

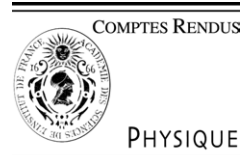


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Strings, gravity, and the quest for unification/Cordes, gravitation, et la quête d'unification

Field theory supertubes

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Abstract

Starting with intersecting M2-branes in M-theory, the IIA supertube can be found by S^1 compactification followed by a boost to the speed of light in the 11th dimension. A similar procedure applied to Donaldson–Uhlenbeck–Yau instantons on \mathbb{C}^3 , viewed as intersecting membranes of $D = 7$ supersymmetric Yang–Mills (SYM) theory, yields (for finite boost) a new set of 1/4 BPS equations for $D = 6$ SYM–Higgs theory, and (for infinite boost) a generalization of the dyonic instanton equations of $D = 5$ SYM–Higgs theory, solutions of which are interpreted as Yang–Mills supertubes and realized as configurations of IIB string theory. **To cite this article:** P.K. Townsend, C. R. Physique 6 (2005).

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Résumé

Théorie des champs des supertubes. Considérant des M2-branes s'intersectant en théorie M, le supertube pour la théorie de type IIA est déduite par une compactification sur S^1 et un boost à la vitesse de la lumière selon la 11ème dimension. Une procédure similaire est appliquée aux instantons de Donaldson–Uhlenbeck–Yau sur \mathbb{C}^3 , qui sont vus comme des intersections de membranes de la théorie supersymétrique de Yang–Mills (SYM) en $D = 7$, donnant (pour un boost fini) un ensemble nouveau d'équations 1/4 BPS pour la théorie de SYM–Higgs en $D = 6$, et (pour un boost infini) une généralisation des équations d'instantons dyoniques de la théorie SYM–Higgs en $D = 5$, solutions qui sont interprétées comme des supertubes de Yang–Mills et réalisées comme des configurations de la théorie des cordes de type IIB. **Pour citer cet article :** P.K. Townsend, C. R. Physique 6 (2005).

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Mots-clés: Théorie des cordes ; Supertube ; Théorie de Yang–Mills

1. Introduction

Starting with the 1/4 supersymmetric intersection of two M2-branes in M-theory, one can obtain other 1/4 supersymmetric configurations. For example, compactify on the 11th dimension to get the 1/4 supersymmetric configuration in which a IIA string ends on a D2-brane. Now boost in the 11th dimension; in ten dimensions this corresponds to adding D0-charge so that the IIA string now ends on a bound state of a D2-brane with dissolved D0-branes. What happens if we boost to the speed of light in the 11th dimension? Consider this question at the level of the effective Dirac–Born–Infeld (DBI) theory for the D2-brane, which

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is just a dual version of the 11-dimensional supermembrane. At finite boost, we have a ‘dyonic BIon’, which is a D2-brane spike carrying constant electric flux and a constant magnetic charge density [1]. As we boost to the speed of light, the spike becomes more tubular and we end up with a supertube [2].

In effect, we have constructed the D2-brane supertube from the 11-dimensional supermembrane (the ‘M-ribbon’ is an alternative starting point [3]), but the worldvolume action for the supermembrane exists in spacetime dimension $D = 4, 5, 7, 11$ [4], and the above (worldvolume) construction works as well for $D = 5, 7$ as it does for $D = 11$, yielding supertubes in $D = 4$ and $D = 6$ in addition to the supertube in $D = 10$. The 10-dimensional supertube is an effective description of a configuration of IIA string theory that has an alternative low-energy description as a supertube solution of IIA supergravity [5]. Is there a similar ‘microscopic’ interpretation of the $D = 4, 6$ supertubes?

As the starting point was a membrane in $D = 5, 7$ we should first ask whether there are supersymmetric theories in these dimensions that admit membrane solutions. For $D = 7$ the obvious candidate is a supersymmetric Yang–Mills (SYM) theory because an instanton solution of the YM equations on R^4 can be interpreted as a $1/2$ supersymmetric membrane. For $D = 5$ there are various candidates, one being a supersymmetric sigma-model because, for an appropriate choice of (necessarily hyper-Kähler) target space, there is a $1/2$ supersymmetric lump soliton that has a 5D interpretation as a membrane.

