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Shared issues of wavefield inversion and illustrations in 3-D diffusive electromagnetics

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Abstract

Electromagnetic non-destructive evaluation of complex objects means that one has to decipher data which result from their interaction with imposed sources. This task is crucial in civil, environmental and medical engineering, to quote obvious fields, as well as for safety and reliability of industrial processes of various kinds in key energy and transportation sectors, for example. This short contribution does not attempt to review the huge variety of themes and the many applications of the *science of inversion*, but aims at emphasizing a number of points that seem common enough to this science to be worthwhile to be reviewed. Two illustrations and a few main references thought of good interest among the ever increasing literature are given. ***To cite this article: D. Lesselier et al., C. R. Physique 6 (2005).***

Résumé

Questions partagées en inversion des ondes et illustrations en régime de diffusion de l'électromagnétisme 3-D. L'évaluation électromagnétique non-destructive d'objets complexes impose que nous soyons capables de déchiffrer les données qui résultent de leur interaction avec des sources imposées. Cette tâche est cruciale en ingénierie civile, environnementale et médicale, pour nommer d'évidents domaines, aussi bien que pour la sécurité et qualité de processus industriels variés dans des secteurs clés de l'énergie et des transports, par exemple. Cette brève contribution ne fait pas revue de la considérable variété de thèmes et des nombreuses applications de la *science de l'inversion*, mais tente de mettre en avant un certain nombre de points qui semblent suffisamment communs à cette science pour être rappelés. Deux illustrations et quelques références principales estimées de bon intérêt parmi la littérature toujours croissante sont données. ***Pour citer cet article : D. Lesselier et al., C. R. Physique 6 (2005).***

Keywords: Wavefield; Inversion

Mots-clés : Champs et ondes ; Inversion

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Qui dit inversion dit synergie d'outils d'analyse théorique, de modélisation, de simulation, d'inversion, et de mesure. Cette mise en synergie doit être validée, éclairée, initiée par la situation terrain, et nécessite clairement des ponts convenables entre disciplines : mathématiques appliquées, théorie, modélisation et simulation (électromagnétiques et élastiques), traitement du signal et de l'image, mesure et instrumentation, science des matériaux, mise en œuvre de dispositifs, etc. Ceci implique une compréhension appropriée de la physique de l'inversion : nous sommes ainsi confrontés à des structures naturelles et artificielles de possible grande complexité physique et géométrique, affectées d'endommagements, de perturbations, d'hétérogénéités voulues ou recherchées, les uns ou les autres étant variés et souvent évolutifs, les structures étant observées par des sources et capteurs non simples. Ceci impose naturellement de déchiffrer des signaux complexes afin de les rendre intelligibles et pertinents par le développement d'inversions élaborées, robustes, rapides, générales ou dédiées, validées en prenant en compte la réalité des moyens d'inspection, sachant qu'un individu au final implémente l'inversion et décide de son résultat, ce qui exige tant aide à la décision appropriée que retour d'expérience. Le tout prend en compte par exemple des problématiques environnementales et on fera typiquement (mais non exclusivement !) référence aux caractérisations de sous-sols (ce qui inclut la caractérisation des interfaces et matériaux), et d'objets enfouis (mines ou munitions non explosées, canalisations, containers, fondations de pylônes et structures, cavités, failles souterraines, réseaux hydrographiques, pollutions, etc.), impliquant des interactions électromagnétiques avec des objets complexes, artificiels ou naturels, affectant des environnements naturels complexes (terrestres, voire planétaires). On se préoccupe aussi de manière semblable des questions d'évaluation non destructive de structures métalliques ou composites de l'industrie ou du génie civil, ce qui implique outils de modélisation et simulation mis en jeu dès le design tant des pièces élémentaires que des structures, afin d'en réduire le coût, accroître l'efficacité, atteindre une meilleure sûreté de fonctionnement, assurer une meilleure prédiction de maintenance. Nombre de travaux d'inversion des ondes existent, et il est vain de prétendre en faire un tour exhaustif. Cette contribution se borne donc à dégager des grandes lignes d'approche en harmonie avec ce qui précède, proposant ensuite deux exemples illustratifs, via travaux des auteurs et collègues alors considérés dans un contexte académique et pré-industriel. Référence est faite si et comme de besoin à des contributions principales estimées de bon intérêt parmi la littérature considérable et toujours croissante.

