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MCAO for very large telescopes/OAMC pour les très grands télescopes

Wavefront sensing issues in MCAO

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Abstract

The problematic of wavefront sensing in Multiconjugate Adaptive Optics (MCAO) is addressed in this article. We first focus on the sky coverage estimation which drives, in particular, the choice between natural and laser guide stars and therefore has a major impact on wavefront sensor design. Then a comparison between star oriented and layer oriented concepts is proposed. Analytical developments and optimization of the concepts are derived in the simplest MCAO case: the ground layer AO system. From this study, advantages and drawbacks of each concept are highlighted. ***To cite this article: T. Fusco et al., C. R. Physique 6 (2005).***

Résumé

Problématique de l'analyse de front d'onde en optique adaptative multiconjuguée. La problématique de la mesure de front d'onde pour l'Optique Adaptative Multiconjuguée (OAMC) est présentée dans cet article. Dans un premier temps, l'estimation de la couverture de ciel pour un systeme d'OAMC est étudiée. Cette estimation permet, entre autre, un choix entre étoiles naturelles et étoiles lasers. Ensuite, une comparaison des concepts de mesures de front d'onde dits « orienté étoiles » et « orienté couches » est effectuées. Des développements analytiques et une optimisation des deux concepts sont proposés dans le cas simplifié d'un systeme de correction de la couche au sol. Cette étude permet de mettre en évidence les avantages et les inconvénients de chaque concept. ***Pour citer cet article : T. Fusco et al., C. R. Physique 6 (2005).***

Keywords: Turbulence; Adaptive Optics; Wavefront sensing

Mots-clés : Turbulence ; Optique adaptative ; Analyse de front d'onde

1. Introduction

In order to extend the field of view (FoV) of the classical adaptive optics (AO) systems, a modification of AO concept is required. Using several Guide Stars (GS) and several deformable mirrors (DM) conjugated at different selected altitudes, the multiconjugate AO (MCAO) [1,2] enables to overcome the limitation imposed by the anisoplanatism of the AO correction [3] and to reach the diffraction limit of the telescope in larger fields of view.

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In the last years, numerous MCAO systems have been proposed to fulfill various scientific requirements (see [4], J.-L. Beuzit, this volume). Their designs as well as their performance are extremely spread and cover a huge spectrum of new ideas and technologies (laser [5]/natural guide stars, ground layer AO [6,7], multi-object AO [8,9], layer oriented approach [10], ...). Therefore, to give a synthetic summary on MCAO systems is rather tricky. This is especially true for wavefront sensing issues. Indeed, the key parameters of the wavefront sensor (WFS) devices should derive from the system performance requirements, the wavefront characteristics to be measured and the available flux to perform such measurements. A possible approach to tackle the complex problem of wavefront sensing in MCAO is to divide it in three distinct topics which will be studied separately but remain highly interconnected.

Firstly, a sky coverage study has to be performed in order to obtain the first trade-offs (in close relation to the scientific needs) in terms of field of view, global performance, percentile of accessible sky and *in fine* to drive the choice between natural and laser guide stars (see Rigaut, [11] this volume). The generalization of classical AO sky coverage to MCAO systems is not obvious [12–14] and a lot of parameters have to be considered if one wants to obtain pertinent results. We propose in Section 2 a algorithm improvement of the sky coverage computation for MCAO. We present a new algorithm which allows to account for the real corrected FoV surface corrected by an MCAO system (depending on GS positions and system characteristics) as well as the type of strategies considered for the wavefront sensing.

The choice of a wavefront sensing concept is the second critical issue to be considered. It determines the way the photons coming from different GS will be recombined and used. Two different concepts have been proposed so far to measure the wavefront in MCAO: Star Oriented (SO) [2,15,16] or Layer Oriented (LO) [14,17]. A presentation of the two concepts as well as the identification of their main advantages and drawbacks is proposed in Section 3. A comparison of both concepts is proposed in the Ground-Layer AO case [6,7].

The last main point to consider in order to define a wavefront sensor for an MCAO system is the wavefront sensor device itself, that is the way the photons will be detected for the phase measurements. A large number of techniques have been proposed in the last thirty years, each one with its own advantages and drawbacks. We will not study in this article all the WFS techniques but a review can be found in [18–20].

2. Sky coverage in MCAO

The notion of sky coverage is rather simple for classical adaptive optics. Indeed, in that case, the system performance can be directly related to the GS magnitude and its separation from the object of interest (for given atmospheric conditions). The AO sky coverage is therefore nothing but a simple count of GS (of magnitude lower than a given limit) in a sky region taking into account the isoplanatic patch size [3] to compute the observable fraction of the sky (because this parameter is used to described the performance decrease outside the area defined by the GS, classical AO isoplanatic angle has to be used). Unfortunately, the generalization to MCAO systems is not obvious. Instead of one single parameter (GS flux), the final system performance is given by a combination of multiple factors in close interaction.

The idea is to link the MCAO global performance to physical information on guide stars and on the system. This performance can be expressed either in terms of average Strehl ratio (or any other MCAO performance estimator) in the FoV, performance uniformity or any combination of these two points. The analysis should allow to determine observational constraints (magnitude ranges, number of GS, maximum distance between GS, maximum magnitude difference between GS) as a function of system characteristics (field of view, telescope diameter, WFS concept and device, number of corrected modes, ...) and expected performance (correction level and uniformity in the field). The goal of such kind of analysis is to give tendencies which are essential to explore a large domain of parameters and to rapidly obtain feeling on general behaviors for various kind of MCAO systems. The output should be the first trade-offs concerning the system design and the adjustment of the scientific requirements. In particular, it could help to select the systems for which laser guide stars are mandatory, which constitutes a first step toward the WFS definition.

