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Electromagnetic modelling/Modélisation électromagnétique

Sub-domain methods for collaborative electromagnetic computations

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Abstract

In this article, we describe a sub-domain method for electromagnetic computations based on boundary element method. The benefits of the sub-domain method are that the computation can be split between several companies for collaborative studies; also the computation time can be reduced by one or more orders of magnitude especially in the context of parametric studies.

The accuracy and efficiency of this technique is assessed by RCS computations on an aircraft air intake with duct and rotating engine mock-up called CHANNEL. Collaborative results, obtained by combining two sets of sub-domains computed by two companies, are compared with measurements on the CHANNEL mock-up. The comparisons are made for several angular positions of the engine to show the benefits of the method for parametric studies. We also discuss the accuracy of two formulations of the sub-domain connecting scheme using edge based or modal field expansion. ***To cite this article: P. Soudais, A. Barka, C. R. Physique 7 (2006).***

Résumé

Méthodes de sous domaines pour calculs électromagnétiques collaboratifs. Dans cet article nous présentons une méthode de sous domaines pour les calculs d'électromagnétisme, basée sur la méthode des équations intégrales. Cette méthode de résolution par sous domaines permet de répartir le calcul entre plusieurs organismes industriels dans le cadre d'études collaboratives. De plus le temps de calcul peut être réduit d'un ou plusieurs ordres de grandeur et particulièrement lors de la réalisation d'études paramétriques.

La précision et l'efficacité de ces techniques sont évaluées pour le calcul de la Surface Equivalente Radar de la maquette de conduit d'air CHANNEL équipée de deux étages d'un compresseur. Des résultats de calculs collaboratifs intégrant des sous domaines calculés par deux organismes sont comparés avec des résultats expérimentaux de SER obtenus sur la maquette CHANNEL. Les validations sont réalisées pour plusieurs positions angulaires de la roue mobile afin d'illustrer l'intérêt de la méthode pour mener efficacement des études paramétriques. Nous discutons également de la précision de deux schémas de calculs par sous domaines utilisant pour l'un des fonctions d'arêtes et pour l'autre des fonctions modales. ***Pour citer cet article : P. Soudais, A. Barka, C. R. Physique 7 (2006).***

Keywords: Radar cross-section; Antenna–platform interaction; Numerical methods; Integral equations

Mots-clés : Surface équivalente radar ; Interaction antenne–structure ; Méthodes numériques ; Équations intégrales

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1. Introduction

The electromagnetic model of an aircraft or of a part of an aircraft often involves several companies. For instance, the electromagnetic modeling of an antenna or of a jet engine mounted on an aircraft requires a model built by several companies (the company building the engine or antenna and the company building the aircraft).

In this article we will describe a sub-domain method for electromagnetic computations based on the boundary element method (BEM). The degrees of freedom internal to each sub-domain are eliminated, i.e., a reduced operator characterizing the sub-domain is computed. Adjacent reduced sub-domains can be assembled together to eliminate degrees of freedom internal to the assembled sub-domains. Finally, the reduced models are assembled to obtain the final solution. If the reduced operators share the same basis functions as the sub-domain solver, the sub-domain scheme is only an algebraic manipulation and does not result in any loss of accuracy.

Sub-domains methods are well-known to the Finite Element Method (FEM) community. Reduced models of sub-structures, often called super elements, can be introduced in a structure computation of a large object [1]. For instance, structure computation of an airplane will often include super element models of components supplied by subcontractors. Models that include super elements are used for parametric and collaborative studies.

In the electromagnetic scattering community, the BEM is often used to deal with unbounded domains. Sub-domains methods for BEM have been introduced to model large air intake scattering problems [2]. The sub-domain method was used as a frontal elimination technique to reduce the computation time of the duct. Hybrid FEM/BEM methods have been developed, the FEM part in conjunction with a frontal elimination technique was used for the engine and duct domains, while the BEM was used on the duct aperture [3]. Dispersion errors of the FEM have been reduced by using high-order finite elements [4] and large air intake problems have been successfully modeled. A more general sub-domain method has been developed [5] for collaborative studies and parametric computations: the sub-domains can be computed independently and in any order and the assembly process can handle sub-domains modifications.

