

Recent advances in crystal optics/Avancées récentes en optique cristalline

## Spatial quantum optical properties of c.w. Optical Parametric Amplification

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### Abstract

We have experimentally studied parametric amplification in the c.w. regime using a type-II parametric medium inserted in a transverse-degenerate resonant optical cavity. We demonstrate that this device is able to amplify either in the phase insensitive or phase sensitive way single mode beams and multimode images. We have also observed various multimode correlation and squeezing effects on the amplified output of this device. **To cite this article:** *L. Lopez et al., C. R. Physique 8 (2007).*

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### Résumé

**Propriétés quantiques spatiales de l'Amplification Paramétrique Optique.** Nous avons étudié expérimentalement l'amplification paramétrique en régime continu dans un milieu paramétrique de type II inséré dans une cavité optique résonnante ayant des modes transverses dégénérés. Nous démontrons que ce dispositif est en mesure d'amplifier, de manière insensible à la phase ou sensible à la phase, aussi bien des faisceaux optiques monomodes que des images multimodes. Nous avons de plus observé sur le faisceau de sortie de ce dispositif différents effets quantiques de corrélation et de compression des fluctuations ayant un caractère multimode. **Pour citer cet article :** *L. Lopez et al., C. R. Physique 8 (2007).*

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### 1. Introduction

Three-wave parametric mixing in a  $\chi^{(2)}$  non-linear crystal provides a very simple way of coherent optical amplification: when the crystal is submitted to a strong pump wave of frequency  $\omega_p$ , any incident signal wave of frequency  $\omega_s < \omega_p$  is amplified during its propagation in the crystal due to energy transfer from the pump wave to the signal wave. In addition, a new wave, named 'idler', of frequency  $\omega_i = \omega_p - \omega_s$  is generated in the same medium. According to the exact configurations of the system, the parametric amplification can be either phase sensitive or phase insensitive. In the simpler case it concerns a single input mode, but it is also possible to amplify in such a way transverse-multimode fields, and in particular to *amplify optical images*.

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The drawback of parametric amplification is its low efficiency. ‘Decent’ gains of the order of  $20 \text{ dB cm}^{-1}$  are obtained for pump radiances typically of the order of  $100 \text{ MW cm}^{-2}$ , which are only reached in high-power Q-switched or mode locked lasers. One then must restrict oneself to pulsed amplification, which has been widely studied in the past decades [1]. In particular pulsed amplification of images has been the subject of very interesting and promising studies [2].

There is a possibility to enhance the efficiency of parametric amplification: it is to take advantage of the build-up effect of a resonant cavity. If one inserts the non linear crystal in a resonant optical cavity, one obtains a ‘super-regenerative’ parametric amplifier (Optical Parametric Amplifier: OPA), which has, at least in principle, an infinite gain when one approaches from below the threshold for Optical Parametric Oscillation. As this threshold is usually in the  $100 \text{ mW}$  range or even less, this amplifier can be operated with c.w. pump lasers. This kind of c.w. parametric amplifier has been the subject of our investigations.

In addition Parametric Amplifiers have very interesting quantum properties, based on the fact that the generated signal and idler photons are quantum correlated photons: in the non-degenerate configuration the signal and idler output fields are in some respect ‘quantum clones’; in the degenerate configuration, the amplified output field is phase squeezed, and the phase-sensitive amplification process can be free of added quantum noise. We will detail the properties, classical and quantum, of single mode intracavity parametric amplification in Section 2, together with experimental results.

An optical cavity is known as an efficient filter of modes, both in the frequency and spatial domains. It is for example usually only resonant for a single  $\text{TEM}_{pq}$  mode. It seems therefore impossible to amplify highly multimode images in such devices. We will show in Section 3 how the use of *transverse-degenerate cavities* allows us to circumvent the problem. We will present the experimental realisation of a cw multimode parametric amplifier, and give experimental evidence for quantum effects in such a device.