We should next ask whether these field theories admit $1/4$ supersymmetric solutions that can be interpreted as intersections of the $1/2$ supersymmetric membranes. If so, the procedure outlined above should yield solutions of the dimensionally-reduced theories (6D SYM or 4D sigma models) that we could call *field theory supertubes*. These would have an effective description in terms of the supertube solution of the DBI action for a 6D or 4D membrane, just as the IIA supergravity supertube has an effective description in terms of the DBI action for the D2-brane (the Born–Infeld vector potential arising, in each case, from dualization of a worldvolume scalar).

In the sigma-model case, the answer to this question is known. One can find an explicit $1/4$ supersymmetric non-singular solution of a 5D sigma model that represents the intersection of two membranes (or 3-branes of the 6D sigma model) [6]. A Scherk–Schwarz-type reduction to 4D then yields a ‘massive’ supersymmetric sigma model, and the intersecting membrane solution of the massless 5D model becomes in 4D, the $1/4$ supersymmetric ‘kink-lump’, which can be interpreted as a lump-string ending on a kink-membrane [1]. A boost in the 5th dimension generalizes this to the ‘Q-kink-lump’ of the massive 4D sigma-model [1], and a boost to the speed of light yields a tubular configuration with a cross-section that is a $1/4$ supersymmetric Q-lump solution of the dimensionally-reduced 3D massive sigma-model [7,8]. Thus, the Q-lump solution of 3D massive supersymmetric hyper-Kähler sigma models is, when viewed as a tubular solution of the 4D sigma model, a *field theory supertube*. In fact, it was this observation that led to the discovery of the string theory supertube, and the above discussion is just a reversal of the logic presented in [2].

In this contribution I explore the same issues for SYM theories. Along the way, we will obtain a new one-parameter set of first-order equations for $1/4$ supersymmetric solutions of 6D SYM-Higgs theory. A limit of these equations, corresponding to a boost to the speed of light in the 7th dimension, yields equations that generalize the dyonic instanton equations [9] of 5D SYM theory. Certain solutions of these equations are interpreted as Yang–Mills supertubes, and a realization of them as IIB string theory configurations is suggested. I conclude with a discussion of some issues raised by these results.

2. Yang–Mills supertubes

Let $F = dA + i[A, A]$ be the YM field-strength 2-form for YM 1-form potential A , which is a traceless hermitian 2×2 matrix for gauge group $SU(2)$. In the gauge $A_0 = 0$, any static bosonic solution of 7D SYM theory is solution of the Euclidean YM equations on \mathbb{R}^6 . The generic solution of this type preserving $1/4$ supersymmetry must satisfy a set of first-order differential equations, and one can choose coordinates x^1, x^2, \dots, x^6 such that these first-order equations are [10]

$$\begin{aligned} F_{13} + F_{42} &= 0, & F_{14} + F_{23} &= 0, \\ F_{15} + F_{62} &= 0, & F_{16} + F_{25} &= 0, \\ F_{35} + F_{64} &= 0, & F_{36} + F_{45} &= 0, \\ F_{12} + F_{34} + F_{56} &= 0. \end{aligned} \tag{1}$$

These equations are equivalent to the Donaldson–Uhlenbeck–Yau equations for Euclidean YM fields on \mathbb{C}^3 , and have been studied previously in the context of SYM theory (e.g., [11]), although not in the context of solitons of 7D SYM theory.

Of course, there will be special solutions of these equations that preserve more than $1/4$ supersymmetry. Apart from the vacuum, these are the solutions for which F is non-zero only on a 4-dimensional subspace of \mathbb{R}^6 ; e.g., the ‘1234’ subspace, in which case the equations reduce to

$$F_{13} + F_{42} = 0, \quad F_{14} + F_{23} = 0, \quad F_{12} + F_{34} = 0, \tag{2}$$

which are equivalent to the self-duality equations

$$F_{ij} + \frac{1}{2}\varepsilon_{ijkl}F_{kl} = 0 \quad (i, j, k, l = 1, 2, 3, 4). \tag{3}$$

The solutions are instantons which, as mentioned above, can be interpreted as 1/2 supersymmetric membrane solitons of the 7D SYM theory. If a 1/4 supersymmetric solution of Eqs. (1) is such that F has support, asymptotically, on some 4-plane then we would interpret this 4-plane as the space transverse to a membrane. Thus, it is reasonable to expect that, for appropriate boundary conditions, solutions of Eqs. (1) represent intersecting membranes.

