)^{1/2}$.

5. No-scale supergravities and the cosmological constant

All the models discussed above exhibit partial super-Higgs around Minkowski vacua. Let us comment on the one-loop corrections to the cosmological constant. We start recalling that the quartic, quadratic and logarithmic divergent parts, in any field theory, are respectively controlled by the following coefficients

$$\mathrm{Str}\big(\mathscr{M}^{2k}\big) = \sum_J (-)^{2J} (2J+1) m_J^{2k}; \quad k = 0, 1, 2. \tag{41}$$

On the other hand, the sum rules

$$\mathrm{Str}\big(\mathscr{M}^{2k}\big) = 0; \quad k < \mathcal{N}, \tag{42}$$

in $\mathcal{N}$-extended supergravity seem to be of general validity for theories where a $\mathcal{N} \to \mathcal{N} - 1$ breaking is possible [18]. This requires long massive gravitino multiplets since the massive gravitino is Majorana and therefore cannot be BPS. On the other hand, for theories with central charges, like the Scherk–Schwarz breaking of $\mathcal{N} = 8$, gravitini are pairwise degenerate and the same sum rules apply only for $k < \mathcal{N}/2$, $\mathcal{N}$ being even. It is important to note that the bulk sector of $\mathcal{N} = 4$ Type IIB orientifold with fluxes does indeed coincide with a $\mathbb{Z}_2$ truncation of the $\mathcal{N} = 8$ Scherk–Schwarz supergravity, as was shown in [20]. Similarly $\mathcal{N} \leqslant 6$ Scherk–Schwarz supergravities, by $\mathbb{Z}_2$ reduction which removes the gravitino degeneracy, satisfy the same sum rules as the parent theory [26].

As an example let us consider the $\mathcal{N} = 4$ no-scale model from Type IIB on $T^6/\mathbb{Z}_2$ orientifold. In this case it was shown that $\mathrm{Str}(\mathscr{M}^2) = \mathrm{Str}(\mathscr{M}^4) = \mathrm{Str}(\mathscr{M}^6) = 0$ while from general arguments one would expect $\mathrm{Str}(\mathscr{M}^8) \propto m_1^2 m_2^2 m_3^2 m_4^2 \neq 0$. The first finite contribution to the cosmological constant would then be:

$$\Lambda \sim \frac{m_1^2 m_2^2 m_3^2 m_4^2}{M_{Pl}^4}. \tag{43}$$

It is intriguing to note that, if the supersymmetry breaking scale is taken to be $m_1 \sim m_2 \sim m_3 \sim m_4 \sim 10$ TeV $\sim 10^{-15} M_{Pl}$ then we would obtain from the above formula $\Lambda \sim 10^{-120} M_{Pl}^4$ which is consistent with the most recent experimental data [40].

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