## 2. Single mode intracavity Parametric Amplification

### 2.1. General properties

Let us consider a type I parametric crystal inserted in a cavity which is resonant for the signal and idler modes of frequencies  $\omega_s$  and  $\omega_i$ , that we assume first to be different (non-degenerate case). It is submitted to an intense pump field of frequency  $\omega_p$  (with  $\omega_p = \omega_s + \omega_i$ ) and power  $P_p$ . Above some pump power threshold  $P_{\text{th}}$ , the system generates signal and idler fields of non-zero mean amplitude, and turns into an optical oscillator [3]. Below  $P_{\text{th}}$ , it is a phase insensitive ‘regenerative amplifier’, of power gain:

$$G = \frac{(P_p + P_{\text{th}})^2}{(P_p - P_{\text{th}})^2} \quad (1)$$

in the undepleted pump approximation (small signal gain).  $G$  tends to infinity when one approaches the threshold  $P_{\text{th}}$  from below. The price to pay to obtain such an infinite gain is a reduction of the bandwidth of the amplifier, which is now restricted to the cavity bandwidth.

In the degenerate case where the signal and idler frequencies ( $\omega_s = \omega_i = \omega_p/2$ ), the signal and idler twin photons are emitted in the same mode. Below threshold the system is now a phase-sensitive amplifier which amplifies the quadrature component of the input signal mode which is in phase with the pump field. The power gain is then:

$$G_1 = \frac{(\sqrt{P_p} + \sqrt{P_{\text{th}}})^2}{(\sqrt{P_p} - \sqrt{P_{\text{th}}})^2} \quad (2)$$

The quadrature component of the input signal mode which is in quadrature with the pump field is de-amplified by a factor  $G_2 = (G_1)^{-1}$ .

Let us now consider a type II parametric crystal inserted in an optical cavity, and let  $Ox$  and  $Oy$  be the directions of polarisation of the signal and idler waves. The device behaves like the type I OPA when the signal and idler frequencies are different, but not when they are equal. A weak field of frequency  $\omega_p/2$  injected in it is amplified at the output of the crystal whatever its polarization. If the input field polarization is aligned either along the  $Ox$  or  $Oy$  axis, the amplification is phase insensitive. In contrast, when the input field is injected at 45 degrees from these axes,

the amplification is phase sensitive: there is either amplification or de-amplification, depending on the relative phase between the pump wave and the injected wave.

## 2.2. Quantum properties

The general quantum aspects of optical amplification are well known [4]: if the amplifier is a usual phase-insensitive amplifier, it is possible to show that it cannot be described at the quantum level as a single mode system. A ‘noise’ mode inevitably enters into the play that introduces excess noise. In the case of amplification by stimulated emission, for example, this is the spontaneous emission noise, and the noise modes are the ones in which the spontaneous photons are emitted. In the case of parametric amplification, this is the parametric fluorescence noise, and the noise mode is the idler mode. As a result the signal-to-noise ratio is degraded at the output of the amplifier. It is equal to the signal-to-noise ratio at the input divided by a quantity  $F$  (the noise figure of the amplifier) which is larger than 1. At the limit of very large gain, one can show that  $F$  is equal to 2: this is the well-known ‘3 dB cost’, present in any large gain amplifier. In contrast, it turns out that a phase-sensitive amplifier which amplifies one quadrature component and ‘de-amplifies’ by the same amount the other quadrature does not need a coupling with a second input mode, and can therefore operate without added noise. It can be characterized by a noise factor equal to 1: the amplifier is then labelled as ‘noiseless’ [5].

Let us go back to the type II parametric interaction where the signal and idler modes have the same frequency ( $\omega_s = \omega_i = \omega_p/2$ ). In the phase insensitive configuration, the noise figure approaches 2 at the high gain limit. In the phase sensitive configuration (input field at 45 degrees from the  $Ox$ – $Oy$  axes), the amplification process is noiseless, at least in the ideal case of no extra-losses, and it generates a squeezed field at the output [6]. Furthermore, if one decomposes the output field on its two polarization components along the signal and idler polarization axes  $Ox$  and  $Oy$ , one gets two intensity correlated outputs, which are in some respect two ‘quantum clones’ in both the phase insensitive and phase sensitive configurations [7]. This is due to the high level of quantum correlation existing between the signal and idler photons produced by the parametric interaction. These effects have already been experimentally demonstrated [8] for a single transverse mode input field in single pass amplification using an intense pulsed laser as a pump.

