Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
C. R. Physique 6 (2005) 663–674
COMPTES RENDUS PHYSIQUE
http://france.elsevier.com/direct/COMREN/

Interaction of electromagnetic fields with the environment/Interaction du champ électromagnétique avec l'environnement

Diffuse reflection by rough surfaces: an introduction

Michel Sylvain

Laboratoire ESYCOM, université de Marne-la-Vallée, bâtiment Copernic, 5, boulevard Descartes, Champs sur Marne, 77454 Marne-la-Vallée cedex 02, France

Available online 8 September 2005

Abstract

In this introductory paper, we present with some details the (mathematically) simplest methods proposed to compute the electromagnetic field scattered by a rough surface separating two homogeneous media. These methods remain largely used both in propagation and remote sensing problems. The methods described in the paper are:

– the geometrical optics approximation, in which the wave is considered as a set of rays obeying the laws of reflection and refraction;
– the small perturbation method, due to Rayleigh and Rice, in which the field is given as an expansion on a set of elementary harmonic plane waves, the coefficients of which are determined so as to satisfy the boundary conditions;
– the Kirchhoff approximation, in which the field is given as an integral on the rough surface; in this method one needs to know some components of the field on the surface, and an approximation is substituted to the unknown true value.

We end with a short discussion of some problems not adequately solved by these methods, namely self-shadowing, multiple scattering and some inadequacies of the Gaussian model for random surfaces. ***To cite this article: M. Sylvain, C. R. Physique 6 (2005).***

Résumé

Réflexion diffuse par les surfaces rugueuses : une introduction. La réflexion (et la réfraction) partielle d'une onde électromagnétique à l'interface plane entre deux milieux semi-infinis homogène est un des rares problèmes pour lesquels on connaît une solution analytique rigoureuse. En pratique les surfaces de séparation entre milieux naturels (surface du sol, surface de la mer, murs et parois, etc.) présentent des irrégularités : ce sont des surfaces rugueuses. La réflexion par une telle surface est diffuse : l'énergie n'est plus renvoyée dans une seule direction (la direction spéculaire) mais diffusée dans toutes les directions. Le calcul du champ ainsi diffusé n'a pas de solution analytique rigoureuse et un grand nombre de solutions ont été proposées pour en donner des solutions approchées. Dans cet article introductif nous présentons de façon relativement détaillée les plus simples (du point de vue mathématique) de ces méthodes qui, du fait de cette relative simplicité, restent encore largement utilisées, tant dans les problèmes de propagation que de télédétection, puis nous discutons rapidement certaines de leurs limitations. La méthode la plus simple est l'utilisation de l'optique géométrique : on représente l'onde incidente par des rayons que l'on suit au cours de leur propagation, chaque rencontre avec la surface rugueuse donnant un rayon réfléchi et (le cas échéant) un rayon réfracté. Il s'agit comme toujours dans les méthodes de rayons d'une approximation haute fréquence. Dans la méthode la plus ancienne, celle des petites perturbations, on se donne le champ diffusé a priori comme une superposition d'ondes harmoniques planes et on identifie les coefficients de ce développement pour satisfaire les conditions aux limites. La convergence de la méthode n'est assurée que si la hauteur et la pente des irrégularités de surface sont assez petites. Dans l'approximation de

E-mail address: michel.sylvain@univ-mlv.fr (M. Sylvain).

1631-0705/$ – see front matter
doi:10.1016/j.crhy.2005.06.014

Kirchhoff (ou de l'optique physique) le champ en tout point est représenté comme une intégrale de surface prise sur la surface rugueuse, en tout point de laquelle on doit connaître certaines composantes du champ et/ou de son gradient : l'approximation s'introduit dans le remplacement de ce champ inconnu par une approximation, le champ que donnerait la réflexion sur le plan tangent à la surface. Parmi les phénomènes dont la prise en compte demande des méthodes plus élaborées, on peut citer les effets d'ombrage et de diffusion multiple, ainsi que la modélisation correcte des surfaces réelles aléatoires qui n'est souvent pas en accord avec le modèle gaussien, modèle prédominant dans les analyses théoriques. ***Pour citer cet article : M. Sylvain, C. R. Physique 6 (2005).***

Keywords: Electromagnetic field; Rough surface scattering; Gaussian model

Mots-clés : Champ électromagnétique ; Réflexion par des surfaces rugueuses ; Modèle gaussien

1. General

The solution of an electromagnetic problem always consists in solving Maxwell equations with appropriate boundary and initial conditions: it supposes in particular that the medium electrical characteristics are everywhere known (here we consider only nonmagnetic media characterized by their (complex) permittivity).

In propagation problems, one generally looks first for mathematically simple field waves able to propagate independently from the sources of the field and giving the solution of a particular problem by linear superposition (for instance, plane harmonic waves are these general solutions in an homogeneous and isotropic medium).

Problems leading to an analytical solution suppose a simple medium and a simple geometry and are very few. In more realistic situations, one has to resort to approximations or to numerical methods, the latter being restricted to such a limited volume in space generally insufficient for propagation problems. Each approximation has its own domain of validity, a determinant parameter being the ratio between the wavelength and the dimensions of the encountered obstacles (the scatterers): quasi-stationary solutions, as Rayleigh diffusion, are thus valid when the scatterers are much smaller than the wavelength, and high frequency approximations derived from optics when they are much larger.

In this paper we summarize the simplest approximations used in the analytical study of scattering by rough surfaces. We limit our discussion to the case of a rough surface consisting of height fluctuations $z = h(x, y)$ around an averaged plane ($z = 0$) and separating two semi-infinite homogeneous and isotropic media (the last hypothesis is to eliminate the occurrence of volume scattering). One of the media is air or vacuum.