In this article, the sub-domain method is used for collaborative studies and to reduce the computation time, especially in the context of parametric studies. This method has been used for collaborative studies involving Dassault Aviation, Thales Airborne Systems [6], Snecma Moteurs [7] and ONERA. The method is not problem specific; it has been applied to different problems such as antenna integration and jet engine inlet scattering.

Recently the multi-domain approach has been assessed and improved for civil applications in the context of antenna interactions with civil airframe structures [8–10].

This method is applied in this paper to a collaborative RCS computation of an air intake mock-up with fans. The RCS is computed for several fan positions in order to obtain the RCS with a rotating fan. The reduced model for the sub-domain with the rotating fan and the final assembly are the only computations to be made for each fan position. This gives an example of a collaborative computation and of the reduction in computation time obtained in parametric studies. Furthermore, in this case, the overall computation time and memory requirement are greatly reduced by splitting the computation into a set of smaller ones.

The electromagnetic scattering from the interior of a complex jet engine inlet contributes significantly to the overall radar cross section (RCS) of a modern jet aircraft. The scattering mechanism in jet or missile inlets is complex and difficult to simulate accurately. The geometry is both complex (engine face, structural obstacles, materials) and electrically large.

ONERA and Dassault Aviation have developed for several years this modular sub-domain scheme for collaborative computations. The first step is to split the geometry between the partners thanks to a set of fictitious coupling surfaces. Each partner may divide again its geometry in order to reduce the computation time and to take advantage of local sub-domain geometrical symmetries. Note that, the sub-domains associated to the external geometry are unbounded. For each sub-domain, a reduced operator is computed (either a generalized scattering matrix S, a generalized Y admittance matrix or a Z impedance matrix) [5]. In this paper, the reduced operators are computed with the BEM based upon integral equations [11]. Sub-domains can also be reduced by the finite element method [5]. Reduced operators can be expanded in terms of several kinds of expansion functions, such as the Rao–Wilton–Glisson (RWG) basis functions used to expand the unknowns in the BEM [12], 'modal' numerical basis functions [13] and 'deltagaps' basis functions [9].

Each partner can assemble adjacent sub-domains to eliminate degrees of freedom internal to the assembled sub-domains. Each partner then ends up with a reduced operator on the coupling surface of its geometry. Only this operator is exchanged between the partners, efficiently protecting each partner's geometrical and material data.

Then, the final solution is computed from the assembly of each partner's operators. In the context of parametric investigations, the scattering matrices of the modified domains have to be re-evaluated, the other are simply re-used in the connection step.

The RCS modulations induced by a rotating fan are obtained by the computation of the RCS for a discrete set of the fan angular position. For each rotation angle, the engine sub-domain is computed and the graph equation is solved providing the new RCS patterns. Similarly, if a local modification of the primary geometry is required, the new RCS pattern can be computed efficiently. This point makes the method very suitable in the conception process.

2. Sub-domain technique

Let us consider an object Ω whose surface is denoted by Γ_0. The sub-domain formulation applied to 3D objects consists in splitting the geometry Ω in N sub-domains or volumes (V_i) and M fictitious surfaces (Γ_j). Note that V_0 is an unbounded volume, and the volumes $V_1, \ldots, V_N$ are bounded. In each bounded or unbounded volume V_i of the decomposition, the boundary Γ is made of n_i fictitious surfaces Γ_j and a part of Γ_0 (Fig. 1).

$$\Gamma = \Gamma_0 + \sum_{j=1}^{M} \Gamma_j \tag{1}$$

The surrounding medium has permittivity ε_0 and permeability μ_0. The scatterer is illuminated by an incident wave $\vec{E}_i, \vec{H}_i$ with wave number $k = \omega/c$.

Each volume may contain scattering objects as metallic parts and materials but also sources as generators and plane waves.

