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C. R. Physique 6 (2005) 640–646
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Interaction of electromagnetic fields with the environment/Interaction du champ électromagnétique avec l'environnement

Inverse characterization of antennas by equivalent sources using spherical harmonics

Azeddine Gati [a,*], Yacine Adane [a], Man-Faï Wong [a], Joe Wiart [a], Victor Fouad-Hanna [b]

[a] *France Télécom R&D, 92794 Issy Moulineaux, France*
[b] *Laboratoire instruments et systèmes, université Pierre et Marie Curie, 75252 Paris cedex 05, France*

Available online 30 August 2005

Abstract

The present paper deals with the analysis of electromagnetic radiations and the interaction of radiated waves with the environment. The characterization by simulation of the phenomena resulting from the interaction of the fields with the surrounding objects requires as a preliminary the synthesis of robust and simple equivalent source models to be integrated in propagation schemes. Methods such as ray-tracing or launched rays coupled with reliable mathematical models such as the Uniform technique of Diffraction UTD then allow a good analysis of the phenomena. The proposed technique is based on the decomposition of a source, of arbitrary geometry and whose dimensions do not allow direct use of the asymptotic methods in its vicinity, into smaller equivalent sources which make possible to calculate the field radiated close to the transmitter. The equivalent elementary sources are represented by their spherical modes. The choice of the number of sources and the number of modes allows us to develop effective models and to account for the objects close to the antennas. ***To cite this article: A. Gati et al., C. R. Physique 6 (2005).***

Résumé

Caractérisation inverse d'antennes par des sources équivalentes en utilisant des modes sphériques. Ce travail s'inscrit dans la démarche de l'analyse des émissions électromagnétiques (EM) et de l'interaction de ces champs avec l'environnement. La caractérisation par la simulation des phénomènes résultant de l'interaction des champs avec les objets environnants nécessite au préalable la synthèse de modèles de sources équivalentes robustes et simples d'utilisation dans des modèles de propagation. Des méthodes telles que le tracé ou le lancé de rayons couplées avec des modèles mathématiques fiables tel que la technique uniforme de diffraction (UTD) permettent alors une bonne analyse des phénomènes. La technique proposée est basée sur la décomposition d'une source, de géométrie quelconque et dont les dimensions ne permettent pas l'utilisation triviale des méthodes asymptotiques en zone proche, en sources équivalentes plus petites qui, elles, permettent de calculer le champ rayonné très près de l'émetteur. Les sources élémentaires équivalentes sont représentées par leurs modes sphériques. Le choix du nombre de sources et du nombre de modes permet l'élaboration de modèles efficaces et la prise en compte des objets proches des antennes. ***Pour citer cet article : A. Gati et al., C. R. Physique 6 (2005).***

Keywords: Equivalent sources; Spherical modes; Ray-tracing; Inverse problem

Mots-clés : Sources équivalentes ; Modes sphériques ; Tracé de rayon ; Problème inverse

* Corresponding author.
E-mail address: azeddine.gati@francetelecom.com (A. Gati).

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doi:10.1016/j.crhy.2005.06.006

1. Introduction

These last few years, much work has been carried out in electromagnetic modelling in order to assess human exposure to electromagnetic waves near radio base station antennas [1]. Many methods of modelling have been proposed [2,3] for this purpose. Full-wave methods, such as FDTD (Finite Difference Time Domain) or MoM (Method of Moments), have proved their efficiency. However, they are rapidly limited by computing resources in time and memory. Moreover, the major drawback is the requirement of a perfect knowledge of the geometrical and electrical properties of the antennas studied. Other methods based on the gain calculation of the base station antennas are easier to set up and faster, but unfortunately they are less accurate [4].

In [5], we have presented a very accurate and fast method for the field characterization of base station antennas. By considering a base station antenna as a linear array of elementary cells, its field can be viewed as the superposition of the fields radiated by the elementary cells (see Fig. 1).

