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Neutrino masses and oscillations: an overview

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Abstract

After an historical introduction showing how our understanding of neutrino properties has improved over time, we focus on the phenomenon of flavor oscillations. The formalism is detailed, first for two neutrino families, then for three; matter effects are explained. We finally give an overview of the present experimental status on oscillations, and indicate the future prospects. ***To cite this article: J. Bouchez, C. R. Physique 6 (2005).***

Résumé

Sur les masses des neutrinos et des oscillations. Après une introduction historique montrant comment notre compréhension des propriétés des neutrinos s'est affinée au cours du temps, nous nous intéressons au phénomène des oscillations de saveur. Le formalisme est expliqué, dans le cas de 2 puis de 3 saveurs, les effets de matière sont décrits, puis la situation expérimentale actuelle et les perspectives futures esquissées. ***Pour citer cet article : J. Bouchez, C. R. Physique 6 (2005).***

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Mots-clés : Neutrino ; Masse ; Oscillation

1. Neutrinos: a chronological overview

As a general introduction to neutrinos, I will briefly overview the early history of this particle, since 1930 when it was postulated until the 1990s where the electroweak standard model was definitely assessed as a successful theory of particle physics. It is, however, clear that there must be some ultimate theory beyond this standard model, and the neutrino mass is certainly a first window opened on this so-called grand unified theory (see [1] for more details). Most of the contributions describe in detail current attempts at a better understanding of neutrinos, and I will just present in a second part the phenomenon of neutrino flavor oscillations, which, as of today, is the only experimental proof that neutrinos have indeed a mass, although it is very small.

1.1. From neutrino hypothesis to neutrino discovery

When in 1914, J. Chadwick measured the beta spectrum of radium E (that is ^{210}Bi), its continuous character came as a big surprise (as other types of radioactivity were characterized by monoenergetic lines, interpreted as the energy difference between

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initial and final states). This led to many speculations which were refuted one after the other by experiments: it was shown in 1924 that beta radioactivity emitted only one electron [2], and in 1927 that the beta energy was the only detectable energy in a calorimeter [3]. Two equally challenging hypotheses were formulated to explain this energy crisis:

- N. Bohr suggested that energy was *not* conserved in beta decays, and that could explain the mysterious (at that time) source of energy in stars.
- W. Pauli, on the contrary, insisted on the necessity of energy conservation and proposed, in a celebrated letter in December 1930 [4], that the missing energy was carried away by a hypothetical, light and very penetrating particle, which he nicknamed neutron (and that Fermi renamed neutrino when the neutral partner of the proton was discovered by Chadwick in 1932).

Many experimenters tried to get a signature of this elusive neutrino and designed very smart experiments. Let me just mention Nahmias [5], who tried to detect the ionisation left by neutrinos; to get rid of the cosmics background, he installed its detector in the deepest station of the London subway: this was the very first *underground* neutrino experiment, followed by many others; Nahmias was able to put a limit of 2×10^{-4} Bohr magneton on the neutrino magnetic moment. I would also like to mention another attempt by Crane in 1939 [6], who put a 1 millicurie source of radium inside a bag filled with 3 pounds of salt (NaCl) with the hope of observing the transmutation of some ^{35}Cl atoms into ^{35}S, which has a half-lifetime of 87 days. After 3 months of irradiation followed by a chemical extraction of sulfur, he found no sign of ^{35}S decay and could put an upper limit of 10^{-30} cm^2 on the capture cross-section: this certainly was the very first *radiochemical* experiment for neutrino detection, and the same principle was later used in the first solar neutrino experiment in 1968.

Things became more favorable for neutrino hunters after the second world war, with the appearance of nuclear reactors. It was soon realized (actually by Fermi) that they were extremely strong sources of neutrinos (a fission dissipates 200 MeV and gives 6 neutrinos through cascade decays of fission products, so that a reactor with a thermal power of 1 GW produces isotropically 2×10^{20} neutrinos per second!). The neutrino was finally discovered by Cowan and Reines [7], as explained elsewhere [8].

1.2. Neutrinos and antineutrinos

After Dirac produced his theory of spin 1/2 particles and predicted the existence of antiparticles with opposite charges, it was natural to wonder if neutrinos and antineutrinos were or were not different particles. As neutrinos have no electric charge, there is a possibility that they are truly neutral and carry no charge of whatever nature. This possibility has been put forward by E. Majorana, after whom self-conjugate neutrinos are now named. If some internal charge (such as a leptonic charge) is carried by neutrinos, antineutrinos will carry the opposite charge and be different: they will be the so-called Dirac neutrinos.

Before answering the question, one has to label what would be a neutrino and what would be an antineutrino: one has decided to call antineutrinos those which are produced together with an electron, while neutrinos are produced by β^+ decays together with a positron (so that β decays produce a lepton-antilepton pair, one electrically charged and the other neutral). In a nucleus, the transition between a neutron and a proton produces an antineutrino. This antineutrino is thus able to turn a proton into a neutron (this is the discovery detection). Now neutrinos will, for the same reason, certainly be able to transform a neutron into a proton inside a nucleus. But will neutrinos be also able to transform a proton into a neutron, or equivalently antineutrinos be able to transform a neutron into a proton? If yes, one would tend to admit that neutrinos are their own antiparticle, while they would be different particles if the answer is no. (However, as we will see later, this reasoning happens to be too naïve and is wrong). Anyhow, an experimental test was done, where a tank filled with chlorine; a prototype made by R. Davis for his solar neutrino experiment, was brought near the Savannah River reactor (used by Reines) to eventually observe the transmutation of ^{37}Cl into ^{37}Ar by the reactor antineutrinos. No such transmutation was observed, and it was concluded that neutrinos and antineutrinos actually were different particles, carrying a leptonic charge (1 for neutrinos and electrons, -1 for antineutrinos and positrons) which was conserved in interactions. This meant that neutrinos were Dirac particles.

However, things became much more complicated when parity conservation was shown to be violated in weak interactions by Miss Wu in her celebrated experiment [9]. This violation was found to be maximal, and this meant that spin was playing an essential role in weak interactions: amplitudes depend upon the helicity[1] of the (anti)neutrino produced in beta decays. As the parity violation was found to be maximal, it meant that only left-handed helicity neutrinos ($h = -1/2$) were produced in β^- decays, and conversely β^+ decays were producing neutrinos of right helicity ($h = +1/2$).

This peculiarity had been guessed by Lee and Yang [10] who modified the Fermi theory by adding a factor $(1 - \gamma_5)$ in the current-current Hamiltonian describing beta decays. This factor actually selects by definition a given 'chirality' for the neutrino,

[1] The helicity h of a particle is, in essence, the measurement of its spin along its direction of propagation.

and the opposite one for the antineutrino. When these neutrinos are relativistic, chirality and helicity are nearly the same, and this so-called V–A theory could explain Wu's observations.

Now, this imposes a reconsideration of the difference between neutrinos and antineutrinos: is the observed difference between them when looking at chlorine transmutation due to an intrinsic difference (leptonic charge) or just a spin effect? It could well be that neutrinos and antineutrinos are the same particle, but that due to V–A, only left-handed neutrinos are produced with electrons and right-handed neutrinos are produced with positrons. As long as neutrinos stay ultrarelativistic, so that chirality and helicity are nearly equivalent, V–A prevents left-helicity neutrinos from interacting with protons and right-helicity neutrinos from interacting with neutrons.

