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Statistical mechanics of non-extensive systems/Mécanique statistique des systèmes non-extensifs

Mean field theory and general relativistic black holes

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Abstract

We review the basics of a newly developed mean field theory of relativistic gravitation. A particularly simple coarse graining of the Schwarzschild space–time is presented as an example. We then use this example to discuss current and near future observations of Sgr A*. ***To cite this article: C. Chevalier, F. Debbasch, C. R. Physique 7 (2006).***

Résumé

Théorie champ moyen et trous noirs en relativité générale. Nous rappelons les bases d'une théorie champ moyen récemment développée pour la gravitation relativiste. Un lissage particulièrement simple de l'espace–temps de Schwarzschild est proposé en tant qu'exemple. Nous utilisons ensuite cet exemple pour discuter des observations actuelles et futures de Sgr A*. ***Pour citer cet article : C. Chevalier, F. Debbasch, C. R. Physique 7 (2006).***

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1. Introduction

Developing a mean field theory for general relativity has long been the subject of active research (see Futamase [1–3], Kasai [4], Zalaletdinov [5] and Buchert [6,7]). This problem is of undeniable theoretical interest, but it is also of real practical importance because finite precision effects in astrophysical observations of relativistic objects and in observational cosmology can only be modeled properly through a mean field theory of relativistic gravitation (Ellis and van Elst [8]).

The last three years have witnessed the construction of the first general mean field theory for Einstein gravitation (Debbasch [9,10]). The aim of the present contribution is to review the basic elements of this theory and to present a simple application of astrophysical interest. The material is organized as follows. Section 2 is devoted to the mean field theory itself. We introduce statistical ensembles of space–times and show how to define in a mathematically consistent and physically meaningful way a mean or coarse grained space–time associated with such an ensemble. Section 3 presents a particular statistical ensemble which can be interpreted as modeling finite precision observations of a single Schwarzschild black hole. The mean space–time corresponding to this ensemble is found to be also a black

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hole, but of different horizon radius. Further investigation of this space–time is presented in Section 4. Contrary to the Schwarzschild black hole, the mean black hole is surrounded by a matter which describes the net effect of the averaged degrees of freedom of the gravitational field on the coarse grained field. This matter is characterized in Section 4.1 by its energy density and pressures. We then show in Section 4.2 that the total mass of the space–time is not modified by the averaging. The averaging thus only redistributes the mass in space–time. The temperature of the mean space–time is finally evaluated in Section 4.3. It is, at fixed total mass of the black hole, an increasing function of the coarse graining. Our results are discussed in Section 5. In particular, the conclusions of the theoretical calculations presented here can be used to interpret, at least qualitatively, the results of current and near future observations of Sgr A*, which is the massive black hole closest to us. We argue that current observations, despite the very poor relative precision (of order 400), probably deliver a correct estimate of the mass M of Sgr A*. As far as near future observations are concerned, we argue that they will probably detect a shadow, but that the relative difference between the size of this shadow and the theoretical estimate $5 \times 2M$ will probably be of order unity. The size of the observed shadow should therefore not furnish a direct, unbiased estimate of the mass of Sgr A*.

2. A mean field theory for general relativity

Let $\mathcal{M}$ be a fixed 'base' manifold and let Ω be an arbitrary probability space. Let $g(\omega)$ be an ω-dependent Lorentzian metric defined on $\mathcal{M}$. Each ω-dependent couple $\mathcal{S}(\omega) = (\mathcal{M}, g(\omega))$ represents a physical space–time and the collection of all couples defines a statistical ensemble Σ of space–times. Each member of Σ is naturally equipped with the Levi-Civita connection $\Gamma(\omega)$ associated with $g(\omega)$ (Wald [11]) and associated with a stress-energy tensor $\mathcal{T}(\omega)$ related to the metric $g(\omega)$ and $\Gamma(\omega)$ through the Einstein equation.