The challenge is then to define the fields of the elementary cells which reproduce by superposition the field of the entire antenna. Therefore, we choose to characterize the fields of the cells by spherical harmonics coefficients since few coefficients lead to the radiating characteristics of any cell [6]. The synthesis problem is stated, assuming that the position and the number of the elementary cells are approximately known. The position of a given cell is needed for calculating a transformation matrix which allows stating the synthesis problem as the solution of the following rectangular matrix equation:

$$[\, M_1 \quad M_2 \quad M_3 \quad \cdots \quad M_{N_{\mathrm{sub}}} \,] \times \begin{pmatrix} \chi_1 \\ \chi_2 \\ \vdots \\ \chi_{N_{\mathrm{sub}}} \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_P \end{pmatrix} \tag{1}$$

where M_i represents the conversion matrix which relates the mode coefficients of an elementary cell i in its local coordinate system to the mode coefficients in the global coordinate system attached to the entire antenna. The matrix M_i is obtained by using the spherical harmonics translation theory which considers only the translation along the z-axis [7].

All elementary cells of the array should be represented by their corresponding conversion matrices which are concatenated as in (1). The unknown of the inverse problem is the vector formed by all the χ_i containing the local-mode coefficients associated to each cell. The right-hand side of (1) is a vector representing spherical modes coefficients of the original antenna provided by measurements for instance. It is worth mentioning that a 3D radiation pattern is sufficient to deduce the spherical modes spectrum and it can be obtained either by calculation or measurement. The fields of the elementary cells are deduced from the spherical harmonics coefficients χ_i obtained by solving (1). Finally, the field of the whole antenna is derived from the superposition of elementary fields.

In the construction of the conversion matrix of each elementary cell it was considered that the cell is translated only via the z-axis. This restriction comes from the spherical harmonics translation theory which makes the translation along the other axis more difficult [7]. As a consequence, our synthesis method is only limited to the array antennas positioned on the z-axis.

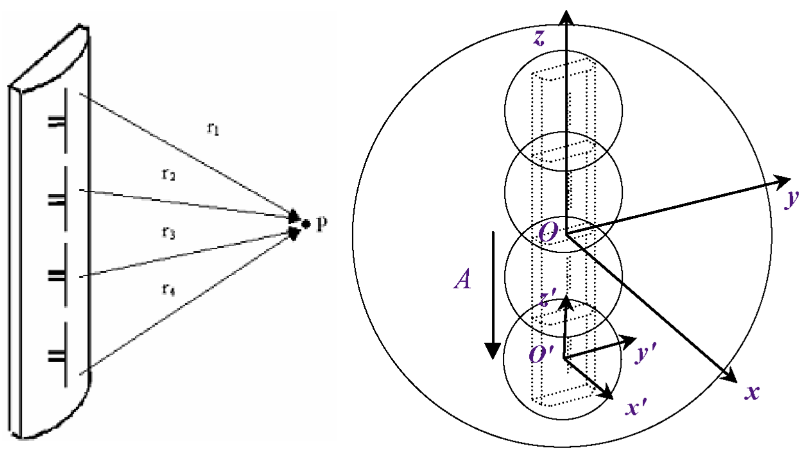

Fig. 1. Antenna decomposition and cell translation along the z-axis.

2. General procedure to characterize an arbitrary positioned cell

Our purpose is to find a procedure to extend this method to all kinds of array antennas which are forming an array in any direction. In other words, the conversion matrix of a cell must be able to express its translation through any direction in space. An arbitrary translation direction can be defined by combining the z-axis translation operation with the rotation. The spherical harmonics can be moved by rotation by using the Euler angles theory [7]. To get the modal information of an arbitrary translation, three steps are required. The first step is to carry out a rotation through the adequate Euler angles for positioning the z-axis of the cell in the right direction. After, a translation is performed via the new z-axis to shift the cell. Finally, the new z-axis must be moved to its previous position by inverse rotation.

Fig. 2 shows a simple example of a cell translation via x-axis, polarized along the z-axis, by following the three operations mentioned earlier. The cell is initially positioned along the z-axis at the centre of the coordinate system. The aim is to translate this cell by a distance A along the x-axis. The procedure begins by cell rotation by -90° around the y-axis. Then, it is translated following z-axis by a distance A. Finally, the inverse rotation of $+90^\circ$ is done to get the cell at the desired location. In summary, these three steps are equivalent to a translation along the x-axis.

