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Towards reconfigurable and cognitive communications/Vers des communications reconfigurables et cognitives

# Cognitive radio: methods for the detection of free bands

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**Abstract**

In contrast to current systems where the spectrum allocation is static, future cognitive radio devices will be able to seek and use in a dynamic way the frequencies for network access; this will be done by autonomous detection of vacant bands in the radio spectrum. In this article, we are interested in various methods of detection of a signal embedded in the noise by specifying their advantages and their drawbacks. Following that, a cyclostationary detection method, called multi-cycles detection, will be proposed. For illustrative purposes, we will apply these methods to the detection of the free channels within the television (TV) bands. ***To cite this article: M. Ghozzi et al., C. R. Physique 7 (2006).***


**Résumé**

**Radio cognitive : des méthodes de détection des canaux libres.** Par opposition aux systèmes actuels où l'allocation de spectre est statique, les terminaux radio cognitive de demain pourront chercher de manière dynamique les fréquences d'accès au réseau par la détection des bandes de fréquences libres dans le spectre radio. Dans cet article, nous nous intéressons aux différentes méthodes de détection d'un signal noyé dans le bruit en précisant leurs avantages et leurs inconvénients. Ensuite, une méthode de détection cyclostationnaire dite détection multi-cycles sera proposée. Pour illustrer notre propos, nous appliquerons ces méthodes à la détection des canaux libres sur les bandes de télévision. ***Pour citer cet article : M. Ghozzi et al., C. R. Physique 7 (2006).***
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*Mots-clés :* Radio cognitive ; Accès opportuniste ; Radiomètre ; Cyclostationnaire

## 1. Introduction

The term 'cognitive radio', defined by Mitola [1], was recently reused by the FCC [2] to define a class of terminals which are able to modify their transmission parameters based on interaction with their environment.

In the majority of cases (excluding the important case of ISM bands), the radio spectrum access is handled by the attribution of a licence to a user, often referred to as primary user. Cognitive radio devices can, nevertheless, access in

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an opportunistic way to the parts of the spectrum held by the primary users when those do not use them and consent to. Hence, cognitive radio technology can facilitate a more intensive and efficient spectrum use.

However, as cognitive radios are considered to be of lower priority or secondary users of spectrum allocated to a primary user, a fundamental requirement is to avoid interference to potential primary users in their vicinity. One of the challenges to take up in such a system is the detection of vacant frequency bands in the radio spectrum.

In the literature, we find several definitions of a vacant frequency band [2,3], but generally we can consider that a frequency band is unoccupied if the filtered radio signal within this band is only composed of noise. In the opposite case, this signal will consist of an unknown nonzero number of telecommunication signals in addition to the noise. This is a binary signal detection problem [4], which can be modelled as a hypothesis testing problem. There are two possible hypotheses, $H_0$ and $H_1$:

$$\begin{aligned} H_0&: \quad x(t) = n(t) \\ H_1&: \quad x(t) = s(t) + n(t) \end{aligned} \tag{1}$$

The solution to this problem, largely studied in the past, depends on the degree of knowledge we have on the signal to be detected and/or the noise. If a stationary Gaussian noise is considered and if one has a sufficient knowledge on the signal, then one can use a matched filter [4] to the shape of the awaited signal. Since the awaited signal $s(t)$ in (1) may be totally unknown to the detector, this solution must be discarded and one is required to use a general-purpose detector, the radiometer [5]. Also known as the energy detector, the radiometer simply relies on detection of changes in the total received energy and consequently gives less importance to the signal structure. Under the well known noise variance condition, the obtained detection results are satisfactory [6]. However, in a realistic situation, the quality of detection is strongly degraded [6,7] due to noise power uncertainty. The main difficulty with this way of detection is to obtain a good estimate of the variance of the noise because of the non cooperative character of the opportunistic access.