Let us now compactify one space dimension on a circle; take it to be the x^6 direction, so that $A_6 = \Phi$, an adjoint Higgs field. Take the YM fields to be independent of x^6 ; this means that $F_{56} = D_5\Phi$, where D_5 is the 5th component of the gauge-covariant derivative. Let D_i ($i = 1, 2, 3, 4$) be the other four components. Eqs. (1) may now be written as

$$F_{ij} + \frac{1}{2}\varepsilon_{ijkl}F_{kl} = -\Omega_{ij}D_5\Phi, \quad F_{i5} = \Omega_{ij}D_j\Phi, \tag{4}$$

where Ω_{ij} are the entries of the 4×4 antisymmetric matrix with non-zero entries

$$\Omega_{12} = -\Omega_{21} = \Omega_{34} = -\Omega_{43} = 1. \tag{5}$$

We are still considering static solutions so it is understood that $A_0 = 0$, and that all fields are time-independent; in gauge-invariant terms,

$$D_0\Phi = 0, \quad F_{05} = 0, \quad F_{0i} = 0 \quad (i = 1, 2, 3, 4). \tag{6}$$

This means that the Gauss-law constraint

$$D_5F_{05} + D_iF_{0i} = 0 \tag{7}$$

is trivially satisfied.

Let us suppose $A_3 = A_4 = 0$, and that all fields become independent of x^3 and x^4 , asymptotically as $(x^3)^2 + (x^4)^2 \rightarrow \infty$. In this case, Eqs. (4) reduce to

$$F_{12} + D_5\Phi = F_{51} + D_2\Phi = F_{25} + D_1\Phi = 0, \tag{8}$$

which are the equations for a magnetic-monopole membrane in the x^3, x^4 plane (assuming that Φ is non-zero in the vacuum). If, on the other hand, $A_5 = 0$ asymptotically, as $(x^3)^2 + (x^4)^2 \rightarrow 0$, such that Φ becomes independent of x^5 , and the fields A_i ($i = 1, 2, 3, 4$) have an x^5 dependence such that $\partial_5 A_i = -\Omega_{ij}D_j\Phi$, then we are left with the self-duality Eqs. (3), and hence an instanton string in the x^5 direction. The instanton core of this string would collapse to a singularity if the string were not attached to the monopole membrane (which induces the x^5 -dependence of the YM fields). Thus, one expects there to exist a non-singular solution of Eqs. (4) with an interpretation as an instanton-string ending on a monopole domain wall.¹

We dimensionally reduced the 7D SYM theory in the x^6 direction, allowing for a non-zero vacuum value for the Higgs field $A_6 \equiv \Phi$. Now, returning temporarily to the 7D perspective, we boost to velocity v along x^6 . This takes Eqs. (4) into the new set of equations

$$\begin{aligned} F_{ij} + \frac{1}{2}\varepsilon_{ijkl}F_{kl} + \sqrt{1-v^2}\Omega_{ij}D_5\Phi &= 0, & F_{i5} - \sqrt{1-v^2}\Omega_{ij}D_j\Phi &= 0, \\ F_{0i} + vD_i\Phi &= 0, & F_{05} + vD_5\Phi &= 0, & D_0\Phi &= 0. \end{aligned} \tag{9}$$

To determine the fraction of the 16 supersymmetries of the 6D SYM-Higgs vacuum that are preserved by solutions of these equations, it is convenient to note that the 7D SYM theory from which we started is the dimensional reduction on T^3 of 10D SYM theory, so any solution of Eqs. (9) is also a solution of 10D SYM theory with $A_6 = \Phi$ but $A_7 = A_8 = A_9 = 0$, and a field strength $F_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, \dots, 9$) that is independent of x^6, x^7, x^8, x^9 . The number of supersymmetries preserved by any such solution is the number of linearly independent real, chiral, constant, 10D spinors ε such that $F_{\mu\nu}\Gamma^{\mu\nu}\varepsilon = 0$, where Γ^μ are the 10D Dirac matrices. Use of Eqs. (9) leads to the conclusion that the independent constraints satisfied by ε are

$$\Gamma^{1234}\varepsilon = -\varepsilon, \quad (\Gamma^{1256} + v\Gamma^{1250})\varepsilon = -(\sqrt{1-v^2})\varepsilon. \tag{10}$$

These constraints imply preservation of 1/4 supersymmetry. Note, that this does not, by itself, imply that the YM field equations are satisfied; for that we must also impose the Gauss-law condition (7).