In terms of modal expansion, the three steps for geometry transformation can be summarized by three matrices: one matrix for rotation in the local coordinate system, one conversion matrix for the translation, and one inverse rotation matrix. The product of these three matrices yields one conversion matrix which expresses the conversion of the local modes of the cell to the modes of an arbitrary coordinate system. Such technique allows synthesizing any antenna with a defined number of cells.

3. Spherical decomposition

According to the theory of the spherical modes, the electromagnetic field radiated by a source can be expressed according to an orthogonal base of spherical harmonics. Consequently, it can be represented by the spherical coefficients of harmonics a_{nm} and b_{nm} ($n = 1, 2, \ldots, \infty$; $m = -n, \ldots, +n$) such as:

$$\vec{E} = k\sqrt{\eta} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[a_{nm} \vec{F}_{1mn}^{(c)}(r, \theta, \phi) + b_{nm} \vec{F}_{2mn}^{(c)}(r, \theta, \phi)\right] \tag{2}$$

$$\vec{H} = -jk/\sqrt{\eta} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[b_{nm} \vec{F}_{1mn}^{(c)}(r, \theta, \phi) + a_{nm} \vec{F}_{2mn}^{(c)}(r, \theta, \phi)\right] \tag{3}$$

where $\vec{F}_{1mn}^{(c)}(r, \theta, \phi)$ and $\vec{F}_{2mn}^{(c)}(r, \theta, \phi)$ are the TE and TM spherical harmonics, respectively. $\eta = 377\ \Omega$ is the free space wave impedance and k the wave number. The expressions above make it possible to determine the fields E and H in all space exterior to the minimum sphere circumscribing the source, once the coefficients a_{nm} and b_{nm} are known.

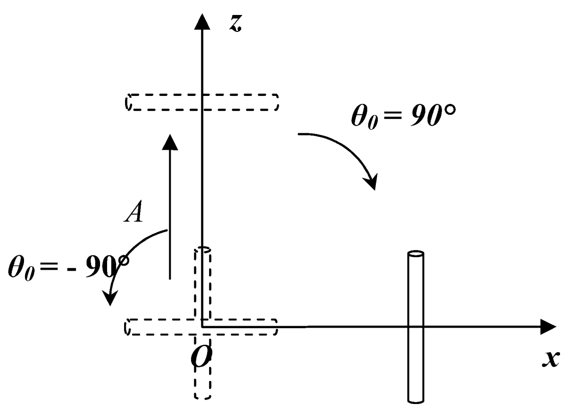

Fig. 2. Cell translation through x-axis.

4. Test case for a base station antenna

We applied this approach in order to model an antenna of 2.7 m height. In this synthesis problem, we decompose the antenna into 8 elementary sources vertically positioned and one wavelength spaced. After resolution of the linear system we obtain the 8 spectrum coefficients representing the spherical harmonics of the 8 elementary cells.

The transfer matrix was built for a number of TE and TM modes corresponding to 3040 lines and $98 \times 8 = 784$ columns. Fig. 3(a) shows the field computed in the vertical plane coming from the spherical modes of the total antenna for a power of 20 Watts. The minimal sphere has a radius of 1.7 m.

Fig. 3(b) shows the field obtained after superposition of the 8 basic cells generated by this approach in the same cut-plane. This figure shows that the field is calculated in all the points even inside the initial minimal sphere. The model is valid a wavelength beyond the cells corresponding to the minimal sphere of the elementary cells.

Fig. 3(c) shows the relative error $(\Delta E/E)$ according to the intensity of the field. This figure shows that the error does not exceed 10% for the points of low intensity generally corresponding to points on the side lobes.

