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Superconductivity and magnetism/Supraconductivité et magnétisme

Effects of magnetic excitations on the I–V characteristic of magnetic superconductors

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Abstract

The coupling between magnetic and superconducting degrees of freedom in magnetic superconductors leads to subgap resonances in the I–V characteristic. We study two mechanisms of coupling: the spin-assisted cotunneling (in layered magnetic superconductors) and the interaction mediated by the ac magnetic field of a moving vortex lattice. The latter mechanism we study in both layered and moderately anisotropic superconductors. At resonance conditions the dynamics of vortices in magnetic superconductors changes drastically, resulting in strong peaks in the dc I–V characteristic at voltages at which the washboard frequency of vortex lattice matches the spin wave frequency $\omega_s(\mathbf{g})$, where $\mathbf{g}$ are the reciprocal vortex lattice vectors. We show that for a high enough washboard frequency, peaks in the I–V characteristic in borocarbides and cuprate layered magnetic superconductors are strong enough to be observed on the quasiparticle background. ***To cite this article: M. Hruška, L.N. Boulaevskii, C. R. Physique 7 (2006).***

Résumé

Effets des excitations magnétiques sur la propriété I–V des supraconducteurs magnétiques. Le couplage des degrés de liberté magnétique et supraconducteur dans les supraconducteurs magnétiques conduit à des résonances sous le seuil dans la caractéristique I–V. Nous étudions deux mécanismes de couplage : le cotunneling induit par spin (dans les supraconducteurs magnétiques lamellaires) et l'interaction transportée par le champ magnétique alternatif d'un réseau de vortex en mouvement. Ce dernier mécanisme est étudié à la fois dans les supraconducteurs lamellaires et modérément anisotropiques. A la résonance, la dynamique des vortex dans les supraconducteurs magnétiques change brutalement, conduisant à des pics très marqués dans la caractéristique I–V continue à des tensions pour lesquelles la fréquence du réseau de vortex atteint celle de l'onde de spin $\omega_s(\mathbf{g})$ ($\mathbf{g}$ désignant le champ de vecteurs position du réseau réciproque de vortex). Nous montrons que, pour une fréquence du réseau de vortex assez haute, les pics dans la caractéristique courant-tension des supraconducteurs magnétiques lamellaires de carbure de bore et cuivriques émergent suffisamment du bruit de fond de quasiparticules pour être observés. ***Pour citer cet article : M. Hruška, L.N. Boulaevskii, C. R. Physique 7 (2006).***

Keywords: Moving vortex lattice; Borocarbides; Cuprates

Mots-clés : Réseau de vortex en mouvement ; Carbure de bore ; Cuivriques

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1. Introduction

Magnetic ordering can coexist with superconductivity without strong interference in the cases where spin density varies on the scale much smaller than the superconducting correlation length and net magnetic moment vanishes (for review see [1,2]). The coexistence of magnetism and superconductivity was observed in many crystals, such as $R\mathrm{Mo_6S_8}$, $R\mathrm{Rh_4B_4}$, $R\mathrm{Ba_2Cu_3O_{7-\delta}}$ and $(R, A)\mathrm{CuO_{4-\delta}}$ ($A = \mathrm{Sr, Ce}$) with the temperatures of magnetic ordering T_M much smaller than the superconducting critical temperature T_c, and also in borocarbides $R\mathrm{T_2B_2C}$ and ruthenocuprate $\mathrm{RuSr_2GdCu_2O_8}$ with T_M of the same order as T_c. Here R is the rare earth element, while $T = \mathrm{Ni, Ru, Pd, Pt}$. In such crystals f-electrons of ions R give rise to localized magnetic moments, while conducting electrons exhibit the Cooper pairing. In most of these crystals (exceptions are $\mathrm{HoMo_6S_8}$ and $\mathrm{ErRh_4B_4}$), magnetic moments order antiferromagnetically below T_M with a magnetic unit cell on a length scale much smaller than the superconducting coherence length and the London penetration lengths.

In this review we consider effects of coupling between magnetic and superconducting excitations on the I–V characteristic in magnetic superconductors. The interaction of the superconducting subsystem with the magnetic one opens up an additional channel of energy dissipation beside the quasiparticles. The additional dissipation (as would be evident from, e.g., the dc current) is the most prominent when conditions for a resonance between superconducting and magnetic excitations are met. Thus, by measuring the I–V characteristics one encounters a new possibility to probe magnetic excitations in magnetic superconductors. In particular we study the following two mechanisms of interaction between superconducting and magnetic degrees of freedom: (1) the spin-assisted co-tunneling in layered magnetic superconductors; and (2) interaction between a moving vortex lattice and magnetic moments via the ac magnetic field induced by moving vortices. In the first mechanism, existing in layered magnetic superconductors, the matrix element for superconducting tunneling from one layer to the next, depends on the spin of magnetic moments present in-between layers. The second mechanism of coupling exists in both isotropic (and nearly isotropic) as well as layered magnetic superconductors. The advantage of this mechanism to the techniques employed to probe nonsuperconducting magnetic conductors (e.g., radio-frequency irradiation) lies in the intrinsic generation of the ac magnetic field inside the superconductor. As a dc current is applied, the static spatially periodic magnetic field of the vortex lattice becomes an ac magnetic field of the same spatial period (determined by the external dc magnetic field), since the vortex lattice starts moving as a whole under the influence of the Lorentz force. The interaction of this intrinsic ac magnetic field and the magnetic moments is strong enough to change drastically the dc I–V characteristic in the superconducting mixed state when the ac magnetic field periodic in space is in resonance with spin waves of a corresponding momentum and frequency. Then moving vortices excite spin waves and energy transfer from vortices to the magnetic system leads to additional dissipation relative to that caused by quasiparticles. This results in strong current peaks in the dc I–V characteristics at voltages at which the washboard frequency of vortex lattice [3,4] matches the spin wave frequency $\omega_s(\mathbf{k})$ and $\mathbf{k}$ matches the reciprocal vortex lattice vector $\mathbf{g}$.

In Section 2 we consider slightly anisotropic superconductors, i.e., all systems mentioned above except layered superconductors with intrinsic Josephson junctions $\mathrm{SmLa_{1-x}Sr_xCuO_{4-\delta}}$ and $\mathrm{RuSr_2GdCu_2O_8}$ crystals, and probably also $\mathrm{Sm_{2-x}Ce_xCuO_{4-\delta}}$ [5–9]. In this section we introduce the idea of using the electromagnetic coupling of magnetic moments to the ac magnetic field induced by the moving vortex lattice to extract information on the magnetic excitations. In Section 3 we study the case of layered magnetic superconductors, where in the first subsection this idea is followed for the case of magnetic field applied parallel to the layers, while in the second subsection the effects of spin-assisted cotunneling are considered. We conclude in Section 4, proposing to probe low-frequency magnetic excitations in magnetic superconductors by measuring the I–V characteristics in the mixed state with a moving vortex lattice.

2. Spectroscopy of magnetic excitations in moderately anisotropic magnetic superconductors using vortex motion

To introduce spectroscopy of magnetic excitations in moderately anisotropic superconductors we follow [3] in this section. We assume, for simplicity, a uniaxial crystal structure with the principal axis along z. The dc magnetic field is applied along the z-axis and we assume that the magnetic induction $\mathbf{B}(\mathbf{r})$, $\mathbf{r} = x, y$, inside the superconductor corresponds to the ideal Abrikosov square vortex lattice (such a lattice is realized in clean borocarbide crystals in field $\mathbf{B} \parallel c$ in some field intervals [1]; our calculations are however trivially modified in the case of a triangular vortex

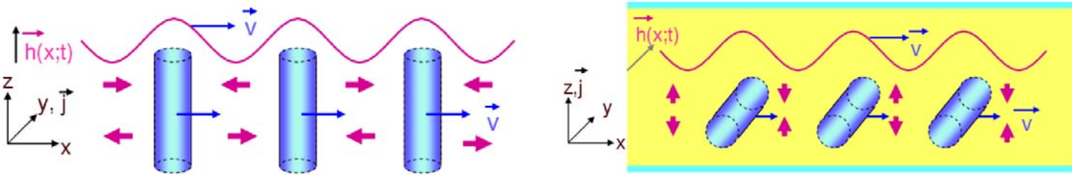

Fig. 1. Vortex lattice moving with the velocity $\mathbf{v}$ induces an ac magnetic field $h(x,t)$ which excites the system of magnetic moments shown by thick arrows. This additional dissipation results in the current peaks in I–V characteristics. Shown are the geometries discussed for nearly isotropic (left) and layered (right) superconductors.

lattice). The sublattice magnetization is assumed to be oriented in the (x, y) plane. The dc transport current with the density $\mathbf{j}$ is along the y-axis which, due to the Lorenz force, causes motion of the vortex lattice with the velocity $\mathbf{v}$ along the x-axis (Fig. 1).