1. Introduction

The general idea, when aiming at the non-destructive evaluation of complex objects hidden within a complex and maybe partially unknown or uncertain environment from a collection of data resulting from their interaction with probing signals driven by known sources, is that this collection contains encoded information on such objects; the inversion is the deciphering procedure by which one is transforming this collection so as it yields pertinent and intelligible information, like an image of parameter distribution in a search space, a surface contour on which some condition is fulfilled, or any other set of characteristic factors that should be of interest to the user.

For example, one may wish to identify a denumerable set of heterogeneities from a few measurements; such factors as limited aspect of the configurations of illumination and reception (e.g., sources and receivers are above an interface and the zone to be searched lies beneath it), and as the limited amount of frequency coverage (e.g., the highly restrictive case of one operation frequency) could be essential.

This identification clearly requires a combination of tools of analysis, modeling, simulation and inversion itself, within a thematic which one can denote as *imaging and inversion—interaction of wave fields and complex structures*, which lies at the heart of the preoccupations of the French and International *wave community*, as is illustrated by the present CNFRS special issue of the Comptes-Rendus.

It goes without saying that this has critical applications in civil engineering and environment, e.g., characterization of perturbations (artificial and natural) of Earth and possibly of planetary subsoils, including their interfaces and their constitutive materials, and the one of buried objects possibly found therein (mines and UneXploded Ordnance—UXO, pipes, containers, foundations, cavities, hydrographical structures, polluted zones, etc.). The above has also obvious parallels and/or extensions to non-destructive evaluation and testing in industry (energy and transportation sectors, notably), e.g., in view of the characterization of metal and composite parts and structures affected by various damages, and to (bio)medical imaging and engineering, e.g., in order to assess anomalous biological objects of and at various scales. Whilst the issue is by no means limited to electromagnetic interrogation, as is exemplified in underwater acoustics by the special section [1] and for material testing at ultrasonic frequencies by the special section [2].

To claim to be able to tackle in a few pages and even in marginal fashion the extraordinary wealth of worthwhile topics, from pure mathematical issues to instrumentation in industrial practice and vice-versa, and the many existing and potential applications of the *science of inversion*, would be preposterous. (The far sighted and deeply expert analysis of Sabatier in [3] and its afterthought [4] are, however, proofs one may succeed in the endeavour at least when focusing onto the upscale aspects.

Let us also mention the clever enumeration of open problems in inverse scattering recently performed in [5]. With emphasis on inversion *at large*, let us also not forget a reference volume published earlier under the patronage of the Office Français des Techniques Avancées [6].)

So, in line with the oral contribution which this short paper is originated from, one has settled here to merely stating a number of points that seem to be shared enough by scientists involved in the field for being worthwhile to be reminded.

However, to help us to connect between such rather general points and practical research issues, two sets of problems are briefly considered, both involving full three-dimensional scattering configurations and being originated from industrial preoccupation, and briefly illustrated—with no equations, specialized papers referenced in due time providing them well enough. In addition, a few general references (books, review papers, special sections of journals) which the authors believe of good interest among the already said huge and ever increasing literature are provided for the sake of completeness (including those already introduced).

2. Examination of shared and pending issues in wavefield inversion

From our experience, four main questions arise, which are listed below:

– Managing complexity: it is necessary to construct hybrid models of appreciation of the real world, integrating uncertainties, and to look for accurate representations of the physics of the interaction both as reference and support of the analysis.
– Near field: it has to be engaged whatever the diversity of scales and of applications, and beyond the traditional disciplinary barriers (as exemplified in between electromagnetics, photonics, acoustics and elasticity).
– Multi-scale and multi-physics: one has to consider both individual objects and structured ensembles and aggregates of individual objects, and the systems within which they are embedded, and this task should cover a vast range of scales, involving various physical phenomena considered independently or in combination.
– Modeling, simulation, and high-performance computation: this should enable us to get a precise and robust numerical model of the most probably multi-dimensional and vector (dyadic) field interaction considered, and should lead to better identification (of objects), better synthesis (of structures), and better use (of probing devices).