2.1. Boundary element method

For simplicity, we write here the integral equations for a single homogeneous domain with Γ_j boundary and n normal; inhomogeneous domains are taken into account by multidomain integral equation (IE) formulation or hybrid IE/partial differential equations [11]. The electromagnetic problem in each sub-domain can be formulated with two equivalent equations, the electric field integral equation (EFIE) or the magnetic field integral equation (MFIE) written in weak form:

$$\langle Z(B-S)\vec{J}, \vec{\Psi}\rangle + \langle (P+Q)(j\vec{K}), \vec{\Psi}\rangle = \langle I\vec{E}_i, \vec{\Psi}\rangle \tag{2}$$

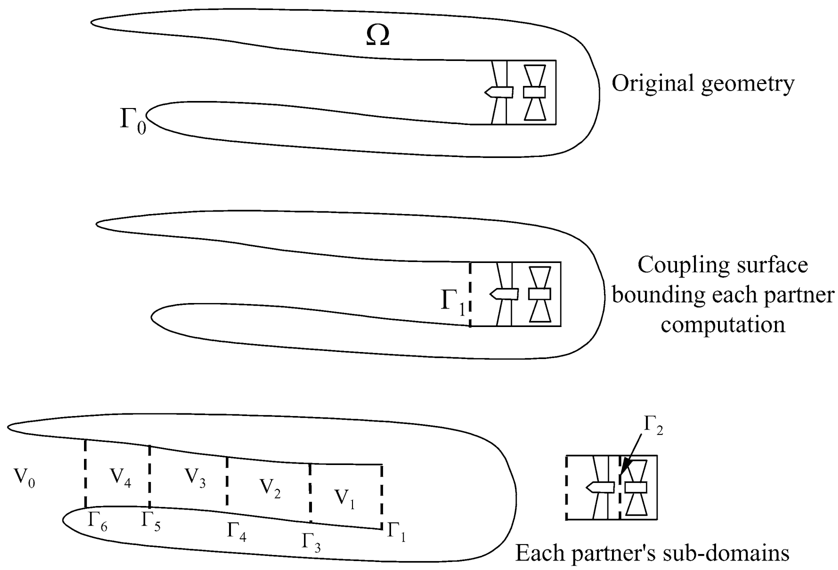

Fig. 1. An example of coupling surface for collaborative computation and of the sub-domains used by each partner.

$$\left\langle (P+Q)\vec{J},\vec{\Psi}\right\rangle + \left\langle \frac{1}{Z}(B-S)\big(j\vec{K}\big),\vec{\Psi}\right\rangle = \left\langle jI\vec{H}_i,\vec{\Psi}\right\rangle \tag{3}$$

with the following notations

$$\left\langle (B-S)\vec{A}',\vec{\Psi}\right\rangle = \frac{j}{4\pi}\iint\limits_{\Gamma_j\times\Gamma_j}\left\{kG\big(\vec{\Psi}\cdot\vec{A}'\big) - \frac{1}{k}G\big(\big(\nabla_S\cdot\vec{\Psi}\big)\big(\nabla_S\cdot\vec{A}'\big)\big)\right\}\mathrm{d}s\,\mathrm{d}s' \tag{4}$$

$$\left\langle (P+Q)\vec{A}',\vec{\Psi}\right\rangle = -\frac{j}{2}\int\limits_{\Gamma_j}\vec{\Psi}\cdot\big(\vec{n}\times\vec{A}\big)\,\mathrm{d}s - \frac{j}{4\pi}\iint\limits_{\Gamma_j\times\Gamma_j/M}(\nabla' G)\cdot\big(\vec{\Psi}\times\vec{A}'\big)\,\mathrm{d}s\,\mathrm{d}s' \tag{5}$$

$$\left\langle I\vec{A},\vec{\Psi}\right\rangle = \int\limits_{\Gamma_j}\vec{\Psi}\cdot\vec{A}\,\mathrm{d}s \tag{6}$$

and k media wavenumber, Z media impedance, $r = \|\overrightarrow{MM'}\|$, $j=\sqrt{-1}$, $G = \mathrm{e}^{-jkr}/r$ the free-space Green function, $\vec{E}_i, \vec{H}_i$ the source field and Ψ the test-functions. The unknowns are related to the total field by $\vec{J} = \vec{n}\times\vec{H}$ and $\vec{K} = \vec{E}\times\vec{n}$.