We use the quasistatic approach assuming that at every moment in time the space structure of the magnetic induction is the same as in the static vortex lattice for the position given at that moment. Thus the magnetic induction $\mathbf{B}$ has the dependence on coordinates and time in the combination $(\mathbf{r} - \mathbf{v}t)$, moving in the same way as the vortex lattice. In the field interval $B \ll H_{c2}$ the magnetic induction is found from the London equations [10,11,2]

$$\operatorname{curl}\mathbf{B} = \frac{4\pi}{c}\mathbf{j}_s + 4\pi \operatorname{curl}\mathbf{M} \tag{1}$$

$$\mathbf{j}_s = \frac{c\Phi_0}{8\pi^2\lambda_\perp^2}\left(\nabla\phi - \frac{2\pi}{\Phi_0}\mathbf{A}\right), \quad \mathbf{B} = \operatorname{curl}\mathbf{A} \tag{2}$$

$$\operatorname{curl}\nabla\phi = \sum_n 2\pi\delta(\mathbf{r} - \mathbf{r}_n) \tag{3}$$

where $\mathbf{j}_s$ is the supercurrent, $\mathbf{A}$ is the vector potential, ϕ is the phase of the superconducting order parameter, $\mathbf{M}$ is the local magnetization, Φ_0 is the flux quantum and $\lambda_\perp = \lambda_x = \lambda_y$ is the London penetration length for currents in the (x, y) plane in the absence of magnetic moments. Further, $\mathbf{r}_n(t) = \mathbf{r}_n(0) + \mathbf{v}t$ are the coordinates of vortices in the xOy plane and $\mathbf{r}_n(0)$ form a regular vortex lattice. From Eqs. (1)–(3) we obtain

$$\operatorname{curl}\operatorname{curl}(\mathbf{B} - 4\pi\mathbf{M}) + \frac{1}{\lambda_\perp^2}\mathbf{B} = \frac{\Phi_0}{\lambda_\perp^2}\sum_n \delta(\mathbf{r} - \mathbf{r}_n) \tag{4}$$

To relate the Fourier components of $M_z(\mathbf{r}, t) \equiv M$ and $B_z(\mathbf{r}, t) \equiv B$ we use the linear response approximation in which supercurrents induce the 'external' magnetic field, $H(\mathbf{k}, \omega) = B(\mathbf{k}, \omega) - 4\pi M(\mathbf{k}, \omega)$, acting on the magnetic moments, where $M(\mathbf{k}, \omega) = \chi(\mathbf{k}, \omega)H(\mathbf{k}, \omega)$ and $\chi(\mathbf{k}, \omega) \equiv \chi_{zz}(\mathbf{k}, \omega)$ is the susceptibility of the magnetic system. This approach is valid for the magnetization harmonics $g_x \neq 0$ satisfying the condition

$$\left|M(\mathbf{g}, g_x v)\right|^2/(\mu n_M)^2 \ll 1 \tag{5}$$

where n_M is the density of magnetic ions and their magnetic moment is μ. For an antiferromagnet with two sublattices the magnetic susceptibility is given [12] by

$$\chi(\mathbf{k}, \omega) = \frac{\omega_M\omega_s(\mathbf{k})}{\omega_s^2(\mathbf{k}) - \omega^2 - \mathrm{i}\omega\nu_s} \tag{6}$$

Here $\omega_M = \mu^2 n_M/(2\hbar)$ at $\mu B \ll k_B T_M$, $\omega_s(\mathbf{k})$ is the magnetically active spin wave dispersion renormalized by superconductivity [2], while ν_s is the relaxation rate of spin waves due to the interaction with phonons. Using Eq. (4), we obtain for the Fourier components $\mathbf{k} = \mathbf{g} = 2\pi(B_0/\Phi_0)^{1/2}(n, m, 0)$ of the magnetic field

$$\left[1 + \frac{\lambda_\perp^2\mathbf{k}^2}{1 + 4\pi\chi(\mathbf{k}, \omega)}\right]B(\mathbf{k}, \omega) = (2\pi)^4\sum_{\mathbf{g}} B_0\delta(\mathbf{k} - \mathbf{g})\delta(\omega - g_x v) \tag{7}$$

where B_0 is the average induction, and n, m are integer. From Eq. (7) we see that magnetic moments renormalize the London penetration length so that the effective penetration length in magnetic superconductors is given [2] by

$\Lambda_\perp(\mathbf{k},\omega) = \lambda_\perp[1 + 4\pi\chi(\mathbf{k},\omega)]^{-1/2}$. Solving Eq. (7) we obtain the Fourier components of the 'external' field H at the washboard frequency $\omega = g_x v$ as

$$H(\mathbf{k},\omega) = (2\pi)^4 B_0 \frac{\delta(\mathbf{k}-\mathbf{g})\delta(\omega - g_x v)}{1 + 4\pi\chi(\mathbf{g},\omega) + \lambda_\perp^2 g^2} \tag{8}$$

Thus the moving vortex lattice induces a spatially periodic ac 'external' magnetic field $h(\mathbf{r},t) = H(\mathbf{r},t) - B_0$ along the z-axis characterized by momenta $\mathbf{g}$ and washboard frequencies $\omega = v g_x$. The moving vortex lattice induces also an electric field $\mathbf{E} = [\mathbf{v}\times\mathbf{B}]/c$ along the current direction.

The alternating magnetic field $h(\mathbf{r},t)$ of a vortex lattice moving with velocity $\mathbf{v}$ is characterized by the washboard frequency in direction $\mathbf{g}$ given by $\omega_w(\mathbf{g}) = \mathbf{g}\cdot\mathbf{v}$. This ac magnetic field excites spin waves of momenta $\mathbf{k} = \mathbf{g}$ and the frequencies $\omega_s(\mathbf{g}) = \omega_w$ provided v exceeds some critical velocity determined by the spin wave velocity, as in the case of the Cherenkov radiation. (We will discuss this condition in more detail at the end of this section.)

Assuming that sublattice magnetizations are almost perpendicular to the applied magnetic field, we obtain for the power per unit volume transmitted from the vortex lattice to the magnetic system [12]

$$P_M = -\left\langle \mathbf{M}(\mathbf{r},t)\cdot\frac{\partial\mathbf{h}(\mathbf{r},t)}{\partial t}\right\rangle = \sum_{\mathbf{g}(g_x>0)} 2g_x v\left|h(\mathbf{g}, g_x v)\right|^2 \operatorname{Im}\left[\chi(\mathbf{g}, g_x v)\right] \tag{9}$$

where brackets denote time and space average and $h(\mathbf{g}, g_x v) = B_0/[1 + 4\pi\chi(\mathbf{g},\omega) + \lambda_\perp^2 g^2]$. At $\chi = 0$ for $\lambda_\perp = 1300$ Å, typical for borocarbides, the amplitude of the main harmonic, $n = 1$, $m = 0$, is about 20 G.

To find the velocity of the vortex lattice at a given transport current density j we equate the power per unit volume performed by the battery, jE, to the sum of the power dissipated by quasiparticles, ηv^2, and that transmitted to the magnetic system, P_M. Here η is the viscous drag coefficient due to quasiparticles in normal vortex cores. It is given by the Bardeen–Stephen expression $\eta = B_0 H_{c2}^* \sigma_n/c^2$, where σ_n is the normal state conductivity, $H_{c2}^* = \Phi_0/(2\pi\xi_\perp^2)$ is the orbital upper critical field and $\xi_\perp$ is the superconducting correlation length in the direction perpendicular to the applied magnetic field. Taking into account that $E = vB_0/c$ and $\omega = v g_x = cEg_x/B_0$, we find v and finally j–E (i.e., I–V) characteristics in the intervals of E, where inequality Eq. (5) is fulfilled:

$$j(E) = \frac{c^2\eta}{B_0^2}E + \sum_{\mathbf{g}\neq 0} \frac{2g_x c B_0 \operatorname{Im}[\chi(\mathbf{g}, cEg_x/B_0)]}{|1 + 4\pi\chi(\mathbf{g}, cEg_x/B_0) + \mathbf{g}^2\lambda_\perp^2|^2} \tag{10}$$

From this equation we see that the current density as a function of E has peaks corresponding to the resonances in magnetic susceptibility, i.e., the resonances between the ac magnetic field and spin waves, when $\omega_s(n,m) = 2\pi v(B_0/\Phi_0)^{1/2} n$.