It has been shown in Debbasch [9] that the statistical ensemble Σ of space–times can be used to define a single, mean or coarse grained space–time $\bar{\mathcal{S}} = (\mathcal{M}, \bar{g})$ where the metric $\bar{g}$ is simply the average of the metrics $g(\omega)$ over ω; one thus has, for all points P of $\mathcal{M}$:

$$\bar{g}(P) = \langle g(P, \omega) \rangle \tag{1}$$

where the brackets on the right-hand side indicate an average over the probability space Ω. The connection of the mean space–time $\bar{\mathcal{S}}$ is simply the Levi-Civita connection associated to the metric $\bar{g}$ and will be conveniently called the mean or coarse grained connection. Since the relations linking the coordinate basis components $g_{\mu\nu}$ of an arbitrary metric g to the Christoffel symbols $\Gamma^{\alpha}_{\mu\nu}$ of its Levi-Civita connection are non-linear, the Christoffel symbols of the mean connection are *not* identical to the averages of the Christoffel symbols associated to the various space–times $\mathcal{S}(\omega)$.

Because the Einstein tensor depends non-linearly on the metric, the Einstein tensor $\bar{\mathcal{E}} = \mathcal{E}(\nabla(\bar{\Gamma}), \bar{g})$ associated to $\bar{g}$ and $\bar{\Gamma}$ *does not* generally coincide with the average of the Einstein tensors $\mathcal{E}(\nabla(\Gamma(\omega)), g(\omega))$. The tensor $\bar{\mathcal{E}}$ is nevertheless the Einstein tensor of the mean space–time. It therefore defines, via Einstein equation, a stress-energy tensor $\bar{\mathcal{T}}$ for the mean space–time. Since $\mathcal{E}_{\mu\nu}(\bar{g}) \neq \langle \mathcal{E}_{\mu\nu}(g(\omega)) \rangle$, $\bar{\mathcal{T}}^{\alpha\beta}$ is generally different from $\langle \mathcal{T}^{\alpha\beta}(\omega) \rangle$. The additional, generally non-vanishing tensor field $\Delta\mathcal{T} = \bar{\mathcal{T}} - \langle \mathcal{T}(\omega) \rangle$, can be interpreted as the stress-energy tensor of an 'apparent matter'. This apparent matter simply describes the cumulative effects of the averaged upon (small scale) fluctuations of the gravitational field on the (large scale) behaviour of the coarse grained field. In particular, the vanishing of $\mathcal{T}(\omega)$ for all ω does not necessarily imply the vanishing of $\bar{\mathcal{T}}$. The mean stress-energy tensor $\bar{\mathcal{T}}$ can therefore be non-vanishing in regions where the unaveraged stress-energy tensor actually vanishes. A general discussion of this and other perhaps unexpected consequences of definition (1) can be found in Debbasch [9]; possible cosmological applications are also discussed in Debbasch [10] and Debbasch and Ollivier [12]; finally, related issues are addressed, albeit in a less general framework, by Kolb et al. [13], and by Ishibashi and Wald (2005) [14]. Let us finally mention that the averaging scheme just presented is the only one which ensures that the motions in a mean field can actually be interpreted, at least locally, as the averages of 'real' unaveraged motions. This very important point is fully developed in Debbasch [10].

3. Example: description of a single Schwarzschild black hole observed with finite precision

3.1. Definition of the statistical ensemble and determination of the mean metric

The statistical ensemble considered in Debbasch and Ollivier [12] describes a single Schwarzschild black hole observed with finite precision measurements of the three spatial Kerr–Schild coordinates. More precisely, this ensemble is defined as an ensemble of space–times $\mathcal{S}(\boldsymbol{\omega})$, each member of the ensemble being equipped with the metric $g(t, \mathbf{r}, \boldsymbol{\omega})$ given by:

$$\mathrm{d}s_{\boldsymbol{\omega}}^2 = \mathrm{d}t^2 - \mathrm{d}\mathbf{r}^2 - \frac{2M}{|\mathbf{r}-\boldsymbol{\omega}|}\left[\mathrm{d}t^2 - \frac{2\,\mathrm{d}t}{|\mathbf{r}-\boldsymbol{\omega}|}\,\mathrm{d}\mathbf{r}\cdot(\mathbf{r}-\boldsymbol{\omega}) + \frac{1}{(\mathbf{r}-\boldsymbol{\omega})^2}\big(\mathrm{d}\mathbf{r}\cdot(\mathbf{r}-\boldsymbol{\omega})\big)^2\right] \tag{2}$$