In the limit where neutrinos are massless, chirality and helicity are intrinsic conserved quantities, and the distinction between the two vanishes: neutrinos are described as Weyl particles (2-component massless spinors) and the Dirac or Majorana descriptions become mathematically equivalent. This is actually the way neutrinos enter the minimal standard model of electroweak interactions: neutrinos are left-handed, antineutrinos are right-handed, and the two other degrees of freedom, which would be anyway perfectly sterile due to V–A, simply do not exist. The apparent lepton number conservation is just a consequence of V–A.

When neutrinos have a mass, the alternative between Majorana and Dirac descriptions could, in principle, be tested: for example, a neutrino beam impinging on a fixed nucleus target will be described as left helicity particles, while a nucleus beam with a higher speed than the neutrino beam, and going in the same direction, would see them as right helicity particles provided we use the same helicity convention in the center-of-mass frame; then Dirac neutrinos would produce electrons in the first case and be sterile in the second case, while Majorana neutrinos would produce electrons in the first case and positrons in the second case (all this being true up to correction factors of order $(m_\nu/E_\nu)^2$). Such a gedanken experiment is evidently totally unrealistic and in practice, only neutrinoless double beta decays [11] would allow us to determine whether neutrinos are Dirac or Majorana particles.

1.3. Three families of neutrinos

Pauli, when postulating the neutrino, increased the elementary bricks of the microscopic world from 2 (proton and electron) to 3. But as we all know, the zoology of 'elementary' particles showed an exponential increase with experimental progress. The neutron and the positron (the first antiparticle) were soon discovered. The muon was seen in cosmic rays, and it took some time to realize it was not the hypothetic pion mediating the nuclear force, although it had the expected mass, but rather a heavy electron. Pions were copiously produced with the start of GeV accelerators, and the study of their decays was puzzling. Why was the main decay into a muon and a neutral light particle (a neutrino so that the lepton number was conserved) rather than electron-neutrino? The V–A theory had the explanation: lepton-neutrino decay is forbidden by V–A for spin 0 particles like pions in the limit of massless leptons. Moreover, this interdiction is only violated due to the helicity-chirality mismatch for massive charged leptons. So, although the phase space for electron-neutrino is much higher than for muon-electron, the V–A rule dominates and the electron-neutrino branching ratio is only 1.2×10^{-4} due to the much lower electron mass. But a question was to be answered: was this ν_π neutrino the same as the ν_β emitted in radioactive decays? If yes, this neutrino, when interacting with nuclei, should produce muons and electrons roughly in equal numbers. If it was a second neutrino variety, specifically related to the muon, it should produce only muons and no electrons. The absence of radiative decays of muons into electrons, but rather in electron and two neutrinos, suggested that there were 2 species of neutrinos associated with two species of charged leptons, with different lepton numbers between the two lepton families. This had to be tested. In 1962, the new accelerator at Brookhaven was used to produce a secondary pion beam which after decay, sent neutrinos to a detector placed behind a very thick steel shielding to absorb all charged particles [12]. 34 interactions producing a muon were observed, while only 6 electron or gamma showers were observed. The ν_π was different from the ν_β, and the two neutrino species were labelled ν_μ and ν_e, referring to their associated charged lepton.

When the third charged lepton (the τ) was discovered at SLAC in the 1970s, it became natural to link it with a third variety of neutrino, the ν_τ. The direct proof of the ν_τ existence was brought only in 2000, when the DONUT experiment [13], using the beam dump technique with the high energy proton beam at Fermilab, could produce a so-called prompt neutrino beam enriched in ν_τ's which were detected in emulsions.

The existence of 3 families of neutrinos had previously been proven indirectly by LEP experiments, which deduced from the width of the Z^0 gauge boson that it has to decay into 3 different varieties of ν–$\bar{\nu}$ pairs, each contributing 110 MeV to the total width.

1.4. Neutrinos and the standard model

The LEP result relies on the so-called standard model of electroweak interactions, which was slowly built from experimental observations and theoretical progress during the 1960s and the 1970s. The success of this theory culminated with the discovery

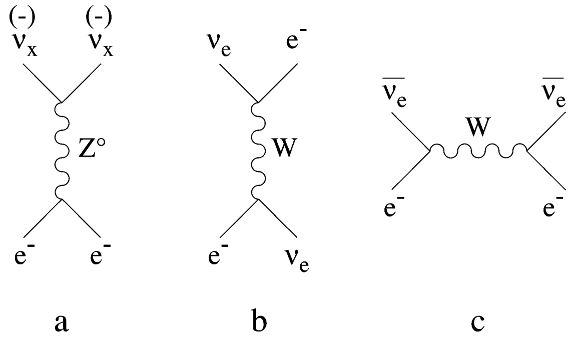

Fig. 1. Exchange diagrams describing the diffusion of neutrinos on electrons: (a) Z^0 neutral current diagram, valid for all neutrino flavors; (b) and (c) W charge current exchange diagrams, only present for (b) ν_e and (c) $\bar{\nu_e}$.

at CERN of the two gauge bosons W and Z. The study of neutrino interactions played an important role in the conception of this standard model, which unifies weak and electromagnetic interactions in a single theoretical frame. An important step has been the discovery of 'neutral currents'.

As we have seen, neutrino species are linked to the charged lepton with which they are associated. This association can be in a decay, a hadron decaying leptonically or semi-leptonically (that is, together with other hadrons) into a charged lepton l and an antineutrino $\bar{\nu}_l$ (or an antilepton $\bar{l}$ and a neutrino ν_l); or it can be in so-called charged current interactions (mediated by W) where a ν_l interacts with a hadron to give a lepton l together with hadrons. This association defines the neutrino flavor as the flavor of the associated charged lepton.

The discovery was made in 1973 when an experiment with the Gargamelle bubble chamber submitted to a ν_μ beam observed first one [14], then several events [15] interpreted as the elastic diffusion of a neutrino upon an electron. Kinematically, these knocked electrons keep the direction of the beam when the neutrino energy is much higher than the electron mass. However, such a diffusion is impossible if the neutrino interactions are charged-current interactions, since necessarily a muon should appear in the final state (as for example $\nu_\mu + e^- \to \mu^- + \nu_e$). This new way of interacting can only be explained if there exists neutral currents, mediated by the boson Z^0, as shown on Fig. 1, where the initial neutrino appears also in the final state.

This observation was an important step towards electroweak unification, since at the time, several scenarios were possible, with or without neutral currents. The existence of these neutral currents was later confirmed in neutrino interactions on nuclei in which no final charged lepton was observed.

1.5. Are neutrinos massless or massive?

It was realized from the beginning that neutrinos had to be light particles. From the difference observed in beta decays between the electron and the missing (that is neutrino) mean energies, F. Perrin suggested that mean momenta were probably equal, and that implied a neutrino mass much lighter than the electron mass. In a 1936 review, Bethe and Bacher wrote that "*the neutrino mass ... was probably zero*".