Yet, it is also indispensable to be able to carry out fast, optimal (minimal) and reproducible measurements on non-canonical geometrical domains in order to characterize complex systems and components.

In managing the above four issues, plus the measurement one just mentioned, three hard tasks are to be faced, now listed:

– To understand the physics of the inversion: it is necessary to handle natural and/or artificial structures of possibly large physical and geometrical complexity, affected by various damages, perturbations, heterogeneities, probed by complex sensors.
– To conceive acquisition systems: they should be electronically smart, most certainly multi-element (1-D and 2-D arrays, planar, conformal), and could be using new GMR, GMI or SQUID-based sensors (those in the eddy-current field at least), etc., all of them (and technologies behind) having to be compatible with mass production (notwithstanding the always precious niche applications) and having to be useful both in the laboratory and in the field.
– To decipher complex signals: be they general or dedicated, the inversion tools must be robust, fast, and validated by accounting for the reality of the inspection means. This implies decision making, based on appropriate rules and support systems, and on proper user's feedback, since, in fine, someone is implementing the inversion and is acting upon its result.

Evidently, proper integration of several simulation tools possibly involving quite different models, as already said, should enable us to achieve what might be denoted as a *numerical mock-up*, notably in the case of the testing of artificial structures; this is exemplified by the CIVA multi-method platform (dedicated to eddy currents and ultrasonic fields at this stage) developed at CEA. Consequently, modeling and simulation instruments should be made available and should be used since the design of the elementary part, of the structure in which it is embedded, and of the sensors employed for testing the said part or structure. Reduced economic costs, increased reliability, improved safety, better maintenance prediction, etc., are expected. To the above, there is a sine qua none condition however: the synergy of theoretical analysis, modeling and simulation, inversion and measurement tools, validated, inspired, initiated by the situation in the field, and involving many disciplines, e.g., electrical engineering evidently, applied and computational mathematics and mathematical physics, Earth sciences, geophysics, remote sensing, civil engineering, material physics, ...

In passing, one might question whether or not it is possible to put forth a generic solution method for inversion? This might be the construction, step by step, starting from a single *bubble* (large or small, issued from back-propagation, or simply hypothesized) set within a prescribed search domain in space, of one or several bubbles (of unknown number, small or extended ones, with simple or complicated shapes, distributed throughout the domain or aggregated, heterogeneous as well), so as a certain criterion be satisfied.

The latter usually involves two results: (i) Reaching the least value of a residual of an observation equation; a misfit between observed data and those computed via the model should be minimized somehow; (ii) Ensuring a compatibility; this itself involves reaching the least value of a residual of a state equation (i.e., the fields should satisfy the Maxwell PDE or integral formulations derived from them, and the necessary boundary conditions), plus proper regularization inspired from prior mathematical or physical knowledge on the solution and fields involved, and from the data structure also.

Naturally, the inversion is doable as far as direct modeling tools are powerful enough (with the traditional and often overinflated controversy between fast, dedicated Green-based approaches and general, yet slower, finite-element methods, see, e.g., [7], and the present-day or potential use of hybrid combinations between brute-force and asymptotic approaches of various kinds. However, the analysis so far leaves aside the question whether or not an exact copy of the sought object is indeed necessary; i.e., the quest for a simplified object often appears worthwhile, and it might even constitute the only realistic aim. Whereas the ability of inversion tools to be ready or even to be required by the end users remains open (the signal is still often seen as the solution ...).