As the required signal is of telecommunication type, an interesting alternative consists in choosing a cyclostationary model [8] rather than a stationary one for the signal. This model is particularly attractive when the noise is of stationary type. The problem of detection of (1) is hence reduced to that of testing for the presence of cyclostationarity in the received signal. Several works [10–12], and in particular [9], are devoted to this kind of problem and propose various tests of cyclostationarity over a given set of cyclic frequencies. Unfortunately, when the cyclic frequency is not specified but may be within a given interval, then it will be required to repeat one of these tests as often as necessary to scan this interval.

In this article, we present a technique of detection called *multi-cycles detector* allowing the test in only one stage of a whole set of cyclic frequencies. This can have more than one use. The most important one will be in tracking unknown cyclostationary signals (with unknown cyclic frequencies) emerged in noise. This technique also has an interesting use when the harmonics of a (known) fundamental cyclic frequency are present. They take part in the improvement of the detection performances compared to the test of only one cyclic frequency. As will be seen later, the possibility of detecting an unknown cyclostationary signal is conditioned by the length of the segment of data.

We apply this technique of *multi-cycle* detection to detect free TV channels and we compare results to the radiometer detector.

## 2. Statistical models for detection

Let $x(t)$ be a zero-mean random process with values in $\mathbb{R}$. $x(t)$ is said to be cyclostationary at order $n_0$ if and only if its statistical properties until order $n_0$ are periodic functions of time. In particular for $n_0 = 2$, the process is called cyclostationary in the wide sense and verifies:

$$c_{xx}(t,\tau) = E\big(x(t)x(t+\tau)\big) = c_{xx}(t+T,\tau) \tag{2}$$

where the parameter $T$ represents the cyclic period. Note that if the process $x(t)$ is stationary then its statistical properties will be independent of time. The covariance function $c_{xx}(t,\tau)$ of (2) admits a Fourier series representation with respect to time $t$, i.e.,

$$c_{xx}(t,\tau) = c_{xx}(\tau) + \sum_{\alpha\in\psi} C_{xx}(\alpha,\tau)\mathrm{e}^{\mathrm{i}2\pi\alpha t} \tag{3}$$

where

$$C_{xx}(\alpha,\tau)=\lim_{Z\to\infty}\frac{1}{Z}\int_{-Z/2}^{Z/2}c_{xx}(t,\tau)\mathrm{e}^{-\mathrm{i}2\pi\alpha t}\,\mathrm{d}t \tag{4}$$

The sum in (3) is taken over integer multiples of the fundamental frequencies, i.e., reciprocals of periods in $c_{xx}(t,\tau)$, such as carrier frequency, symbol rate, guard interval, and their sums and their differences. $\alpha$ is said to be the cyclic frequency, $\psi$ is the total set of cyclic frequencies, and $C_{xx}(\alpha,\tau)$ is called the cyclic covariance function. In the case of a stationary process, $\psi$ is reduced to an empty set.

We also define the cyclic spectrum density as the Fourier transform of the cyclic covariance function according to the variable $\tau$:

$$S_{xx}(\alpha,f)=\int C_{xx}(\alpha,\tau)\mathrm{e}^{-\mathrm{i}2\pi f\tau}\,\mathrm{d}\tau \tag{5}$$

Note that for $\alpha=0$, the cyclic covariance and the cyclic spectrum density reduce to the conventional covariance and the power spectral density respectively.

Let us take an example of a baseband telecommunication signal:

$$x(t)=\sum_k s_k q(t-kT_s-t_0) \tag{6}$$

with

- $\{s_k\}_{k\in\mathbb{Z}}$: i.i.d. symbols with values in $\mathbb{C}$,
- $q(t)$: the transmitting filter,
- $T_s$: symbol period,
- $t_0$: initial phase in $[0,T_s[$.