The standard model was built with the explicit hypothesis of massless neutrinos; the grand unified theories based on SU(5) made the same hypothesis, since there was no room for a ν_R in the 15-plets, while SO(10) unified theories could accommodate massive neutrinos in the 16-plets [1].

Starting from 1968, the solar neutrino problem (see [16]) was the main reason for reconsidering the massless feature of the neutrino. As anticipated by Pontecorvo and others [17–19], flavor oscillations, by analogy to the K–$\bar{K}$ oscillations, was a possible explanation for the solar ν_e deficit, but this implied massive neutrinos. These oscillations were actively searched for, specially near nuclear reactors, after Reines claimed in 1980 a positive signal from a CC/NC anomaly in neutrino interactions on deuterium [20], which was later refuted.

A non-zero mass for the neutrinos has a strong impact for cosmology, as these particles could then explain the dark matter in the Universe. For some time, the best models for the apparition of large scale structures preferred a mixture of cold dark matter (weakly interacting heavy particles, or WIMPs) and hot dark matter (for which neutrinos with few eV masses were perfect candidates). This is no longer true after a nonzero cosmological constant has been introduced in these models. However, this triggered experiments in the 1990s searching for ν_μ–ν_τ oscillations in the few eV range, such as NOMAD [21] and CHORUS [22] at CERN.

The see-saw mechanism, proposed in 1979 [23,1], brought a natural explanation for light neutrinos in grand unified theories. More recently, the CP violation induced by a complex neutrino mixing matrix is considered as the best candidate to explain matter-antimatter asymmetry in our Universe, through leptogenesis [24]. This explains why neutrino properties can shed light on physics at the grand unification energy scale, and why the determination of neutrino properties is considered nowadays of fundamental importance.

2. The phenomenon of flavor oscillations

Attempts at a direct proof of neutrino masses have been up to now unsuccessful [25]. There is, however, an indirect way to prove their massive character, which consists in looking for flavor oscillations. This phenomenon is predicted by standard quantum mechanics, and based on the fact that if neutrinos are massive, the 3 flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ need not coincide with the 3 mass eigenstates (ν_1, ν_2, ν_3). We have then 2 distinct bases connected through a unitary 3×3 matrix. The Schrödinger equation describing the free propagation of a neutrino predicts the appearance of different flavors with time. We will start with the 2 flavor case, technically simpler, first in vacuum then in matter, and finally address the 3 flavor case, which is more intricate.

2.1. Two flavor formalism

Let us restrict to a world where only two flavors (ν_e, ν_μ) are present so that we have two mass eigenstates (ν_1, ν_2) with masses m_1 and m_2. The unitary matrix linking the two bases is just a rotation by an angle θ.

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle, \qquad |\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

Let us consider a ν_e produced with momentum p at $t = 0$. After a time t, it will be:

$$\big|\nu(t)\big\rangle = \cos\theta \mathrm{e}^{-\mathrm{i}E_1 t}|\nu_1\rangle + \sin\theta \mathrm{e}^{-\mathrm{i}E_2 t}|\nu_2\rangle$$

with

$$E_i = \sqrt{p^2 + m_i^2}$$

The probability to interact as a ν_μ at time t is given by:

$$\big|\big\langle\nu_\mu\big|\nu(t)\big\rangle\big|^2 = 4\sin^2\theta\cos^2\theta\sin^2\frac{(E_1 - E_2)t}{2}$$

If the neutrino is relativistic (which is always the case) then $E_1 - E_2 = \frac{m_1^2 - m_2^2}{2p}$ and we can write:

$$P(\nu_e \to \nu_\mu, t) = \sin^2 2\theta \sin^2\frac{\Delta m^2}{4p}t$$

As can be seen on Fig. 2(a), the ν_μ component and the ν_e component (which add up to 1) oscillate sinusoïdally with time with a period $T = \frac{4\pi p}{\Delta m^2}$, corresponding to an oscillation length

$$L_{\mathrm{osc}} = cT = 2.5\ [\mathrm{m}] \times \frac{E_\nu([\mathrm{MeV}])}{\Delta m^2([\mathrm{eV}]^2)}$$

the maximal amplitude of the oscillation being given by $\sin^2 2\theta$. Note that the frequency of oscillation is proportional to Δm^2.

Actually, the correct way to derive this formula implies the description of localized neutrinos as wave packets, but the result is the same [26]. As a bonus, one finds, however, that the oscillation pattern fades away after p/σ_p oscillations, where σ_p is the width in momentum of the wave packet. In practice, the pattern is not experimentally observable after p/σ_{exp} oscillations, where σ_{exp} is the experimental resolution on the neutrino momentum, or the natural width of the source if this momentum is not measured. After this damping has occurred, the transition probability becomes constant at $0.5\sin^2 2\theta$.

2.2. Oscillation experiments, exclusion plots

As we have seen, there are two possible ways to look for an oscillation. Flavors are observable through charge currents on nuclei, the produced charge lepton identifying the flavor of the interacting neutrino. Either one looks for a deficit in the initial flavor (disappearance experiment), or for the appearance of a flavor initially absent (appearance experiment). For small oscillation amplitudes, appearance is certainly better since in case of no background and a pure flavor source (for example ν_e), a single interaction producing a muon will prove the oscillation, while for disappearance the sensitivity to the oscillation amplitude, $\sin^2 2\theta$, is limited by statistical fluctuations on the number of ν_e interactions.

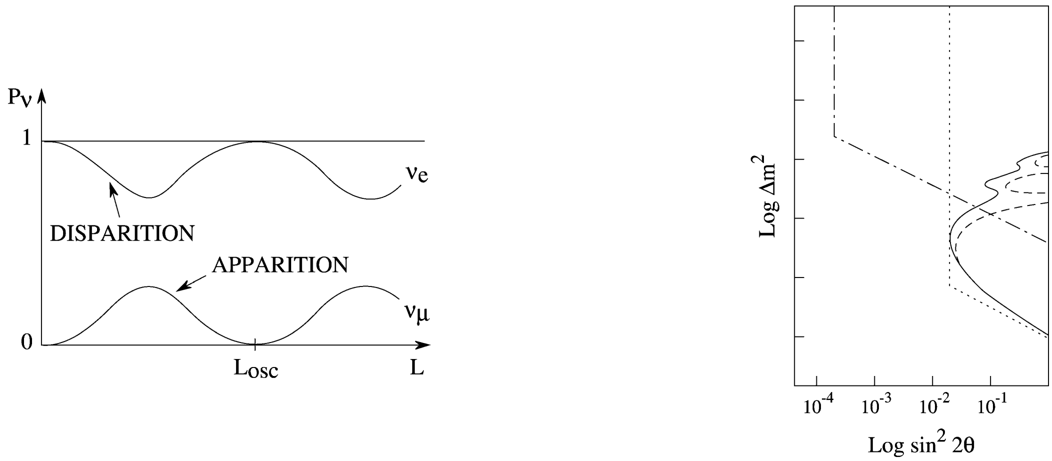

Fig. 2. (a) Left: oscillation pattern between two neutrino flavors for a given neutrino energy. (b) Right: Typical result of an oscillation experiment with a negative result. The dash-dotted curve corresponds to an appearance experiment, the dotted curve to a disappearance experiment, showing the better sensitivity to small oscillations for appearance experiments. The vertical line at high Δm^2 corresponds to oscillation lengths smaller than the distance between source and detector, while the lower line with a slope corresponds to oscillation lengths larger than this distance. The continuous curve shows a typical exclusion domain obtained from the comparison of a near and a far detector, and shows a loss in sensitivity for high Δm^2 when the oscillation length becomes much smaller than the distance between source and near detector.