In the following sections, we will present an improved algorithm for sky coverage computation in MCAO. It requires the definition of different types of FoV which drive the final performance of MCAO systems and are essential to obtain a correct estimation of the sky coverage.

2.1. Varieties of field of view in MCAO

The definition of the relevant FoV is essential to derive a sky coverage estimation. In the MCAO case, four FoV (see Fig. 1) have to be defined in order to fully characterize the system and its performance. It is therefore essential to account for these various FoV in the sky coverage estimation. They are:

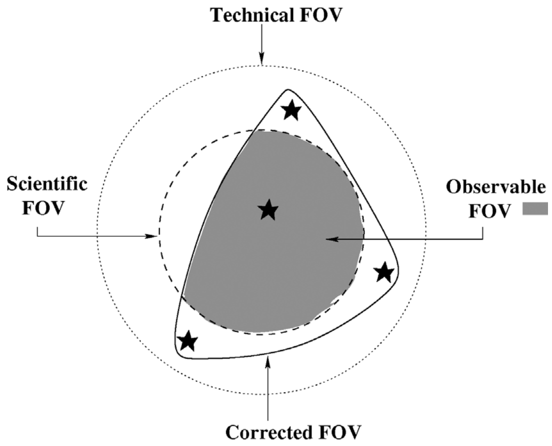


Fig. 1. Definition of the various notions of FoV in MCAO: The technical FoV (larger one) in which the GS have to be found, the scientific FoV ($\leqslant$ the technical FoV) in which the required MCAO performance should be fulfilled, the corrected FoV in which, for a given GS configuration the MCAO performance are actually achieved and the observable FoV which represents the part of the scientific FoV actually corrected for a given GS configuration.

- The technical FoV (Tech-FoV) which is the field in which GS have to be found. It depends on technical constraints (related to the telescope and system design) and scientific constraints. In particular, the size of the technical field directly impacts on the performance of MCAO system since an increase of the technical FoV implies a larger volume of turbulence to be sensed and thus a possible increase of wavefront correction error due to anisoplanatism effects.
- The scientific FoV (Sci-FoV) corresponds to the field one wants to correct, that is the field in which the scientific targets are located and thus in which the system performance has to be fulfilled.
- The corrected FoV (Corr-FoV) represents, for a given GS configuration, the part of the technical FoV actually corrected by the MCAO system. It is equal to the convolution of the surface defined by the stars within the technical FoV with the isoplanatic field (see Fig. 1). We have assumed here the MCAO system is able to interpolate the wavefront inside the surface defined by the GS in the wavefront reconstruction process [21]. The quality of this reconstruction is directly linked to the telescope diameter, the number of corrected modes and the Cn^2 profile [21,22].
- The observable FoV (Obs-FoV) is the last, but the truly important FoV (from an astronomer point of view). It corresponds to the part of the scientific FoV actually corrected by the MCAO system for a given GS configuration. It is nothing but the intersection of the corrected FoV with the scientific FoV.

$$\text{Obs-FoV} = \text{Corr-FoV} \cap \text{Sci-FoV} \tag{1}$$

It is interesting to note here that when the technical FoV is equal to the scientific FoV, it automatically implies an equality between corrected and observable FoV.

2.2. The surface sky coverage

In the classical sky coverage computation approach for MCAO [12,14], it is implicitly assumed that the whole scientific FoV is corrected if the right number of GS is found in the technical FoV. In other words, it does not account for the relative position between GS within the technical FoV.

Accounting for the GS geometry is essential to well describe the system performance in a given FoV. This leads us to introduce the four notion of FoV presented in Section 2.1 in the sky coverage computation to be as close as possible to the actual MCAO performance.

The fraction of observable FoV surface is introduced in the sky coverage computation and is defined by the ratio between the surface of the observable FoV over the surface of scientific FoV. The new computed sky coverage is called ‘Surface’ Sky coverage.

The main step of our ‘Surface’ sky coverage computation algorithm consists in the estimation of the observable FoV. It is obtained using the GS distribution in the technical FoV and their magnitude, the turbulence parameters (isoplanatic angle) and system characteristics:

- limiting magnitude, maximum magnitude difference between GS for the WFS,

- minimum number of GS ($N_{\mathrm{WFS,min}}$) mandatory to achieve a good wavefront reconstruction from the $N_{\mathrm{WFS,min}}$ measurements,
- maximum number of GS ($N_{\mathrm{WFS,max}}$) limited by the number of available WFS in the system.

From these data, one can compute the corrected surface from the available GS (fulfilling flux conditions) and the scientific FoV. We generate random GS fields using a stellar population model [23] in order to perform statistical computation of the observable surface[1] for a given technical FoV size. Three cases have to be considered for each occurrence of GS in the technical FoV:

- the number of GS is smaller than $N_{\mathrm{WFS,min}}$, in that case the corrected FoV value is set to 0;
- the number of GS is larger than $N_{\mathrm{WFS,min}}$ but smaller than $N_{\mathrm{WFS,max}}$, in that case we compute the corrected FoV as the largest possible polygon including all the GS;
- the number of GS is larger than $N_{\mathrm{WFS,max}}$. In that case we compute all the possible corrected FoV including the maximum number of GS and we define the global corrected FoV as the union of all the computed FoV.