The IE weak form is discretized with divergence conforming RWG basis and test functions.

2.2. *Field expansion on fictitious surfaces*

On each coupling surface Γ_j of the volume V_i the electromagnetic currents $\vec{J}$ and $\vec{K}$ are expanded on a set of basis functions $\vec{f}_i$ and $\vec{g}_i$. On each coupling surface, these basis functions may be chosen as wave-guided modes basis functions [13], the BEM RWG basis functions [12] or 'deltagap' basis functions [9]:

$$\vec{J} = \vec{n}\times\vec{H} = \sum_i J_i\vec{f}_i \tag{7}$$

$$\vec{K} = \vec{E}\times\vec{n} = \sum_i K_i\vec{g}_i \tag{8}$$

Then, by introducing the following a_j and b_j coefficients:

$$\begin{aligned} J_j &= a_j - b_j \\ K_j &= a_j + b_j \end{aligned} \tag{9}$$

the current expansions become:

$$\vec{J} = \vec{n}\times\vec{H} = \sum_i (a_j - b_j)\vec{f}_i \tag{10}$$

$$\vec{K} = \vec{E}\times\vec{n} = \sum_i (a_j + b_j)\vec{g}_i \tag{11}$$

2.3. *Sub-domain reduction*

We introduce the coupling surface current expansion (7), (8) in the Boundary Element Method (BEM) used for the solution of the Maxwell equations in the frequency domain. The BEM linear system is solved for each test function of the coupling interface. We obtain a reduced linear system relating the basis functions $\{J_b; K_b\}$ to the test functions $\{J_t; K_t\}$ of the coupling surface of the volume in one of the two following forms or in a combination of:

$$\begin{aligned} J_t &= YK_b + B \\ K_t &= ZJ_b + C \end{aligned} \tag{12}$$

The matrices Y and Z are respectively the Admittance and Impedance matrix characterizing the volume. Matrices B and C correspond to the sources internal to the volume.

When wave guide modes are used in relations (10), (11) as expansion and test functions, the BEM linear system is solved for each modal test function and a scattering matrix S is computed for each sub-domain [5]. The sub-domains are then characterized by a relation:

$$b = Sa + b_{\text{source}} \tag{13}$$

for which vectors a, b and b_{source} correspond respectively to the incoming, outgoing waves on the coupling surfaces of the volume V_i, and to outgoing waves created by sources internal to the volume.

2.4. Condensed operators assembling

When the sub-domains are computed and characterized by the previous relation (12) in the case of edge basis functions expansion on the interfaces or by a generalized scattering matrix (13) associated with wave guide modes the sub-domains can be connected in a final step.

In the method developed by Dassault Aviation, further reduction can be achieved by assembling adjacent reduced operators two by two:

$$\begin{bmatrix} I & -Y \\ -Z & I \end{bmatrix} \begin{bmatrix} J \\ K \end{bmatrix} = \begin{bmatrix} B_J \\ B_M \end{bmatrix} \tag{14}$$

Reduced operators are finally assembled yielding the equivalent currents and RCS.

The sub-domains assembling technique performed at ONERA consists in solving the following 3D graph equation involving the scattering matrices of the sub-domains:

$$(I - S)x = b_{\text{source}} \tag{15}$$

Vector x coefficients are the unknown amplitude coefficients of incoming and outgoing wave on each fictitious surface of the structure. Then, once the vector x has been determined, the total electromagnetic field is recombined on the boundary of the exterior domain for all the incident waves.

2.5. Transformation of the condensed operators

For real life collaborative RCS computations of military aircraft involving radar antennas, weapons systems, air-intake and engine structures, all the manufacturers need to exchange data characterizing their own sub-domain. With the numerical technique described in this paper, each manufacturer calculates the Y admittance or S scattering matrix of his sub-domain. If the current basis functions on a shared fictitious surface are different, it is necessary to perform a Y or S matrix transformation before assembling the sub-domains.