However, appearance is not always possible. In the previous example, if the neutrino energy is below the muon production threshold, it will not be possible to sign ν_μ appearance and we have to limit ourselves to disappearance experiments: this will be the case for low energy ν_e like those from the Sun or from nuclear reactors.

The principle of oscillation searches is to use a detector far from the source. This distance is to be compared to the oscillation length, which goes like the inverse of Δm^2. Any experiment will then be sensitive to Δm^2 values above a lower limit defined by the distance of the detector and the energy of the neutrinos. An experimental complication arises when the flux, or the flavor composition of the source (in neutrino beams, mainly ν_μ from π decays, there is always a small ν_e component from μ or K decays) is not perfectly known. In this case, a remedy consists in using two detectors, one near and the other far from the source, and compare observations at the 2 locations. Any difference in flavor content will prove the presence of flavor oscillations. This will, however, work only if the oscillation length is higher than the near location distance from the source; this means that such comparisons will be blind to oscillations of too high frequency, that is too high Δm^2. The sensitivity on Δm^2 will then be limited from below and from above.

In the design of an experiment looking for neutrino oscillations, one always has to think beforehand to all the possible backgrounds which could mimic the signal. To decrease the background, shieldings have to be used. The best shielding against cosmic rays is to go deep underground. Furthermore, local backgrounds due to radioactivity, and specially gammas and neutrons, impose in the case of low energy neutrinos to design passive and/or active shieldings surrounding the detector. In some cases, the remaining background can be measured when the neutrino source (beam, reactor) is off. This is not always possible (think of the Sun!).

Finally, when the result is obtained, it is translated into an acceptance domain (in case of a positive result) or into an exclusion domain (in case of negative result) in the plane of the two physical parameters, $\sin^2 2\theta$ and Δm^2 (see Fig. 2(b)).

One should, however, keep in mind that there are 3 neutrino flavors in nature, so that actual oscillations are governed by more than 2 parameters. Fortunately, nature has been kind enough so that these 2-flavor parameters are easily re-interpreted in the 3-flavor case.

2.3. Oscillations and matter effects

The first hint at neutrino oscillations came from the solar neutrino deficit (see [16]), and was suggested by several authors [18,19]. Solar neutrinos begin their travel to the Earth inside very dense matter, and it was realized by Wolfenstein in 1978 [27] that the presence of electrons would modify the oscillation pattern of ν_e compared to what happens in vacuum. Although neutrinos have negligible interactions with matter, these interactions will, however, generate an index of refraction, linked to the elastic amplitude in the forward direction. All flavors have the same amplitude on nuclei, but not on electrons (see Fig. 1). ν_μ and ν_τ will be subjected to the same refractive index, but this index will be different for ν_e. The effect of this refractive index, which acts as a potential to be added to the vacuum Hamiltonian, has to enter the Schrödinger equation. This potential is diagonal in flavor basis, while the free Hamiltonian is diagonal in the mass basis. Adding both will define propagation eigenstates which are different from any one of these 2 bases, and are labelled ν_{1m}, ν_{2m} and ν_{3m}. This matter basis will be constant in matter of constant electronic density, but will vary with time when the electronic density varies along the path of the neutrino (as it is the case inside the Sun from its center to its surface). We will study in the following the two cases of constant and varying electronic densities, restricted to the two flavor case.

2.3.1. Constant density

In vacuum, the Hamiltonian is diagonal is the mass basis:

$$H_V|\nu_1\rangle = E_1|\nu_1\rangle, \qquad H_V|\nu_2\rangle = E_2|\nu_2\rangle$$

When neutrinos go through matter, a potential has to be added to the vacuum Hamiltonian. This potential is diagonal in the flavor basis:

$$V|\nu_e\rangle = \left(C + \sqrt{2}G\rho_e\right)|\nu_e\rangle, \qquad V|\nu_\mu\rangle = C|\nu_\mu\rangle$$

The term C describes the neutral current interactions on nuclei (or nucleons) and electrons; it is common to all flavors (if only C was present, oscillations would not be modified). The extra term for ν_e, proportional to the Fermi constant G and the electron number density ρ_e corresponds to charge currents of ν_e on electrons. Please note that this extra term changes its sign when going from neutrinos to antineutrinos.

The total Hamiltonian is diagonal in a new basis ν_{1m} and ν_{2m} deduced from the flavor basis by a rotation θ_m given by[2]

$$\tan(2\theta_m) = \frac{(E_2 - E_1)\sin(2\theta)}{(E_2 - E_1)\cos(2\theta) - \sqrt{2}G\rho_e}$$

When the electron density is constant, the oscillation formula has the same structure as in vacuum, but the mixing angle θ is replaced by θ_m and the oscillation length is multiplied by $\sin(2\theta_m)/\sin(2\theta)$.

One sees immediately that oscillation amplitudes will be enhanced with respect to vacuum for neutrinos and damped for antineutrinos when $E_2 > E_1$, that is $m_2 > m_1$. If $m_2 < m_1$, oscillations will be enhanced for antineutrinos and damped for neutrinos. Notice also that the oscillation length increases with respect to vacuum for the enhanced oscillation and decreases for the damped oscillation. Thus matter effects create an asymmetry between neutrinos and antineutrinos which should not be confused with CP violation; it is just due to the fact that this matter is not CP symmetric. This effect, if detected [28], gives access to the mass hierarchy between m_1 and m_2.

One can also compute the density of electrons for which the enhanced oscillation becomes maximal; it is

$$\rho_R = \Delta m^2 \cos(2\theta)/2\sqrt{2}GE_\nu$$

2.3.2. Varying density

When neutrinos travel through matter with a varying electron density, the propagation equations cannot be solved analytically in the general case, so that one has to resort to numerical simulations. However, there is a special case, called the *adiabatic* case, where the solution is simple. It happens when variations of density are small over one oscillation length, so that evolution equations can be rewritten in the *variable* basis of instantaneous propagation eigenstates while neglecting terms induced by its varying character. This is legitimate when the rotation speed of these eigenstates in the fixed basis of mass or flavor is negligible compared to the oscillation frequency. We actually are familiar with this simplification, when studying how the spin of a particle at rest evolves in space when the particle is submitted to a *slowly varying* (in direction and in strength) magnetic field; one knows that the spin will precess around the magnetic field (Larmor precession) and the axis of this precession will stay aligned with the magnetic field direction: this is how one rotates the polarisation direction of a polarized target. The analogy is perfect when instead of using the standard orthogonal bases ν_1, ν_2 or ν_e, ν_μ, one uses the so-called Poincaré representation, where a neutrino state $|\nu\rangle = \cos\theta|\nu_e\rangle + \mathrm{e}^{-\mathrm{i}\phi}\sin\theta|\nu_\mu\rangle$ is ascribed a point on a sphere of unit radius with a polar angle 2θ and an azimuth ϕ. One notices that orthogonal states (like ν_e, ν_μ or ν_1, ν_2) will be represented by 2 points opposite on the sphere, so that any orthogonal basis corresponds to a given direction on the sphere. (Furthermore, the probability for a state P to be observed in the state M is just $(1 + \overrightarrow{OP}.\overrightarrow{OM})/2$, O being the center of the sphere.) The equations for neutrino evolution become the same as the evolution of a spin in a magnetic field, with the following correspondences: the field direction corresponds to the direction of the instantaneous neutrino propagation eigenstates, and the strength of the magnetic field (multiplied by the particle magnetic moment) is replaced by the difference in energy eigenvalues of the two neutrino propagation eigenstates. To summarize, in the adiabatic approximation, a neutrino propagating through matter with slowly varying density will precess (on the sphere) around the axis of instantaneous eigenstates and follow it (see Fig. 3).