To discuss the behavior of $j(E)$ near the resonances we introduce the frequency deviation $\Delta\omega = \omega_s(\mathbf{g}) - \omega$ such that $\nu_s \ll \Delta\omega \ll \omega_s(n,m)$. Then we obtain $\chi(\mathbf{g},\omega) \approx \omega_M/(2\Delta\omega)$ and $\operatorname{Im}[\chi(\mathbf{g},\omega)] \approx \omega_M\nu_s/(2\Delta\omega)^2$. We consider the interval of frequency deviations $\Delta\omega$ where $\lambda_\perp^2 g^2 \gg 4\pi\chi(\mathbf{g},\omega)$. In this interval we estimate

$$\frac{M(\mathbf{g},\omega)}{\mu n_M} \approx \frac{\mu\Phi_0}{16\pi^2(n^2+m^2)\lambda_\perp^2\hbar\Delta\omega} \tag{11}$$

Due to the condition Eq. (5) our approach is valid for $\hbar\Delta\omega > \mu\Phi_0/(4\pi\lambda_\perp)^2$. The ratio of the additional current caused by spin waves over the current background is given as

$$\frac{\Delta j(n,m)}{j} \approx \frac{\omega_M\nu_s\Phi_0 B_0}{8\pi^2\omega\eta(\Delta\omega)^2\lambda_\perp^4}\frac{n^2}{(n^2+m^2)^2} \tag{12}$$

In the frequency interval $\hbar\Delta\omega > \mu\Phi_0/(4\pi\lambda_\perp)^2$ we derive

$$\frac{\Delta j(n,m)}{j} < \frac{16\pi^2\hbar n_M\nu_s B_0}{\omega\eta\Phi_0}\frac{n^2}{(n^2+m^2)^2} \tag{13}$$

In magnetic insulators ν_s is typically of order 10^6 s^{-1}. One can anticipate the same value in magnetic superconducting crystals, as conducting electrons are gapped. For $HoNi_2B_2C$, taking $H_{c2}^* \approx 10$ T, $n_M = 10^{22}$ cm^{-3},

$\sigma_n = 10^5\ \Omega^{-1}\,\mathrm{cm}^{-1}$ at $\omega = 10^{10}\ \mathrm{s}^{-1}$ we derive $\Delta j(n,m)/j < 0.8n^2/(n^2+m^2)^2$. Thus, the peak $n = 1$, $m = 0$ is observable even in the frequency interval where our linear response approach is valid. Here the magnetic system deviates only slightly from equilibrium as energy is transformed further to the phonon bath.

Closer to the resonance the linear response approach breaks down. Here the dominant contribution to dissipation comes from generation of spin waves by vortices which leads to a strong deviation of the magnetic system from equilibrium. For quantitative description of the j–E characteristics close to resonances the full dynamic approach for vortices and magnetic system is necessary.

Based on Eq. (10) we see that measurements of the I–V characteristics at different magnetic fields and currents may provide information on the spin wave dispersion $\omega_s(\mathbf{g})$. The washboard frequency ω and the reciprocal vortex lattice vectors $\mathbf{g}$ may be changed independently, but an important question is what are limitations on their variations. Momentum $k \sim 2\pi(B_0/\Phi_0)^{1/2}$ is of order $10^6\ \mathrm{cm}^{-1}$ in fields $B \leqslant 1$ T and increases as one approaches H_{c2}, but then harmonic amplitudes $h(\mathbf{g},\omega)$ drop. Limitations on frequency are due to limitations on j, which should be lower than the depairing current density, and also should not lead to excessive heating. From Eq. (10), to reach frequency ω one needs current density $j(\omega) \geqslant \sigma_n \omega H^*_{c2}/cg_x$ and the electric field $E(\omega) = \omega B_0/cg_x$. For $B = 1$ T we obtain $j(\omega) \approx 10^7 n^{-1}(\hbar\omega/1\mathrm{K})\ \mathrm{A/cm^2}$ if the lowest harmonics are used, while for higher harmonics higher frequencies may be reached. The depairing current density for borocarbides is of order $10^7\ \mathrm{A/cm^2}$ and thus spin waves with energies $\hbar\omega \lesssim 1$ K may be probed. For the dissipation power per unit volume, $P_{\mathrm{dis}} = jE \geqslant \sigma_n \omega^2 \Phi_0 H^*_{c2}/4\pi^2 c^2$, we estimate $P_{\mathrm{dis}} \sim 10^8 n^{-2}(\hbar\omega/1\ \mathrm{K})^2\ \mathrm{W/cm^3}$. To diminish heating the pulse technique may be used as in I–V measurements by Kunchur [13].

As energies that may be probed by I–V measurements in this way are limited from above, an important question is what the minimum spin wave energy is. Neglecting effects of magnetic field (which are small for $\mu B \ll SJ$, where J is the exchange interaction of order of the Neel temperature), the spectrum of magnetic excitations is determined solely by the direct exchange of magnetic ions, by their RKKY interaction via the conducting electrons and by the magnetic anisotropy. When the magnetic anisotropy is absent, the dispersion in a two-sublattice antiferromagnet is linear at small momenta ($k \ll 1/a$), $\omega_s(\mathbf{k}) = v_s k$, where $v_s = Ja/\hbar$ z is the coordination number, S is the ion spin and a is the magnetic correlation length (of order of the nearest neighbour spacing). Then the generation of spin waves by the moving vortex lattice occurs if $v \geqslant v_s$, as in the case of generation of sound by the moving vortex lattice due to the ac electric field [23,24]. The magnetic anisotropy introduces a gap Δ giving the dispersion of the form $\omega_s(\mathbf{k}) = \sqrt{(\Delta/\hbar)^2 + (v_s k)^2}$. Then the condition for spinwave excitation by the ac magnetic field of the moving vortex lattice is given by $v^2 > v_s^2 + (\Delta a/\hbar)^2$ (see also Fig. 2). Neither experimental nor theoretical information on the strength of magnetic anisotropy or the structure of excitations in these materials is available to date. Thus we cannot predict yet whether resonance conditions for the lowest harmonics will be fulfilled in borocarbides. However, we can anticipate that higher harmonics will be effective in the case of weak pinning.

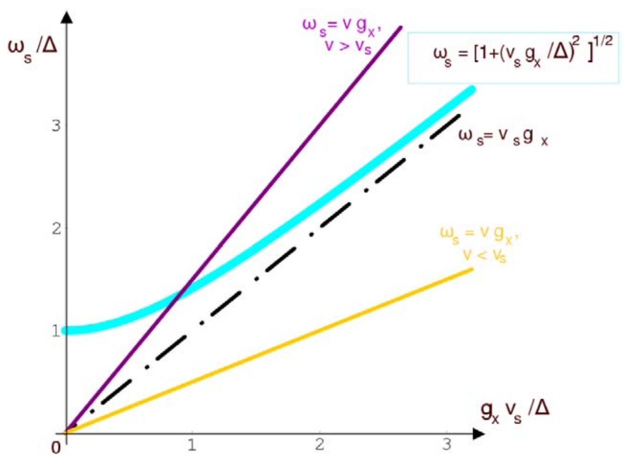

Fig. 2. (Rescaled) momentum dependence of the spinwave frequency (the light curve, with a dashed asymptote) and the washboard frequency at large (dark straight line) and small (light straight line) velocities.

3. Spectroscopy of magnetic excitations in layered magnetic superconductors

The c-axis transport in many layered superconductors (intercalated dichalcogenides, cuprates and organic superconductors) is well described by modeling them as a stacked array of two-dimensional superconductors coupled by Josephson tunneling between adjacent layers [25,2]. The intrinsic Josephson effect between the copper-oxide layers has been demonstrated in several ways. Josephson plasma resonance measurements [26] for the direction perpendicular to the layers show that the energy of this collective mode is much smaller than the superconducting gap in these materials. This is in contrast to the high energy plasma mode for the electric field along the layers and in bulk isotropic superconductors [27]. The difference is due to the fact that plasma oscillations are gapless when electrons are almost constrained to move in 2D layers causing the screening of their Coulomb interaction to be strongly anisotropic [28]. In the case of weak interlayer coupling plasmon for the electric field perpendicular to the layers acquires small gap known as the Josephson plasma frequency. Another confirmation of the Josephson type of interlayer coupling comes from the multibranch structure of the current-voltage characteristic, with each branch corresponding to a well defined number of Josephson junctions in the resistive state [29]. In addition, one can calculate the coherence length in the direction perpendicular to the layers (along the c-axis) from the knowledge of the anisotropy ratio γ and thus demonstrate the applicability of the Lawrence–Doniach tunneling model [25] which is valid when the coherence length perpendicular to the layers is much smaller than the interlayer spacing s.

The Josephson oscillations generate time-dependent electromagnetic field which couples to the electromagnetic cavity modes of the junction structure. As a result, the current-voltage characteristic exhibits current peaks or multiple branches at voltages related to the corresponding mode frequencies by the Josephson relation [31,32].

In addition to excitation of electromagnetic cavity modes, other degrees of freedom which interact with the superconducting phase difference could influence the shape of the I–V characteristic of the junction in the same way. Recently, specific subgap structures in the form of current anomalies at some voltages were observed in the I–V characteristics of intrinsic Josephson junctions in $Bi_2Sr_2CaCu_2O_8$ and $Tl_2Ba_2Ca_2Cu_3O_{10}$ with the tunneling current in the c-direction [30]. The specific subgap structures in the I–V characteristics of $Bi_2Sr_2CaCu_2O_8$ and $Tl_2Ba_2Ca_2Cu_3O_{10}$ were explained as stemming from two different mechanisms of coupling between Josephson oscillations and phonons: the electromagnetic interaction between the ionic charges of the dielectric layer and the charges of conduction electrons [16] and the phonon assisted tunneling as due to the dependence of the tunneling matrix element on lattice displacements [17]. It was demonstrated that also two-level systems may be excited inside the junction due to coupling to the phase oscillations [33,34].