¹ This interpretation was developed in unpublished work with Jerome Gauntlett and David Tong, following the work in [6] in which it was shown that a similar interpretation is indeed realized by the kink-lump solution of the analogous sigma-model equations.

Given a solution of the unboosted 1/4 BPS Eqs. (4) representing an instanton-string ending on a monopole domain wall, there should exist a corresponding solution of the boosted 1/4 BPS equations and Gauss law constraint that represents an instanton-string ending on a *dyon* domain wall. This is because the limit that led previously to the equations for a 1/2 BPS monopole now leads to the equations for a 1/2 BPS dyon. Suppose that we have such a solution for any v and that we take $v = 1$; i.e., we boost to the speed of light. In this case, we will have a solution of the equations

$$F_{ij} + \frac{1}{2}\varepsilon_{ijkl}F_{kl} = 0, \quad F_{0i} + D_i\Phi = 0, \quad D_0\Phi = 0, \quad (11)$$

and

$$F_{i5} = 0, \quad F_{05} + D_5\Phi = 0, \quad (12)$$

which are obtained from (9) by setting $v = 1$. Eqs. (11) are 1/4 BPS equations for a *dyonic instanton* [9]. Given an instanton solution of the self-dual YM equations, the other dyonic instanton equations are solved by setting $A_0 = \Phi$ for time-independent Higgs field Φ , in which case (12) is equivalent to

$$\partial_5 A_i = D_i A_5, \quad \dot{A}_5 = 0, \quad (13)$$

and the Gauss law constraint becomes

$$\left(\sum_{i=1}^4 D_i^2 + D_5^2 \right) \Phi = 0. \quad (14)$$

That is, Φ must solve the covariant 5D Laplace equation in a YM background provided by a 4D instanton, with x^5 -dependence given in terms of A_5 by (13).

If we suppose that $A_5 = 0$ then $\partial_5 A_i = 0$. Assuming that Φ is also independent of x^5 , we get a string-like solution of 6D SYM-Higgs theory, with a dyonic instanton core that carries ‘electric’ charge Q in addition to instanton number N (it would be interesting to investigate whether more general solutions are possible, but that will not be done here). For the sigma-model supertube, the core is a Q-lump, and a Q-lump has an interpretation as a charged closed loop of kink-string [8]; the analogous interpretation of the dyonic instanton would be as a charged closed loop of monopole-string, and this interpretation is also suggested by various other arguments [12,13]. However, the number of monopoles in a given solution of the 1/2 BPS equations of 4D SYM-Higgs theory is determined by the number of zeros of the Higgs field, and the positions of these zeros are the positions of the monopole. A single monopole, or dyon, has a single zero of the Higgs field, which will lift to a line of zeros in 5D; a monopole, or dyon, loop will thus be associated with a closed loop of zeros of the Higgs field. In contrast, dyonic instantons corresponding to instantons found by the ‘t Hooft ansatz have only *isolated* zeros of the Higgs field [14].

It therefore appeared, until recently, that the interpretation of dyonic instantons as charged loops of monopole-string could not be correct. However, recent work of Kim and Lee [15] has shown that the locus of zeros of the Higgs field for a *generic* dyonic instanton with instanton number $N \geq 2$ is a closed curve, exactly as one would expect for a loop of monopole-string. Already for $N = 2$, for which the general instanton solution can be found from the Jackiw–Nohl–Rebbi ansatz, there is an additional parameter as compared to the ‘t Hooft ansatz, and this yields a one-parameter family of closed curves that degenerate to two points in the ‘t Hooft ansatz limit. Lifting to $D = 6$ we have a configuration of 6D SYM-Higgs theory in which the Higgs zero lie on a tube. This is a *field theory supertube*; as I have argued here, it is related to an intersecting membrane solution of 7D SYM theory in the same way that the D2-brane supertube of IIA string theory is related to a configuration of intersecting M2-branes of M-theory.