The parameter M represents the mass of the black hole and $\mathbf{r}$ stands for the set of three 'spatial' coordinates x, y, z. The set Ω of possible values for $\boldsymbol{\omega}$ is taken to be the Euclidean 3-ball of radius a: $\Omega = \{\boldsymbol{\omega} \in \mathbb{R}^3;\ \omega^2 \leqslant a^2\}$. The probability measure on Ω is defined by its density $p(\boldsymbol{\omega})$ with respect to the Lebesgue measure $\mathrm{d}^3\omega$ and we take this density to be uniform, i.e., $p(\boldsymbol{\omega}) = 1/V_a$, where $V_a = 4\pi a^3/3$. In (2), $\mathbf{u}.\mathbf{v}$ and $\mathbf{u}^2$ are short-hand notations for the usual Euclidean scalar products of elements $(\mathbf{u},\mathbf{v}) \in \mathbb{R}^3$ and we also use the notation u for the Euclidean norm $|\mathbf{u}| = (\mathbf{u}\cdot\mathbf{u})^{1/2}$ of an element of $\mathbb{R}^3$.

The exact expression of the mean metric $\bar{g}$ corresponding to this ensemble can be obtained for every $a < r$. The mean metric expressed in Kerr–Schild coordinates is given by:

$$\begin{aligned}\langle \mathrm{d}s^2\rangle = {}& \left(1-\frac{2M}{r}\right)\mathrm{d}t^2 - \left(\frac{2M}{r} - \frac{6a^2M}{5r^3}\right)\left(\frac{\mathbf{r}}{r}\cdot\mathrm{d}\mathbf{r}\right)^2 - \left(1+\frac{2a^2M}{5r^3}\right)\mathrm{d}\mathbf{r}^2 \\ & + \left[-\frac{3M}{2r} - \frac{3Mr}{2a^2} + \frac{3M}{4a^3r^2}\big(a^2-r^2\big)^2\ln\!\left(\frac{r+a}{r-a}\right)\right]\frac{\mathbf{r}}{r}\cdot\mathrm{d}\mathbf{r}\,\mathrm{d}t\end{aligned} \tag{3}$$

One can construct a new set of coordinates $(\tau, \rho, \theta, \phi)$ which makes the static and spherically symmetric character of the mean space–time apparent. Using these so-called Schwarzschild coordinates for the coarse grained space–time, the metric then takes the simpler form:

$$\langle \mathrm{d}s^2\rangle = F(\rho)\,\mathrm{d}\tau^2 - G(\rho)\,\mathrm{d}\rho^2 - \rho^2\,\mathrm{d}\Gamma^2 \tag{4}$$

with

$$F(\rho) = 1 - \frac{2M}{r} \tag{5}$$

and

$$\begin{aligned}G(\rho) = {}& \frac{-2a^2M - 5r^3}{64a^6(2M-r)(a^2M-5r^3)^2}\Big[4\big(64a^8M(2M-r) + 90a^4M^2r^4 + 45a^2M^2r^6 \\ & + a^6\big(-275M^2r^2 + 80r^4\big)\big) - 180aM^2r\big(a^2-r^2\big)^2\big(a^2+r^2\big)\ln\!\left(\frac{r+a}{r-a}\right) \\ & + 45M^2\big(a^2-r^2\big)^4\ln\!\left(\frac{r+a}{r-a}\right)^2\Big]\end{aligned} \tag{6}$$

In the above expression, the old radial coordinate r is the function of ρ implicitly defined by $\rho(r) = r\sqrt{1+\frac{2a^2M}{5r^3}}$ and $\mathrm{d}\Gamma^2$ stands for the usual volume element on the unit sphere S^2.