2. Qualitative considerations

2.1. A reference solution: specular reflection from a plane

The reflection and refraction of a wave by a plane interface is one of the problems for which we have at hand an analytical solution. If an harmonic plane wave propagating in medium 1 (the incident wave) encounters a plane interface separating this medium from medium 2 (Fig. 1), both media being homogeneous and isotropic, two new waves are produced, one reflected into medium 1, the other transmitted (or refracted) into medium 2. These two waves are also plane harmonic, with the same frequency as the incident one. Their directions of propagation are given by Snell–Descartes laws and their amplitudes by Fresnel formulas. It appears that the reflection and transmission coefficients are functions of the permittivities of both media (therefore generally of the frequency), the incidence direction and the incident wave polarization. In particular, if the incident electric field is not in the plane of incidence (TM polarization) neither perpendicular to it (TE polarization) the reflected and transmitted waves have not the same polarization as the incident wave and are elliptically polarized.

Let us now consider the same problem from the point of view of geometrical optics. The waves are now represented by rays, which are orthogonal to the wave surfaces (according to the Malus and Dupin theorem) and represent the trajectories followed by the radiated energy. At an interface they are reflected and refracted according to the Descartes laws, a result extended to curved surfaces (if their radius of curvature is much larger than the wavelength) by simply substituting to the surface its tangent plane at the point of incidence. The fact that rays and plane waves obey to the same laws of reflection and transmission is a milestone on the way leading from Maxwell equations to geometrical optics and is the reason why Fresnel formulae are used in geometrical optics. When an incident ray gives a single reflected ray in a specified direction, the reflection is specular.

A physically more intuitive understanding is delivered by physical optics. According to the Huygens–Fresnel principle, each point of a wave front (or more generally each point reached by the incident wave) may be considered as the source of a secondary

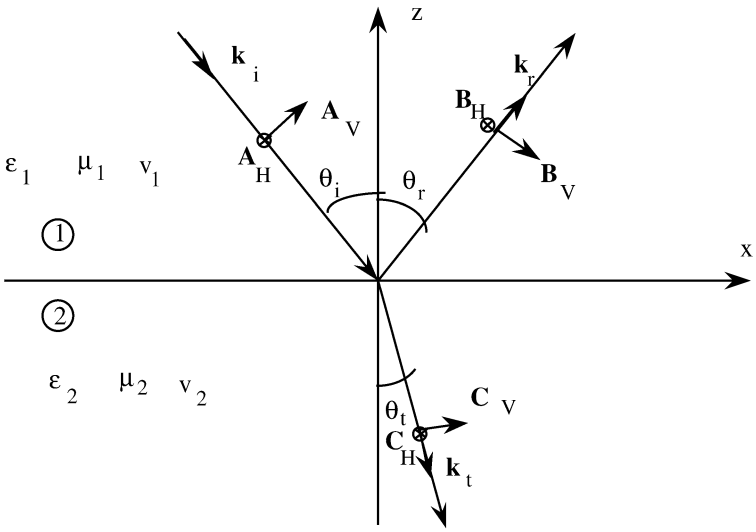

Fig. 1. Reflection and refraction of a plane wave by a plane discontinuity.

Fig. 1. Réflexion et réfraction d'une onde plane par une discontinuité plane.

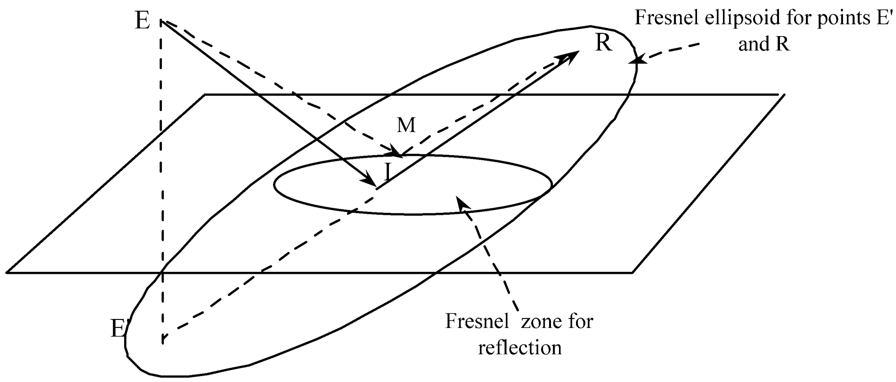

Fig. 2. Fresnel ellipsoid and Fresnel zone for reflection.

Fig. 2. Ellipsoïde de Fresnel et zone de Fresnel pour la réflexion.

spherical wave. When an incident plane wave interacts with a plane, all these secondary waves interfere constructively only in the specular direction and destructively in all other directions.

In real situations, a reflective plane has not an infinite extent. It can be shown that the previous results remain valid if the reflective region covers entirely what is called the Fresnel zone, the inside of the ellipse given by $\mathrm{EM} + \mathrm{MR} = \mathrm{EI} + \mathrm{IR} + \lambda/2$ (Fig. 2). The size of the Fresnel zone decreases with the wavelength; in optics, only the immediate surrounding of the point of incidence needs to be considered (principle of locality).

2.2. *Diffuse reflection*

An experimental approach to diffuse reflection can be gained from observations in visible light. With a polished plane light is specularly reflected and gives an image of the source. When the rugosity of the plane increases, one observes that the specularly reflected energy decreases: the energy is scattered in other directions. A wall generally diffuses visible light in all directions; the scattering is less important at radiofrequencies.

Scattering is not always a drawback. Vision is adapted to it. In radio propagation, scattering decreases the specularly reflected ray, which is generally an advantage, but it induces delayed replicas of the signal which can be detrimental to high data rate digital transmissions (Fig. 3).