3. String theory realizations

We have found a 6D Yang–Mills supertube by a procedure that is analogous to one that can be used to find the 10D string theory supertube. However, there is also a *direct* connection between the two that arises from the interpretation of SYM-Higgs theory, for gauge group $SU(2)$, as the effective field theory on a pair of parallel D-branes in type II string theory.

Let us first consider the 5D $SU(2)$ SYM-Higgs theory on a pair of parallel D4-branes. A line of zeros of the Higgs field, corresponding to a monopole-string, would have a natural interpretation as the endpoint of a planar D2-brane since T-duality in a direction parallel to the line yields a D1-string stretched between two D3-branes, which is the standard D-brane realization of a BPS magnetic monopole. A closed loop of zeros of the Higgs field therefore represents the (common) boundary of a tubular D2-brane on the two D4-branes. The non-zero instanton number N indicates N dissolved D0-branes, but D0-brane charge is magnetic charge on a D2-brane. Moreover, the fraction of supersymmetry preserved by the total D-brane configuration is

expected to be 1/8, which translates to 1/4 of the supersymmetry of the SYM-Higgs theory vacuum. Thus, the generic $SU(2)$ dyonic instanton has a string theory interpretation [15] as a supertube stretched between two D4-branes.² By T-duality we can convert this to a 1/8 supersymmetric IIB string configuration in which a D3-brane with $S^1 \times R$ boundary having N dissolved D1 strings, is stretched (along with some number of dissolved IIB strings) between two D5-branes. The common boundary on the D5-branes is the tubular locus of zeros of the Higgs field of 6D SYM-Higgs theory; in other words, *we have a IIB string theory description of the 1/4 supersymmetric field-theory supertube of 6D SYM-Higgs theory.*

If the supertubular D3-brane suspended between the D5-branes collapses, ‘precipitating’ out the dissolved IIB and D1 strings then we end up with a configuration represented by the array

$$\begin{array}{l}
 \text{D5: } 1 \ 2 \ 3 \ 4 \ 5 \ - \ - \ - \ - \\
 \text{D1: } 1 \ - \ - \ - \ - \ - \ - \ - \ - \\
 \text{F1: } - \ - \ - \ - \ - \ 6 \ - \ - \ -
 \end{array} \tag{15}$$

where the first row represents the two D5-branes, in which the ‘precipitated’ D1-branes are actually still dissolved, and ‘F1’ indicates the ‘fundamental’ IIB strings. By adding angular momentum, one can reverse the collapse and blow up the IIB and D1 strings to the configuration described previously. Let us now increase the IIB coupling and pass to the S-dual IIB string theory, and then T-dualize in the x^6 direction; this yields a IIA configuration represented by the array

$$\begin{array}{l}
 \text{KK: } - \ - \ - \ - \ - \ o \ x \ x \ x \\
 \text{F1: } 1 \ - \ - \ - \ - \ - \ - \ - \ - \\
 \text{D0: } - \ - \ - \ - \ - \ - \ - \ - \ -
 \end{array} \tag{16}$$

where ‘KK’ (for Kaluza–Klein monopole) indicates a 4-dimensional ALE space, and the circle in the 6th position indicates that ∂_6 is the $U(1)$ Killing vector field; since we started with two D5-branes, this Killing vector field should have two isolated singularities. However, we could view this ALE space as a local description of a compact K3-manifold. Expansion of the IIA string and D0-branes by the addition of angular momentum then yields a supertube in what is effectively a 6D spacetime obtained by K3 compactification of the IIA theory. This should be related to the M2-brane of K3-compactified M-theory in the same way that the original supertube is related to the M2-brane in the 11-dimensional Minkowski vacuum. As M-theory on K3 is dual to the heterotic string theory on T^3 , this membrane is dual to the membrane of T^3 -compactified heterotic string theory, which is just the heterotic five-brane wrapped on the T^3 . We may conclude from this there must exist a supertube of T^4 -compactified heterotic string theory dual to the D2-brane supertube of IIA string theory. The corresponding supergravity/SYM solution is presumably the heterotic dyonic instanton [14].