The calculation of the covariance of $x(t)$ gives:

$$\begin{aligned}c_{xx}(t,\tau)&=E\left\{\sum_{k,k'}s_k s_{k'}q(t-kT_s-t_0)q(t-k'T_s-t_0-\tau)\right\}\\&=\sum_k\sigma_s E\left\{q(t-kT_s-t_0)q(t-kT_s-t_0-\tau)\right\}\end{aligned} \tag{7}$$

If we suppose $t_0$ fixed and unknown, then $c_{xx}(t,\tau)$ becomes $c_{xx}(t,\tau)=\sum_k\sigma_s q(t-kT_s-t_0)q(t-kT_s-t_0-\tau)$ and $c_{xx}(t+T_s,\tau)=c_{xx}(t,\tau)$. Consequently, $x(t)$ is a cyclostationary signal with a cyclic period equal to $T_s$. On the other hand, if $t_0$ is taken randomly and uniformly distributed in the interval $[0,T_s[$, then $c_{xx}(t,\tau)=c_{xx}(\tau)$ will be independent of $t$ and $x(t)$ is being rather a realization of a stationary random process.

## 3. Energy detection or radiometer

The radiometer was first proposed by Urkowitz [5]. It relies on discriminating between the binary hypothesis of (1) based on the difference in energy levels of the signal of interest and noise. The signal is considered to be deterministic, although unknown in detail. The spectral region to which it is approximately confined is, however, known. The noise is assumed to be Gaussian and additive with zero mean and known power density spectrum $\sigma_0$.

Fig. 1 depicts a block-diagram of an energy detector. The input band-pass filter selects the centre frequency and bandwidth $W$ of interest. It is easy to show [5] that the test statistics $V$ follows a central chi-square law ($\chi^2$) with

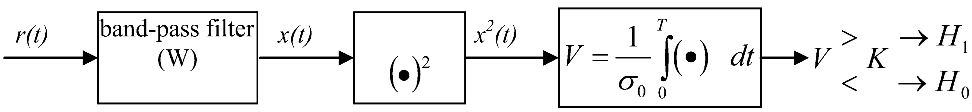


Fig. 1. Typical block diagram of an energy detector.

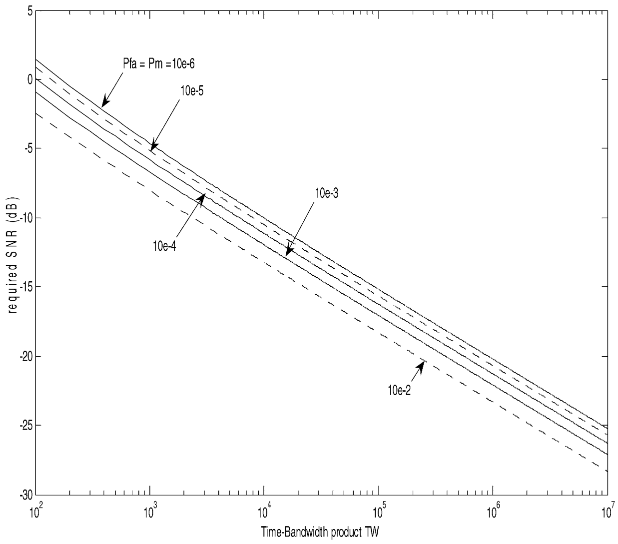


Fig. 2. Required SNR: known noise.

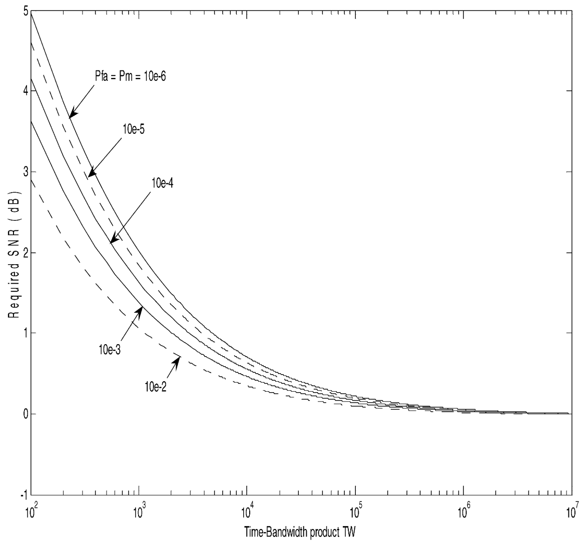


Fig. 3. Required SNR in uncertain noise; $U = 3$ dB.