For each case, the observable FoV surface ($S_{\text{obs-fov}}$) is obtained by the intersection of the scientific FoV with the corrected FoV finally computed using the convolution of the polygon formed by the GS with a disk of size the isoplanatic angle. It is the scientific FoV really corrected by the GS. More details on the algorithm can be found in [24]. For each GS geometry, the fraction of the observable FoV surface is defined by:

$$\mu_s = \begin{cases} \dfrac{S_{\text{obs-fov}}}{S_{\text{sci-fov}}} & \text{if } S_{\text{sci-fov}} > S_{\text{obs-fov}}, \\ 1 & \text{if } S_{\text{sci-fov}} \leqslant S_{\text{obs-fov}} \end{cases} \tag{2}$$

where $S_{\text{sci-fov}}$ is the surface of the scientific FoV. The smaller the science FoV is in comparison to the technical FoV, the closer to 1 μ_s should be. The "Surface" sky coverage ($P^{\text{Surface}}_{\text{N stars}>X}$) defines the percentage of the sky which can be observed at a given galactic coordinate for given technical and scientific FoV as well as the system constraints:

$$P^{\text{Surface}}_{\text{N stars}>X} = \langle \eta \cdot \mu_s \rangle \tag{3}$$

where η is a boolean number which is set to 0 when conditions on GS in the FoV are not fulfilled and set to 1 otherwise. $\langle\cdot\rangle$ stands for a statistical average on random realizations. It is interesting to note here that if μ_s is always set to 1, $P^{\text{Surface}}_{\text{N stars}>X}(m,r)$ is nothing but the classical sky coverage for MCAO. Let us now demonstrate the interest of the new Surface sky coverage algorithm considering the MOAO (Multi-Object AO) system of the FALCON instrument.

2.3. An example of an application: the FALCON project

The FALCON (Fiber-spectrograph with Adaptive-optics on Large-fields to correct at Optical and Near-infrared) project [8,25] aims at studying a concept of 3D multi-object spectrograph for the next generation of VLT instrumentation. FALCON will combine high spectral and spatial resolution in order to study the morphology as well as the dynamic behavior of distant galaxies (red-shift around 1–2). FALCON will operate in a 25×25 arcmin^2 FoV in which approximately 10 to 15 galaxies have to be found. From scientific requirements, at least an improvement of a factor two of ensquared energy in a 0.25×0.25 pixel^2 have to be obtained for typical observing conditions at Paranal.

Achieving high angular resolution on very dim targets requires a MCAO-like system, called hereafter Multi-Object AO (MOAO) system which will be composed of several WFS devices (observing on GS) to obtain a tomographic reconstruction of the turbulent wavefront and a dedicated correcting device in each galaxy direction. Each correcting device is composed of a DM controlled by the information coming from the closest (at least 3) GS.

In the following we will focus on one individual sub-system of the global FALCON instrument that is 3 WFS for tomographic reconstruction and one pupil plane DM for the wavefront correction in the galaxy direction. In the FALCON case, only on-axis optimization will be considered since we are just interested in a particular area of the field containing the Galaxy. Nevertheless such an optimization can be performed for any position in the technical FoV and in that sense, one can observe a Galaxy anywhere in the technical FoV. The whole FoV is not covered in a single shot but the whole FoV remains accessible to the system. Hence, the scientific FoV is equal to the technical one.

We compare in Fig. 2 the new surface sky coverage versus the classical sky coverage for various limit magnitudes, technical FoV and galactic latitude. The various plots highlight the importance of the surface parameters which weight the classical sky coverage in MCAO and allow us to account for GS positions in the technical FoV. Fig. 2 demonstrates that a classical algorithm

[1] Note that real astronomical field can be easily substituted to the simulated one if one wants to account for real part of the sky.

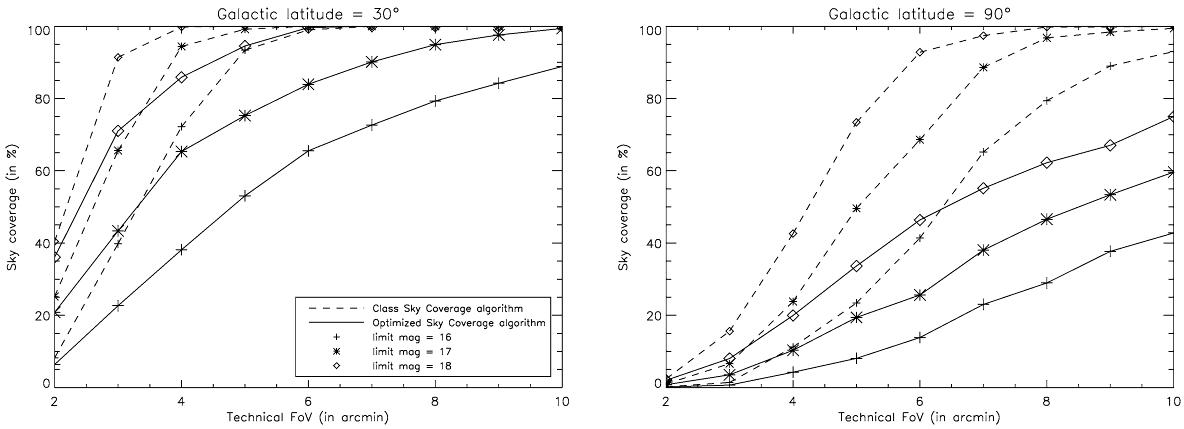


Fig. 2. Comparison of classical [dashed line] and surface [solid line] sky coverage for various technical FoV diameter and various limit magnitude. Two galactic latitude (30 and 90 degrees) are considered.