4. Fuzzy conjectures

We have seen that a D2-brane supertube can be suspended between two D4-branes, and that there is a T-dual of this configuration for which the interpretation within an effective 6D SYM-Higgs theory is as a field theory supertube. Can the latter be suspended, preserving supersymmetry, between other branes? Yes, because the two D5-branes can themselves be suspended between two NS5-branes. Consider the ‘precipitated’ configuration represented by the 1/16 BPS array

$$\begin{array}{l}
 \text{D5: } 1 \ 2 \ 3 \ 4 \ 5 \ - \ - \ - \ - \\
 \text{NS5: } - \ 2 \ 3 \ 4 \ 5 \ 6 \ - \ - \ - \\
 \text{D1: } 1 \ - \ - \ - \ - \ - \ - \ - \ - \\
 \text{F1: } - \ - \ - \ - \ - \ 6 \ - \ - \ -
 \end{array} \tag{17}$$

where the second row represents the additional pair of NS5-branes, and the low energy effective field theory is the $(1 + 4)$ -dimensional theory on the D5/NS5 intersection. The IIB strings are parallel to the NS5-branes and the D1-strings are suspended between them (like the D5-branes in which they are dissolved). Adding angular momentum to blow up the IIB and D1 strings to a supertubular D3-brane would yield a configuration of the type sought, which would presumably correspond to some ‘Q-lump-type’ supertube of the 5D effective field theory. However, the relationship of this construction to the explicit M-theory description of the sigma-model Q-lump given in [1] is not obvious. This may be because there are other possible field theory realizations of the 5D supertube.

² In the one instanton case, the loop of Higgs zeros degenerates to a point, and the interpretation is as a IIA string stretched between two D4-branes carrying D0-brane charge [16].

As reviewed above, the work of Kim and Lee [15] has shown that the generic dyonic instanton for instanton number N can be interpreted as a D2-brane supertube suspended between two D4-branes. The instanton number N corresponds to the number of dissolved D0-branes; this would be infinite for an infinite supertube but can be finite for one suspended between D4-branes. For $N = 2$ there is a dyonic instanton solution for which the Higgs field zeros lie on a circle. This corresponds to a supertube with circular cross-section. The limiting procedure explained here for finding a D2-brane supertube from the 11-dimensional supermembrane yields a supertube of this type (because until the limit is taken 1/4 supersymmetry implies a circular cross-section). However, 1/4 supersymmetry allows supertubes with other cross-sectional shapes [17]; in fact, any curve, even a non-planar one, is permitted [18]. In contrast, the locus of Higgs field zeros for the generic dyonic instanton with instanton number N involves only a finite number of parameters, so it cannot correspond to the generic classical supertube cut-off by D4-branes. On the other hand, it seems likely that any cross-sectional curve will be possible in the limit as $N \rightarrow \infty$, so that this should be viewed as a semi-classical limit. In effect, the Yang–Mills supertube for finite N is a fuzzy supertube, analogous to the M(atrrix) model supertube of [19] but constructed from a finite number of D0-branes. This expectation suggests a purely mathematical conjecture about the locus of zeros of an adjoint Higgs field satisfying a covariant Laplace equation on \mathbb{R}^4 in a YM instanton background with instanton number N . Specifically, although imprecisely, it suggests that *this locus can be chosen to approximate any closed curve in \mathbb{R}^4 with an error that goes to zero as $N \rightarrow \infty$.*

Finally, it should be noted that there are Donaldson–Uhlenbeck–Yau equations for Euclidean YM theory on \mathbb{E}^8 which are equivalent to static soliton equations for $D = 9$ SYM theory that preserve 1/8 of the supersymmetry of the SYM vacuum, and there are other first-order equations (comprehensively analysed in [10]) that imply preservation of 1/16 supersymmetry. It is possible that there exist non-singular solutions of these equations that could be interpreted as triple or quadruple intersections of orthogonal 4-branes with instanton cores, in analogy with the sigma-model case [6]. These intersecting-instanton solutions will be unstable against collapse to a singularity in the Higgs phase, but in that case we have 1/2 supersymmetric monopole 5-branes, on which an (otherwise singular) instanton 4-brane could have a boundary (this being the lift to 9D of the 5D instanton string ending on the 5D monopole membrane). This configuration of $D = 9$ SYM–Higgs theory would preserve 1/4 supersymmetry but one can envisage many more complicated intersecting soliton configurations that preserve only 1/8 or 1/16 supersymmetry. There is still a lot to learn about supersymmetric intersections of field theory solitons. Field theory supertubes constitute just one element in a much larger, and still emerging, picture.