5. Test case for Yagi antenna

The Yagi antenna shown in Fig. 4 is used to test the performance of the general formulation of the synthesis problem. This antenna is formed by a reflector, a feeder and 23 directors [8]. Each element is characterized by its corresponding conversion matrix calculated according to the procedure described in Fig. 2. The conversion matrix is calculated by considering that 6 spherical modes are enough to describe the field induced by a unit cell. The spherical harmonics coefficients of the whole

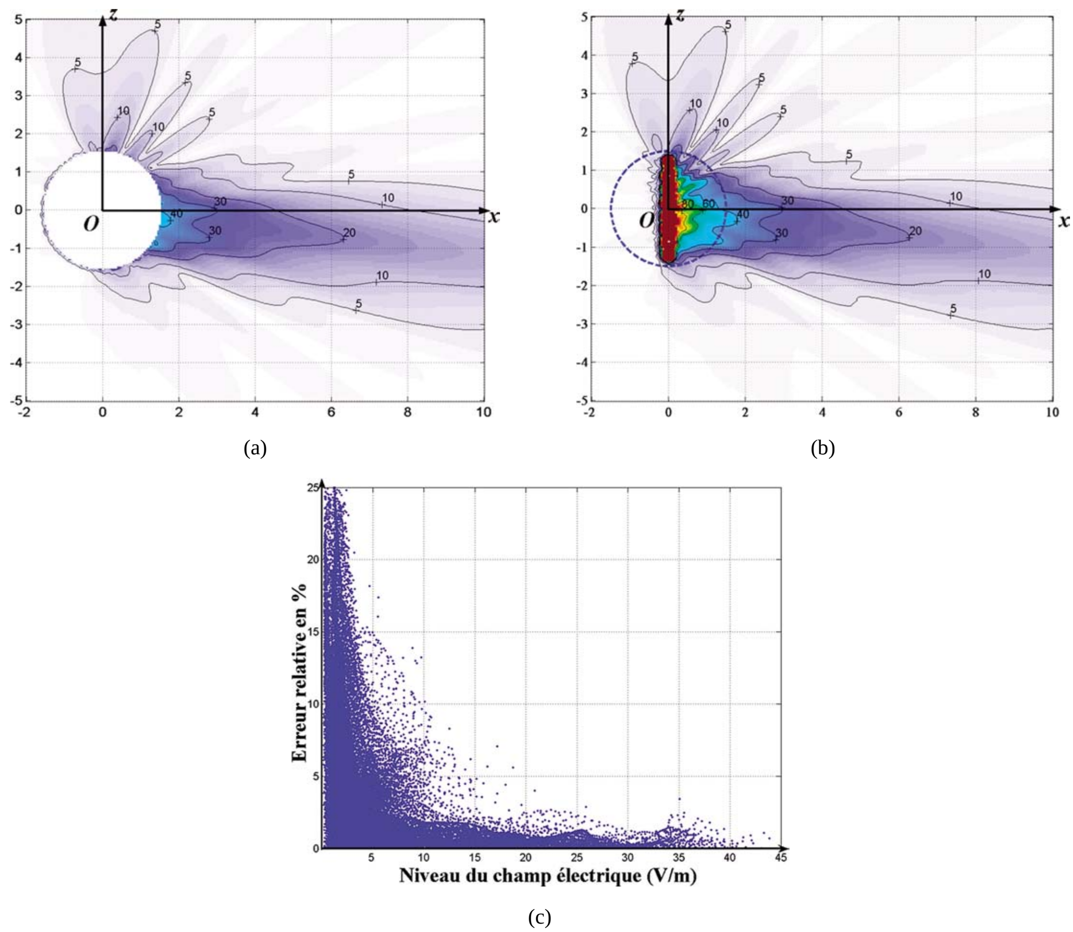

Fig. 3. Model of an antenna: (a) vertical cut-plane; (b) equivalent model; (c) relative error.

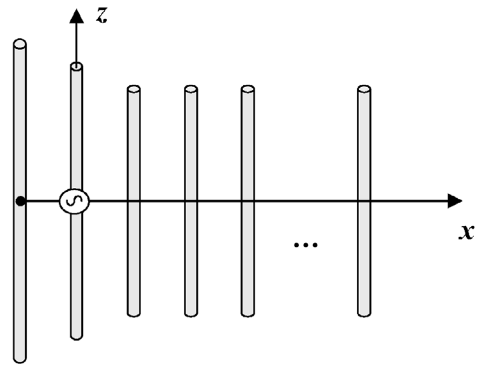

Fig. 4. Scheme of the Yagi antenna used for the test case.