## 2.3. Experimental results

They are detailed in [9]. Our setup uses a very stable, single mode, frequency-doubled cw Nd-YAG laser system delivering intense beams at 532 nm, used as a pump, and at 1064 nm, used to inject the OPA. The phase between the pump and the injection is controlled by a mirror placed on a piezo-electric transducer. We use a walk-off compensated KTP crystal cut for non-critical type-II collinear phase-matching and frequency degeneracy, and temperature stabilized within 1 mK. The cavity is made by two mirrors with radii of curvature  $R = 100$  mm, separated by a distance of approximately 100 mm, i.e., close to confocality, but not exactly tuned to it. The input mirror reflects 90% of the 532 nm light and almost all the 1064 nm light. The output mirror reflects almost all the 532 nm light and 99% of the 1064 nm light. In these conditions, the oscillation threshold of this OPO is approximately 20 mW.

The operating point of the OPA was taken as close as possible to the threshold for higher parametric gain. Both pump and injection were spatially matched to the  $TEM_{00}$  cavity mode. As the refractive indices for signal, idler and pump are different, the three modes do not in general resonate for the same length of the cavity. Triple resonance was achieved by precisely acting on various cavity parameters: temperature tuning, which gives simultaneous resonance of signal and idler modes, and crystal tilt, which allow us to reach the triple resonance configuration. We have observed a strong amplification in the phase insensitive configuration (injection along the signal or idler polarisation), while sweeping the cavity length through the triple resonance point. We have measured a maximum gain in such a ‘transient’ c.w. Optical Parametric Amplifier of 23 dB. We have then rotated the injected field polarization by 45 degrees and stabilized the OPO length on the triple resonant point. By scanning the relative phase between the pump and the injected field, we have observed phase sensitive amplification with gain up to 6 dB. To our knowledge, this is the first observation of such an effect in the c.w. regime and in a type II configuration.

In order to get a more stable operation of the device, we have used a new set-up, based on a dual cavity configuration, in which the signal and idler modes resonate in a different cavity from the pump cavity, but where the three modes overlap in the crystal. This kind of cavity, introduced in [10], has been used in c.w. [11] or pulsed [12,13]

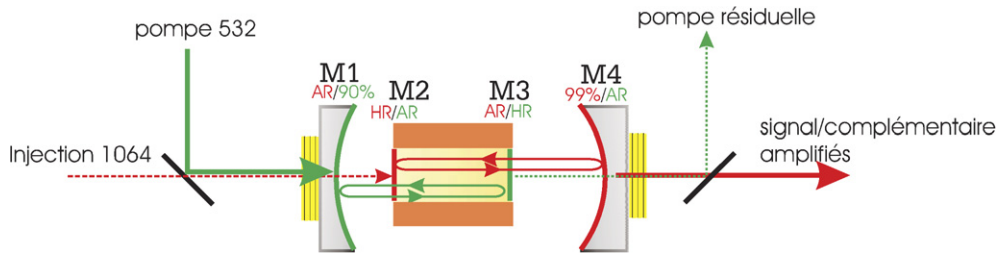


Fig. 1. Scheme of the experimental set-up using a dual cavity.