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References

- [1] J.P. Gauntlett, R. Portugues, D. Tong, P.K. Townsend, D-brane solitons in supersymmetric sigma-models, Phys. Rev. D 63 (2001) 085002, hep-th/0008221.
- [2] D. Mateos, P.K. Townsend, Supertubes, Phys. Rev. Lett. 87 (2001) 011602, hep-th/0103030.
- [3] Y. Hyakutake, N. Ohta, Supertubes and supercurves from M-ribbons, Phys. Lett. B 539 (2002) 153, hep-th/0204161.
- [4] E. Bergshoeff, E. Sezgin, P.K. Townsend, Supermembranes and eleven-dimensional supergravity, Phys. Lett. B 189 (1987) 75; E. Bergshoeff, E. Sezgin, P.K. Townsend, Properties of the eleven-dimensional super membrane theory, Ann. Phys. 185 (1988) 330.
- [5] R. Emparan, D. Mateos, P.K. Townsend, Supergravity supertubes, JHEP 0107 (2001) 011, hep-th/0106012.
- [6] R. Portugues, P.K. Townsend, Sigma-model soliton intersections from exceptional calibrations, JHEP 0204 (2002) 039, hep-th/0203181.
- [7] R.A. Leese, Q lumps and their interactions, Nucl. Phys. B 366 (1991) 283; E. Abraham, Nonlinear sigma models and their Q lump solutions, Phys. Lett. B 278 (1992) 291.
- [8] E.R.C. Abraham, P.K. Townsend, More on Q kinks: A (1 + 1)-dimensional analog of dyons, Phys. Lett. B 295 (1992) 225.
- [9] N.D. Lambert, D. Tong, Dyonic instantons in five-dimensional gauge theories, Phys. Lett. B 462 (1999) 89, hep-th/9907014.
- [10] D.-S. Bak, K.-M. Lee, J.-H. Park, BPS equations in six and eight dimensions, Phys. Rev. D 66 (2002) 025021, hep-th/0204221.
- [11] J.M. Figueroa-O’Farrill, A. Imaanpur, J.G. McCarthy, Supersymmetry and gauge theory on Calabi–Yau 3-folds, Phys. Lett. B 419 (1998) 167, hep-th/9709178; O. Lechtenfeld, A.D. Popov, R.J. Szabo, Noncommutative instantons in higher dimensions, vortices and topological K-cycles, JHEP 0312 (2003) 022, hep-th/0310267.
- [12] M. Kruczenski, R.C. Myers, A.W. Peet, D.J. Winters, Aspects of supertubes, JHEP 0205 (2002) 017, hep-th/0204103.
- [13] D.-S. Bak, K.-M. Lee, Supertubes connecting D4 branes, Phys. Lett. B 544 (2002) 329, hep-th/0206185.
- [14] E. Eyras, P.K. Townsend, M. Zamaklar, The heterotic dyonic instanton, JHEP 0105 (2001) 046, hep-th/0012016.
- [15] S. Kim, K.-M. Lee, Dyonic instanton as supertube between D4 branes, JHEP 0309 (2003) 035, hep-th/0307048.
- [16] M. Zamaklar, Geometry of the nonabelian DBI dyonic instanton, Phys. Lett. B 493 (2000) 411, hep-th/0006090.

- [17] D.-S. Bak, A. Karch, Supersymmetric brane–antibrane configurations, *Nucl. Phys. B* 626 (2002) 165, hep-th/0110039;
D.-S. Bak, N. Ohta, Supersymmetric D2 anti-D2 strings, *Phys. Lett. B* 527 (2002) 131, hep-th/0112034.
- [18] D. Mateos, S. Ng, P.K. Townsend, Tachyons, supertubes and brane/anti-brane systems, *JHEP* 0203 (2002) 016, hep-th/0112054.
- [19] D.-S. Bak, K.-M. Lee, Noncommutative supersymmetric tubes, *Phys. Lett. B* 509 (2001) 168, hep-th/0103148;
D.-S. Bak, N. Ohta, M.M. Sheikh-Jabbari, Supersymmetric brane anti-brane systems: Matrix model description, stability and decoupling limits, *JHEP* 0209 (2002) 048, hep-th/0205265;
Y. Hyakutake, Fuzzy BIon, *Phys. Rev. D* 68 (2003) 046003, hep-th/0305019.