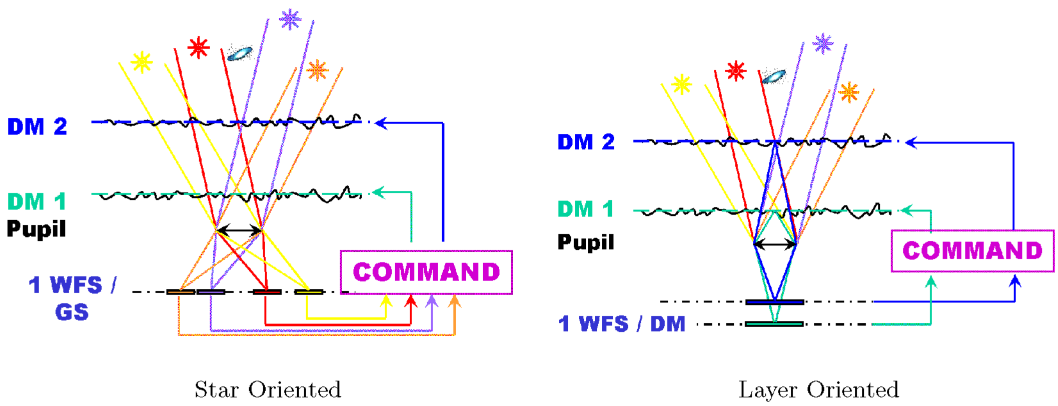


Fig. 3. Star and Layer Oriented concepts in MCAO. [Left] SO concept: one WFS per GS to control all the DM. [Right] LO concept: one WFS per DM combining all the GS.

over-estimates the sky coverage and that accounting for GS positions in the technical FoV is essential to obtain a more accurate estimation. It shows that a sky coverage close to 50% can be achieved for GS of magnitude 17 in a technical FoV of 8×8 arcminutes^2 and for a 90° galactic latitude. This is fully coherent with end-to-end simulations performed by F. Assemat [25] in the framework of the FALCON project.

3. Wavefront measurement concepts

The number and the magnitude of the available GS is an essential input to derive the final performance of an MCAO system. But the way the photons coming from these GS are recombined and used by the wavefront sensors (WFS) is also critical. In the following, because of the linearity of the WFS devices, we will assume that they provides a noisy measurement of the phase. Two main concepts (Star Oriented and Layer Oriented) have been proposed to deal with the turbulent volume measurement in a MCAO scheme. They are presented on Fig. 3 (for more details, see [26] in this volume).

3.1. The star oriented concept

In the SO case (see Fig. 3(left)) the wavefront ($\Phi_{\mathrm{SO}}^{\alpha_k}(\boldsymbol{r})$) in each GS direction ($\alpha_k$) is measured by a dedicated WFS. The whole set of measurement is given by:

$$\{\Phi_{\mathrm{SO}}^{\alpha_k}(\boldsymbol{r})\}_{k=1}^{\mathcal{K}_\star} = \left\{\sum_{l=1}^{n_l} \varphi_{\mathrm{true}}^{l}(\boldsymbol{r} - \alpha_k h_l) + \mathcal{N}oise(\overline{N}_k)\right\}_{k=1}^{\mathcal{K}_\star} \tag{4}$$

where $\overline{N}_k$ is the flux per GS, $\mathcal{K}_\star$ the number of GS, n_l the number of layers used to estimate the true phase φ^l_{true} in the layer l at the altitude h_l. r stands for the pupil coordinates. The function $\mathcal{N}oise(\cdot)$ depends on the WFS characteristics.

The reconstruction process consists in a tomographic reconstruction of the turbulent volume from the wavefronts measured in the GS directions and a projection onto the deformable mirrors [21]. Hence, the quality of this process is related to the noise of each WFS, that is to the magnitude of each GS. From a photometric point of view, the only constraint for the SO concept is the limiting magnitude per GS (m_{lim}). m_{lim} is the highest GS magnitude which makes the MCAO performance requirements fulfilled within the corrected FOV, considering a given GS distribution and given system characteristics (sampling frequency, sub-aperture number, detector noise).

3.2. The layer oriented concept

In the LO case (see Fig. 3(right)), the phase measurement is performed using one WFS and the correction using one DM per turbulent layer, each WFS (and DM) being optically conjugated to the altitude h_j of its corresponding layer. Hence only one resultant phase is measured per layer. In that case the measured phase ($\varphi_{\text{LO}}^{\text{WFS}_{\text{layer}_j}}(\boldsymbol{\rho})$) is much more complex than in the SO case, as shown in Eq. (5). In the WFS process the phase information is coded in term of intensity, so that the measured phase is the average of the phases in each GS direction *weighted by the corresponding GS flux*:

$$\left\{\varphi_{\text{LO}}^{\text{WFS}_{\text{layer}_j}}(\boldsymbol{\rho})\right\}_{j=1}^{N_{\text{WFS}}} = \left\{ \frac{\sum_{k=1}^{\mathcal{K}_\star}[\lambda_k \overline{N}_k \cdot \sum_{l=1}^{n_l} P(\boldsymbol{\rho}-\alpha_k h_j)\varphi^l_{\text{true}}(\boldsymbol{\rho}-\alpha_k[h_j-h_l])]}{\sum_{k=1}^{\mathcal{K}_\star}[\lambda_k \overline{N}_k P(\boldsymbol{\rho}-\alpha_k h_j)]} + \frac{\mathcal{N}oise(\gamma_j \sum_{k=1}^{\mathcal{K}_\star}\lambda_k \overline{N}_k[P(\boldsymbol{\rho}-\alpha_k h_j)])}{\gamma_j \sum_{k=1}^{\mathcal{K}_\star}[\lambda_k \overline{N}_k P(\boldsymbol{\rho}-\alpha_k h_j)]} \right\}_{j=1}^{N_{\text{WFS}}} \tag{5}$$

where P stands for the pupil function and $\{\gamma_j\}_{j\in[1,N_{\text{WFS}}]}$ represent the flux separation between the WFS ($\sum_{j=1}^{N_{\text{WFS}}}\gamma_j = 1$). N_{WFS} stands for the number of WFS.