In electron-doped cuprates $Ln_{2-x}Ce_xCuO_{4-\delta}$ with Ln = Nd, Sm, the antiferromagnetic ordering of the rare-earth ions has been found to coexist with superconductivity at low temperatures within a narrow range of doping near $x = 0.15$ [9,35]. The $SmLa_{1-x}Sr_xCuO_{4-\delta}$ compound is especially interesting because its T^* structure leads to a two-dimensional character of the magnetic system. Here magnetic Sm_2O_2 and nonmagnetic $La_{2-x}Sr_xO_{2-\delta}$ layers alternate in the barriers between the superconducting CuO_2 layers. The Josephson nature of the interlayer coupling in this crystal has been confirmed by observation of the double Josephson plasma resonance stemming from two layers in a unit cell [5,6]. According to specific heat measurements [7], magnetic ordering is absent down to a temperature of 0.3 K and a magnetic gap, if any, lies below 0.3 K. They reveal a broad peak near the temperature 1 K and the height of this peak indicates the presence of competing interactions that might be described by the two-dimensional J_1–J_2 Heisenberg model with $J_2/J_1 > 0.4$ [7,14]. Such a model has very complex dynamics and contains a variety of transitions down to zero temperature, making it an ideal testing ground for the theory of quantum phase transitions. The most interesting part of the phase diagram is in the region $0.4 \lesssim J_2/J_1 \lesssim 0.55$, where a gapped phase without magnetic ordering is likely to be taking place. However, its characterization has been one of the most intriguing puzzles of the physics of strongly correlated systems [15].

We discern two main mechanisms of coupling between the superconducting phase difference and additional degrees of freedom:

(1) the electromagnetic mechanism, when the ac magnetic or electric field of Josephson oscillations couples to other degrees of freedom, as in the case of the ac electric field of the Josephson oscillations coupling to the optically active phonons [16]; and
(2) the cotunneling mechanism, when the tunneling matrix element depends on the additional degrees of freedom, e.g., on lattice displacements in the phonon-assisted tunneling [17] or on spins.

The latter mechanism we study in Section 3.2. whereas in Section 3.1. we study the electromagnetic mechanism of coupling in the presence of a strong magnetic field applied parallel to the layers. We will show that the spin-assisted cotunneling leads to enhancement of the dc current at voltages corresponding to singlet magnetic excitations, and that the moving vortex lattice leads to peaks in the dc I–V characteristic at voltages corresponding to triplet spinwave excitations with momenta determined by the applied magnetic field and the interlayer spacing. These two mechanisms of coupling of the phase difference to the spin degrees of freedom provide an alternative tunneling spectroscopy that may be important in the cases when the inelastic neutron scattering spectroscopy cannot be applied (as for example is the case for Sm compounds due to a high scattering cross-section for neutron capture).

3.1. Spectroscopy using vortex motion in layered superconductors

If the magnetic field is applied perpendicular to the layers (along the c-axis), it induces pancake vortices which do not form regular lattice in the magnetic fields above 20 G as they order along the c-axis only due to weak Josephson and magnetic interactions [18]. This makes excitation of spin waves ineffective by moving vortex lattice induced by a perpendicular magnetic field. When a magnetic field is applied parallel to the layers (in the ab-plane, along the y-axis), the situation is drastically different, because now Josephson vortices [19–22] are induced. In high fields they form a lattice which is quite regular in the x-direction (parallel to the layers). Josephson vortices do not have normal cores and so only thermally induced quasiparticles (or those near the nodes in the case of d-wave pairing) cause dissipation. A weak interlayer tunneling transport current, which leads to vortex motion in the x-direction, cannot destroy superconductivity and produces much less heating than in the case of isotropic or weakly anisotropic superconductors. An ac magnetic field in the direction of the applied dc magnetic field parallel to the layers results naturally from the motion of a regular vortex lattice. The spatial periodicity of this field is determined by the periodicity of the vortex lattice while the time periodicity is determined by the Josephson frequency.

In the absence of magnetic moments, the distribution of the magnetic induction $B(\mathbf{r})$ inside intrinsic Josephson junctions has been found as a solution to coupled finite-difference differential equations for the phase difference φ_n and for the magnetic induction B_n inside the junction n between layers n and $n+1$ [22,20]. These equations can be derived from Maxwell equations expressing fields and currents in terms of the gauge-invariant phase-difference between layers, $\varphi_n = \phi_{n+1} - \phi_n - (2\pi s/\Phi_0)A_{zn}$ and the in-plane superconducting momentum $p_{xn} = \partial\phi_n/\partial x - (2\pi/\Phi_0)A_{xn}$. In the presence of magnetic moments, we account for magnetization M_n of ions inside intrinsic Josephson junction n in the linear response approximation. Taking the direction of the magnetic induction to be parallel to the layers (along y-axis) and the applied current along the c-axis in the z-direction, the local magnetic field between layers n and $n+1$ can be expressed as

$$H_n = \frac{A_{x,n+1} - A_{x,n}}{s} - \frac{\partial A_{zn}}{\partial x} - 4\pi M_n = \frac{\Phi_0}{2\pi s}\left(\frac{\partial\varphi_n}{\partial x} - p_{x,n+1} + p_{x,n}\right) - 4\pi M_n \tag{14}$$

The components of the electric field can be approximated as

$$E_{xn} \simeq \frac{\Phi_0}{2\pi c}\frac{\partial p_{xn}}{\partial t}, \qquad E_{zn} \simeq \frac{\Phi_0}{2\pi cs}\frac{\partial\varphi_n}{\partial t} \tag{15}$$

The components j_{xn} and j_{zn} of electric currents containing the normal and superconducting currents are given by

$$j_{xn} = \sigma_{ab}\frac{\Phi_0}{2\pi c}\frac{\partial p_{xn}}{\partial t} + \frac{c\Phi_0}{8\pi^2\lambda_{ab}^2}p_{xn}, \qquad j_{zn} = \sigma_z\frac{\Phi_0}{2\pi cs}\frac{\partial\varphi_n}{\partial t} + j_J\sin\varphi_n \tag{16}$$

where σ_c and σ_{ab} are quasiparticle conductivities along the c-axis and in the ab-plane, respectively, λ_c and λ_{ab} are the corresponding London penetration lengths and $j_J = c\Phi_0/8\pi^2 s\lambda_c^2$ is the Josephson current density. Using these relations we can rewrite the z and x components of the Maxwell equation $\nabla\times\mathbf{H} = (4\pi/c)\mathbf{j} + \partial\mathbf{D}/\partial t$:

$$\frac{2\sigma_c\Phi_0}{c^2 s}\frac{\partial\varphi_n}{\partial t} + \frac{4\pi}{c}j_J\sin\varphi_n + \frac{\varepsilon_c\Phi_0}{2\pi c^2 s}\frac{\partial^2\varphi_n}{\partial t^2} = \frac{\partial H_n}{\partial x} \tag{17}$$

$$\frac{2\sigma_{ab}\Phi_0}{c^2}\frac{\partial p_{xn}}{\partial t} + \frac{\Phi_0}{2\pi\lambda_{ab}^2}p_{xn} = -\frac{H_n - H_{n-1}}{s} \tag{18}$$

where in the second equation the displacement current was neglected (as its typical frequency is assumed to be much smaller than the in-plane plasma frequency c/λ_{ab}) and the derivative $\partial H/\partial z$ is replaced by the discrete derivative

$(H_n - H_{n-1})/s$ instead by $(H_{n+1} - H_n)/s$. (This procedure prevents a drift of the discrete derivative indices to the neighbouring layer indices in the following step of the derivation when the second derivative is taken.) Taking a discrete derivative of Eq. (18) and making use of Eq. (14) we obtain

$$\left(\frac{4\pi\sigma_{ab}}{c^2}\frac{\partial}{\partial t} + \frac{1}{\lambda_{ab}^2}\right)\left(\frac{\Phi_0}{2\pi s}\frac{\partial\varphi_n}{\partial x} - H_n - 4\pi M_n\right) = -\frac{H_{n+1} + H_{n-1} - 2H_n}{s^2} \tag{19}$$