$2TW$ degrees of freedom under hypothesis $H_0$, and a non-central chi-square law ($\chi^2$) with $2TW$ degrees of freedom and a non-centrality parameter $\lambda$ given by $E_s/\sigma_0$ under hypothesis $H_1$, $E_s$ being the signal's energy.

As mentioned above, the unknown signal is considered to be deterministic; however, the result also applies if the signal is random provided the probability of detection is considered a conditional probability of detection, where the condition is a given amount of signal energy [5]. For instance, in [13] and [14] the probability of detection expressions are determined when the signal is of random amplitude.

With $TW$ increasing, the statistics $V$ will instead be a normal random variable [6]. For several values of the false alarm probability ($P_{\text{fa}}$), theoretical graphs of the minimum required SNR (Signal to Noise Ratio) ($E_s/\sigma_0$) versus the time-bandwidth product ($TW$) are plotted in Fig. 2.

For a fixed bandwidth, the SNR required to achieve the desired detection probability ($P_d = 1 - P_{\text{fa}}$) is proportional to $T^{-1/2}$ [6]. Note that signals can be detected at a SNR as low as desired, provided the detection interval is long enough and the noise power spectral density ($N_0$) is known. However, realistic limitations on the detector's knowledge of the noise level produce serious degradation in the detector's performance.

In almost all practical situations, $N_0$ would need to be estimated by the detector. Denote this estimate by $\hat{N}_0$ and assume the error in estimating $N_0$ is bounded by

$$(1 - \varepsilon_1)N_0 \leqslant \hat{N}_0 \leqslant (1 + \varepsilon_2)N_0 \tag{8}$$

with $0 \leqslant \varepsilon_1 < 1$ and $\varepsilon_2 \geqslant 0$.

Theoretical graphs in Fig. 3 show that, whilst increasing $TW$ indefinitely, detection cannot be made at low SNR [6]. Here, $U$ denotes the peak-to-peak uncertainty and is defined as:

$$U\ (\text{dB}) = 10\log_{10}\left[\frac{1 + \varepsilon_2}{1 - \varepsilon_1}\right]$$

In current telecommunication systems, channel estimation routines also facilitate the estimation of the noise level due to known reference pilot sequences. However, in an opportunistic system, it is not very likely that the cognitive radio has access to the nature of the primary users' emitted signal, hence rendering noise estimation impossible.

## 4. Cyclostationary detection

When a cyclostationary model is selected for the searched signal, the detection problem of vacant bands in the spectrum is transformed to the following hypotheses testing problem on the received radio signal $x(t)$:

- under $H_0$ $x(t)$ is of stationary type and the band is regarded as free;

- under $H_1$ $x(t)$ is of cyclostationary type and the band is said occupied.

This can be looked upon as a test for presence of cyclostationarity rather than a detection of a signal in noise. Theoretically, the obtained solution will be noise-knowledge independent but some knowledge about the searched signal will be required.

In [9], a statistical test for the presence of cyclostationarity over a candidate cyclic frequency is given. Although it is computationally extensive, this test exhibits good performances and can be applied when the transmission parameters of the primary user are known to the cognitive radio device. As for now, however, let us suppose the worst case of unknown primary user transmission parameters. Then we do not face a single but rather an interval of candidate cyclic frequencies. Following the line of reasoning of [9], the test will have to be carried out frequency by frequency, which makes the algorithm more computationally complex. In the following, we will present an extension to this *mono-cycle* test [9] aiming at simultaneously testing an increasingly important set of cyclic frequencies. Indeed, the more important the number of samples in a segment of data is, plus the set of cyclic frequencies tested is large, the better the performance of the detection. We refer to this test as the *multi-cycles* test.