[2] We need a convention to label m_1 and m_2; here we decide that the m_1 component is the dominant mass component in ν_e, or equivalently that θ is between 0 and $\pi/4$.

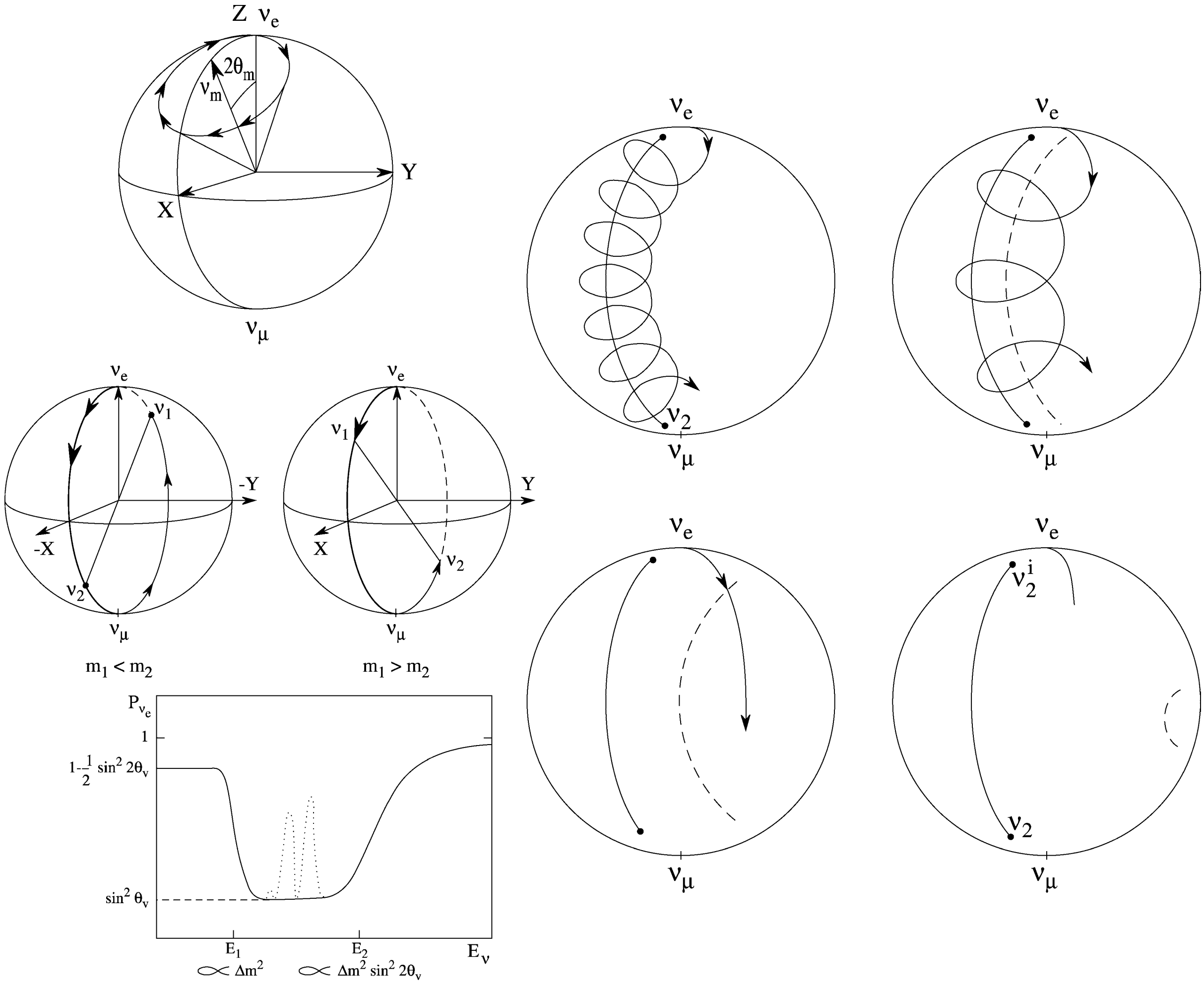

Fig. 3. Left, upper: matter with constant density. The neutrino state precesses around the fixed axis of propagation eigenstates in matter.

Left, middle: evolution of propagation eigenstates, starting from ν_e at infinite density and ending for null density either as ν_2 if $m_1 < m_2$ or ν_1 if $m_1 > m_2$.

Right: matter with decreasing density (as in Sun): evolution of an initial ν_e state between Sun center and surface for $m_1 < m_2$ when the adiabaticity condition is less and less satisfied, from top left to bottom right; the driving towards ν_2 becomes less and less efficient.

Left, lower: neutrino spectral distorsion when the MSW effect is fully active between E_1 and E_2; below E_1, the central solar density is too small for matter effects to be sizeable, and above E_2, we have a slow loss of the adiabatic condition; the dotted line is for detection during the night when ν_e's are partially regenerated in the Earth.

2.3.3. Interpretation of solar and atmospheric data

The matter in the Sun corresponds to the case of varying electron density. The adiabatic approximation will hold when the 'Larmor frequency' on the Poincaré sphere is higher than the rotation speed of the direction of the propagation eigenstates. Taking into account the known exponential decrease of electron density with solar radius, this condition will hold when:

$$\Delta m^2\big([\mathrm{eV}^2]\big)\sin 2\theta \tan 2\theta \gg 5\times 10^{-9} E_\nu\big([\mathrm{MeV}]\big)$$

We produce ν_e near the center of the Sun, and the matter effect will dominate over Δm^2 in the energy splitting if:

$$\Delta m^2\big([\mathrm{eV}^2]\big)\cos 2\theta \ll 1.5\times 10^{-5} E_\nu\big([\mathrm{MeV}]\big)$$

If this second condition holds, ν_e and ν_μ are the propagation eigenstates at the production point.