There is no reconstruction process needed in that case since the measured phase $\varphi_{\text{LO}}^{\text{WFS}_{\text{layer}_j}}(\boldsymbol{\rho})$ can be used to directly control the DMs. The main interest of such an approach is the light co-addition before the detection which increases the signal to noise ratio (SNR) per WFS when noisy CCD are considered [27]. Nevertheless, this co-addition has a drawback: as shown in Eq. (5), the wavefronts coming from different directions in the FOV are mixed and weighted by the GS flux, leading to information loss and prevailance of the brighter stars in the phase measurement process [27,28]. A way to deal with GS flux differences is to optically attenuate the flux [17] of the brighter stars before the detection (λ_k coefficients in Eq. (5), see [28]). Using these coefficients, one reduces the effects of the magnitude differences on the LO measurement. However, this leads to a reduction of the integrated flux over the whole FOV and to an increase of the noise in the WFS measurements.

From a photometric point of view the relevant parameters in the LO case are therefore the integrated flux over all the GS (driving the WFS noise) and the magnitude differences between the GS. The interest of using optical densities ($0 \leqslant \lambda_k \leqslant 1$) to perform a trade-off between these two points will be demonstrated in the following sections.

3.3. SO and LO comparison: the simplest case of GLAO

GLAO [6] systems aim at compensating only for the boundary layer of the atmosphere which is indeed the location of most of the atmospheric turbulence [29] and whose correction remains valid on a wide FOV. As for MCAO the optimal way to get a knowledge on the ground layer turbulence is to perform an optical tomography [15,21] of the atmosphere. A simpler way is to take benefit of the angular decorrelation of high altitude turbulence by simply averaging the phase perturbations over all the GS directions. In this case, the phase one wants to estimated is:

$$\Phi^{\text{glao}}(\boldsymbol{r}) = \frac{1}{\mathcal{K}_\star}\sum_{k=1}^{\mathcal{K}_\star}\phi_k(\boldsymbol{r}) \tag{6}$$

where $\phi_k(\boldsymbol{r})$ is the phase of the wavefront in pupil plane coming from the direction $\boldsymbol{\alpha}_k$ of the kth GS: $\phi_k(\boldsymbol{r}) = \int_0^\infty \varphi(\boldsymbol{r} - h\boldsymbol{\alpha}_k, h)\,\mathrm{d}h$.

If a large number of GS are available then the Φ^{glao} measurement is mainly sensitive to the phase perturbations due to the ground layer. In that framework, the two WFS concepts (SO or LO) can be studied by minimizing a criterion corresponding to the difference between the phase to be measured Φ^{glao} (Eq. (6)) and its noisy version provided by the WFS concept (SO or LO) $\Phi^{\text{glao}}_{\text{so/lo}}$:

$$QC_{\text{WFS}} = \iint_{PUP} \left\langle \left(\Phi^{\text{glao}}(\boldsymbol{r}) - \Phi^{\text{glao}}_{\text{so/lo}}(\boldsymbol{r})\right)^2 \right\rangle \mathrm{d}^2\boldsymbol{r} \tag{7}$$

3.3.1. Optimized SO and LO measurements for GLAO

In the concept of SO, the $\mathcal{K}_\star$ measurements are recombined during the reconstruction process to drive the only DM conjugated to the ground. The reconstruction does not necessarily consist in a simple average on the GS directions; it has to be described more generally by a linear combination of the measurements ϕ_k:

$$\Phi^{\text{glao}}_{\text{SO}} = \sum_{k=1}^{\mathcal{K}_\star} \eta_k \cdot \left(\phi_k + \mathcal{N}oise(\overline{N}_k)\right) \tag{8}$$

In the LO approach, the light of several stars is co-added and sensed simultaneously with one unique WFS device conjugated to the ground layer. In that case, Eq. (5) is simplified as follow:

$$\Phi^{\text{glao}}_{\text{LO}} = \eta_{\text{LO}} \cdot \left[\frac{\sum_{k=1}^{\mathcal{K}_\star} \lambda_k \overline{N}_k \cdot \phi_k}{\sum_{\ell=1}^{\mathcal{K}_\star} \lambda_\ell \overline{N}_\ell} + \mathcal{N}oise\left(\sum_{\ell=1}^{\mathcal{K}_\star} \lambda_\ell \overline{N}_\ell\right)\right] \tag{9}$$

λ_k represents a possible flux attenuation (using optical density) in order to deal with flux dispersion between GS and η_{LO} is a numerical coefficient that can be used for optimizing the phase correction with respect to the overall WFS signal to noise ratio (SNR), as done with a closed loop gain. It is important to note that η_{LO} has the same value for all the GS directions and thus cannot deal with GS flux difference.

3.3.2. Expression of the quality criterion for SO and LO

Using the quality criterion defined in Eq. (7) it is possible to evaluate and compare the performance of both SO and LO concepts in the case of GLAO.

In the case of the SO concept, combining Eqs. (7) and (8) leads to following results:

$$QC_{\text{SO}} = \sum_{j=1}^{\mathcal{K}_\star} \sum_{k=1}^{\mathcal{K}_\star} \left[\left(\frac{1}{\mathcal{K}_\star} - \eta_j\right)\left(\frac{1}{\mathcal{K}_\star} - \eta_k\right)\mathcal{C}_{jk}\right] + \sum_{k=1}^{\mathcal{K}_\star} \eta_k^2 \cdot \left(\frac{\zeta}{\overline{N}_k} + \frac{\xi\,\sigma^2_{\text{det}}}{\overline{N}_k^2}\right) \tag{10}$$

where $\mathcal{C}_{jk} = \iint_{PUP} \langle \phi_j(\boldsymbol{r})\phi_k(\boldsymbol{r})\rangle\,\mathrm{d}\boldsymbol{r}$ is the covariance of the turbulence between the directions $\boldsymbol{\alpha}_j$ and $\boldsymbol{\alpha}_k$, ζ (resp. ξ) is the coefficient of propagation of the photon (resp. detector) noise and σ_{det} is the RMS CCD readout noise (assumed to be identical for all WFS devices).