Eqs. (17) and (19) together with the linear response approximation describe the phase dynamics in terms of phases, magnetic field and magnetization. In terms of dimensionless variables φ_n, $b_n = B_n 2\pi\lambda_{ab}\lambda_c/\Phi_0$, $m_n = M_n 2\pi\lambda_{ab}\lambda_c/\Phi_0$ and $h_n = b_n - 4\pi m_n$, we have:

$$\frac{\partial^2\varphi_n}{\partial\tau^2} + \nu_c\frac{\partial\varphi_n}{\partial\tau} + \sin\varphi_n - \frac{\partial h_n}{\partial u} = 0 \tag{20}$$

$$\nabla_n^2 h_n - \frac{b_n}{\ell^2} + \frac{\partial\varphi_n}{\partial u} + \nu_{ab}\frac{\partial}{\partial\tau}\left(\frac{\partial\varphi_n}{\partial u} - \frac{b_n}{\ell^2}\right) = 0 \tag{21}$$

where $u = x/\lambda_J$, $\tau = t\omega_p$, $\lambda_J = \gamma s$, s is the interlayer distance, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy ratio, $\omega_p = c/(\lambda_c\sqrt{\varepsilon_c})$ is the Josephson frequency, ε_c is the dielectric function along the c-axis, $\nu_c = 4\pi\sigma_c/(\omega_p\varepsilon_c)$ and $\nu_{ab} = 4\pi\sigma_{ab}/(\gamma^2\varepsilon_c\omega_p)$. We use the linear response approximation

$$m_{\kappa,q,\omega} = \chi_{\kappa/\lambda_J,q,\omega\cdot\omega_p} b_{\kappa,q,\omega}/(1 + 4\pi\chi_{\kappa/\lambda_J,q,\omega\cdot\omega_p}) \tag{22}$$

where we introduced Fourier transformed quantities $m_{\kappa,q,\omega}$ and $b_{\kappa,q,\omega}$ with respect to dimensionless variables u, n and τ (thus the indices κ and ω are also dimensionless, obtained from the usual momentum and frequency by rescaling by $1/\lambda_J$ and ω_p), while the Fourier transform of magnetic susceptibility $\chi \equiv \chi_{yy}$ is done with respect to usual variables of space and time coordinates (and thus given by expression (6)) to keep the usual form of the linear response approximation. Substituting Eq. (22) into the Fourier-transformed Eq. (21) we obtain how the magnetic field depends on the phase:

$$b_{\kappa_x,0,q,\omega} = \frac{\mathrm{i}\kappa_x l^2(1 + \mathrm{i}\nu_{ab}\omega)\varphi_{\kappa_x,0,q,\omega}}{1 + \mathrm{i}\nu_{ab}\omega + 2(1 - \cos q)l^2/(1 + 4\pi\chi_{\kappa_x/\lambda_J,0,q,\omega\cdot\omega_p})} \tag{23}$$

and see that $h_{\kappa_x,0,q,\omega} = b_{\kappa_x,0,q,\omega}/(1+4\pi\chi_{\kappa_x/\lambda_J,0,q,\omega\cdot\omega_p})$ satisfies the same equations as $b_{\kappa_x,0,q,\omega}$ at $\chi = 0$, but with the renormalized parameter $\tilde{\ell}^{-2} = (1 + 4\pi\chi_{\kappa_x/\lambda_J,0,q,\omega\cdot\omega_p})\ell^{-2}$. For $SmLa_{1-x}Sr_xCuO_{4-\delta}$ we estimate $\tilde{\ell}^{-2} \ll 1$ because $\omega_M \approx 1.8\times10^8\ \mathrm{s}^{-1}$, $\ell^2 \approx 2\times10^4$ at $\mu = 0.8\mu_B$, $n_M = 5\times10^{21}\ \mathrm{cm}^{-3}$ and $\lambda_{ab} \approx 2000$ Å.

It is essential to keep the terms describing normal currents or dissipation in the spin system (which we will include in the expression for the magnetization via the magnetic susceptibility) in the equation for the phase because a nonvanishing DC current appears only if the dissipation is accounted for. We neglected deviations of the electron distribution from equilibrium (for more details see Ref. [20]).

The magnetic induction (23) acting on the localized moments has a DC component accounting for the applied dc magnetic induction B and also the AC component induced by the moving vortex lattice. Substituting (23) into the Fourier transformed Eq. (20) we obtain the equation of motion for the Fourier transformed phase difference of a layered superconductor containing magnetic moments in parallel magnetic field and applied current perpendicular to the layers:

$$\left[\omega^2 - \mathrm{i}\nu_c\omega - \frac{\kappa_x^2(1 + \mathrm{i}\nu_{ab}\omega)/(1 + 4\pi\chi_{\kappa_x/\lambda_J,0,q,\omega\cdot\omega_p})}{(1 + \mathrm{i}\nu_{ab}\omega)/l^2 + 2(1 - \cos q)/(1 + 4\pi\chi_{\kappa_x/\lambda_J,0,q,\omega\cdot\omega_p})}\right]\varphi_{\kappa_x,0,q,\omega} = (\sin\varphi_n)_{\kappa_x,0,q,\omega} \tag{24}$$

In the following we consider large enough fields $B > B_J \equiv \Phi_0/(2\pi s\lambda_J)$. Then the Josephson vortices fill all intrinsic junctions, overlap strongly and form a regular triangular lattice [19–22]. In this regime the Josephson coupling can be treated perturbatively. Dropping the sin term in the RHS of (24) the phase is given by

$$\varphi_n^{(0)}(x,t) = \omega_E\tau + k_B u + \phi_n \tag{25}$$

where $\omega_E = 2eV/(\hbar\omega_p) = 2\pi c E_z s/(\Phi_0\omega_p)$, $k_B = 2\pi B s\lambda_J/\Phi_0$ and ϕ_n is a constant phase shift between the layers. We substitute this zeroth-order result into the Josephson term and solve the resulting equation by the Green's function method:

$$\varphi_n^{(1)}(u,\tau) = \int \mathrm{d}u' \int \mathrm{d}\tau' \sum_m G_{n-m}(u - u', \tau - \tau') \times \sin(\omega_E\tau' + k_B u' + \phi_m) \tag{26}$$

where the Green's function is given as a Fourier transform

$$G_n(u,\tau)=\iint\frac{\mathrm{d}\kappa\,\mathrm{d}\omega}{(2\pi)^2}\int_{-\pi}^{\pi}\frac{\mathrm{d}q}{2\pi}\mathrm{e}^{\mathrm{i}\kappa x+\mathrm{i}qn+\mathrm{i}\omega\tau}G_{\kappa q\omega}$$
$$G_{\kappa q\omega}=\left\{\omega^2-\mathrm{i}\nu_c\omega-\frac{\kappa^2(1+\mathrm{i}\nu_{ab}\omega)}{(1+\mathrm{i}\nu_{ab}\omega)(1+4\pi\chi_{\kappa/\lambda_J,q,\omega\cdot\omega_p})/l^2+2(1-\cos q)}\right\}^{-1} \tag{27}$$

Since the magnetic susceptibility is a real quantity, we have $\chi_{k,q,\omega}=\chi^*_{-k,-q,-\omega}$. We will also assume that the magnetic system possesses the symmetry under z-axis inversion, so $\chi_{k,q,\omega}=\chi_{k,-q,\omega}$. Hence $\chi_{k,q,\omega}=\chi^*_{-k,q,-\omega}$ and we obtain that the expression (26) is a difference of two quantities complex conjugate to each other, giving

$$\begin{aligned}\varphi_n^{(1)}(u,\tau)&\\&=\sum_m\int_{-\pi}^{\pi}\frac{\mathrm{d}q}{2\pi}\mathrm{e}^{\mathrm{i}q(n-m)}\,\mathrm{Im}\left\{\frac{\mathrm{e}^{\mathrm{i}(k_Bu+\omega_E\tau+\phi_m)}}{(\omega_E^2-\mathrm{i}\nu_c\omega_E-k_B^2l^2/[1+4\pi\chi_{k_B/\lambda_J,q,\omega_E\omega_p}+2(1-\cos q)l^2/(1+\mathrm{i}\nu_{ab}\omega_E)]}\right\}\\&\equiv\sum_m\int_{-\pi}^{\pi}\frac{\mathrm{d}q}{2\pi}\mathrm{e}^{\mathrm{i}q(n-m)}\left[C_{k_Bq\omega_E}\cos(\omega_E\tau+k_Bu+\phi_m)+S_{k_Bq\omega_E}\sin(\omega_E\tau+k_Bu+\phi_m)\right]\end{aligned}$$

In the absence of magnetic moments, the triangular lattice ($\phi_n=n\pi$) was found to take place in an infinite sample at low lattice velocities $v=cE_z/B$ [36]. (The case of finite samples has also been investigated recently [37].) In the presence of magnetic moments, the triangular lattice is still stable in an infinite sample in the static case, because the Zeeman energy is only of order $1/l^2$ of the energy difference of the triangular and rectangular ($\phi_n=0$) configuration. For simplicity, we limit ourselves here to the study of the triangular configuration.