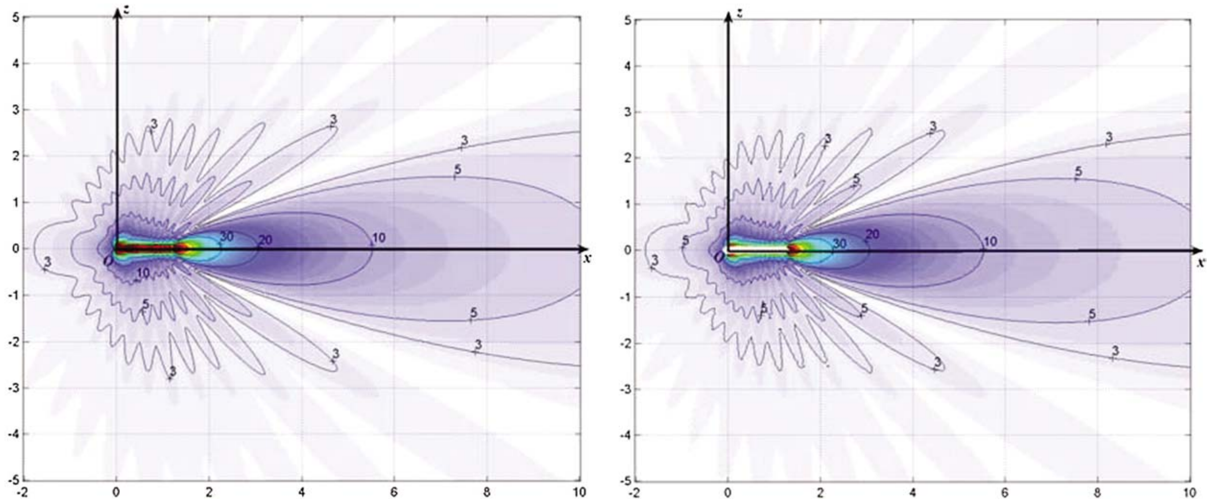

Fig. 5. E-field of the Yagi antenna calculated by (left) NEC and (right) our model, both at 1400 MHz. The field is calculated in the vertical cut plane (the distances are given in meters).

antenna should be determined to constitute the left-hand side of (1). The Yagi antenna is also modelled by NEC (Numerical Electromagnetic Code) and the magnitude of the radiated E-field at 1400 MHz is displayed in Fig. 5 (left).

The Yagi antenna is synthesized respecting carefully the position of the 25 elements forming the antenna. Therefore, 25 conversion matrices are needed to establish the Eq. (1). After solving (1) and calculating the fields due to the 25 elements, the Yagi antenna field is derived by superposing them. The field of our synthesized model which is displayed in Fig. 5 (right), agrees very well with the one obtained from NEC. Fig. 6. shows the relative error between the results of our model and those obtained by NEC never exceeds 9%. Furthermore, the power distribution of the 25 elements given by the model is plotted in Fig. 7. This figure shows that the cells obtained from the model appear to reproduce a realistic power distribution. The higher power corresponds the second cell, which is the feeder. We can observe that the nearest elements from the feeder have also a significant power. These results are coherent since the power of the elements decay when we move away from the feeder.

The general model shows a remarkable ability to synthesize antennas which are different from the classical vertical array of base station antenna. This enables us to synthesize antennas which are not linear arrays such as parabolic or 2-D arrays of patch antennas. These antennas could be viewed as a network of equivalent sources obtained by solving (1). The main parameters to be determined are the location of the cells, their number, and their modes.

The field shown in Fig. 5 (right) is obtained by considering all the 25 cells for the synthesis problem. Such a model is highly resource consuming since the number of the considered cells is very important and the field of each one must be calculated. Moreover, the synthesis problem solved with 25 cells gives results close to those provided using NEC as all the positions of the cells are well known. In practice, such precise information about the position of the cells is difficult to obtain. This limitation could be overcome by modelling the antenna with a reduced number of equivalent sources.