3.2. *The coarse grained space–time describes a black hole for $a < 2M$*

Let us introduce the adimensionalized coarse graining parameter $x = a/M$. For all values of x inferior to 2, the first singularity of G encountered when coming from infinity in ρ-space is located at $r = r_+ = 2M$, i.e., $\rho = \rho_+ = 2M\sqrt{1+x^2/20}$. This singularity of G is also a zero of F; this shows that the coarse grained space–time is a spherically symmetric black hole with horizon radius $\rho_H = \rho_+ = 2M\sqrt{1+x^2/20}$. Kruskal coordinates for this black hole can be constructed in the usual way. One first expands the components of $\bar{g}$ given by (4) in the neighbourhood of $\rho = \rho_+$ and gets:

$$\langle \mathrm{d}s^2\rangle = (\rho-\rho_+)F'(\rho_+)\,\mathrm{d}\tau^2 - \frac{1}{(\rho-\rho_+)P'(\rho_+)}\,\mathrm{d}\rho^2 - \rho^2\,\mathrm{d}\Gamma^2 \tag{7}$$

One then considers the class[1] of coordinate systems $(\tau_K, X_K, \theta, \phi)$ which verify, for $\rho = \rho_+ + 0^+$:

$$X_K^2 - \tau_K^2 = \frac{\rho}{\rho_+} - 1; \quad \ln\left(\frac{X_K + \tau_K}{X_K - \tau_K}\right) = \tau\sqrt{F'(\rho_+)P'(\rho_+)} \tag{8}$$

By extension of the usual terminology, any of these coordinate systems can be called a Kruskal coordinate system for $\bar{\mathcal{S}}$. Indeed, when ρ approaches ρ_+ by positive values, the components of $\bar{g}$ in any of these coordinate systems read

$$\langle \mathrm{d}s^2 \rangle = \frac{4\rho_+}{P'(\rho_+)}\left(\mathrm{d}\tau_K^2 - \mathrm{d}X_K^2\right) - \rho^2(X_K, \tau_K)\,\mathrm{d}\Gamma^2 \tag{9}$$

with $\rho(X_K, \tau_K) = \rho_+ + \rho_+(X_K^2 - \tau_K^2)$. In this form, the metric $\bar{g}$ can be extended through the surface $\rho = \rho_+$ to values of ρ inferior to ρ_+. Equations (8) and (9) also make it clear that the surface $\rho = \rho_+$, over which the Killing field ∂_τ is null, is a bifurcate Killing horizon. The coarse grained space–time thus describes a black hole and its geometry around the horizon $\rho = \rho_H$ is completely similar to the geometry of the original, unaveraged Schwarzschild solution around $\rho = 2M$.

4. Some properties of the coarse grained space–time

4.1. Stress-energy tensor

It is interesting to further investigate the properties of this coarse grained black hole by determining the stress-energy tensor in the region lying 'outside' the horizon, where the Killing field ∂_τ is time-like. The exact expressions for the Schwarzschild components of the mean stress-energy tensor are too complicated to warrant reproduction in this review. We just recall the approximate expressions of these components already given in Debbasch and Ollivier [12], which are valid when $a \ll r$:

$$\begin{aligned} 8\pi \bar{\mathcal{T}}_0^{\ 0} &= \varepsilon \approx -\frac{6a^2M^2}{5\rho^6}; \qquad 8\pi \bar{\mathcal{T}}_1^{\ 1} = -p_1 \approx -\frac{6a^2M^2}{5\rho^6} \\ 8\pi \bar{\mathcal{T}}_2^{\ 2} &= -p_2 \approx \frac{12a^2M^2}{5\rho^6}; \qquad 8\pi \bar{\mathcal{T}}_3^{\ 3} = -p_3 \approx \frac{12a^2M^2}{5\rho^6} \end{aligned} \tag{10}$$

This shows that the coarse graining procedure endows the original vacuum space–time with a non-vanishing stress-energy tensor $\bar{\mathcal{T}}$. This tensor describes how the averaged upon (small scale) degrees of freedom of the Schwarzschild gravitational field can be viewed as an apparent matter which acts as the effective 'source' of the coarse grained (large scale) field. The apparent matter is characterized by a negative energy density and an anisotropic pressure tensor. Note that all energy conditions (i.e., the weak, strong and dominant energy conditions (Wald [11])) are violated by the mean stress energy tensor $\bar{\mathcal{T}}$. Finally, by taking the trace of Einstein's equation, the scalar curvature $\bar{\mathcal{R}}$ of the mean space–time outside the horizon can be obtained directly from the exact components of $\bar{\mathcal{T}}$; one finds, at second order in a/ρ:

$$\bar{\mathcal{R}} = -8\pi \bar{\mathcal{T}}_\mu^{\ \mu} \approx -\frac{12a^2M^2}{5\rho^6} \tag{11}$$

The coarse graining thus endows the space–time with a negative scalar curvature. This inevitably evokes the recent observations (Spergel et al. [15]) of a positive, non-vanishing cosmological constant Λ, which also endows vacuum regions of space–time with a negative scalar curvature (Peebles [16]) $\mathcal{R}_\Lambda = -4\Lambda$. The similarity and differences between the coarse grained space–time constructed here and space–times of cosmological interest are further explored in Debbasch and Ollivier [12].