For $\mathrm{SmLa_{1-x}Sr_xCuO_{4-\delta}}$ we have $\gamma\approx 500$, $\omega_p\approx 10^{12}\ \mathrm{s^{-1}}$ and $B_J\approx 0.5$ T. In a Josephson system the washboard frequency is the Josephson frequency $\omega=\omega_J=2eV/\hbar$, where V is the voltage between neighboring layers. For a triangular lattice at frequencies and magnetic fields satisfying the conditions $\ell^2\gg(1+4\pi\chi)$ and $|2\tilde{\omega}-\tilde{b}|\gtrsim 1$, where $\tilde{\omega}=\omega/\omega_p$ and $\tilde{b}=B_0/B_J$, the magnetic field Eq. (23) has the form

$$h_n(u,\tau)\approx -h\cos(\tilde{\omega}\tau-\tilde{b}u+\pi n),\qquad h\approx\frac{\tilde{b}}{4\tilde{\omega}^2-\tilde{b}^2}$$

neglecting ν_c and ν_{ab}. We estimate $h=\Phi_0/(2\pi\lambda_{ab}^2\gamma)\approx 0.16$ G at $\omega=0.1\omega_p$ and $B=B_J$. Near the Eck resonance, $2\tilde{\omega}\approx\tilde{b}$, the amplitude of the magnetic field h is larger. For the reciprocal lattice vector we have $\mathbf{g}=(2\pi sB/\Phi_0,0,\pi/s)$. So $g_x=1/\lambda_J\approx 10^4\ \mathrm{cm^{-1}}$ at $B=B_J$.

Assuming that sublattice magnetization is almost perpendicular to the applied magnetic field or that magnetic ordering is absent we obtain for the I–V characteristics

$$j(V)=\sigma_{\mathrm{eff}}\frac{V}{s}+\frac{esh^2}{\hbar}\,\mathrm{Im}\left[\chi_{yy}\left(\mathbf{g},\frac{2eV}{\hbar}\right)\right] \tag{28}$$

where $\sigma_{\mathrm{eff}}=\sigma_c+2\sigma_{ab}B_J^2/(\gamma B)^2$ describes dissipation due to quasiparticles. At the resonance, $\omega_J=\omega_s(\mathbf{g})$, we estimate $\Delta j/j\approx 2\pi^2c^2s^2h_0^2\omega_M/(\omega_J\sigma_{\mathrm{eff}}\nu_s\Phi_0^2)$. This gives $\Delta j/j\approx 4$ and $|M(\mathbf{g},\omega_J)|/(\mu n_M)\approx 0.3$ at $\omega=10^{12}\ \mathrm{s^{-1}}$ and $B=B_J$ and bigger values near the Eck resonance. Certainly, such frequencies are sufficient to probe almost complete spectrum in $\mathrm{SmLa_{1-x}Sr_xCuO_{4-\delta}}$.

3.2. Tunneling spectroscopy of magnetic excitations via spin-assisted cotunneling

While the electromagnetic mechanism leads to triplet magnetic excitations of a small nonzero momentum determined by the applied magnetic field and the interlayer spacing, we show that the spin-assisted cotunneling gives rise to singlet magnetic excitations (since the Cooper pair carries no spin) of small momentum determined by the period of the vortex lattice, The spin-assisted cotunneling between adjacent layers is described by the following tunneling Hamiltonian:

$$\widehat{H}_T = \sum_{ni\alpha\beta} \gamma^{\dagger}_{n\alpha}(\mathbf{r}_{ni})(T_0 + T_S \boldsymbol{\sigma}_{\alpha\beta} \cdot \mathbf{S}_{ni})\gamma_{n+1,\beta}(\mathbf{r}_{ni}) + \text{h.c.}$$

Here $\gamma^{\dagger}_{n,\alpha}(\mathbf{r}_i)$ ($\gamma_{n,\alpha}(\mathbf{r}_i)$) creates (annihilates) an electron in the plane n at point ($\mathbf{r}_i$) with spin α, i runs over spin sites, $\mathbf{S}_{ni}$ is the spin operator at site i of the magnetic sublattice in layer n, T_0 and T_S are the tunneling coefficients due to spin-independent and spin-dependent tunneling, respectively, and $\boldsymbol{\sigma}$ are three Pauli matrices representing the spin of a tunneling electron; we assumed for simplicity that the magnetic atom lattice coincides with the conduction layer one. The interaction H_T can be treated perturbatively to exclude all degrees of freedom except the phase difference $\varphi_n(\mathbf{r}, t)$ and spins. Thus an effective Hamiltonian $\widetilde{H}_{ct}$ is obtained, after averaging over electronic degrees of freedom with characteristic frequencies much higher than the superconducting energy gap Δ [38],

$$\widetilde{H}_{ct}(t) = -\frac{2\mathrm{i}}{\hbar}\sum_{n,i,j} \Lambda_{ij}\left(-T_0^2 + T_S^2 \mathbf{S}_{ni}\cdot\mathbf{S}_{nj}\right)\cos\varphi_n(\mathbf{r}_{ni}, t), \quad \Lambda_{ij} = \int \mathrm{d}t\, F^{+}(\mathbf{r}_i - \mathbf{r}_j, t)F(\mathbf{r}_i - \mathbf{r}_j, t) \tag{29}$$

where indices i, j run over all spins in the junction n, the kernel Λ_{ij} couples the spins on the range of the superconducting coherence length and $F_n(\mathbf{r}, \mathbf{r}'; t - t') = -\mathrm{i}\langle \mathbf{T}\gamma_{n\uparrow}(\mathbf{r})\gamma_{n\downarrow}(\mathbf{r}')\rangle$ is the anomalous Green's function (in momentum space given by $F_{\mathbf{p}}(t) = -\mathrm{i}a^2 \frac{\Delta}{2\varepsilon_p}\mathrm{e}^{-\mathrm{i}\varepsilon_p|t|}$, where the momentum $\mathbf{p}$ resides in the plane of conduction layers, a is the lattice constant, $\varepsilon_p \equiv \sqrt{\Delta^2 + \xi_p}$ and $\xi_p = p^2/2m - \mu$ measures the energy from the chemical potential μ). Here $\mathbf{T}$ denotes time-ordering. The spin operators were taken in (29) at the same moment in time because their dynamics is slow on the characteristic timescale $\hbar/\Delta$ of oscillations in the product of anomalous Green's functions.

The interaction described by H_T and H_{ct} preserves the total spin and hence leads to singlet spin excitations because the ground state in an antiferromagnet or a spin-liquid is a singlet. While neutron scattering techniques and the electromagnetic mechanism yield, e.g., information on single spinwave excitations, the spectroscopy of singlet magnetic excitations based on spin-assisted cotunneling provides a comparison of the energy of singlet magnetic excitations to the energy of two spinwaves.

The dissipated power at given voltage can be found for the tunneling mechanism as the probability per unit time for transition under the perturbation $\widetilde{H}_{ct} = H_{ct}\cos(2eVt/\hbar)$ (from the ground state to an excited state $|\Phi_j^{(0)}\rangle$ of energy ε_j and momentum κ) times the energy (ε_j) absorbed in each transition. Taking into account that the excitation energy is connected to the voltage by the Josephson relation, we obtain the dc current

$$I_{dc} \simeq \frac{4\pi \mathrm{e}}{\hbar N_n}\sum_j \left|\left\langle \Phi_j^{(0)}\middle|\widetilde{H}_{ct}\middle|\Phi_0^{(0)}\right\rangle\right|^2 \delta(\varepsilon_j \pm 2eV) \tag{30}$$

where N_n is the total number of layers and the momentum selection rule $k_x = \kappa$ is contained in the matrix element. We see that only the spin-phase interaction that does not commute with the unperturbed Hamiltonian can give a nonzero contribution to the dc current. This effect is of fourth order in tunneling amplitude, since it represents a back-action of magnetic excitations on the tunneling current.

We describe the magnetic system by the anisotropic antiferromagnet Heisenberg model in the absence of magnetic field, by

$$H_s = \sum_{nn'ij} J^{ij}_{nn'}\mathbf{S}_{ni}\cdot\mathbf{S}_{n'j} - \sum_{ij} D^{ij}_{nn'} S^z_{ni} S^z_{n'j} \tag{31}$$

where the site-dependent nearest-neighbour antiferromagnetic coupling J is of order 1 K. The nearest-neighbour interaction H_s does not commute with the large-range interaction H_{ct}, and thus the spin-phase interaction via the cotunneling mechanism can excite the magnetic system.