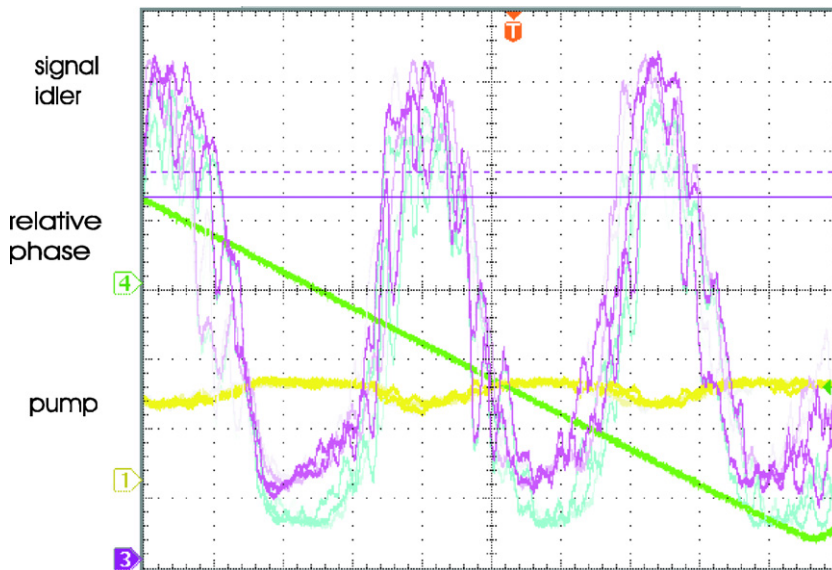


Fig. 2. Oscilloscope traces of phase sensitive single-mode parametric amplification.

OPOs. The linear dual cavity is shown in Fig. 1. The pump cavity is limited by mirrors M1 and M3, and the signal-idler cavity by mirrors M2 and M4. The advantage of such a dual cavity is that it is now possible to control and stabilize independently the length of each cavity. There is no need in this configuration to tilt the crystal to get the triple resonance condition, which is much easier to obtain in a stable way. M2 and M3 being parallel, the parallelism of the optical axes of the two cavities is automatically ensured. The coincidence of these axes is finally adjusted by slightly tilting the mirrors M1 or M4 to maximize the mode overlap, so as to obtain the minimum threshold possible, of the order of 30 mW. Because of the imperfect antireflection coatings deposited on the crystal faces, unavoidable with dual-wavelength coatings, the M1–M2 and M3–M4 systems constitute low finesse secondary cavities. When the dual cavity is injected at  $45^\circ$  from the crystal axes, and when the pump and injected modes are the  $TEM_{00}$  modes of their respective cavities, we have measured below the oscillation threshold c.w. stable phase-sensitive amplification with typical gains of 5–6 dB (Fig. 2). The precise gain value is not known because of the secondary cavity effect. It was not possible to observe higher gains because the intracavity pump intensity variations due to the secondary cavity prevented us from operating very close to threshold. We have also observed phase sensitive amplification with typical gain of 4–6 dB in this configuration.

We have finally measured the correlation between the intensities of the signal and idler components of the amplified output beam: for a parametric gain of the order of 6 dB, the noise on the difference between the two intensities has been measured 35% below the standard quantum limit. These results show that the signal and idler components of the output beams are quantum correlated, and can be considered as (imperfect) clones of the input field.

### 3. Multimode intracavity Parametric Amplification

Single mode pulsed amplification in single pass through a parametric crystal readily extends to the multimode case of images [14]. Noiseless amplification has been experimentally observed both on the temporal [15] and pure spatial fluctuations [16]. We will see in this section in which respect these properties extend to the case of intracavity parametric amplification.

#### 3.1. Multimode optical cavities

As already mentioned, an optical cavity is a spatial filter, which projects the input field on its own eigenmodes. If one wants to keep the spatial information of the input field, one must therefore use a cavity which is ‘maximally degenerate’, as far as its transverse modes are concerned. We will briefly describe here the imaging properties of degenerate cavities, which have been studied in detail in [17].