When both conditions are fulfilled, the ν_e born as a propagation eigenstate will stay at all times a propagation eigenstate up to its exit from the Sun, so that it will leave the Sun either as a ν_1 (if $m_1 > m_2$) or a ν_2 (if $m_1 < m_2$). The second case is the most interesting (remember that the main mass component in ν_e is ν_1) and is called the MSW effect, after the name of the two

Russian physicists (Mikheyev and Smirnov) who first noticed this effect [29], the W standing for Wolfenstein who had exhibited the importance of matter effects. It corresponds to an adiabatic *driving* in the Sun of a ν_e into a ν_2 (actually no oscillation takes place), which will be predominantly a ν_μ (the smaller θ is, the bigger the ν_μ component will be). The neutrino exiting the Sun as a ν_2 will arrive at Earth as a ν_2, since it is a propagation eigenstate in vacuum. This MSW effect will be effective for a span in neutrino energy given by the 2 above conditions (see Fig. 3), and can explain naturally as big a ν_e deficit as requested, while vacuum oscillations would be at pain to explain large deficits (that is, factors around 3).

When fitting data from all solar experiments, matter effects both inside the Sun and through the earth during nights being properly taken into account, a single scenario emerges where the MSW effect is unambiguously present in the Sun [16] (see [30] for details).

Atmospheric neutrinos (see [31]) have exhibited a large (compatible with maximal) oscillation between ν_μ and ν_τ, but in this case matter effects are negligible (they are only important for the undiscovered subleading oscillation between ν_μ and ν_e).

2.4. The mixing for three families

We now address the case of three-family flavor oscillations. In this case, three different masses will induce three oscillation frequencies proportional to $(m_i^2 - m_j^2)$, thus the biggest frequency is the sum of the two others. However, we already know that solar and atmospheric frequencies are in a ratio of 30 or so (see later in the present paper); it means that the two bigger frequencies are roughly equal to the atmospheric one, while the smallest is the solar one. We will use this fact in the following.

The unitary matrix linking mass and flavor eigenstates can be written:[3]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{1}$$

This unitary matrix U can be decomposed as the product of 3 rotations, complemented with extra phases responsible for CP violation:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c23 & s23 \\ 0 & -s23 & c23 \end{pmatrix} \begin{pmatrix} c13 & 0 & \mathrm{e}^{\mathrm{i}\delta} s13 \\ 0 & 1 & 0 \\ -\mathrm{e}^{-\mathrm{i}\delta} s13 & 0 & c13 \end{pmatrix} \begin{pmatrix} c12 & s12 & 0 \\ -s12 & c12 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi_1} & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where cij and sij stand for cosine and sine of θ_{ij}.

The rightmost matrix is only present if neutrinos are Majorana particles, but these phases ϕ_1 and ϕ_2 do not enter oscillation formulae, so they are irrelevant for oscillation experiments. They are, however, important for other processes such as neutrinoless double beta decays. The 3 other matrices are the quasi standard representation of a rotation in 3-D space with 3 Euler angles, corresponding to successive rotations (from right to left) around ν_3 axis by θ_{12}, then around the transform of ν_2 by θ_{13}, and finally around the transform of ν_1, (that is ν_e) by an angle θ_{23} (see Fig. 4). One sees, however, that the θ_{13} rotation matrix is modified by a phase δ which will enter oscillation formulae and induce, if nonzero, a CP violation in oscillations. When switching from neutrinos to antineutrinos, it is enough to change the sign of the CP phases. By convention, mass indices 1 and 2 will be used for solar oscillations, while the mass index 3 is used for atmospheric oscillations. This leaves open two scenarios: either m_3 is the heaviest mass (so-called normal mass hierarchy) or the lightest (inverted hierarchy), an alternative presently unsolved.

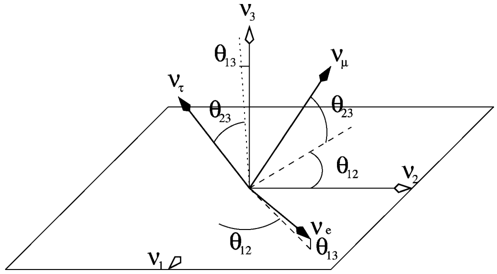

Fig. 4. Definition of the 3 angles used in the neutrino mixing matrix.

[3] This matrix is called MNSP, following the pioneering work of Maki, Nakagawa, Sakata [18] and of Pontecorvo [17].

From this matrix, one can derive oscillation formulae between an initial flavor l and a final flavor l'; we note Δ_{ij} the oscillating term $(m_i^2 - m_j^2)t/(4E_\nu)$:

$$P_{ll'} = \delta_{ll'} - 4\,\mathrm{Re}\sum_{i<j} U_{l'i}^* U_{l'j} U_{li} U_{lj}^* \sin^2\Delta_{ij} + 2\,\mathrm{Im}\sum_{i<j} U_{l'i}^* U_{l'j} U_{li} U_{lj}^* \sin 2\Delta_{ij}$$

We deduce the formulae for the already observed oscillations, taking into account the fact that the solar frequency is much smaller than the atmospheric one and the smallness of θ_{13} as deduced from CHOOZ [32]:

- For atmospheric oscillations, neglecting Δ_{12} terms and equating Δ_{13} and Δ_{23}:

  $$P_{\mu\tau} = 4c_{13}^4 s_{23}^2 c_{23}^2 \sin^2\Delta_{23}$$

  which is exactly the 2-family formula, except for the c_{13}^4 factor which is bigger than 0.92 according to CHOOZ. The ν_e disappearance relevant for CHOOZ can also be written with the same approximations:

  $$P_{ee} = 1 - 4s_{13}^2 c_{13}^2 \sin^2\Delta_{23}$$

  that is exactly the 2-family formula (taking into account the very beginning of the solar oscillation would modify this probability by less than half a percent).
- For solar oscillations in vacuum (relevant for KamLAND, see [16]), we must first take into account the damping of the fast (atmospheric) oscillations and replace $\sin^2\Delta_{23}$ and $\sin^2\Delta_{13}$ by 0.5:

  $$P_{ee} = (1 - 2s_{13}^2 c_{13}^2) - 4c_{13}^4 s_{12}^2 c_{12}^2 \sin^2\Delta_{12}$$

  which, taking into account the fact that θ_{13} is small can be rewritten:

  $$P_{ee} = (1 - 2\theta_{13}^2)(1 - \sin^2 2\theta_{12} \sin^2\Delta_{12})$$

  that is, apart from an overall factor between 0.92 and 1, the same formula as in the two-family case.

One sees that the strong frequency hierarchy and the smallness of one mixing angle both contribute to the fact that up to now, two-family formulae, widely used by experimentalists, happened to be good approximations to the more correct three-family formalism.

3. Present status of oscillations and global fits

As described in detail in the other contributions to this special issue on neutrinos, flavor oscillations have now been firmly established:

– First from the study of solar neutrinos, for which a unique solution taking into account matter effects in the Sun has finally emerged. The corresponding oscillation in vacuum has also been observed (with no matter effects) by the KamLAND experiment detecting the antineutrinos emitted by the Japanese nuclear reactors [16].
– Second from atmospheric neutrinos, the most precise results being obtained by SuperKamiokande [31]. Here also, an independent confirmation recently came from the K2K experiment using accelerator neutrinos [31].
– A third result, which does not fit into the standard three-flavor scenario, comes from the LSND experiment [33] which observed a $\bar{\nu_\mu}$ to $\bar{\nu_e}$ transition which would require a fourth neutrino with a mass around 1 eV, as the oscillation frequency in case of an oscillation interpretation is very high. There exists no other experimental evidence for such a neutrino, which would have to be 'sterile', as its interactions with matter should be much smaller than those of the 3 standard neutrinos.