The 'edge to mode' transformations are necessary before assembling two sub-domains expressed in two different basis (edge and modes). We denote Y the sub-domain admittance expressed relatively to an edge based expansion $\vec{f}^{\text{edges}}$ and P the matrix transforming basis $\vec{f}^{\text{modes}}$ to $\vec{f}^{\text{edges}}$ basis:

$$\vec{f}^{\text{modes}} = P.\vec{f}^{\text{edges}} \tag{16}$$

The admittance matrix expressed in the $\vec{f}^{\text{modes}}$ basis is derived by:

$$\widetilde{Y} = P.Y.P^T \tag{17}$$

The fan rotation in the engine sub-domain is a particular case of S 'mode to mode' matrix transformation. The challenge is to obtain the RCS for all the angular positions of the engine from the computation of a reference position. The scattering matrix S of the reference position expresses the incoming waves in function of the outgoing waves on each interface of the sub-domain relatively to a basis $\vec{f}_1$. The algorithm consists in writing the scattering matrix in a new basis $\vec{f}_2$ [14]. If P is the matrix transforming basis $\vec{f}_2$ in basis $\vec{f}_1$, the admittance and scattering matrix expressed in the $\vec{f}_2$ basis are derived by:

$$\widetilde{Y} = P.Y.P^T \tag{18}$$

$$\widetilde{S} = P.S.P^T \tag{19}$$

The modeling of the engine rotation is then realized thanks to a simple rotation of the modal functions $\vec{f}_1$.

3. CHANNEL mock-up RCS computations

The main advantages of the sub-domain methods are discussed in the following sections in the particular case of the RCS computations of air intake and engine structures. The efficiency of these schemes is described in the context of parametric investigations and especially for the RCS modulations prediction induced by fan rotations.

3.1. Inlet and engine specifications

The air inlet and engines structures of military aircraft are significant contributors to RCS in the forward sector. The diffraction phenomena are complex due to the strong interaction between the aircraft structure the air intake and the engine face. Furthermore an efficient and accurate modeling of the RCS modulation is necessary as it contributes to the aircraft radar detection.

The inlet/engine subset is split in several sub-domains characterized by an Admittance or Scattering matrix computed separately by each partner and connected in a final step. Within each volume planar or cyclic symmetry are taken into account to reduce storage and CPU requirements [15]. The computation associated to several angles of the rotating fan is carried out locally in the engine volume before assembling the domains.

3.2. Computation on the CHANNEL mock-up

We describe validations obtained on the 1/4 scaled CHANNEL mock-up at 16 GHz (Fig. 2). This mock-up is a slowly varying air-intake enclosed in a circular cylinder and terminated by a two wheels engine. The mock-up engine is an 8 blades fixed wheel followed by a 16 blades rotating fan.

The total length of the mock-up and the fan diameter are respectively 79λ and 10λ at 16 GHz. Computations and measurements patterns are provided for a scan angle θ from 80° to 135° for both $\Phi\Phi$ and $\theta\theta$ polarization. The RCS measured patterns have been measured in the ONERA's facilities in Palaiseau.

A fictitious surface is introduced between the 8 blades fixed wheel and the 16 blades rotating fan (Fig. 2). Thanks to this fictitious surface, two engine sub-domains are defined with different rotational symmetry (here 8 and 16). This is an efficient way to compute a set of wheels with different rotational symmetry. Furthermore, the computation of the RCS for many wheel positions requires only many computation of the sub-domain with the rotating wheel and to re-play the assembly process, rather than re-computing the whole geometry anew for each wheel position.

These computations were made to validate the previous collaborative sub-domain techniques combining several volumes solved by different codes belonging to ONERA and Dassault Aviation. We also wanted to compare the accuracy and efficiency of the edge based and mode based sub-domain schemes.

Source and observation antenna were set at 8.475 m with 4° bistatic angle to model the measurement conditions.

The first sub-domain scheme was computed by Dassault Aviation. It involved 8 sub-domains connected by 7 interfaces. Reduced RWG based Y and Z operators are computed with the BEM. The currents on the surface separating the two wheels sub-domains are expanded with 15 000 edge functions. The rotational symmetry of order 16 or 8 is used to reduce the storage of the operators by a factor of 16 or 8, respectively.