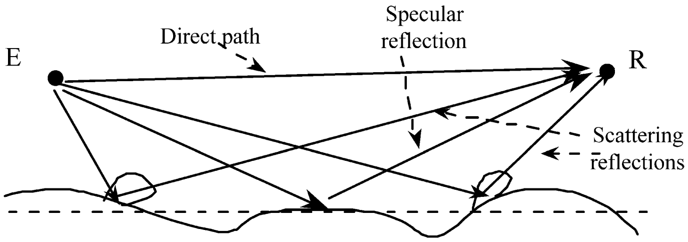

Fig. 3. Specular and scattering reflections.

Fig. 3. Réflexions spéculaire et diffuse.

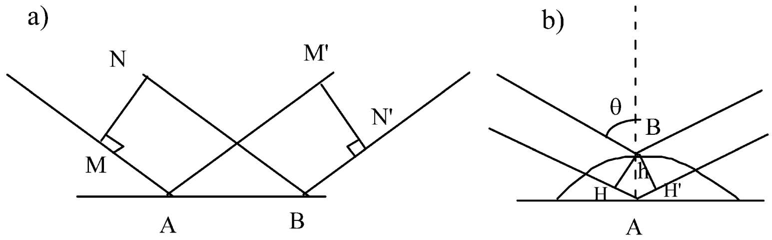

Fig. 4. Rayleigh criterion.

Fig. 4. Critère de Rayleigh.

2.3. Rayleigh criterion

The Rayleigh criterion discriminates between polished and rough surfaces. The situation is summarized in Fig. 4.

Fig. 4(a) shows the case of a polished plane: MAM′ and NBN′, the optical paths are equal, giving coherent waves in the specular direction. If an irregularity of height $\mathrm{AB} = h$ is present (Fig. 4(b)) however, these optical paths are no longer equal. Their path difference is $\Delta L = \mathrm{HA} + \mathrm{AH}' = 2h\cos\theta$, leading to a phase difference $\Delta\Phi = 2\pi\Delta L/\lambda = 4\pi h\cos\theta/\lambda$. If this difference is small enough the surface can be considered as polished. If it becomes large, the surface must be considered as rough. The Rayleigh criterion uses, somewhat arbitrarily, a threshold value of $\pi/2$ considering a surface as rough if

$$4\pi h\cos\theta/\lambda \geqslant \pi/2 \quad \text{or} \quad h \geqslant \lambda/(8\cos\theta)$$

In practice, h should be some average value of the irregularities sizes on the Fresnel zone. Roughness does not appear as an intrinsic characteristic of the surface but also depends on the wavelength and on the direction of propagation of the incident wave: the effect of the irregularities of a specified surface increases with frequency and as incidence approaches to the normal.

3. How to compute the effects of roughness

3.1. The problem

The problem to solve is described in a very general manner on Fig. 5. An incident wave (wave vector $\mathbf{k}_\mathrm{i}$) interacts with the rough surface. Energy is scattered in all directions and one wishes to find (at great distance) the electromagnetic field (wave vector $\mathbf{k}_\mathrm{s}$) in the direction defined by θ_2 and θ_3 angles. Let $\mathbf{E}_\mathrm{i}$ and $\mathbf{E}_\mathrm{s}$ be the incident and scattered electric fields, i.e.,

$$\mathbf{E}_\mathrm{s} = \left[\exp(-\mathrm{i}\mathbf{k}_\mathrm{d}.\mathbf{r})/r\right]\left[M(\mathbf{k}_\mathrm{i}, \mathbf{k}_\mathrm{s})\right]\mathbf{E}_\mathrm{i} \tag{1}$$

where $[M(\mathbf{k}_\mathrm{i}, \mathbf{k}_\mathrm{s})]$ is a tensor, the scattering matrix once a coordinates system has been chosen.

The solution is easier to obtain if medium 2 is a perfect conductor. It is then possible to derive directly the electric field (orthogonal at each point of the interface) and to deduce from it the magnetic field (in the far field approximation). It is why this assumption is often made. If not, there is a transmitted field which must also be computed and the boundary conditions involve both electric and magnetic fields.

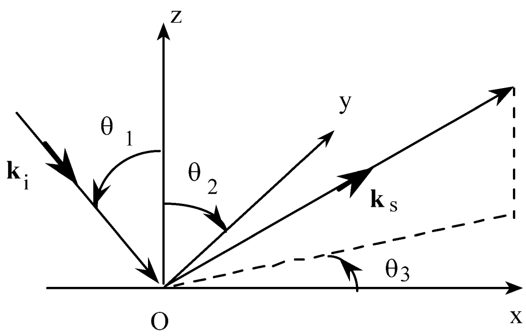

Fig. 5. Scattering geometry.

Fig. 5. Géométrie de la diffusion.

3.2. Rough surface description

According to the solution method, a more or less detailed knowledge of the surface is required. Some methods need a complete description of the surface, that is its equation $z = f(x, y)$. Such a determinist description is generally an academic model for which an analytical solution is tractable.

Real surfaces are not known with a sufficient degree of detail to be given a determinist equation. A statistical (or random) description is then adequate, and the result is a statistical description of the field, in general limited to the second order. The average field is called the coherent field, and its standard deviation is the incoherent field. If the random surface is ergodic, the field statistical averages should correspond to observations averaged over a large number of points.

Several models of random rough surfaces have been proposed. In some of them, obstacles of given shape and size are placed randomly on the plane (Twersky [1,2]). A more general method consists in specifying the probability density of z (considered as a stationary and zero average random variable) and its autocorrelation function (homogeneous and often isotropic). A second order description is limited to the knowledge of the variance and of the correlation length: this is equivalent to a complete description in the case of a Gaussian surface.

If the surface is represented by its Fourier spectrum, the coefficients of this spectrum are considered as random variables with a known joint probability distribution.

In most cases a random rough surface also exhibits temporal variations: it is particularly so in the case of the sea surface.

3.3. Solution methods

A great number of methods have been proposed to compute the reflected field from a rough surface. Probably the simplest is the geometrical optics approximation (Section 4). It is straightforward to apply but its results are not quite good (except perhaps in the visible range).