The round trip propagation of a monochromatic field in an empty optical cavity is characterized by a linear integral transform that we denote  $T$ , given by:

$$E_{\text{roundtrip}}(\vec{r}) = e^{ikL} \mathbf{T}(E_{\text{in}}(\vec{r})) \quad (3)$$

where  $E_{\text{in}}$  and  $E_{\text{roundtrip}}$  are respectively the fields at point  $\vec{r}$  of a given reference transverse plane before and after one cavity round trip, and  $L$  is the cavity round-trip optical length. The integral transform  $\mathbf{T}$  only depends on the coefficients of the geometrical optics Gauss matrix  $T_{\text{cav}}$  describing the transformation of a light ray over one round trip [18]. The on-axis phase shift of a Hermite–Gauss mode  $\text{TEM}_{mn}$  over one cavity round trip is equal to  $kL + (m + n + 1)\Phi_{\text{roundtrip}}^{\text{Gouy}}$ , where  $\Phi_{\text{roundtrip}}^{\text{Gouy}}$  is the roundtrip Gouy phase shift which can be easily calculated from the round trip Gauss matrix  $T_{\text{cav}}$  [19,20], because the eigenvalues of  $T_{\text{cav}}$  are equal to  $\exp(\pm i\Phi_{\text{roundtrip}}^{\text{Gouy}})$ .

The cavity eigenmodes will then be the Hermite–Gauss modes which have a total round-trip phase-shift equal to  $2p\pi$ , with  $p$  integer. This occurs only for a comb of cavity length values  $L_{mnp}$  given by:

$$L_{mnp} = \frac{\lambda}{2} \left( p + (m + n + 1) \frac{\Phi_{\text{roundtrip}}^{\text{Gouy}}}{2\pi} \right) \quad (4)$$

Apart from the ‘normal’ degeneracy ( $m + n = cst$ ) related to the cylindrical symmetry of the problem, the cavity will be therefore ‘transverse degenerate’ when  $\Phi_{\text{roundtrip}}^{\text{Gouy}}/2\pi$  is a rational fraction  $K/N$ . In this case  $(T_{\text{cav}})^N = 1$ , and after exactly  $N$  round trips any input geometrical optics light ray, as well as any input field configuration  $E_{\text{in}}(\vec{r})$ , is transformed into itself (within a phase factor for the electric field). In this case, there are only  $N$  values of resonant cavity lengths  $L_{mnp}$  inside an interval of size  $\lambda/2$ , i.e.,  $N$  different ‘families’ of resonant modes. Let us give three examples of degenerate cavities:

- the confocal cavity (two identical mirrors separated by a distance equal to their radius of curvature) has a round trip Gouy phase equal to  $\pi$  ( $K = 1$ ;  $N = 2$ ). The round trip field transformation  $\mathbf{T}$  gives the field at the symmetrical point  $-\vec{r}$ . The transmitted field through the cavity is the odd or even part of the input field, depending on choice of the family of modes;
- the semi-confocal cavity (a plane mirror and a curved mirror separated by a distance equal to half its radius of curvature  $R$ ) has a round trip Gouy phase equal to  $\pi/2$  ( $K = 1$ ;  $N = 4$ ). One finds that the transmitted field, depending on the four families of modes, is a combination of the even or odd part of the input field and of the even or odd part of its Fourier transform;
- the ‘self imaging cavity’ has a Gauss matrix equal to the identity and a Gouy phase equal to zero. Any intracavity ray is retraced onto itself after one round-trip; any field configuration, whatever its shape, is perfectly transmitted, and still benefits from the build up effect of the cavity. It is thus an ideal cavity for studies on cavity-enhanced imaging. Self-imaging cavities have been extensively studied in [19]. They cannot be built using only two spherical mirrors, and can consist for example of a plane end-mirror, an intracavity lens and a curved end-mirror, separated by appropriate distances.