3. On the exploration of generic subsoil structures via low-frequency (induction) models

The exploration of the Earth subsoil, at high depths (beyond about a few hundred meters) in view of mining applications, or at lesser depths (say, in the meter to decameter range), in view of military, civil engineering and environmental applications, assumes the ability to handle the response of complex objects (complex by geometry or by physics, or by both) in a frequently variable and poorly known medium.

The authors and colleagues have carried out a range of works on the modeling and numerical simulation of electromagnetic fields due to homogeneous, simply-shaped, three-dimensionally bounded objects, these fields being induced by sources placed in their neighborhood and radiating at frequency low enough to consider that the phenomenon is mostly of the inductive type. The main goal at this time is to interpret in robust and fast fashion the magnetic fields recorded in a frequency range of a few dozens Hz to a few hundred Hz by a probe (like the three-component magnetic ones, ARLETT and BORIS, developed these recent years by BRGM) displaced along a borehole in order to infer characteristic features of the sought objects, the source itself being set at the surface, in another borehole nearby or in the same one. Further investigation has now been started on equally demanding situations in which the (natural, artificial) objects to be characterized are hidden at shallow depths and both sources and receivers are displaced above the ground, and operated (with specific technological challenges and new electromagnetic models) at usually higher frequencies (a few dozens kHz to a few MHz).

The main idea beyond this scenario is that, due to lack of knowledge on the complex subsoil environment and scarce collected data in many situations (particularly in mining cases), in addition to the pressing need to get a quick assessment of the objects involved, one has to figure out simple approaches of the electromagnetic interaction and of the three-dimensional field inversion, with respect to direct approaches based on a fine division of the objects into voxels of sub-wavelength sides (or sub-skin-depth in view of the induction phenomena) and reconstruction of the electrical parameters of those voxels via iterative solution methods (refer to a number of specialized publications in [8,9] for more on this subject) at often high computational price.

Research in that direction, led by the authors and colleagues within a cooperative endeavor involving ICEHT-FORTH Patras and BRGM since a number of years, follows two lines of investigation, each one tailored to our prior knowledge on the order of magnitude of the static conductivity contrast between the object to retrieve and the embedding subsoil. (The dielectric permittivity contrast matters only near the MHz range, magnetic permeability at this stage is assumed to be the same as free space, which is a realistic assumption, with the exception of some metallic ores and, unfortunately, of most UXO.)

For small or average contrasts (say, less than a hundred), the object is penetrable and the secondary fields observed outside appear as induced by fictitious electrical currents that are distributed throughout its volume, these currents being the result of a depolarization tensor acting on the primary electric field at same location—this approach is based on the so-called localized nonlinear approximation and variants, often termed as the extended Born approach, all involving domain integral formulations and dyadic Green's function machinery. Though this tensor can itself be computed by brute force via volumetric voxel decomposition, one is expressing it via Rayleigh-like, low-frequency series expansion in powers of jk, k complex wave number in the embedding medium, j the imaginary unit. The first three terms (integer powers 0, or the static component, 2 and 3) have been exhibited for a general ellipsoid and its degenerate counterparts (prolate and oblate spheroid, sphere); the secondary field for a magnetic dipole source or a circular electric loop (a usual model of a source in the considered frequency range) follows [10,11]. The whole analysis has been validated by a method of moments or standard Mie expansion (for the spherical case in homogeneous space).

For large contrasts (those may reach millions and beyond) one conversely assumes that the object is impenetrable due to infinite conductivity. Once established a closed-form low-frequency expansion of the primary field (again in the specific yet standard hypothesis of a magnetic dipole source) the (magnetic) field exterior to the object is constructed via a succession of solutions of boundary value problems (homogeneous and inhomogeneous Laplace solutions) coupled in ascending fashion to one another, each one yielding one term of the low-frequency series of the sought field; this term is itself a closed-form series expansion involving vector or scalar spherical harmonics (spherical case) or scalar ellipsoidal harmonics (ellipsoidal case) [12,13]. The real part (in-phase) of the field is observed to be well reproduced by the static term (power 0 in jk). The imaginary part (quadrature) appears mostly provided by the power 2 in jk, with possibly a small yet useful contribution of the third-order (power 3 in jk). This calculation has been fully validated at this time only for a spherical scatterer. In contrast, the ellipsoidal shape has led to excellent static solutions but poor ones for the quadrature term. As for the spheroidal case it is amenable to complete expansions using scalar spheroidal harmonics, and is presently under study.