4.2. Mass of the coarse grained space–time

We have just seen that the coarse graining changes the repartition of energy in space–time. What about the total mass-energy of the black hole? The total mass of an asymptotically flat space–time is encoded in the behaviour of that

[1] More precisely, (8) defines a jet of coordinate systems.

space–time at spatial and/or null infinity (Wald [11]). A standard analysis shows that the coarse grained space–time described by (4) is asymptotically flat and that its mass is identical to M, the mass of the unaveraged Schwarzschild black hole. This result is true for all values of a (including those superior to $2M$).

4.3. Temperature of the coarse grained black hole

The thermal properties of the usual Schwarzschild black hole are perhaps most simply derived by studying the natural topology of the Euclidean Schwarzschild space time (Wald [11]). Let us therefore introduce 'outside the horizon' the so-called imaginary time $\psi = \mathrm{i}\tau$ and the new radial coordinate: $U(\rho) = 2\sqrt{\frac{\rho-\rho_H}{P'(\rho_H)}}$ which tends towards 0^+ as one approaches the horizon. Near the horizon, the metric $\bar{g}$ (see equation (7)) takes the simple form:

$$\langle \mathrm{d}s^2\rangle = -\left[U^2\,\mathrm{d}\left(\frac{\psi}{2/\sqrt{F'(\rho_H)P'(\rho_H)}}\right)^2 + \mathrm{d}U^2\right] - \rho^2(U)\,\mathrm{d}\Gamma^2 \tag{12}$$

with $\rho(U) = \rho_H + U^2 P'(\rho_H)/4$. In these coordinates, the apparent singularity of the (ψ, U) part of the metric on the horizon at $U = 0$ is clearly similar to the singularity displayed by the 2-D Euclidean metric in polar coordinates at the origin. It is therefore natural to consider the Euclidean coarse grained space–time as periodic in the imaginary time ψ, of period $\beta = 4\pi/(\sqrt{F'(\rho_H)P'(\rho_H)}\,)$. This periodicity in imaginary time is characteristic of a thermal density matrix [11] of temperature

$$T = \frac{1}{4\pi}\sqrt{F'(\rho_H)P'(\rho_H)} \tag{13}$$

The temperature T can be expressed in terms of M and x:

$$T(x, M) = -\frac{1}{12\pi M}g(x) \tag{14}$$

where

$$g(x) = \frac{x^3}{-x(1+\frac{x^2}{4}) + (1-\frac{x^2}{4})^2\ln(\frac{1+x/2}{1-x/2})} \tag{15}$$

Expanding the above expression at order two in x, one finds:

$$T(x, M) = \frac{1}{8\pi M}\left(1 + \frac{x^2}{20}\right) \tag{16}$$

This confirms that $T(0, M)$ coincides with the Hawking temperature $1/8\pi M$ [11] of the Schwarzschild black hole.

5. Discussion

5.1. Discussion

We would like now to discuss what the results presented so far suggest about the interpretation of current or near future observations of astrophysical black holes. To make what follows specific, we will focus on the super massive black hole candidate Sgr A*, which is located at the Galactic Center, i.e., at approximately $D = 8$ kpc from the Sun (Schödel et al. [17]).