The most ancient method is the perturbation method developed by Rayleigh (1895) to study the scattering of an acoustic wave by a sinusoidal surface and extended by Rice [3] to electromagnetic waves and random surfaces (Section 5). It consists in guessing the field components as expansions over a basis of well-chosen functions and obtaining the coefficients of these developments so as to satisfy the boundary conditions.

The physical optics approximation (also called Kirchhoff approximation), very commonly used, is based on an integral representation of the field (Section 6). The field everywhere is expressed as an integral over the rough surface; the field (or some of its components) on the surface should be known, which is not the case, and it has to be estimated in some manner. We present first the scalar version, as developed by Beckmann and Spizzichino [4] from the Kirchhoff–Helmholtz integral. The method is rather simple but does not allow a direct computation of the polarization effects. We then briefly describe the vectorial formulation based on the Stratton–Chu integral formula (Stratton [5]).

4. The geometrical optics approximation

4.1. The 'shining points' method

In this method, illustrated in Fig. 6, a set of rays is associated to the incident wave. Each ray is then followed independently as it is reflected (or refracted) by the surface. For instance, to a plane incident wave is associated a parallel beam of rays. In

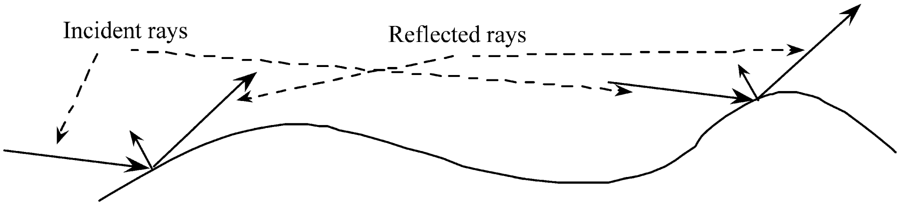

Fig. 6. Geometrical optics approximation.

Fig. 6. Approximation de l'optique géométrique.

this case all rays reflected in a given direction intersect the rough surface at points where the tangent plane has a specified orientation (given by the values of $\partial z/\partial x$ and $\partial z/\partial y$). A random description of both the densities of probability of $z = f(x, y)$ and its partial derivatives are thus needed. In the special case where z is a Gaussian random variable, it is known that $\partial z/\partial x$ and $\partial z/\partial y$ are also Gaussian variables.

4.2. The facets model

The weakness of the 'shining points' method is to use Fresnel formulas when it is not justified because the reflecting surface has not a sufficient extension. This difficulty can be overtaken by dividing the rough surface into plane facets (Ulaby et al. [6]). If a ray is reflected on a large enough facet, the Fresnel coefficients can be used; otherwise, the reflection coefficient can be modified, using the physical optics approximation, but considering only the field reflected into the specular direction and not the scattered components. In a random modelling the probability distributions of both the sizes and orientations of the facets are thus needed.

4.3. Applications

The geometrical optics approximations are very crude. In particular, they excluded backscattering at large incidences, contrarily to experimental evidence. They are, however, the only way to take roughness into consideration in ray tracing algorithms, a reason why they are often used (see, for instance, Didascalou et al. [7] for use of the shining points method, Novel et al. [8] for use of the facet method).

5. The small perturbation method

5.1. Principles of the method

One starts with a rough surface periodic in both Ox and Oy directions with period L (this period can be extended to infinity at the end of the computation). We suppose first that the medium below is a perfect conductor, in which the electromagnetic field is zero everywhere. A plane harmonic incident wave, with wave vector $\mathbf{k}_\mathrm{i} = (k\sin\theta, 0, -k\cos\theta)$ is considered. At each point above the surface, the total field is supposed to be the sum of the incident field, the field that would be reflected specularly from the average surface, and a scattered L period periodic field expressed as a superposition of plane harmonic waves (solutions of the dispersive equation of the medium). This total field must satisfy the divergence equation and must be, at each point, normal to the rough surface. As the surface is periodic it can be developed in a Fourier series as

$$z = f(x, y) = \sum P(m, n)\exp\bigl[-\mathrm{i}2\pi(mx + ny)/L\bigr] \tag{2}$$

The details of the computation depend on the polarization of the incident wave.

5.2. TE polarization

In this case, the incident field is

$$\mathbf{E}_\mathrm{i} = \cos(\omega t - k\sin\theta x + k\cos\theta z)\mathbf{e}_y$$

the specularly reflected field is given by the following relation

$$\mathbf{E}_\mathrm{r} = -\cos(\omega t - k\sin\theta x - k\cos\theta z)\mathbf{e}_y$$

and the total field, using complex notation and eliminating the common time dependence, is given by the following relations

$$E_x = \sum A_{mn} E(m,n,z) \tag{3a}$$

$$E_y = 2\mathrm{i}\sin(k\cos\theta z)\exp(-\mathrm{i}k\sin\theta x) + \sum B_{mn} E(m,n,z) \tag{3b}$$

$$E_z = \sum C_{mn} E(m,n,z) \tag{3c}$$

where

$$E(m,n,z) = \exp\bigl[-\mathrm{i}a(mx+ny) - \mathrm{i}b(m,n)z\bigr], \quad a = 2\pi/L,\ a^2(m^2+n^2) + b^2(m,n) = k^2 \tag{4}$$

It appears that the scattered field is composed of two sets of waves:

- those for which $a^2(m^2+n^2) \leqslant k^2$ with b real positive (propagating modes),
- those for which $a^2(m^2+n^2) > k^2$ with b negative imaginary (evanescent modes).