To illustrate how the above analyses may end up in useful inversion, one displays in Fig. 1 results inferred from Géo-Nickel data at 1120 Hz collected on a Finnish mining site and courtesy of BRGM ([12], in short a large current source is placed on the ground and a 3-component magnetic probe is moved along one single borehole as schematically displayed). Only the in-phase part of the field is used for our inversion purpose. An ellipsoid, whose reliability has been duly validated by the end user and which provides a very small data misfit on every secondary field component, is retrieved. By the way, it suffices to produce its center, three semi-axis lengths and three Euler angles, the underlying model being the approximate

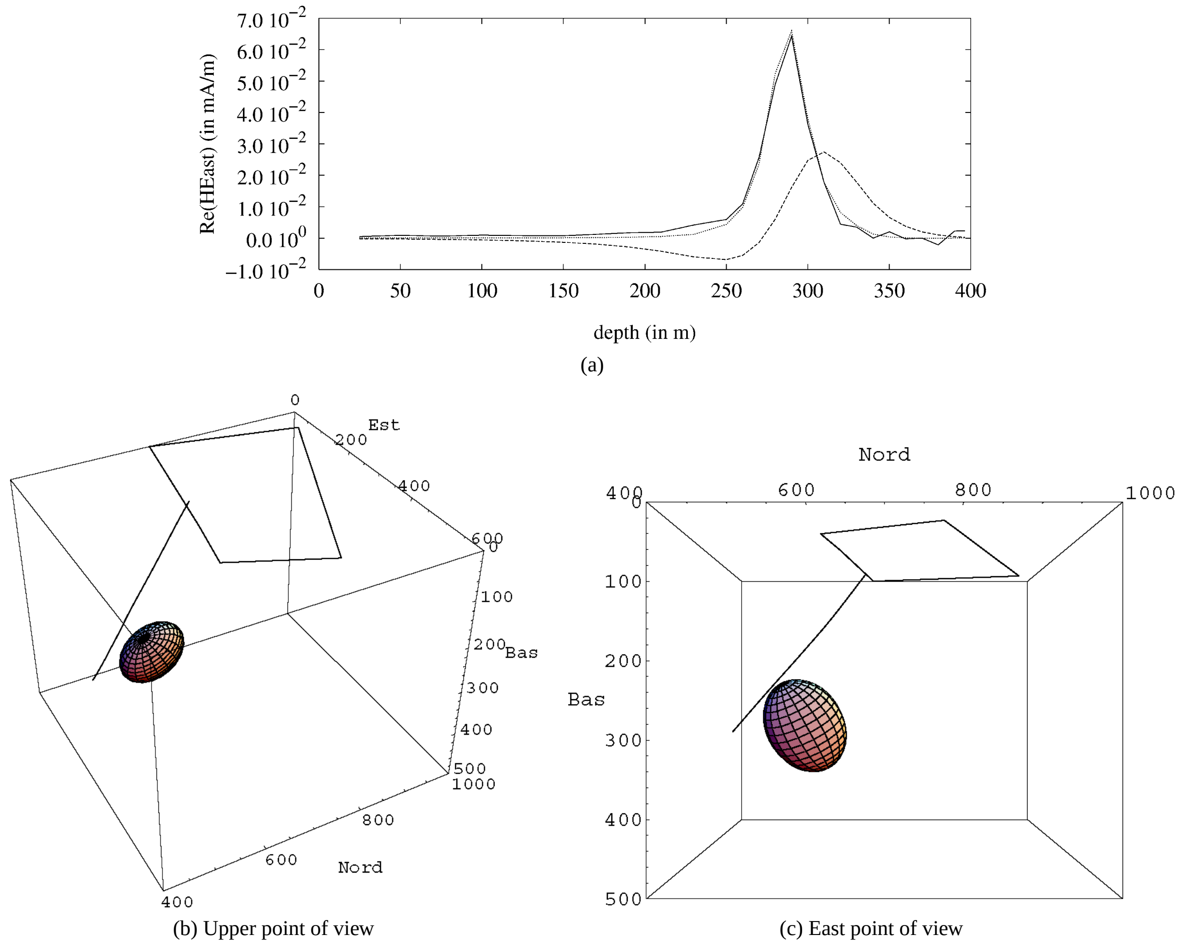

Fig. 1. Exploration of the Earth subsoil (see text for further details). The in-phase part of the so-called East component of the secondary magnetic field along the borehole is shown in (a) as a function of the depth of investigation: experimental data (solid line) acquired on a well-known mining place at 1120 Hz, courtesy of BRGM, are compared to the static secondary field attributed to the best equivalent objects of spherical (dashed line) or ellipsoidal (dotted line) shape illuminated by a vertical magnetic dipole. In (b) and (c), one shows two views of the mining configuration with the nonvertical borehole, the large trapezoidal loop lying on the ground, and the equivalent ellipsoidal object found after inversion.

low-frequency one (with a dipole source instead of the large trapezoidal loop sketched in the figure), which task is fast and rather easy to achieve via a quasi-Newton code. Let us then notice that a properly small data misfit cannot be obtained by a sphere model; in short, the simplified object must account for elongation and orientation, in particular meaning field depolarization.

4. Non-destructive evaluation—NdE—of metal structures at eddy current frequencies

The non-destructive evaluation of artificial structures is led by electromagnetic means at eddy current frequencies (as previously, one is in the induction domain save low-MHz analyses of less conductive composite structures), such works being inserted within an academic and (pre)industrial framework in evaluation and testing. As already underlined this type of problem is of recognized industrial and societal importance. It aims at optimal and safe design and use of industrial systems and processes, in energy, transport and civil engineering domains in particular. It requires the