Let us first address infra-red observations. These study stellar motions in the immediate vicinity of the Galactic Center and they have led to an estimation of the central dark mass of roughly $4 \times 10^6\ M_\odot$ (Schödel et al. [18]; Ghez et al. [19,20]). The associated length is given by $L_M = GM/c^2 = 5$ µas at $D = 8$ kpc. The stars that have been studied so far are located within a region of radius 0.01 pc (250 mas at 8 kpc) around the Galactic Center and some approach 10^{-4} pc from the Galactic Center. In these observations, the relative positions of the stars with respect to the central mass are determined with an angular resolution δ of order 2 mas (Ghez et al. [19]). In the context of the example presented in this contribution, the finite angular resolution δ translates, as a first approximation, into a finite resolution $\Delta = \delta D$ in position measurements around Sgr A*. The realistic order of magnitude of an adimensionalized coarse graining a/M corresponding to these observations is therefore $a/M \sim \delta D/L_M$. The numerical values given above for δ and L_M/D lead to:

$$\frac{a}{M} \simeq \frac{2\times 10^{-3}}{5\times 10^{-6}} \simeq 400 \tag{17}$$

Since $r \gtrsim 10^{-4}$ pc and $a \simeq 10^{-5}$ pc, the condition $r \gg L_M$ is approximately realized in these observations. This means all these observations concern the motions of test objects situated 'at infinity' with respect to Sgr A*. The mass estimate derived from the study of these motions is therefore a mass estimate 'at infinity'. The result presented in Section 4.2 strongly supports the fact that this estimate is correct and free of any systematic bias, *whatever the finite value of the angular resolution δ may be*. Thus, $4\times 10^6\ M_\odot$ should be a realistic estimate of the true mass of Sgr A*, even though the angular resolution of the observations from which this result is deduced is actually quite poor from a theoretical point of view (the corresponding value of the adimensionalized coarse graining parameter a/M being approximately 400).

Let us now address high resolution observations of Sgr A* using Very Large Baseline Interferometry (VLBI). The current resolution of these observations is 20 µas, i.e., 2 Schwarzschild radii $2M$ at 7 mm wavelength (Bower [21]). VLBI imaging resolution is however expected to be narrowed down to 20 µas at 1.3 mm wavelength during the next decade (Doeleman and Bower [22]). In the context of the example presented here, this resolution corresponds roughly to:

$$\frac{a}{M} \sim \frac{\delta D}{L_M} \simeq 4 \tag{18}$$

Thus, a coarse graining parameter a taking into account the typical finite resolution of the near future observations would be of the same order of magnitude as the Schwarzschild radius $2M$ of Sgr A*.

The material developed in Section 3.2 supports the fact that a black hole observed with a finite precision a smaller or comparable to its horizon radius should still appear as a black hole. In particular, near future VLBI observations, which will have a resolution comparable to the theoretical Schwarzschild radius $2M$ of Sgr A*, should be able to detect a shadow (Doeleman and Bower [22]). But the apparent size of the shadow should be compared with a theoretical estimate which takes into account the finite resolution of the observations by using a mean or coarse grained gravitational field to compute the motion of stars and photons orbiting the black hole. Typically, one can expect the size of the apparent shadow to be comparable to a few ρ_H. In the context of the example presented in Section 3.2, ρ_H depends on both the mass M and on the coarse graining parameter a tracing the finite resolution of the observations. Since the projected observational situation corresponds to a value of $x = a/M$ of order unity, the difference between ρ_H and the 'theoretical' horizon radius $2M$ of the unaveraged black hole cannot be expected to be small, and will probably be of order unity. So will also be the difference between the observed size of the shadow ($\sim 5\times \rho_H$) and the 'theoretical' estimate $5\times 2M$. Thus, contrary to what is usually expected (Miyoshi [23]), the apparent size of the shadow will probably not provide a direct, unbiased estimate for the mass of Sgr A*.

Let us conclude by mentioning a few extensions of the work presented in this contribution. On the astrophysical side, one should now systematically consider ensembles of black holes which do take into account the real (current and future) observational conditions and evaluate, from each of these ensembles, a realistic mean or coarse grained black hole. Naturally, for a given observational procedure, one should not only construct ensembles of Schwarzschild black holes, but also ensembles of Kerr black holes if one wants to extract from the data information about the angular momentum of the observed object.

Possible cosmological implications of the mean field theory introduced in (Debbasch [9]) should also be explored systematically, in both perturbative (Kolb et al. [13]) and fully non-linear regimes.

On the theoretical side, the fact that a purely classical averaging modifies the temperature of a black hole suggests a link between the mean field theory developed in Section 2 and string theory; this link obviously demands investigation. A first step would be the computation of the total entropy of coarse grained black holes; in particular, does the apparent matter surrounding the mean black hole carry a non-vanishing entropy?

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