The two conditions to satisfy are:

- the divergence equation $\operatorname{div}\mathbf{E} = 0$ which gives

$$amA_{mn} + anB_{mn} + b(m,n)C_{mn} = 0 \quad \text{for all } (m,n) \text{ pairs} \tag{5}$$

- the boundary orthogonal relation which writes

$$\mathbf{E} - (\mathbf{E}.\mathbf{N})\mathbf{N} = 0 \tag{6}$$

  where $\mathbf{N}$ is the unit vector normal to the surface.

To solve analytically, we make the small perturbation assumption which consists in supposing that kf, f'_x and f'_y are all of the same order of smallness as $\mathrm{o}(\varepsilon)$. In this case, the unknown coefficients appearing in (3) are expanded in increasing powers of ε, and a solution can be obtained to any order. To obtain the second order solution, we limit the expansions to

$$A_{mn} = A_{mn}(1) + A_{mn}(2) + \mathrm{o}\bigl(\varepsilon^3\bigr)$$

We have similar relations for B_{mn} and C_{mn}.

Under the same assumption, Eq. (6) writes

$$E_x - N_x E_z = E_x - f'_x E_z = 0 \tag{7a}$$

$$E_y - N_y E_z = E_y - f'_y E_z = 0 \tag{7b}$$

The algorithm leading to the solution, which makes alternatively use of the boundary condition and of the divergence equation, is described in Fig. 7.

It appears naturally during the resolution that the parameters (and therefore the field) are given (by intricate formulas) as functions of the wavelength (through the wave number k), the angle of incidence θ and the parameters $P(m,n)$ of the Fourier expansion of the surface. It is thus possible to randomize the rough surface by considering the coefficients $P(m,n)$ as random variables. Assuming their joint probability distribution, it is possible to obtain the average values of the electromagnetic field amplitude and power.

5.3. Additional remarks on the small perturbation method

The method equally well applies to TM polarization. The main difference is that the incident and specularly reflected electric fields have two components differing from zero. The principle of the computation is given in Rice [3]. Probably the solution could be obtained more simply in that case considering first the magnetic field.

We have explained the method in the case when the second medium is perfectly conducting. It can be extended to a more general situation of a dielectric or a real conductor. The difference is that the field has now to be determined simultaneously in both media, the field in the second medium being the sum of the specularly refracted field and a scattering field analogous to the reflected one. The coefficients of the expansions expressing the reflected and the transmitted diffuse fields are then determined in order to satisfy the boundary conditions on the surface (continuity of the tangential components of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$): the mathematics involved are heavier in that case.

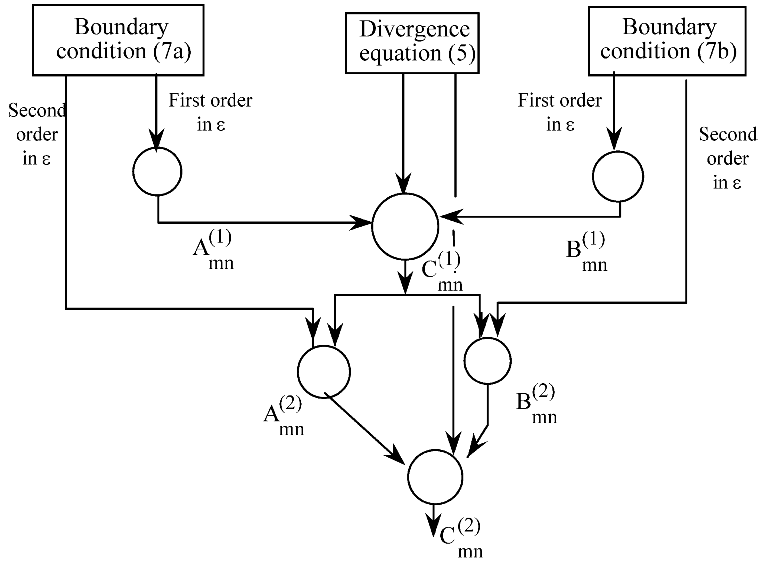

Fig. 7. Algorithm for the second order solution (circles represent mathematical formulas).

Fig. 7. Algorithme de résolution au second ordre ; les cercles représentent des formules.

6. Physical optics approximation

6.1. Principles of the physical optics approximation (scalar theory)

Let U be a scalar harmonic solution of the wave equation in an homogeneous and isotropic medium. U is then solution of Helmholtz equation: $\Delta U + k^2 U = 0$ ($k = \omega/v$ is the wave number). Let S be a closed surface such that the sources of the field are all on the same side of it. It can be shown, from Green's theorem, that the field everywhere on the source-free side of the surface is given by the Kirchhoff integral

$$U(\mathbf{r}) = \frac{1}{4\pi} \iint\limits_S \left[U(\mathbf{r}') \frac{\partial \Psi(\mathbf{r}, \mathbf{r}')}{\partial \mathbf{n}} - \Psi(\mathbf{r}, \mathbf{r}') \frac{\partial U(\mathbf{r}')}{\partial \mathbf{n}} \right] \mathrm{d}\mathbf{r}' \tag{8}$$

where

$\Psi(\mathbf{r}, \mathbf{r}') = \exp[-\mathrm{i}k_\mathrm{S}.R]/R$, $\quad \mathbf{R} = \mathbf{r} - \mathbf{r}'$, $\mathbf{k}_\mathrm{S}$ is the wave vector in the direction of the point of observation $\mathbf{r}$

The field can then be computed if it is known, as well as its normal derivative, at each point of the surface S.

Relation (8) is the basis of the usual method of computing the diffracted field in optics (Kirchhoff diffraction). It can be applied to the problem of the reflection by a rough surface by identifying surface S with the rough surface (which must be of infinite extent). What makes difficulty in applying the method is that the field on the surface is a priori not known and has to be guessed. In the Kirchhoff approximation, the field at each point of the surface is computed as the sum of the incident field and that which would be specularly reflected on the plane tangent to the surface at that point. It is clear that this approximation can be valid only if the surface does not differ too much from its tangent plane on the Fresnel reflection zone, which implies that its radii of curvature are much larger than the wavelength.