understanding, optimization and application of pertinent diagnostic tools, and the impact is felt from early design of complex parts (without waiting for the manufactured parts) to on-site inspection of multi-component systems (most often satisfying proper inspection rules set by others), and it evidently requires to work on the effectiveness of the sensor technologies themselves since it goes without saying that lack of good data impedes inversion success, and at the end affects economic performance and/or system operability.

The field of electromagnetic (and elastic) NdE at least in the academic world seems well-covered by a recent special section [2] which one should refer the reader to, whilst a general analysis of eddy current formulations and inversion thereof is proposed in [7]. The aforementioned CIVA platform, and related investigations like those led within the 2003–2005 European consortium VERDICT, *Virtual Evaluation and Robust Detection for engIne Component non destructive Testing*, provide a good example (which we believe are not yet that frequent, but by no means unique) of industrial software achievements obtained in synergy. Here, one will only mention one main result, which appears significant enough yet can be summarized with the least amount of mathematical material.

Development and validation of exact models and approximate ones (the latter of the extended-Born type) for three-dimensional volumetric defects and cracks in metal plates have enabled us to put together so-called binary specialized iterative contrast-source solution methods. These are based on a rigorous Green-type volumetric integral formulation of the field, and a conjugate-gradient-based method-of-moments solution thereof, involving as usual fictitious currents $\mathbf{J}$ existing in the defect domain; a binary space representation is assumed in practice, i.e., the objects sought are of contrast 1, the embedding medium is of contrast 0. Yet, since the parameters are to be evolved via gradient search, necessarily meaning derivability of both data and state misfits, this binary hypothesis is *mollified* via the introduction of a secondary, real-valued parameter τ which becomes the object descriptor whose unknown finite value is sought within every voxel (centred at $\mathbf{r}$) of the prescribed search domain; τ is such that when $\tau \lesssim 0$ the color of the corresponding voxel is close to white (i.e., there is no damage at this location) and when $\tau \gtrsim 0$ that color is close to black (i.e., there is damage at this location).