## 5. Multi-cycles test

For a fixed lag $\tau$, the covariance function of (3) can be expressed with respect to $t$ as the sum of two components:

$$c_{xx}(t,\alpha) = CC + CPP \tag{9}$$

The first component $CC = c_{xx}(\tau)$ corresponds to the continues part of $c_{xx}(t,\alpha)$. The second component $CPP = \sum_{\alpha\in\psi} C_{xx}(\alpha,\tau)\mathrm{e}^{\mathrm{i}2\pi\alpha t}$ corresponds to the part of $c_{xx}(t,\alpha)$ which is (poly)periodic in time. Under the two assumptions $H_0$ and $H_1$, the component $CC$ exists whereas the component $CPP$ is null only under $H_0$. Let us define $\bar{c}_{xx}(t,\tau) = c_{xx}(t,\tau) - c_{xx}(\tau)$. Then, the hypotheses test on $x(t)$ given in (1) becomes rather a hypotheses test on $\bar{c}_{xx}(t,\tau)$ as follows:

$$\begin{aligned} H_0:&\quad \bar{c}_{xx}(t,\tau) = 0 \\ H_1:&\quad \bar{c}_{xx}(t,\tau) \neq 0 \end{aligned} \tag{10}$$

Now let $x(n)$ be the sampled version of $x(t)$ and $T_e$ the sampling period. Applying the technique of synchronized averaging [8] to the lag-product $x(t)x(t+\tau)$, expression (2) can be expressed by:

$$\hat{c}_{xx}^{(S)}(n,\tau) = \frac{1}{S}\sum_{s=0}^{S-1} x(n+sN)x(n+sN+\tau), \quad n \in [0, N-1] \tag{11}$$

where $N$ is any period and $(S.N+\tau)$ is the total data segment length. One important advantage of this estimator is that, for any given $N$, $\hat{c}_{xx}^{(S)}(n,\tau)$ is the appropriate estimator of $c_{xx}(t,\tau)$ for cyclostationary signals with cyclo-period equal to $N$ or one from the integer fraction of $N$. In other words, in the Fourier transform series expansion (3) of $c_{xx}(t,\tau)$, the cyclic frequencies set $\psi$ is now:

$$\psi = \frac{1}{N} \leqslant T_e.\alpha < 1; \quad \text{where } \alpha = \frac{k}{NT_e};\ k \in \mathbb{N} \tag{12}$$

In addition, the larger $N$, the greater the set $\psi$ will be. Consequently, the choice of $N$ will be of great importance when designing the multi-cycles detector for testing vacant frequency bands.

Let $x(n)$ be a realization of the discrete zero-mean random process $X(n)$ and assuming that:

(A1) $X(n)$ is either cyclostationary with cyclo-period in the set $\{N, \frac{N}{2}, \ldots, \frac{N}{N-1}\}$ or stationary,

(A2) $$\sum_{\xi_1,\ldots,\xi_m=-\infty}^{\infty} \sup_n \left|\xi_l \operatorname{cum}\left\{X(n), X(n+\xi_1), \ldots, X(n+\xi_m)\right\}\right| < \infty, \quad l \in \{1,\ldots,m\},$$

(A3) $X(n)$ is *a-dependent* and $N \gg \alpha$,

where cum is the cumulant function. The second assumption is referred to as the mixing condition [15] and implies that samples of the process $X(n)$ that are well separated in time are approximately independent. This assumption is confirmed especially as the third one is verified. Assumption (A3) cannot be a restrictive assumption since $N$ will be chosen as large as possible for an increased detection reliability.