Two terms can be identified in the expression of QC_{SO}. The first term depends on the covariance of the phases measured in different directions. The second term gathers all the error variances per GS due to photon and detector noises in the different WFS devices; faint GS make this term increase.

The first term can be nulled by treating all the GS directions identically in the average process, that is by setting all the η_k equal to $1/\mathcal{K}_\star$. This solution is however not optimal because it does not manage properly WFS SNR. In practice one can minimize QC_{SO} with respect to η_j. Since Eq. (10) is quadratic, an analytical solution exists and allows to find the optimum coefficients at all the GS directions.

For LO performance study, we have to combine Eqs. (7) and (9). Then we get:

$$QC_{\text{LO}} = \frac{1}{(\sum_{\ell=1}^{\mathcal{K}_\star} \lambda_\ell \overline{N}_\ell)^2} \sum_{j=1}^{\mathcal{K}_\star} \sum_{k=1}^{\mathcal{K}_\star} \left[\left(\langle \lambda_\ell \overline{N}_\ell\rangle_{\mathcal{K}_\star} - \eta_{\text{LO}}\lambda_j \overline{N}_j\right)\left(\langle \lambda_\ell \overline{N}_\ell\rangle_{\mathcal{K}_\star} - \eta_{\text{LO}}\lambda_k \overline{N}_k\right)\mathcal{C}_{jk}\right] + \eta^2_{\text{LO}} \cdot \left(\frac{\zeta}{\sum_{\ell=1}^{\mathcal{K}_\star} \lambda_\ell \overline{N}_\ell} + \frac{\xi\,\sigma^2_{\text{det}}}{(\sum_{\ell=1}^{\mathcal{K}_\star} \lambda_\ell \overline{N}_\ell)^2}\right) \tag{11}$$

As for QC_{SO}, we recognize two terms in the expression of QC_{LO}. The first one, related to the covariance of the phases from one GS direction to another is also depending on the flux difference between the GS. The second term gathers all the noise effects (photon and detector noises), which depend only on the total co-added flux.

Assuming that the numerical coefficient η_{LO} has the same value for all the GS directions (see the previous section) the only way to mitigate the importance of the first term of Eq. (11) is to use optical attenuators located in each GS optical path before the flux co-addition. But this flux attenuation also reduces the overall number of photons and increases the noise effects (second term in Eq. (11)). It is nevertheless possible to optimize the choice of λ_k for a given GS configuration and turbulence profile. It is not as simple as in the SO case because there is no quadratic dependency of QC_{LO} with respect to the $\{\lambda_k\}$. A numerical multi-variable minimization algorithm is required.

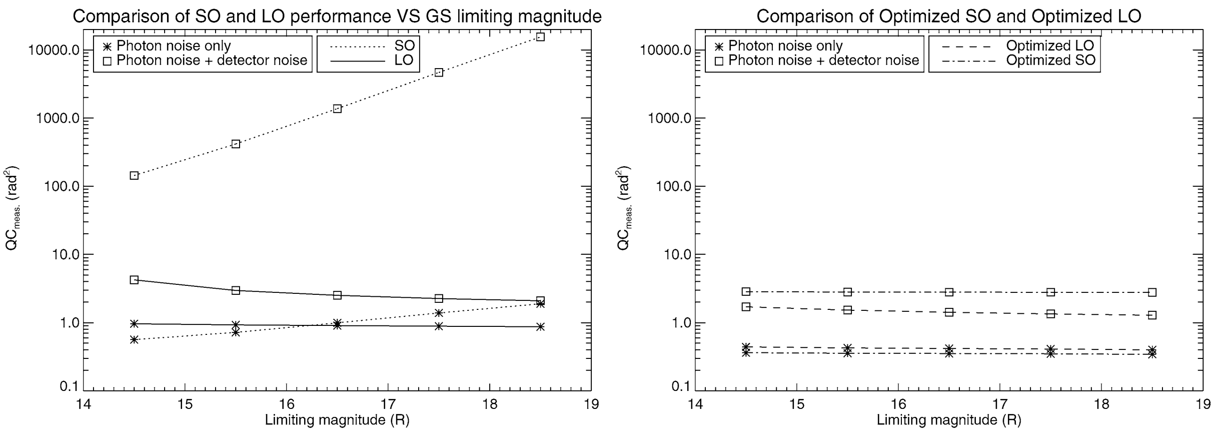
3.3.3. Comparison of the two concepts

In order to compare the behaviors of SO and LO, and to estimate the relevance of the optimization performed above, the results of numerical simulations are presented here. This study has been performed with the following assumptions:

- The system is characterized by a 8 meter pupil and a 8×8 arcmin^2 FOV.
- The turbulence profile is a test profile, with a seeing of 0.9 arcsec @ 0.5 µm, 60% of the turbulence in the pupil plane and 40% in altitude.
- The WFS devices are all identical Shack–Hartmann WFS, characterized by 14×14 sub-apertures. Simulation accounts for photon and/or detector noises with $\sigma_{\mathrm{det}} = 3\,\mathrm{e}^-/\mathrm{pix}$. Phases are decomposed onto Zernike modes and a modal optimization is performed to avoid the noise propagation onto low SNR modes. Even for the LO case, behavior of a SH WFS is assumed.
- GS fields simulation is based on a statistical model of the Galactic population [23] for the galactic latitude $b = +30$ and longitude $l = 0$. The presented measurement errors are averages computed over one thousand sets of FoV;
- For a given FoV, the SO and LO measurement errors are computed for growing GS limiting magnitude; increasing the limiting magnitude means including in the WFS process fainter and fainter GS, but still accounting for the bright ones. Hence increasing the limiting magnitude means increasing the number of GS and the integrated flux. There must be at least 4 GS in the FoV to consider the GLAO correction possible. For the considered FOV and galactic coordinates, a limiting magnitude of 18.5 corresponds approximatively to an average number of 30 GS.