Of course, negative searches constrain equally the oscillation scenarios (for example, the negative result of CHOOZ puts an upper limit on the mixing angle θ_{13} [8,32]).

All these experimental results are used by several groups of phenomenologists who check the consistency between data and perform global fits to extract the relevant oscillation parameters. This task needs some care, specially in the treatment of systematic errors affecting either the experimental data or the models describing the source of neutrinos (solar models, cosmic ray fluxes, ...). These fits most often use the formalism for 3 neutrino mixing (except when LSND is included), and are regularly updated when new data become available [34–36]. Results are now basically stable and agree well between the different groups.

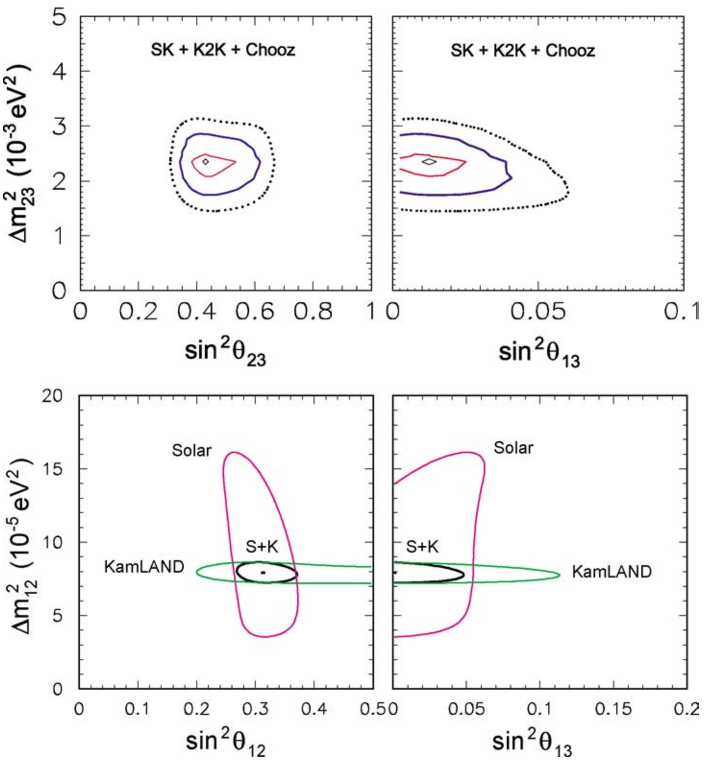

Fig. 5. Main results obtained from 3-family fits on oscillation data (taken from [34]).

Top line: acceptance contours at 1, 2 and 3 sigmas for Δm^2_{23}, $\sin^2(\theta_{23})$ and $\sin^2(\theta_{13})$ using data from SuperKamiokande (atmospheric neutrinos), K2K (ν_μ beam) and Chooz (nuclear reactor).

Bottom line: acceptance contours at 2 sigmas for Δm^2_{12}, $\sin^2(\theta_{12})$ and $\sin^2(\theta_{13})$ using only solar data (red), only reactor data from KamLAND (green) and their combination (black), showing the strong complementarity of solar and reactor data.

The main results of these fits are illustrated in Fig. 5 and can be summarized as follows (all values are given for 95% CL, and taken from [34], unless otherwise indicated):

- The solar oscillation corresponds to Δm^2_{12} between 7.2 and 8.6×10^{-5} eV^2, while the corresponding mixing angle θ_{12} is 38 ± 4 degrees. One sees that a maximal mixing ($\theta_{12} = 45$ degrees) is strongly excluded.
- The atmospheric oscillation corresponds to Δm^2_{23} between 1.8 and 2.9×10^{-3} eV^2 and the mixing angle θ_{23} between 40 and 58 degrees. Notice that the sign of Δm^2_{23} is unknown, while θ_{23} may well be maximal (one actually measures $\sin^2 2\theta_{23} > 0.9$); if not, whether θ_{23} is smaller or bigger than 45 degrees can only be determined from the subleading oscillations of ν_e to ν_μ and ν_τ (occurring in the ratio $\tan^2\theta_{23}$) if observable.
- The third angle θ_{13} is found compatible with 0, and smaller than 11 degrees.
- The mixing of a 4th sterile neutrino with ordinary neutrinos is already strongly constrained, but furthermore its addition fails to give a satisfactory 4-family global fit when LSND data are added [36].

Recent analyses [34] have also studied the impact of the oscillation results on other (non oscillation) experiments, such as the search for neutrinoless double beta decays or the study of large structures in our Universe (which is sensitive to the sum of the neutrino mass eigenvalues [24,37]). The main result here is a marginal compatibility with the recent claim [38] of a positive signal for neutrinoless double beta decay of ^{76}Ge.

Oscillation data have also been used to constrain more exotic scenarios, going from neutrino decays, neutrino decoherence, CPT violation to violations of the equivalence principle or Lorentz invariance, and gave significant limits on these potential effects [36].

To conclude this section, a perfectly coherent description of oscillation data is obtained in the standard 3-flavor framework, as long as the LSND result is discarded. Future experiments will bring more precise estimates of the oscillation parameters (amplitudes and frequencies), and the present consistency will then be put to more severe tests. Whether or not our present understanding will have to be revised is, of course, an open question.

4. The third mixing angle and the CP violating phase

All present data are compatible with θ_{13} being zero, while the CP phase δ creates no sizeable effect in any of the presently observed oscillations. The only way to determine these 2 parameters is to look for oscillations involving ν_e at the atmospheric oscillation frequency. The formula for ν_μ to ν_e transition, which in practice will be the searched for oscillation, at distances where the atmospheric oscillation is fully developed while the solar oscillation is just at its very beginning will be more intricate, as on the one hand the terms for atmospheric and solar frequencies will be both small and may well be of the same order of magnitude, and on the other hand the CP violating phase δ will explicitly appear and may create big asymmetries. An additional complication is due to matter effects which will be important for neutrino beams above 1 GeV, since the detectors will then be located at typical distances of 1000 km or more.

General formulae can be found in the literature [39]. We will here restrict to the vacuum case, and write the formula developed in the 2 small parameters $\varepsilon = \theta_{13}$ and $\varepsilon' = \Delta_{12}$, while Δ will be a shorthand for Δ_{13} and Δ_{23} (the derivation needs some care as the difference between Δ_{13} and Δ_{23} is ε')

$$P_{\mu e} = (2\varepsilon s_{23} \sin \Delta)^2 + (2\varepsilon' c_{12} s_{12} c_{23})^2 + 2(2\varepsilon s_{23} \sin \Delta)(2\varepsilon' c_{12} s_{12} c_{23}) \cos(\Delta + \delta)$$

this formula exhibits the positivity of $P_{\mu e}$ which is of the form: $X^2 + Y^2 + 2XY \cos(\phi)$.