At low temperatures and in the absence of magnetic field, the spin lattice is assumed to be in a state that differs little from the classical Néel state and can be described by spinwave excitations. (In the case of $Sm_{1.85}Ce_{0.15}CuO_{4-\delta}$ under the above assumptions on the relative strength of the coupling constant among nearest neighbours in the same and different layers, atoms in one layer are ferromagnetically ordered and belong to the same sublattice.) In the presence of external magnetic field, the equilibrium directions of spins on the two sublattices are not antiparallel but both tilted with respect to the easy axis. To find the spectrum of spinwave excitations we diagonalize the Hamiltonian (31) with

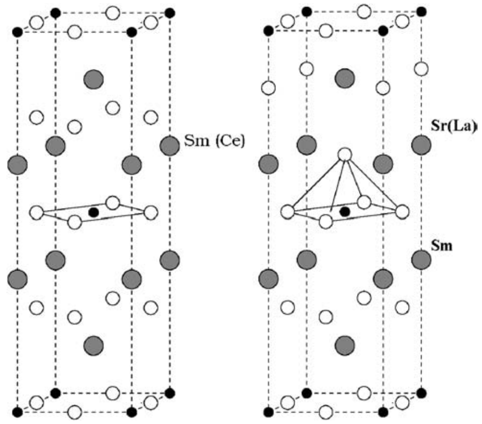

Fig. 3. The structure of $Sm_{1.85}Ce_{0.15}CuO_{4-\delta}$ (left) and $SmLa_{1-x}Sr_xCuO_{4-\delta}$ (right). Assuming the antiferromagnetic coupling between nearest neighbours from separate layers is much stronger than that between the neighbours in the same plane, the lattice of magnetic atoms can be described by a triclinic Bravais lattice with a two-point basis (the elements of which are one atom from the A and one from the B sublattice).

respect to small deviations of spins (as classical vectors) from their equilibrium directions. We first transform spin operators on A and B sublattice to two respective kinds of Holstein–Primakoff bosons [39], keeping only the terms of lowest order in $1/S$:

$$S'^{+}_{Aj} = \sqrt{2S}a_j, \qquad S'^{-}_{Aj} = \sqrt{2S}a_j^{+}, \qquad S''^{+}_{Bl} = \sqrt{2S}b_l^{+}$$
$$S''^{-}_{Bl} = \sqrt{2S}b_l, \qquad S'^{z}_{Aj} = S - a_j^{+}a_j, \qquad S''^{z}_{Bl} = -S + b_l^{+}b_l \tag{32}$$

where the prime and double prime denote tilting of the preferred orientation axis. This axis for the tilted vectors $\mathbf{S}'_{\mathbf{A}}$ and $\mathbf{S}''_{\mathbf{B}}$ is obtained from the easy axis z by a rotation around $\hat{y} = \hat{z} \times (\mathbf{B}/B)$ axis for angle θ and $-\theta$ respectively. Thus the original spin vectors are represented via the tilted ones by a transformation

$$S_A^x = S_A'^x \cos\theta + S_A'^z \sin\theta, \qquad S_A^z = -S_A'^z \sin\theta + S_A'^x \cos\theta$$
$$S_B^x = S_B''^x \cos\theta - S_B''^z \sin\theta, \qquad S_B^z = S_B''^z \sin\theta + S_B''^x \cos\theta$$

After these transformations the Hamiltonian (31) is represented as

$$H_s = E_0(\theta) + S\sum_{j\in A}\sum_{\delta\langle nn\rangle}\Big\{g_j\big(a_j + a_j^{+} + b_j + b_j^{+}\big) + \big(a_j^{+}a_j + b_j^{+}b_j\big)\Big[J_z^{(\delta)} + \frac{\mu B}{\nu S}\sin\theta - \big(J_{xy}^{(\delta)} + J_z^{(\delta)}\big)\sin^2\theta\Big]$$
$$+ \Big(\big\{a_j b_{j+\delta}\big[J_{xy}^{(\delta)} - \big(J_z^{(\delta)}/2 + J_{xy}^{(\delta)}/2\big)\sin^2\theta\big] - a_j^{+} b_{j+\delta}\big(J_z^{(\delta)}/2 + J_{xy}^{(\delta)}/2\big)\sin^2\theta\big\} + \text{h.c.}\Big)\Big\}$$

where ν is the number of nearest neighbours, the coupling is dependent on the nearest neighbour relative position vector as

$$J_{xy}^{(\delta)} = J' \quad \text{for n.n. in shifted layers and } J \text{ otherwise}$$

(and similarly for $J_z^{(\delta)}$), and we omitted the competing smaller antiferromagnetic coupling among nearest neighbours in the same plane. In the classical ground state, the angle θ satisfies the minimization condition $\partial E_0/\partial\theta = 0$:

$$\sin\theta = \frac{\mu B}{S\sum_\delta (J_z^{(\delta)} + J_{xy}^{(\delta)})}$$

and the terms linear in a, b, a^{+}, b^{+} vanish. After performing a Fourier transform

$$c_k^{+} = \frac{1}{\sqrt{N}}\sum_{j\in A} \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{j}} a_j^{+}, \qquad d_k^{+} = \frac{1}{\sqrt{N}}\sum_{j\in B} \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{j}} b_j^{+} \tag{33}$$

where N is the total number of spins on each sublattice in the layer and $\mathbf{k}$ are reciprocal lattice vectors for each sublattice, the Hamiltonian for small deviations around the classical ground state is represented as

$$H_s = E_0(\theta) + x \sum_{\mathbf{k}} \left[c^+_{\mathbf{k}} c_{\mathbf{k}} + d^+_{\mathbf{k}} d_{\mathbf{k}} + \eta_{\mathbf{k}} \left(c_{\mathbf{k}} d_{-\mathbf{k}} + c^+_{-\mathbf{k}} d^+_{\mathbf{k}} \right) - \mu_{\mathbf{k}} \left(c^+_{-\mathbf{k}} d_{-\mathbf{k}} + c_{\mathbf{k}} d^+_{\mathbf{k}} \right) \right], \tag{34}$$

where

$$x = S \sum_{\delta \in \langle \mathbf{nn} \rangle} J_z^{(\delta)} = S(4J'_z + J_z), \qquad \eta_{\mathbf{k}} = \frac{S}{x} \sum_{\delta} \left[J_{xy}^{(\delta)} - \frac{J_z^{(\delta)} + J_{xy}^{(\delta)}}{2} B_1 \right] \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\delta} \equiv \gamma_{\mathbf{k}} - \mu_{\mathbf{k}} \quad \text{and}$$

$$\mu_{\mathbf{k}} = \frac{S}{x} \sum_{\delta} \frac{J_z^{(\delta)} + J_{xy}^{(\delta)}}{2} B_1 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\delta}, \qquad B_1 = \left[\frac{\mu B}{S \sum_{\delta} (J_{xy}^{(\delta)} + J_z^{(\delta)})} \right]^2 \tag{35}$$

We diagonalize H_s by a canonical transformation

$$\alpha_{\mathbf{k}} = u_{\mathbf{k}} c_{\mathbf{k}} + w_{\mathbf{k}} c^+_{-\mathbf{k}} - t_{\mathbf{k}} d_{\mathbf{k}} - v_{\mathbf{k}} d^+_{-\mathbf{k}}, \qquad \beta_{\mathbf{k}} = u'_{\mathbf{k}} c^+_{-\mathbf{k}} - w'_{\mathbf{k}} c_{\mathbf{k}} + t'_{\mathbf{k}} d^+_{\mathbf{k}} - v'_{\mathbf{k}} d_{-\mathbf{k}} \tag{36}$$

(with analogous expressions for $\alpha^+_{\mathbf{k}}, \beta^+_{\mathbf{k}}$) where we require $[\alpha_{\mathbf{k}}, H_s] = \lambda^+_{\mathbf{k}} \alpha_{\mathbf{k}}$ and $[\alpha_{\mathbf{k}}, H_s] = \lambda^-_{\mathbf{k}} \alpha_{\mathbf{k}}$; $[\alpha_{\mathbf{k}}, \alpha^+_{\mathbf{k}'}] = \delta_{\mathbf{k},\mathbf{k}'}$ (with the analogous commutation relation for $\beta_{\mathbf{k}}, \beta_{\mathbf{k}'}$). Here $\lambda^{\pm}_{\mathbf{k}}$ is the excitation energy of a spinwave of momentum $\mathbf{k}$ (with the sign denoting the spectrum branch) and it is determined by the condition for the existence of nontrivial solutions for the coefficients of the canonical transformation:

$$\lambda^{\pm}_{\mathbf{k}} \equiv \hbar\omega^{\pm}_{s\mathbf{k}} = x \sqrt{1 - |\eta_{\mathbf{k}}|^2 + |\mu_{\mathbf{k}}|^2 \pm \sqrt{2} \sqrt{2|\mu_{\mathbf{k}}|^2 - |\eta_{\mathbf{k}}|^2 |\mu_{\mathbf{k}}|^2 + Re\left(\eta^2_{\mathbf{k}} \mu^2_{\mathbf{k}}\right)}} \tag{37}$$

In the absence of external magnetic field the canonical transformation (36) simplifies to

$$\alpha_{\mathbf{k}} = u_{\mathbf{k}} c_k - v_{\mathbf{k}} d^+_{-\mathbf{k}}, \qquad \alpha^+_{\mathbf{k}} = u^*_{\mathbf{k}} c^+_{\mathbf{k}} - v^*_{\mathbf{k}} d_{-\mathbf{k}}$$
$$\beta_{\mathbf{k}} = u'_{\mathbf{k}} d_k - v'_{\mathbf{k}} c^+_{-\mathbf{k}}, \qquad \beta^+_{\mathbf{k}} = u'^*_{\mathbf{k}} d^+_{\mathbf{k}} - v'^*_{\mathbf{k}} c_{-\mathbf{k}}$$

where $|u_{\mathbf{k}}|^2 - |v_{\mathbf{k}}|^2 = 1$, $|u'_{\mathbf{k}}|^2 - |v'_{\mathbf{k}}|^2 = 1$ (to preserve bosonic commutation relations) and

$$u_{\mathbf{k}} = \frac{\eta_{\mathbf{k}}}{\lambda_{\mathbf{k}}/x - 1} v_{\mathbf{k}} \equiv y_{\mathbf{k}} v_{\mathbf{k}}, \qquad u'_{\mathbf{k}}/v'_{\mathbf{k}} = u_{-\mathbf{k}}/v_{-\mathbf{k}} = (u_{\mathbf{k}}/v_{\mathbf{k}})^*, \qquad \eta_{\mathbf{k}} = \frac{S}{x} \sum_{\delta} J_{xy}^{(\delta)} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\delta} \tag{38}$$

and the excitation energy is given by

$$\lambda^+_{\mathbf{k}} = \lambda^-_{\mathbf{k}} = x\sqrt{1 - |\eta_{\mathbf{k}}|^2} \tag{39}$$