Within the above framework the inversion itself is performed in a succession of two-step loops (using conjugate-gradient for minimization purpose), (i) the retrieval of $\mathbf{J}$ in $\mathbf{r}$ at fixed τ, from which the electric field $\mathbf{E}$ follows via the integral formulation hypothesized, (ii) the retrieval of τ in $\mathbf{r}$ at fixed $\mathbf{J}$ and $\mathbf{E}$, until both state and data residuals are considered to be small enough with respect to both data and model uncertainty. (Notice that the quite worthwhile case of perfectly thin cracks has been successfully modeled now via Green-type boundary integral formulations [14], yet still lies out of our reach at this stage for inversion purpose using similar methods, and is not considered here.)

An example (refer to [15], from which the example is drawn, for further details) is given in Fig. 2. Here one is preoccupied by the retrieval of either one or two parallelepiped voids (0.5 mm and 1 mm deep, fully embedded for the single one or opening in air for the pair), that are affecting a non-magnetic horizontal metal plate 2 mm thick and of 1 MS/m conductivity. The source is a thick pancake coil operated at 150 kHz which is located above the plate at close distance (1.3 mm) from it, and one is assuming the availability of samples of the vertical magnetic field 1.55 mm above, each 0.1 mm (availability which is certainly a challenge per se in terms of eddy-current instrumentation, and in particular claims for multi-element, high-resolution, high-sensitivity sensors).

As expected, in view of the poor resolution which the skin effect allows (the skin depth in the plate is about 1.27 mm at the operation frequency), the results are not superb, yet it is possible to separate and locate the voids in meaningful fashion, with good improvement if enough iterations are carried out (notice that the inversion could be improved by using several frequencies of operation). Evidently, each iteration loop is itself computationally costly (and here one is carrying out 4500 of them though one could settle for far less, of the order of 500, if only a quick appraisal was needed), yet pre-calculation of all samples of the Green dyads involved and their proper storage means that only discrete summations, involving fast Fourier transforms to a good extent in order to deal with convolution products whenever those are arising, are to be performed.