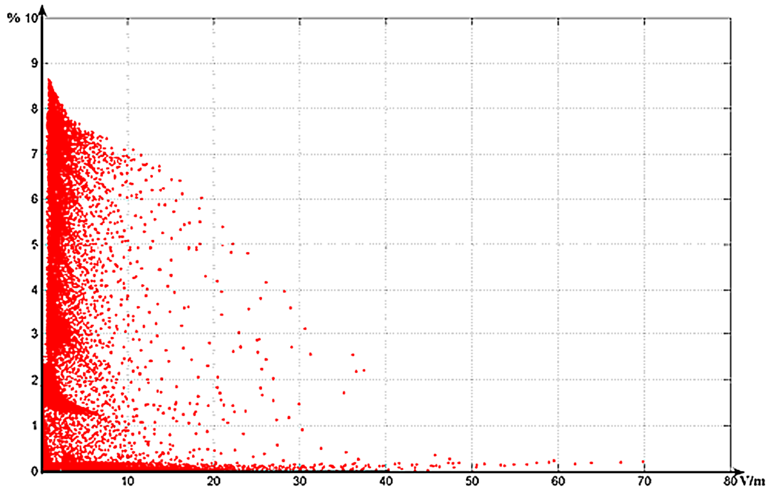

Fig. 6. Relative error between the field of NEC model and synthesis model of Yagi antenna.

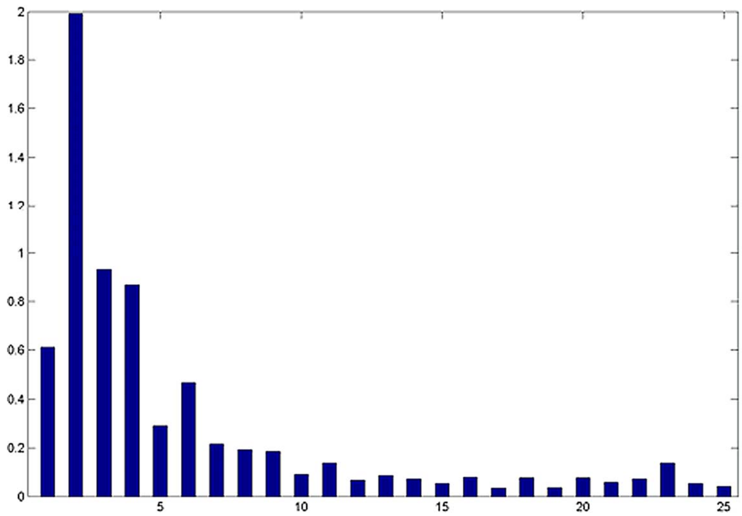

Fig. 7. Power distribution of the 25 elements of the model.

6. Model reduction

The reduced equivalent source model derives from the assumption that the characteristics of the cell and the number of cells are either not necessary known or need to be reduced. Let us consider the previous example of a Yagi antenna and assume that the only information we have about this antenna is its complex 3D gain pattern. To synthesize this antenna, we first need its spherical harmonics coefficients which can be easily derived from the gain pattern [6]. However, the information regarding the elementary cells is unknown. The only way to synthesize this antenna is to consider equivalent cells whose number and positions are determined after a series of tests. For our example, we find that only 5 equivalent cells are needed with one wavelength spacing between them. Each cell requires 336 spherical modes for synthesizing the Yagi antenna.

Fig. 8 shows the E-field of the Yagi antenna calculated with the 5 equivalent sources. The results of this model agree very well with the reference E-field obtained by NEC displayed in Fig. 5 (left). This model could be called ‘economical model’ since only 5 cells are required to characterize the antenna, instead of the 25 cells of the previous model.