Since the effective Hamiltonian $\widetilde{H}_{ct}$ does not commute with the spin Hamiltonian H_s (as the domain of the kernel Λ_{ij} is not restricted to nearest neighbours), it is not diagonalized in the representation (36). In the spinwave representation in the absence of magnetic field, the effective operator $\widetilde{H}_{ct}$ can be written as

$$\begin{aligned}
\widetilde{H}_{ct} = {} & -(T_S)^2 S \sum_{\mathbf{k}} \Bigg\{ \left({\sum_{\delta_{\mathrm{j}}}}' \Lambda_{\delta_{\mathrm{j}}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\delta_{\mathrm{j}}} \right) \left(c_{\mathbf{k}} d_{-\mathbf{k}} + c^+_{-\mathbf{k}} d^+_{\mathbf{k}} \right) \\
& + \left[\left({\sum_{\delta_{\mathrm{j}}}}' \Lambda_{\delta_{\mathrm{j}}} \right) - 2 \left({\sum_{\delta_{\mathrm{j}}}}'' \Lambda_{\delta_{\mathrm{j}}} \right) \right] \left(c^+_{\mathbf{k}} c_{\mathbf{k}} + d^+_{\mathbf{k}} d_{\mathbf{k}} \right) \Bigg\} \cos(2Vt) \\
= {} & \cos(2Vt) \sum_{j \in A,B} \sum_{\mathbf{k}} \left(A_{kj} \alpha^+_{\mathbf{k}} \alpha_{\mathbf{k}} + B_{kj} \beta^+_{\mathbf{k}} \beta_{\mathbf{k}} + C_{kj} \alpha^+_{\mathbf{k}} \beta^+_{-\mathbf{k}} + D_{kj} \alpha_{\mathbf{k}} \beta_{\mathbf{k}} \right)
\end{aligned} \tag{40}$$

where only the magnetic excitation dependent contribution was kept and summations with a single and double prime are taken over relative position vectors of all spins from opposite and the same sublattice in one junction, respectively.

Here $\Lambda_{\delta_j} = \Lambda_{i,i+\delta_j}$. The sum over spins in the same sublattice is taken twice so as to account for both A–A and B–B spin interactions. Therefore Eq. (30) amounts to

$$I_{dc} \simeq \frac{4\pi \mathrm{e}}{\hbar N_n} \sum_{\mathbf{k}} \left| {\sum_j}' C_{kj} \right|^2 \delta\left(\lambda^+_{\mathbf{k}} + \lambda^-_{-\mathbf{k}} - 2V\right)$$

where we approximated the double-prime sum by a corresponding single prime one. Using the expressions

$$c^+_{\mathbf{k}} = \frac{u'_{-\mathbf{k}}\alpha^+_{\mathbf{k}} + v_{\mathbf{k}}\beta_{-\mathbf{k}}}{u_{\mathbf{k}}u'_{-\mathbf{k}} - v_{\mathbf{k}}v'_{-\mathbf{k}}}, \qquad d^+_{\mathbf{k}} = \frac{v'_{\mathbf{k}}\alpha_{-\mathbf{k}} + u_{-\mathbf{k}}\beta^+_{\mathbf{k}}}{u_{-\mathbf{k}}u'_{\mathbf{k}} - v_{-\mathbf{k}}v'_{\mathbf{k}}}$$

we obtain

$$I_{dc} \simeq \frac{4\pi \mathrm{e} T_S^4}{\hbar N_n} \sum_{\mathbf{k},\lambda_{\mathbf{k}}+\lambda'_{\mathbf{k}}=2V} \left| {\sum_{\delta_\mathbf{j}}}' \Lambda_{\delta_j} \left[\frac{u'_{-\mathbf{k}}v_{\mathbf{k}}}{|u^*_{\mathbf{k}}u'_{-\mathbf{k}} - v^*_{\mathbf{k}}v'_{-\mathbf{k}}|^2} + \frac{u_{\mathbf{k}}v'_{-\mathbf{k}}}{|u^*_{-\mathbf{k}}u'_{\mathbf{k}} - v_{-\mathbf{k}}v'^*_{\mathbf{k}}|^2} - \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\delta_\mathbf{j}} v_{\mathbf{k}}v'_{-\mathbf{k}} + \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\delta_\mathbf{j}} u_{\mathbf{k}}u'_{-\mathbf{k}}}{|u^*_{\mathbf{k}}u'_{-\mathbf{k}} - v^*_{\mathbf{k}}v'_{-\mathbf{k}}|^2} \right] \right|^2$$

where the first term stems from $S^z_i S^z_j$ and the second one from $S^+_i S^-_j + S^-_i S^+_j$.

Using (38) we have $|u^*_{\mathbf{k}}u'_{-\mathbf{k}} - v^*_{\mathbf{k}}v'_{-\mathbf{k}}|^2 = |u_{-\mathbf{k}}(u'_{-\mathbf{k}})^* - v_{-\mathbf{k}}(v'_{-\mathbf{k}}) * |^2 = 1$ and $u'_{-\mathbf{k}}v_{\mathbf{k}} = u_{\mathbf{k}}v'_{-\mathbf{k}}$, and thus obtain

$$I_{dc} \simeq -T_S^4 \frac{4\pi \mathrm{e}}{\hbar N_n} \sum_{\mathbf{k}} f_{\mathbf{k}} \delta\left(\lambda^+_{\mathbf{k}} + \lambda^-_{-\mathbf{k}} - 2V\right)$$

where

$$f_{\mathbf{k}} = \left| {\sum_{\delta_\mathbf{j}}}' \Lambda_{\delta_\mathbf{j}} \frac{-2y_{\mathbf{k}} + \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\delta_\mathbf{j}} + y^2_{\mathbf{k}}\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\delta_\mathbf{j}}}{y^2_{\mathbf{k}} - 1} \right|^2$$

The smallest voltage at which a DC component appears in the Josephson current is given by the energy gap for magnetic excitations, which is given by

$$\Delta_s = x\sqrt{1 - |\eta_{\mathbf{k}}|^2}\Big|_{\mathbf{k}=0}$$

In the absence of magnetic field we obtain a jump in the dc current density J_{dc} that appears at the spin gap Δ_s,

$$J_{dc}|_{V\to\Delta_{s+}} = \frac{2\pi \mathrm{e}}{\hbar} \rho(\Delta_s)\left[N_2(0)T_S^2\right]^2 \tag{41}$$

where $N_2(0)$ is the two-dimensional electronic density of states per spin and $\rho(\Delta_s)$ is the two-dimensional spin density of states at the spin gap.

Expressed in terms of the amplitude of the spin-independent current density J_0, the contribution of magnetic excitations to the excess dc current density is given by

$$J_{dc} = 4\pi J_0 \left(\frac{T_S}{T_0}\right)^4 \varepsilon_J \rho_a(\Delta_s) \tag{42}$$

where $\varepsilon_J = a^2\Phi_0 J_0/2\pi c$ and $\rho_a = a^2\rho(\Delta_s)$. Experimental data on T_S are unavailable to date. Taking $T_S \simeq 0.4T_0$, we estimate $J_{dc} \sim 10^{-2}$ A/cm^2 which gives a measurable effect comparable to the effect of phonons [30]. This effect is of fourth order in tunneling amplitude, since it represents a back-action of magnetic excitations on the tunneling current.

4. Conclusion

In conclusion, we propose to probe low-frequency magnetic excitations in magnetic superconductors by measuring I–V characteristics in the mixed state with a moving vortex lattice. Coupling of such a lattice to magnetic moments is due to an ac magnetic field which is inherent to vortex motion. The energy interval of spin waves which can be probed in isotropic and moderately anisotropic superconductors is limited by the depairing current and heating. If spin wave energies fall in this interval, they affect the vortex motion strongly and should be easily seen in the I–V characteristics as current peaks at corresponding voltages. Such an effect may be observed in borocarbides if they have spin waves with energies below 1 K. For highly anisotropic layered superconductors in parallel magnetic fields, higher spin wave energies may be probed by use of moving Josephson vortices. This is sufficient to study almost complete spin wave spectrum in $SmLa_{1-x}Sr_xCuO_{4-\delta}$ with exotic magnetic ordering. For obtaining solely the information on the magnetic gap in layered superconductors one can also use tunneling spectroscopy in the absence of magnetic field. Using the Josephson relation, the gap is then obtained from the voltage at which the dc current experiences a jump.

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