Fig. 4(a) shows the comparison between the unoptimized SO ($\forall k,\ \eta_k = 1/\mathcal{K}_\star$) and the unoptimized LO ($\eta_{\mathrm{LO}} = 1$ and $\forall k$, $\lambda_k = 1$) when adding fainter and fainter GS in the WFS process. In presence of only photon noise, SO and LO performance are roughly the same. The SO performance is degraded when the GS limiting magnitude increases, due to faint GS that correspond to WFS with low SNR. The LO performance is slightly improved when the limiting magnitude increases due to the increasing of the integrated flux on the single WFS device, even if the degradation induced by the turbulence related term makes this improvement lower and lower when fainter GS are added. SO and LO keep the same relative behaviors in presence of detector noise, but their sensibility to the limiting magnitude increases. Moreover the SO performance is dramatically degraded, so that it becomes obvious that SO optimization is required, at least in presence of detector noise.

Fig. 4(b) shows the comparison of the optimized SO and the optimized LO performance (Eqs. (10) and (11)). In presence of only photon noise, the slight degradation of LO performance with compared to the SO one is the price to pay for the impossibility to set the GS attenuations with all the precision it would require (only bins of 1 magnitude have been considered). In detector noise regime, the optimized LO is better than the optimized SO, due to its much smaller number of WFS. The curves of the Fig. (4) shows all the interest brought by optimizing both SO and LO WFS concepts. When no detector noise is considered,



(a) unoptimized SO and LO

(b) optimized SO and LO

Fig. 4. Comparison of SO and LO performance, with or without optimization. Both cases of photon noise only or photon + detector noise are considered. When accounted for, the detector noise is characterized by $\sigma_{\mathrm{det}} = 3\,e^-/\mathrm{pix}$.

Table 1
Pros and Cons of the SO and LO WFS concepts

	Pros	Cons
SO	• In photon noise regime: – More information in SO – LO data can be obtained from SO ones [27] • WFS optimization per GS (nb. of reconstructed modes, exposure time) • Tomographic reconstruction ⇒ Easily adaptable to C_n^2 evolution	• Performance dependent on the detector noise • Detector number (1 per GS) • Command algorithm complexity
LO	• Increase of SNR per WFS (flux co-addition) ⇒ access to very faint stars with detector noise • Adaptability of WFS characteristics to the turbulent layer parameters (sub-aperutre diameter, exposure time) • Simplicity of the command process	• Information loss in co-addition before the detection ⇒ attenuation of brightest stars required: increase of noise • Flux separation per WFS (i.e. per measured layer) • Sensitivity to C_n^2 evolution: modification of the conjugation to follow up C_n^2 strongest layers • Complexity in the implementation of the LO concept

both optimized concepts lead to roughly the same performance. Then only the system complexity could drive a choice between one WFS concept or the other. In presence of detector noise, the optimized LO concept is still better than the optimized SO, but both performances are quite comparable, which is obviously not the case when no optimization is performed.

3.4. Advantages and drawbacks of each concepts

The conclusions derived from the study performed before in the case of GLAO can be generalized to more complex MCAO systems.One has to keep in mind that GLAO is probably the most favorable case for LO concept since only one WFS has to be considered (no flux splitting). The two approaches can be summarized by the PROS/CONS shown in Table 1.

It seems clear that there is no absolute choice and the best strategy has to be determined with respect to system constraints, complexity and performance requirements. The final choice will highly depend on the WFS device performance and in particular its behavior in detector noise regime.

4. Conclusion

Hence, the final choice of a WFS in MCAO is a multiple step process with several interactions between the sky coverage study, the wavefront measurement concept and the WFS device.

The sky coverage studies determine the nature and the characteristics of the GS which can be used by the WFS. It has been shown that information on GS photometry and GS spatial repartition combined with system characteristics are crucial to derive a pertinent estimation of the sky coverage in MCAO. Accounting for all these parameters leads to a sky coverage estimation typically two times smaller than the one obtained with more classical approaches.

The choice of the wavefront measurement concept determines the way the photons coming from different GS will be used. Two concepts have been proposed so far to deal with wavefront measurements in MCAO. It has been shown that, in any case, an optimization is required to reach the ultimate performance of each concept. These optimizations make the concepts equivalent in case of photon noise and comparable in detector noise regime. Hence, the final choice of a concept should be driven not only by the WFS concept expected performance but also by the system complexity.

Finally the WFS device determines the way the photons will be detected for the phase measurement. In the case of MCAO systems, the final choice of the WFS device will depend on a complex trade-off between the three following main characteristics: its sensitivity to deal with faint GS and therefore increase the sky coverages; its linearity to deal with variable conditions, variable system performance and its dynamics range to deal with very partial wavefront correction or even with open loop system (FALCON-like systems for instance).