To best determine θ_{13}, a detector should be placed in a neutrino beam so that the distance roughly corresponds to the first atmospheric oscillation maximum; this optimal distance is approximately 500 km $\times E_\nu$ (GeV). It is then interesting to rewrite the formula when the atmospheric oscillation phase Δ is exactly $\pi/2$, and use the fact that $\theta_{23} = \pi/4$:

$$P_{\mu e}^{\max} = A^2 + S^2 + 2AS \sin(\delta)$$

where $A = \sqrt{2}\theta_{13}$ is the 'atmospheric' term, and $S = \Delta_{12} \sin(2\theta_{12})/\sqrt{2}$ is the 'solar' term.

One clearly sees how the CP violating term comes from an interference between the 2 oscillation amplitudes (solar and atmospheric). When these two terms are equal (which happens for $\theta_{13} \simeq 1^\circ$ as $S \simeq 0.03$), a maximal CP violation will totally cancel one of the oscillations (either ν or $\bar{\nu}$), while the other is twice the expected value without CP violation. When A and S are different, the maximal asymmetry becomes smaller. From this it follows that the sensitivity on δ is roughly constant as soon as A is bigger than S (the bigger is A, the higher the statistics but the smaller the asymmetry, and both variations compensate each other).

In order to extract the 2 unknown parameters θ_{13} and δ, it is necessary to make several measurements, for example with ν_μ and $\bar{\nu_\mu}$. More generally, redundant measurements, using different beams and/or atmospheric neutrinos, will be necessary to completely solve some remaining ambiguities, such as the mass hierarchy (normal or inverted) or the octant ambiguity (whether θ_{23} is smaller or bigger than 45° in case it is not exactly at this value). Several strategies are studied, but will have to be adapted to future results and facilities (beams, detectors). Details can be found in [28].

5. Conclusions

Of course, oscillations will not solve all the open questions on neutrinos. Two main questions will have to be answered:

– what is the absolute mass scale, and the exact ordering of mass eigenstates?
– are neutrinos Dirac or Majorana particles?

Concerning the mass hierarchy, matter effects on the subleading oscillation between ν_μ and ν_e at the atmospheric frequency can bring the answer. It will then most probably come from a combination of atmospheric data with superbeams [40], or from a neutrino factory if θ_{13} is very small.

However, the absolute mass scale will stay unknown, and only neutrinoless double beta decays [11] or a possible direct determination of the mass [25] (if bigger than 0.3 eV) can bring the answer. Cosmological constraints should not be forgotten, although they depend on the cosmological model, which has still to be firmly established (specially in view of the recent discovery of dark energy).

To conclude, neutrino flavor oscillations, an unescapable consequence of quantum mechanics for massive neutrinos, have proven to be a very powerful and effective tool in our understanding of neutrino masses. It will continue to be so in the next decades, but the complete understanding of neutrino masses will require other approaches, the double beta decay being the most promising.

References

[1] P. Ramond, C. R. Physique 6 (2005), in this issue.
[2] K.G. Emeléus, Proc. Camb. Phil. Soc. 22 (1924) 400.
[3] C.D. Ellis, Proc. Roy. Soc. A 117 (1927) 109.
[4] W. Pauli, in: Friedr (Ed.), Physik und Erkenntnistheorie, Vieweg und Sohn, Braunschweig, 1984, p. 156;
See also for example http://www.ethbib.ethz.ch/exhibit/pauli/neutrino_e.html, or http://wwwlapp.in2p3.fr/neutrinos/aplettre.html, or http://www.laradioactivite.com/pages/00_phenomene/03_lettrepauli.htm.
[5] M.E. Nahmias, Proc. Camb. Phil. Soc. 31 (1935) 99.
[6] H.R. Crane, Phys. Rev. 55 (1939) 501.
[7] C.L. Cowan Jr, F. Reines, F.B. Harrison, H.W. Kruse, A.D. McGuire, Science 124 (1956) 103;
F. Reines, C.L. Cowan Jr, Nature 178 (1956) 446.
[8] Th. Lasserre, H. Sobel, C. R. Physique 6 (2005), in this issue.
[9] C.S. Wu, et al., Phys. Rev. 105 (1957) 1413.
[10] T.D. Lee, C.N. Yang, Phys. Rev. 104 (1956) 254;
T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671.
[11] S. Jullian, C. R. Physique 6 (2005), in this issue.
[12] G. Danby, et al., Phys. Rev. Lett. 9 (1962) 36.
[13] K. Kodama, et al., Nucl. Instrum. Methods A 493 (2002) 45.
[14] F.J. Hasert, et al., Phys. Lett. B 46 (1973) 121.
[15] J. Blietschau, et al., Nucl. Phys. B 114 (1976) 189.
[16] M. Cribier, T. Bowles, C. R. Physique 6 (2005), in this issue.
[17] B. Pontecorvo, Sov. Phys. JETP 7 (1958) 172;
B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984.
[18] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
[19] V. Gribov, B. Pontecorvo, Phys. Lett. B 28 (1969) 493.
[20] E. Pasierb, et al., Phys. Rev. Lett. 43 (1979) 96;
F. Reines, et al., Phys. Rev. Lett. 45 (1980) 1307.
[21] P. Astier, et al., Nucl. Phys. B 611 (2001) 3;
J. Altegoer, et al., Nucl. Instrum. Methods A 404 (1998) 96.
[22] E. Eskut, et al., Phys. Lett. B 497 (2001) 8;
E. Eskut, et al., Nucl. Instrum. Methods A 401 (1997) 7.
[23] M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, D. Freedman (Eds.), Supergravity, North-Holland, Amsterdam, 1979, p. 315;
T. Yanagida, in: Proc. of the Workshop on Unified Theory and the Baryon Number in the Universe, KEK, Japan, 1979;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[24] W. Buchmüller, C. R. Physique 6 (2005), in this issue.
[25] C. Weinheimer, C. R. Physique 6 (2005), in this issue.
[26] B. Kayser, Phys. Rev. D 24 (1981) 110;
For field theoretic approaches, see for example C. Giunti, hep-ph/0409230, hep-ph/0302026, and references therein.
[27] L. Wolfenstein, Phys. Rev. D 17 (1978) 2369.
[28] D. Autiero, Y. Déclais, C. R. Physique 6 (2005), in this issue.
[29] S.P. Mikheyev, A.Yu. Smirnov, Nuovo Cimento C 9 (1986) 17.
[30] A.Yu. Smirnov, hep-ph/0412391.
[31] T. Kajita, P. Lipari, C. R. Physique 6 (2005), in this issue.
[32] M. Apollonio, et al., Eur. Phys. J. C 27 (2003) 331.
[33] A. Aguilar, et al., Phys. Rev. D 64 (2001) 112007.
[34] G.L. Fogli, et al., hep-ph/0506083.
[35] S. Goswami, et al., hep-ph/0409224.
[36] M.C. Gonzales-Garcia, hep-ph/0410030.
[37] A.D. Dolgov, Phys. Rep. 370 (2002) 333, hep-ph/0202122.
[38] H.V. Klapdor-Kleingrothaus, et al., Nucl. Instrum. Methods A 522 (2004) 371.
[39] V. Barger, D. Marfatia, K. Whisnant, Int. J. Mod. Phys. E 12 (2003) 569, hep-ph/0308123.
[40] P. Huber, M. Maltoni, Th. Schwetz, Phys. Rev. D 71 (2005) 053006.