6.2. Application to electromagnetic problems

The application of the method to visible incoherent light is rather easy, the scalar quantity to consider being the time-average of the Poynting vector (Born and Wolf [9]). In the case of electromagnetic fields however, the application is not so straightforward due to the importance of the wave polarization. It is true that relation (8) can be applied to any component of the electromagnetic field, in particular to the electric component of a TE field or to the magnetic component of a TM field. If the polarization would be conserved after reflection there would be no problem. Unfortunately, it is not the case with a general rough surface because the local polarization (related to the local normal to the surface) varies from point to point and differs from the average polarization (related to the average plane).

The only case of direct application of the method is that of a surface rough only in one dimension ($z = h(x)$) because the polarization is then the same at each point. For a general surface, rough in two dimensions, the Kirchhoff scalar approximation is possible only if the reflection coefficient is the same for both polarizations, in practice when the second medium is a perfect conductor giving total reflection. It is then possible to compute the modulus of the scattered field, its polarization being obtained specifically in a second step.

We do not have the place here to develop the method in details and we refer the interested reader to the book by Beckmann and Spizzichino [4]. We content ourselves with the major lines of the method and some important results of the solution.

Let us define the scattering coefficient as

$$\rho = E(\mathbf{r})/E_{\mathrm{r}}(\mathbf{r}) \tag{9}$$

where $E_{\mathrm{r}}(\mathbf{r})$ is the field specularly reflected by a polished plane.

After some computation, it is found that, for a rectangular surface ($-X \leqslant x \leqslant X$ and $-Y \leqslant y \leqslant Y$), and at great distance

$$\rho(\theta_1;\theta_2;\theta_3) = F(\theta_1;\theta_2;\theta_3)\frac{1}{A}\int_{-X}^{X}\int_{-Y}^{Y} \mathrm{e}^{-\mathrm{i}\mathbf{v}\cdot\mathbf{r}}\,\mathrm{d}x\,\mathrm{d}y + \frac{e(X,Y)}{A} \tag{10}$$

where

- the angles are defined on Fig. 5.
- $A = 4XY$ is the area of the surface.
- $\mathbf{v} = \mathbf{k}_{\mathrm{i}} - \mathbf{k}_{\mathrm{s}}$.

$$F(\theta_1;\theta_2,\theta_3) = [1 + \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\theta_3]/\big[\cos\theta_1(\cos\theta_1 + \cos\theta_2)\big]. \tag{11}$$

- $e(X,Y)$ is an edge effect term, which can be assumed to be negligible if X and Y are large enough compared to the wavelength λ.

If the surface is considered as random, the only random term in (10) is $\exp(-\mathrm{i}v_z z)$. Average values can be calculated, their expressions depending on the probability law of the random variable z. As the phase is not known, the $\langle\rho\rangle$ average is essentially an intermediate in the calculation. Most interesting are the quantities $\langle\rho\rho^*\rangle = \langle|\rho|^2\rangle$, proportional to the scattered power, and the variance $V(\rho) = \langle\rho\rho^*\rangle - \langle\rho\rangle\langle\rho^*\rangle$, indicative of the field fluctuations. We obtain

$$\langle\rho\rho^*\rangle = \frac{2\pi F^2}{A}\int_0^{\infty} \mathbf{J}_0(v_{xy}l)\Phi(v_z)\Phi^*(v_z)l\,\mathrm{d}l \tag{12a}$$

$$V(\rho) = \frac{2\pi F^2}{A}\int_0^{\infty} \mathbf{J}_0(v_{xy}l)\big[\Phi_2(-v_z,v_z) - \Phi(v_z)\Phi^*(v_z)\big]l\,\mathrm{d}l \tag{12b}$$

where

- $v_{xy} = (v_x^2 + v_y^2)^{1/2}$.
- Φ and Φ_2 are the characteristic functions (Fourier transforms) of the simple and double probability densities $p(z)$ and $p_2(z,z')$ of the surface irregularities $z = f(x,y)$ and $z' = f(x',y')$ at points (x,y) and (x',y') which are τ apart.
- $\mathbf{J}_0(\cdot)$ is the Bessel function of first kind and of order 0.

To go further, the probability densities have to be defined. The calculation can be ended analytically in the case of a Gaussian surface, with zero average, standard deviation σ and correlation function $C(\tau) = \exp(-\tau^2/L^2)$. In that particular case, the expression of $\langle\rho\rho^*\rangle$ is obtained as a power expansion in terms of

$$g = \sigma^2 v_z^2 = \big[2\pi(\sigma/\lambda)(\cos\theta_1 + \cos\theta_2)\big]^2 \tag{13}$$

It is worth noting that in the specular direction ($\theta_2 = \theta_1$) we simply have

$$g = \big[4\pi(\sigma/\lambda)\cos\theta_1\big]^2$$

which is the parameter previously encountered in the Rayleigh criterion and

$$\langle\rho\rangle_{\mathrm{spec}} = \exp(-g/2)$$

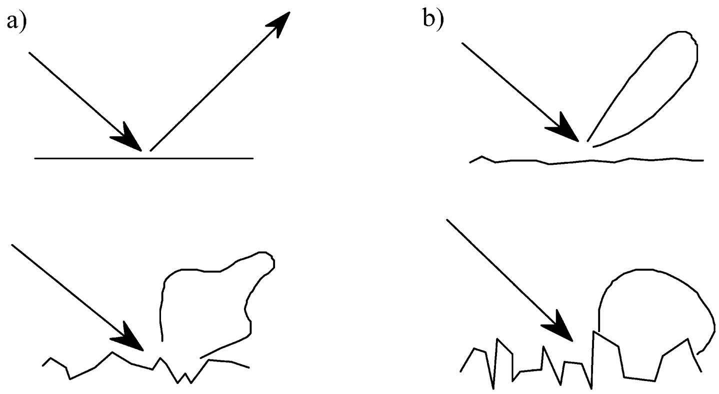

Fig. 8. Evolution of the scattering with the degree of roughness: (a) $g = 0$: specular reflection; (b) $g \ll 1$: diffusion near the specular direction; (c) and (d) $g \gg 1$: transition to purely diffuse reflection.