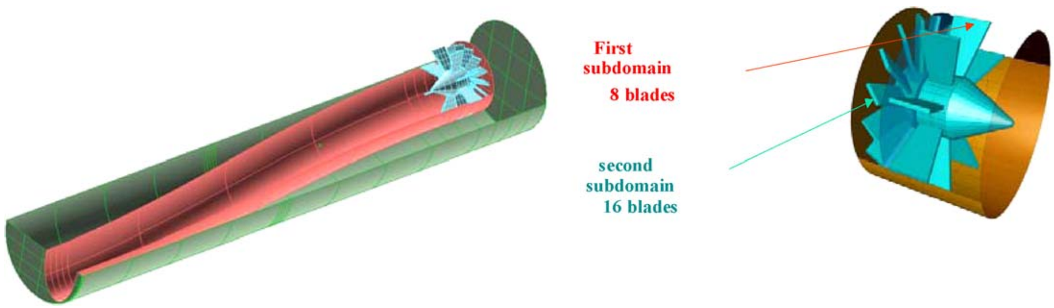

Fig. 2. CHANNEL mock-up.

The second sub-domain scheme was computed by ONERA. The target was split in 18 volumes separated by 17 fictitious surfaces. The 18 scattering matrix sub-domains have been computed with the EFIE. The tangent fields on the air-intake fictitious surface were expanded with 540 modal functions. On the interface between the engine wheels 550 modal functions were used. The 8 blades and 16 blades sub-domains were meshed respectively with 451 752 and 115 376 unknowns and reduced with respectively a 8 order and 16 order rotational symmetry. We observe on Fig. 3 that the results are in good agreement with the measurements.

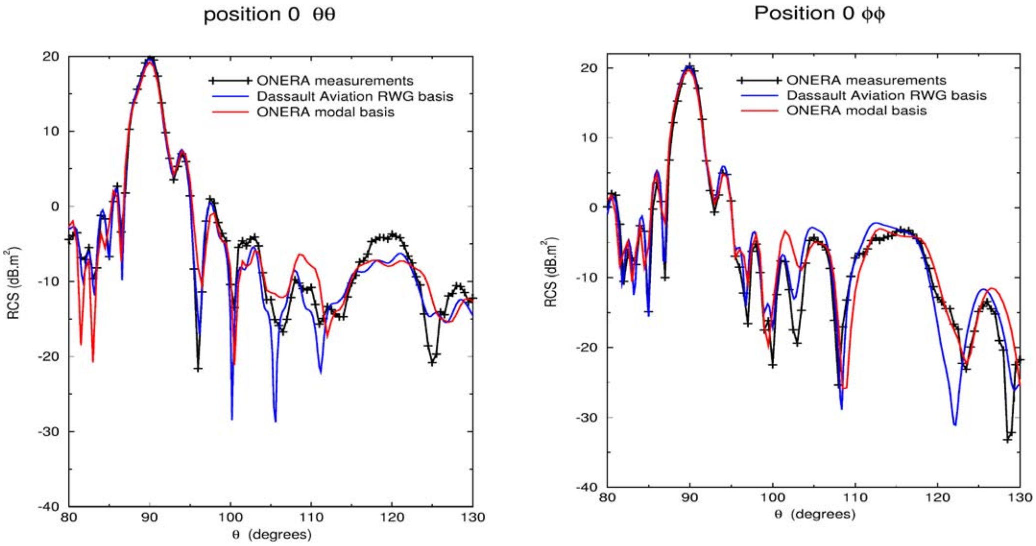

Fig. 3. Comparison of computed and measured RCS of the CHANNEL mock-up at 16 GHz.