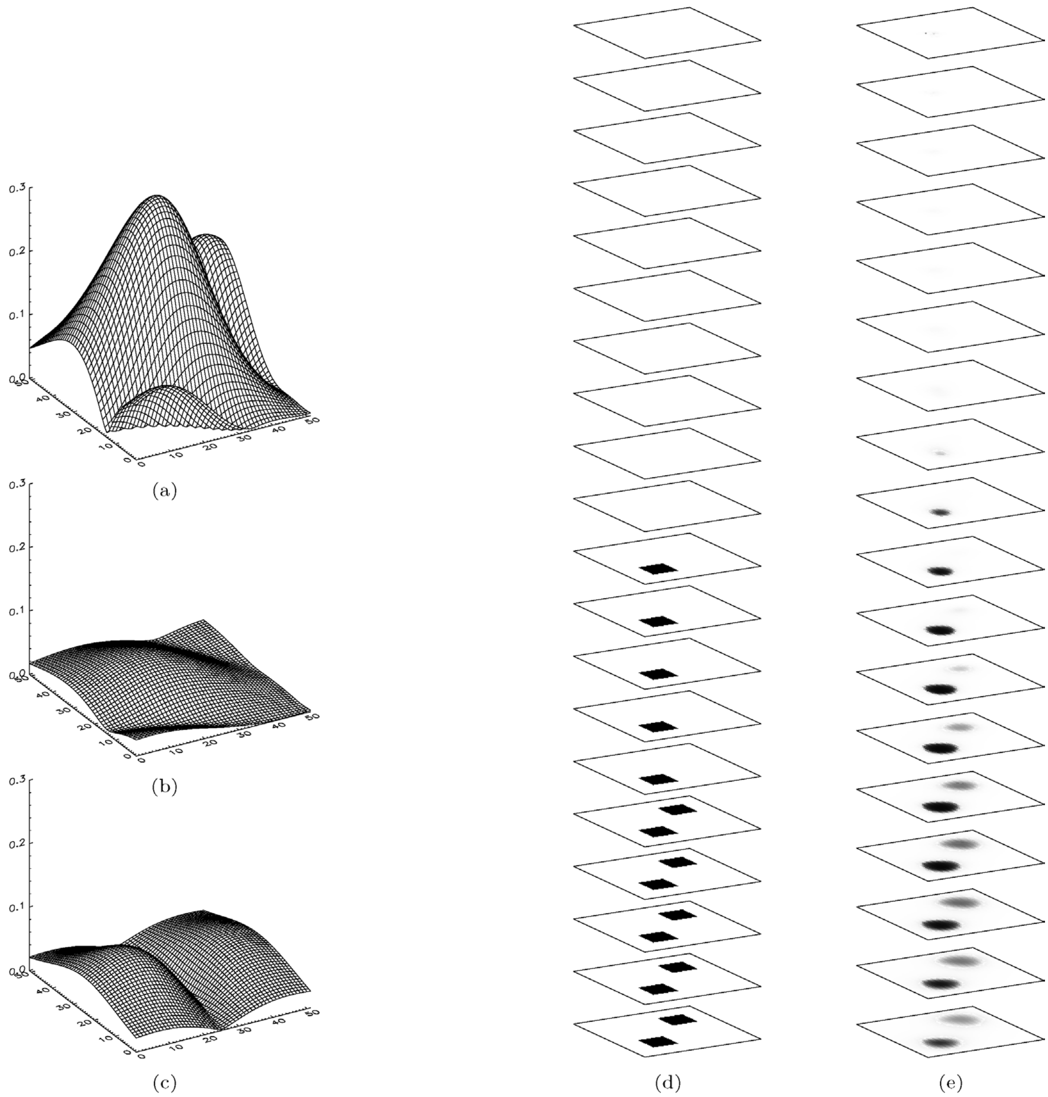

Fig. 2. Eddy-current Non-destructive Evaluation of a metal plate (see text for further details). Amplitude of the vertical component of the anomalous (secondary) magnetic field collected at 150 kHz on a planar surface parallel to the plate for three cases: (a) two parallelepiped void defects opening in air at the top (with respect to the probe location) surface of the plate, (b) the same two defects opening in air at the bottom of the plate, and (c) a single parallelepiped void defect fully embedded in the middle of the plate. Cross-sectional grey-level maps of the conductivity contrast at discretized depths retrieved from the vertical component are proposed for defect (b) only. Exact defect maps are given in (d) for comparison. Results are shown at iteration 4500 (e).

5. Conclusion

To conclude, one certainly needs to put together, today more than yesterday, novel methods which are mathematically sound and which account for and have knowledge of the form and limitations of datasets. It is expected that these methods should often integrate pertinent approximations of the fields involved, at least at some levels in the models, then yielding most probably hybrid formulations involving, for example, generalized polarization tensors of the objects modeled (one can refer to [16] for a corresponding high-end analysis) – these formulations then involve several orders of development, as it is well known already about the low-frequency analysis illustrated in the above. One key question is how to put up with coupling between objects and/or with coupling of the objects to the interfaces of the embedding media, be this coupling weak or strong. Also, the solution algorithms should be expected to work in robust manner in the presence of uncertainties on the embedding environments, be those expressed by clutter or by slow variations, and/or even though the physical parameters of these environments are partially unknown (e.g., the general shape of the pertinent Green dyads is expected to be known, their details may not). Finally, fast numbering and localization of the objects, and, to some extent at least, evaluation of shapes and orientations of objects (e.g., via polarization tensors) equivalent to the said objects, and hopefully some quantification of their electromagnetic parameters, are issues which already are highly challenging in many canonical situations [17], the solution of which in real-world cases should be already a giant step forward in terms of practical use and benefits.

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