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References

[1] R.H. Dicke, Phase-contrast detection of telescope seeing and their correction, Astron. J. 198 (1975) 605–615.
[2] J.M. Beckers, Increasing the size of the isoplanatic patch with Multiconjugate Adaptive Optics, in: Very Large Telescopes and their Instrumentation, ESO Conference and Workshop Proceedings, Garching, Germany, 1988, pp. 693–703.
[3] D.L. Fried, Anisoplanatism in adaptive optics, J. Opt. Soc. Am. 72 (1982) 52–61.
[4] J.-L. Beuzit, D. Mouillet, C. Moutou, K. Dohlen, P. Puget, T. Fusco, A. Boccaletti, e.a., A planet Finder instrument for the VLT, in: Direct Imaging of Exoplanets: Science & Techniques, IAUC 200, Nice, France, October 2005.
[5] M. Tallon, R. Foy, Adaptive telescope with laser probe: isoplanatism and cone effect, Astron. Astrophys. 235 (1990) 549–557.
[6] F. Rigaut, Ground-conjugate wide field adaptive optics for the ELTs, in: R. Ragazzoni, S. Esposito (Eds.), Beyond Conventional Adaptive Optics, Padova, Italy, 2001.
[7] N. Hubin, Ground Layer Adaptive Optics for 8–10 m telescopes, C. R. Physique 6 (2005), in this issue.
[8] F. Hammer, F. Sayde, E. Gendrion, T. Fusco, D. Burgarella, V. Cayatte, J.-M. Conan, F. Courbin, H. Flores, I. Guinouard, L. Jocou, A. Lanon, G. Monnet, M. Mouchine, F. Rigaud, D. Rouan, G. Rousset, V. Buat, Zamkotsian, The FALCON concept: multi-object spectroscopy combined with MCAO in near-IR, in: Scientifique Drivers for ESO Future VLT/VLTI Instrumentation, Garching, Germany, 2001.
[9] E. Gendron, FALCON: Multi-object AO, C. R. Physique 6 (2005), in this issue.
[10] R. Ragazzoni, J. Farinato, E. Marchetti, Adaptive optics for 100-m-class telescopes; new challenges require new solutions, in: P.L. Wininowitch (Ed.), Adaptive Optical Systems Technology, 2000, pp. 1076–1087.
[11] F. Rigaut, Applied LGS MCAO, C. R. Physique 6 (2005), in this issue.
[12] M. Le Louarn, R. Foy, N. Hubin, M. Tallon, Laser Guide Star for a 3.6 m and 8 m telescopes: performance and astrophysical limitations, Mon. Not. R. Astron. Soc. 295 (1998) 756.
[13] C. Arcidiacono, E. Diolaiti, R. Ragazzoni, J. Farinato, E. Vernet-Viard, Sky coverage for layer-oriented MCAO: a detailed analytical and numerical study, in: D. Bonaccini Calia, B. Ellerbreok, R. Ragazzoni (Eds.), Advancements in Adaptive Optics, Bellingham, Washington, 2004, in: Proc. SPIE, vol. 5490, 2004, pp. 563–574.
[14] E. Marchetti, R. Ragazzoni, E. Diolaiti, Which range of magnitudes for layer oriented MCAO?, in: P.L. Wizinowich, D. Bonaccini (Eds.), Adaptive Optical System Technologies II, in: Proc. SPIE, vol. 4839, 2002, pp. 566–577.
[15] M. Tallon, R. Foy, J. Vernin, 3-d wavefront sensing for multiconjugate adaptive optics, in: M.-H. Ulric (Ed.), Progress in Telescope and Instrumentation Technologies, ESO Conference and Workshop Proceedings Garching, Germany, 1992, pp. 517–521.
[16] B.L. Ellerbroek, First-order performance evaluation of adaptive-optics systems for atmospheric-turbulence compensation in extended-field-of-view astronomical telescopes, J. Opt. Soc. Am. A 11 (1994) 783–805.
[17] R. Raggazzoni, E. Dialaiti, J. Farinato, E. Fedrigo, E. Marchetti, M. Tordi, D. Kirkman, Multiple field of view layer-oriented adaptive optics. Nearly whole sky coverage on 8 m class and beyond, Astron. Astrophys. 396 (2002) 731–744.
[18] G. Rousset, Wave-front sensors, in: F. Roddier (Ed.), Adaptive Optics in Astronomy, Cambridge University Press, Cambridge, 1999, pp. 91–130, Chapter 5.
[19] R. Ragazzoni, Pupil plane wavefront sensing with an oscillating prism, J. Mod. Opt. 43 (1996).
[20] R. Angel, Ground-based imaging of extrasolar planets using adaptive optics, Nature 368 (1994) 203–207.
[21] T. Fusco, J.-M. Conan, G. Rousset, L.M. Mugnier, V. Michau, Optimal wavefront reconstruction strategies for Multiconjugate Adaptive Optics, J. Opt. Soc. Am. A 18 (2001) 2527–2538.
[22] A. Tokovinin, M. Le Louarn, M. Sarazin, Isoplanatism in multi-conjugate adaptive optics system, J. Opt. Soc. Am. A 17 (2000) 1819–1827.
[23] A. Robin, C. Reyl, S. Derrire, S. Picaud, A synthetic view on structure and evolution of the Milky Way, Astron. Astrophys. 409 (2003) 523–540.
[24] T. Fusco, A. Blanc, M. Nicolle, V. Michau, G. Rousset, J.-L. Beuzit, N. Hubin, Improvement of sky coverage estimation for MCAO systems: strategies and algorithms, Astron. Astrophys., submitted for publication.
[25] F. Assemat, Apport de la haute résolution angulaire sur létude des galaxies lointaine: imagerie, optique adaptative et spectroscopie 3D, Ph.D. thesis, Université de PARIS VII, Observatoire de Meudon, 2004.
[26] S. Esposito, Introduction to multiconjugate adaptive optics systems, C. R. Physique 6 (2005), in this issue.
[27] D. Bello, J.-M. Conan, G. Rousset, R. Ragazzoni, Signal to noise ratio of layer oriented measurements for multiconjugate adaptive optics, Astron. Astrophys. 410 (2003) 1101–1106.
[28] M. Nicolle, T. Fusco, V. Michau, G. Rousset, A. Blanc, J.-L. Beuzit, C. Petit, Ground layer adaptive optics: wavefront sensing concepts, in: Advancements in Adaptive Optics, Glasgow, UK, June 2004, in: Proc. SPIE, vol. 5490, 2004.
[29] J. Vernin, C. Munoz-Tunon, Optical seeing at La Palma Observatory. 2: Intensive site testing campaign at the Nordic Optical Telescope, Astron. Astrophys. 284 (1994) 311–318.