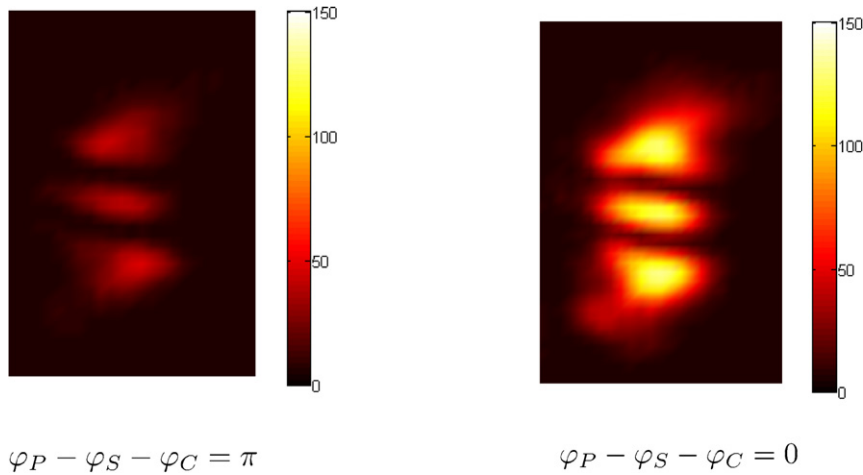


Fig. 3. Phase sensitive amplification of a double-slit ( $\varphi_P, \varphi_S, \varphi_C$  are respectively the pump, signal and idler phases).

In contrast, the planar cavity, limited by two plane mirrors, a configuration widely studied theoretically, is not a transverse degenerate cavity.

### 3.2. Experimental results

The previous set-up was modified for optimum multimode image amplification. By using a mirror having a large radius of curvature, the pump cavity  $TEM_{00}$  mode has a larger waist, and is likely to pump many transverse modes of the signal-idler cavity. The choice of a dual cavity, imposed by our needs of high stability of the degenerate configuration, prevented us to use a confocal cavity for the signal and idler beams, and compelled us to use the semi-confocal cavity instead.

Fig. 3 shows the experimental results when a double slit is used as an input image. As explained above, a semi-confocal cavity transmits the even part of the input image, i.e., the double slit itself, plus the even part of its Fourier transform, which is the well-known fringe pattern. This latter part is actually what can be seen more easily in Fig. 3, as it turns out to have a more intense central feature than the transmitted double slit. The difference in intensity is a clear evidence of the regime of phase sensitive amplification of an image. By performing a point by point division of the two images, we obtain an estimation of the local gain and of its spatial variation. One obtains a gain slightly varying around 5 dB over all the area where the local intensity of the image has a significant value, i.e., on a zone much larger than the size of the  $TEM_{00}$  cavity mode. The fact that one gets significant amplification well beyond the extension of this fundamental mode is another indication of the multimode character of the present amplifier.

Finally, we investigated the quantum properties of these amplified images. We studied in particular the two relative phase positions corresponding to amplification and de-amplification. In the amplification configuration, we could measure quantum intensity correlation (‘twin beams’) between the signal and idler images: the noise on the difference between the intensities of the two images was 30% below the standard quantum noise limit. This value is very close to the single mode noise reduction and shows that the multimode operation of our OPO does not degrade its quantum properties. The regime of ‘quantum cloning’ of the two polarisation components of the amplified image was thus reached in the experiment. Furthermore, the noise reduction could be observed for different input images, consisting of double slits of variable size and orientations. In the de-amplification regime we have measured the total intensity noise of the image and observed an intensity squeezing of 30% below the shot noise limit. Once again, this could be observed for any input image. Here also, this noise reduction was effective for various input images, which is an evidence of the truly multimode character of the device.

## 4. Conclusion

Pure quantum spatial effects, predicted to take place in OPOs using degenerate optical cavities, have been experimentally demonstrated. The experiments turn out to be rather difficult, as many stringent conditions must be fulfilled

and optimized at the same time. This is the reason why, so far, the observed quantum effects are small, and neither non-local squeezing [21] nor local quantum entanglement has been observed. These first c.w. quantum imaging effects are, nevertheless, encouraging, and will be undoubtedly improved in the years to come. Let us also mention that preliminary experiments seem to show that the ‘self-imaging cavity’, described in Section 3.1, has very interesting potentialities for intracavity quantum imaging studies. This system will be actively studied in the near future.

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