Fig. 8. Évolution de la diffusion avec le degré de rugosité : (a) $g = 0$: réflexion spéculaire ; (b) $g \ll 1$: diffusion voisine de la direction spéculaire ; (c) et (d) $g \ll 1$: transition vers de la réflexion purement diffuse.

When g is small enough, the first term (or a few terms) of the power expansion is sufficient and the computation is possible analytically.

When the roughness is large, the power expansion in g does not converge fast enough and approximations such as the steepest descent have to be used. The evolution of the scattering with g (that is to say with the degree of roughness) is illustrated schematically on Fig. 8. When the roughness increases, the power reflected into the specular direction decreases, and more and more energy is scattered in all directions, spreading progressively from directions near the specular direction to the whole half-space.

6.3. *Principles of the physical optics approximation (vectorial theory)*

If polarization effects are to be considered, it is better to use a vectorial formulation of the field. The principles of such an approximation are the same as those used in the scalar approximation except that the Stratton–Chu integral is substituted to the Kirchhoff–Helmholtz, writing

$$\vec{E}(r) = \frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}k_\mathrm{s}R}}{4\pi kR}\vec{k}_\mathrm{s} \wedge \iint\limits_S \left[\vec{n} \wedge \vec{E}(r') - \sqrt{\mu/\varepsilon}\,\frac{\vec{k}_\mathrm{s}}{k} \wedge \left(\vec{n} \wedge \vec{H}(r')\right)\right] \mathrm{e}^{\mathrm{i}\vec{k}_\mathrm{s}.r}\,\mathrm{d}r' \tag{14}$$

It is now necessary to know both electric and magnetic tangential components of the field at each point of the rough surface. The second medium needs to be homogeneous and isotropic but has otherwise arbitrary electric characteristics. Moreover, Eq. (14) can be used to compute the transmitted field as well as the reflected one (coefficients ε and μ being relevant to the medium under consideration). This equation, however, cannot be integrated analytically and only approximate solutions are attainable. The stationary phase approximation, for instance, is often used (Ulaby et al. [6]).

7. Discussion

We have described in the previous sections some analytically manageable methods devoted to resolution of the scattering problem of an electromagnetic field by a rough surface. Although the simplest, these methods are nevertheless mathematically cumbersome. Moreover, some of their assumptions are not compatible with experiment. We conclude by a mention of some of these difficulties (more details can be obtained in Ogilvy [10] and Voronovich [11]). More involved methods have been proposed to overtake them but they fall outside the scope of this introductory paper.

(i) In all the methods we have described, each point of the rough surface is considered (in the physical optics language) as a source of a secondary wavelet. Two phenomena are thus deliberately excluded, multiple scattering by the surface and shadowing of some parts of the rough surface. Both effects are illustrated in Fig. 9 using a ray representation. Clearly, these effects become more important as the roughness increases, and as the incidence becomes tangential.

Both effects are easily taken into account by geometrical optics, but we have said that this approximation is not very good, anyway.

In the Kirchhoff approximation shadowing is taken into consideration by introducing a shadowing function by which the scattered field is multiplied: this function represents the fraction of the surface which is illuminated by the incident wave and it

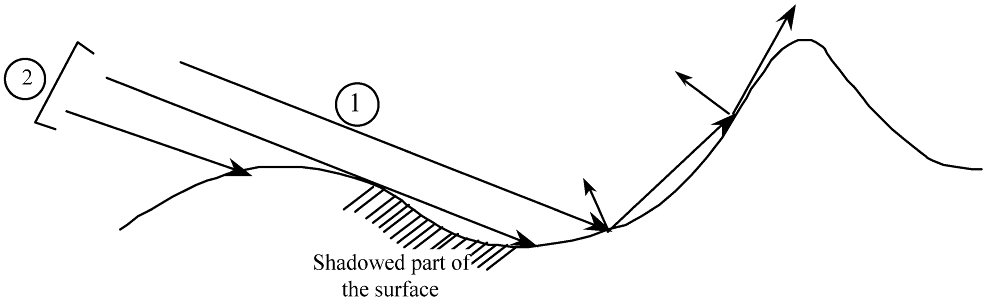

Fig. 9. Ray 1 is doubly reflected; rays 2 illustrate the shadowing effect.

Fig. 9. Le rayon 1 est réfléchi deux fois ; les rayons 2 illustrent l'effet d'ombrage.

depends on the incidence direction. When the surface is described as random, the shadow function depends on the probability density of both the height and slopes of the surface. It is often supposed that the surface is Gaussian, implying that heights and slopes are uncorrelated (Wagner [12], Smith [13]). When the surface is not Gaussian, the correlation between heights and slopes must be considered (Kapp and Brown [14], Bourlier et al. [15]).

Eqs. (8) and (14) can be considered as integral equations. From that point of view, the Kirchhoff approximation can be considered as a first order approximation. It is possible to get better approximations, for instance, by an iterative process, but with the counterpart of higher order integrals. These integral equation techniques give a promising way to process multiple scattering (McCammon and McDaniel [16], DeSanto and Brown [17]).

(ii) We have supposed that the rough surface was around a plane (z being a random variable with zero average). This assumption becomes questionable for long links near the Earth surface, but taking into account the earth sphericity is clearly a mathematically heavy task.