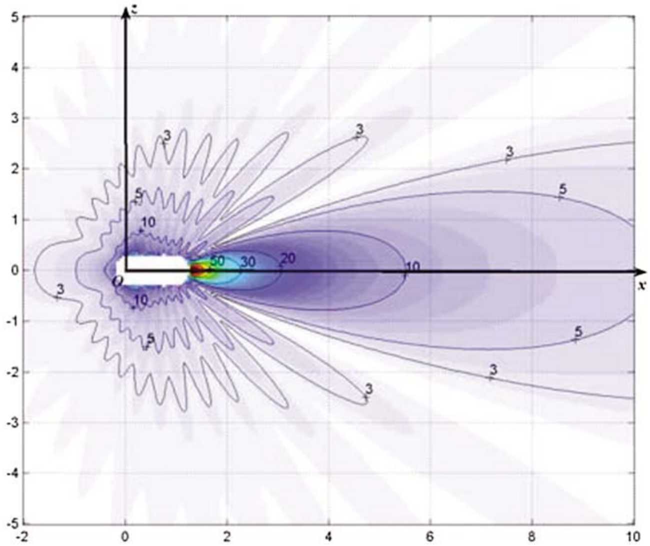

Fig. 8. E-field of Yagi antenna calculated by the economical model at 1400 MHz.

7. Choice of the number of cells and modes

The choice of the number of cells and the number of modes for each cell is an entangled problem. In fact, the solution of the inverse problem is non-unique depending on the model precision. From a physical point of view, the cells have to cover the entire physical antenna. However, the size of each cell depends on the number of modes. One may use the heuristic relationship:

$$N = kr \tag{4}$$

where N is the truncation order of the series expansions in Eqs. (2) and (3) and r is the radius of the minimum sphere surrounding the unit cell. The total number of modes is $2N(N+2)$. Consequently, the number of the modes can be reduced by increasing the number of cells. This enables us to determine the propagating fields at closer distances from the antenna.

8. Conclusion

The spherical harmonics rotation theory was applied to translate a given elementary cell along an arbitrary direction. Our synthesis method was extended to analyze antennas which are not formed as an array along the z-axis. The 25-element Yagi antenna was chosen as simple case to test the ability of our method to synthesis antennas which are designed as the classical base station antennas. An economical model based on equivalent sources was also developed. With this model only little information about the antenna is needed to get a very powerful and efficient model. These source models coupled to ray-tracing or ray-launching techniques will enable us to realistically simulate antennas in their environment. The near field can be obtained including the interaction with the surrounding objects. Finally, the source model is derived from measurement which induces a realistic model.

References

[1] IEEE Std C95.1-1991, IEEE standard for safety levels with respect to human exposure to radio frequency electromagnetic fields, 3 kHz to 300 GHz.
[2] A. Karwowski, Comparison of simple models for predicting radiofrequency fields in the vicinity of base station antennas, Electronics Letters 36 (10) (May 2000) 859–861.
[3] R. Faraone, X. Tay, K. Joyner, Q. Balzano, Estimation of the average power density in the vicinity of cellular base-station collinear array antennas, IEEE Trans. on Vehicular Technology 49 (3) (May 2000) 984–996.
[4] Z. Altman, B. Begasse, C. Dale, A. Karwowski, J. Wiart, M.F. Wong, L. Gattoufi, Efficient models for base station antennas for dosimetric analysis, IEEE Trans. Electromagn. Compat. 44 (2) (2002) 588–592.
[5] Y. Adane, A. Gati, M.-F. Wong, C. Dale, J. Wiart, V. Fouad Hanna, Optimal modelling of real radio base station antennas for human exposure assessment using spherical-mode decomposition, IEEE Antennas and Wireless Propagation Letters 1 (2002) 215–218.
[6] J. Hald, F. Jensen, F.H. Larsen, in: J.E. Hansen (Ed.), Spherical Near-Field Antenna Measurements, Peregrinus, London, 1988.
[7] A.R. Edmonds, Angular Momentum in Quantum Mechanics, Princeton Univ. Press, Princeton, NJ, 1974.
[8] R.M. Fishenden, E.R. Wiblin, Design of Yagi aerials, in: Proc. IEE, vol. 96, January 1949, pp. 5–12.