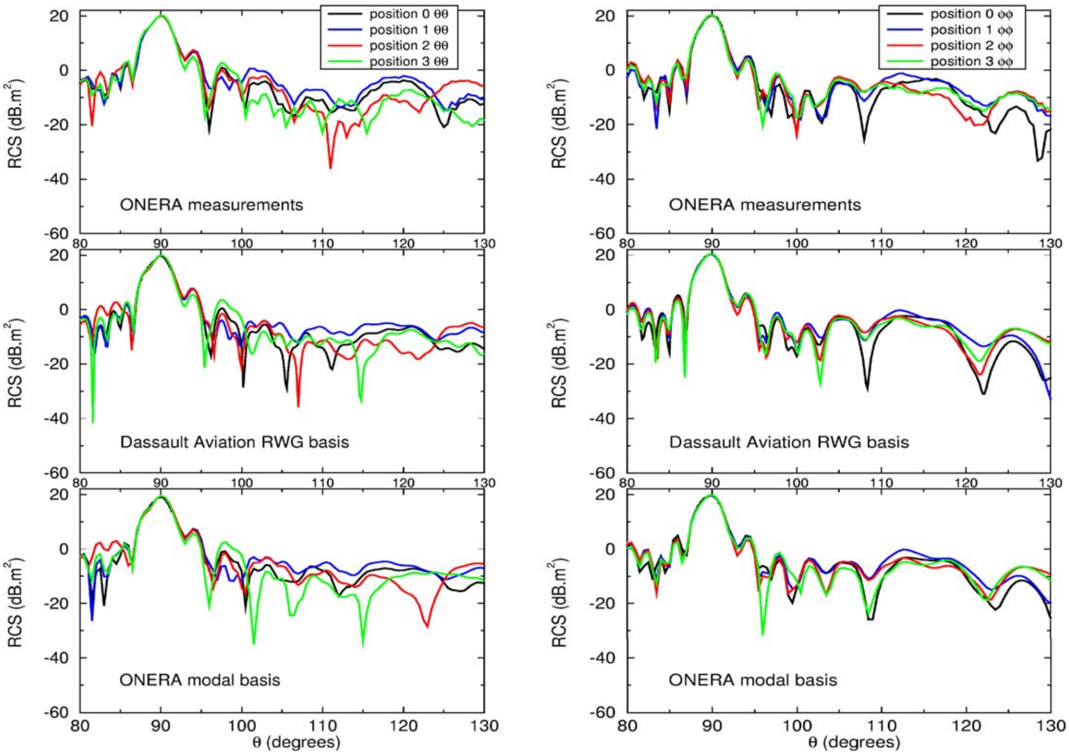

Fig. 4. Comparison of computed and measured RCS for several fan positions of the CHANNEL mock-up at 16 GHz.

The computation time for a single domain solution of this case has been estimated to be approximately 100 times more important than with either of the edge-based or mode-based sub-domain methods.

Another important point is the efficiency of the sub-domain method to analyse the RCS modulations induced by the fan rotations. The inter blades distance of the fan is $360/16 = 22.5°$, the RCS has been measured in the CAMERA2 chamber of ONERA for 4 positions (0°, 5.625, 11.250°, 16.875°) with $22.5/4 = 5.625°$ step.

Dassault Aviation has computed the rotating fan sub-domain for each position and assembled each reduced operator with the reduced operator accounting for the other part of the geometry. The computation time for each new position was 1% of the computation time of the initial position.

Furthermore, ONERA has used S matrix transformation to perform the fan rotations, the computation time to obtain the RCS modulations patterns was less than 0.01% of the computation time for the initial fan position. We observe on Fig. 4 a good agreement between computed and measured modulation patterns. The level of modulation is approximately 5 dB sm for both the measured and computed results.

This shows the efficiency of the method for parametric computations including modifications in a subset of the sub-domains.

4. Conclusion

In this paper we have discussed a class of collaborative sub-domains technique based on BEM for analyzing the EM scattering from 3D complex structures. The BEM is used to compute the reduced operator of the sub-domains. For Air-Inlet applications wave-guided modes and edge based functions are used for the reduced operators basis functions. RCS patterns of the CHANNEL mock-up including modulations were presented and compared to measurements to show the accuracy of this method, the reduction in computation time and the advantages of this method for parametric studies and collaborative computations.

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