(iii) When considering the surface as random, analytical solutions generally assume a Gaussian model. Unfortunately, natural rough surfaces (particularly that of the sea) are generally not Gaussian. An approach to a non Gaussian model is presented in Beckmann [18] but the computations become rapidly untractable.

(iv) Concerning the autocorrelation function of the surface, we have considered it was characterized by a single correlation length. An extension to the case of an anisotropic surface, with two different correlation lengths in directions Ox and Oy, should not be too difficult. But some real surfaces have in fact more than one correlation length in the same direction. It is for instance the case of the sea surface which presents several kinds of waves with different correlation lengths. It can be considered that small waves (capillarity waves) are superimposed to much larger ones (waves due to the wind effect); an approximate solution is then given by mixed methods in which the effects of both types of waves are processed by different methods, for instance, Kirchhoff approximation for long waves and the small perturbation method for short waves (Wright [19], Zavorotny and Voronovich [20]). In the case where there are a great number of correlation lengths, other methods consider the rough surface as a fractal (Berizzi and Dalle-Mese [21], Franceschetti et al. [22]).

(v) In the theoretical methods we have described, the source is generally a plane incident wave. In real situations the incident field is emitted by an antenna and its directivity diagram has to be taken into consideration. This is relatively easily done in the geometric optics approximation or in the physical optics approximation because the scattered field originating from each point of the surface is proportional to the incident field. In more sophisticated methods, such as the perturbation one, the incident field should be first expanded into plane harmonic waves, the reflected fields associated to each of these being then computed and finally added.

8. Conclusion

A great number of methods have been proposed to solve the problem of electromagnetic wave scattering by a rough surface, each method having its own domain of validity. In this paper we have presented the simplest in some detail, and given some information on more elaborate methods in Section 7.

Two important topics have not been considered at all. The first is a comparison of the results of these analytical methods with observations, a necessity to validate them. A lot of publications do that. For instance, a set of both experimental and theoretical papers concerned with low-grazing angle backscatter from rough surfaces, particularly by the sea-surface, can be found in a special issue of IEEE Transactions on Antennas and Propagation (issue of January 1998).

The second is the development of many numerical methods made possible by the progress of computer performances (see, for instance, Ogilvy [10, Chapter 8]).

References

[1] V. Twersky, On the non-specular reflection of electromagnetic waves, J. Appl. Phys. 22 (1951) 825–835.
[2] V. Twersky, On the scattering and reflection of electromagnetic waves by rough surfaces, Trans I.R.E. AP-5 (1957) 81–90.
[3] S.O. Rice, Reflection of electromagnetic waves from slightly rough surfaces, Comm. Pure Appl. Math. 4 (1951) 351–378.
[4] P. Beckmann, A. Spizzichino, The Scattering of Electromagnetic Waves from Rough Surfaces, Pergamon Press, Elmsford, NY, 1963.
[5] J.A. Stratton, Electromagnetic Theory, McGraw-Hill, New York, 1941; traduction française sous le titre Théorie de l'électromagnétisme, Dunod, Paris, 1961.
[6] F.T. Ulaby, R.K. Moore, A.K. Fung, Microwave Remote Sensing, vol. 2, Addison–Wesley, Reading, MA, 1982 (Chapters 11 and 12).
[7] D. Didascalou, M. Döttling, N. Geng, W. Wiesbeck, An approach to include stochastic rough surface scattering into deterministic ray-optical wave propagation modelling, IEEE-AP 51 (7) (2003) 1508–1515.
[8] J.F. Novel, A. Herique, W. Kofman, Radar signal simulation: surface modeling with the facet method, Radio Science, 39, paper RS1013, 2004.
[9] M. Born, E. Wolf, Principles of Optics, Pergamon Press.
[10] J.A. Ogilvy, Theory of Wave Scattering from Random Rough Surfaces, Institute of Physics Publ. Ltd, 1991.
[11] A.G. Voronovich, Wave Scattering from Rough Surfaces, second updated ed., Springer, Berlin/New York, 1999.
[12] R.J. Wagner, Shadowing of randomly rough surfaces, J. Acoust. Soc. America 41 (1) (1966) 138–147.
[13] B.G. Smith, Geometrical shadowing of a random rough surface, IEEE-AP 15 (9) (1967) 668–671.
[14] D.A. Kapp, G.S. Brown, Effect of correlation between shadowing and shadowed points in rough surface scattering, IEEE-AP 42 (1994).
[15] C. Bourlier, J. Saillard, G. Berginc, Effect of correlation between shadowing and shadowed points on the Wagner and Smith monostatic one-dimensional shadowing functions, IEEE-AP 48 (3) (2000) 437–445.
[16] D.F. McCammon, S.T. McDaniel, Surface reflection: on the convergence of a series solution to a modified Helmholtz integral equation and the validity for the Kirchhoff approximation, J. Acoust. Soc. America 79 (1986) 64–70.
[17] J.A. DeSanto, G.S. Brown, Analytical techniques for multiple scattering from rough surfaces, in: E. Wolf (Ed.), Progress in Optics, vol. XXIII, North-Holland, Amsterdam, 1986, pp. 1–62.
[18] P. Beckmann, Scattering by non-Gaussian surfaces, IEEE-AP 21 (2) (1973) 169–175.
[19] J.W. Wright, A new model for sea clutter, IEEE-AP 16 (1968) 217–223.
[20] V.U. Zavorotny, A.G. Voronovich, Two-scale model and ocean radar Doppler spectra at moderate- and low-grazing angles, IEEE-AP 45 (1) (1998) 84–92.
[21] F. Berizzi, E. Dalle-Mese, Fractal analysis of the signal scattered from the sea surface, IEEE-AP 47 (2) (1999) 324–338.
[22] G. Franceschetti, A. Iodice, M. Migliaccio, D. Riccio, Scattering from natural rough surfaces modeled by fractional Brownian motion two-dimensional processes, IEEE-AP 47 (9) (1999) 1405–1415.