Proceeding as in [9], we can show (see Appendix) that the time-varying covariance estimator $\hat{c}_{xx}^{(S)}(n,\tau)$ as defined in (11) is mean-square consistent; i.e., $\lim_{S\to\infty}\hat{c}_{xx}^{(S)}(n,\tau)=c_{xx}(n,\tau)$. Additionally, $\sqrt{S}[\hat{c}_{xx}^{(S)}(n,\tau)-c_{xx}(n,\tau)]$ is asymptotically normal with covariance given by:

$$\mathrm{Cov}\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\}=\frac{1}{S}\mathrm{Cov}\{z^{n,\tau}(s),z^{m,\gamma}(s)\}=\frac{1}{S^2}\sum_{s=0}^{S-1}z^{n,\tau}(s)z^{m,\gamma}(s) \tag{13}$$

where

$$z^{n,\tau}(s)=x(n+sN)x(n+sN+\tau)\quad\text{and}\quad z^{m,\gamma}(s)=x(m+sN)x(m+sN+\gamma) \tag{14}$$

Let us return now to the testing hypothesis of (10). Since we do not have access to the exact quantity $\bar{c}_{xx}(t,\tau)$ but rather to an estimate of it ($\hat{\bar{c}}_{xx}^{(S)}(n,\tau)$ is exactly $\hat{c}_{xx}^{(S)}(n,\tau)$ from which we subtract its mean), it is not evident to have $\hat{\bar{c}}_{xx}^{(S)}(n,\tau)$ exactly zero even if the tested signal $x(n)$ is only constituted of noise. Thus, we can write (11) as:

$$\hat{\bar{c}}_{xx}^{(S)}(n,\tau)=\bar{c}_{xx}(n,\tau)+\varepsilon_{xx}^{(S)}(n,\tau) \tag{15}$$

where $\varepsilon_{xx}^{(S)}(n,\tau)$ represents the estimation error which vanishes asymptotically as $S\to\infty$ and $\bar{c}_{xx}(n,\tau)$ is the asymptotic true value of $\hat{\bar{c}}_{xx}^{(S)}(t,\tau)$. Hence, it is more appropriate to put the hypothesis testing of (6) in the following manner:

$$\begin{aligned}H_0&:\quad \hat{\bar{c}}_{xx}^{(S)}(n,\tau)=\varepsilon_{xx}^{(S)}(n,\tau)\\ H_1&:\quad \hat{\bar{c}}_{xx}^{(S)}(n,\tau)=\bar{c}_{xx}(n,\tau)+\varepsilon_{xx}^{(S)}(n,\tau)\end{aligned} \tag{16}$$

Because $\bar{c}_{xx}(n,\tau)$ is not random, the distribution of $\hat{\bar{c}}_{xx}^{(S)}(t,\tau)$ under $H_0$ and $H_1$ differs only in the mean. Consequently the test of assumption (16) is asymptotically equivalent to a test for nonzero of the unknown mean of a multivariate normal random variable [9].

### 5.1. Algorithm

- For a given lag $\tau$, we consider the fixed set of times $0, T_e, \ldots, (q-1)T_e$; $q\leqslant N$ and we calculate from (11) the row vector:

$$\hat{c}_{xx}^{(S)}\triangleq\left[\hat{c}_{xx}^{(S)}(0,\tau),\ldots,\hat{c}_{xx}^{(S)}(q-1,\tau)\right] \tag{17}$$

- From this we subtract its mean to obtain the row vector $\hat{\bar{c}}_{xx}^{(S)}$.
- Compute the covariance matrix $\hat{\Sigma}$ as given in (13).
- Compute the value of the test statistic $\ell$ as:

$$\ell=S.\hat{\bar{c}}_{xx}^{(S)}.\hat{\Sigma}^{-1}.\hat{\bar{c}}_{xx}^{(S)\prime} \tag{18}$$

  where the prime denotes transpose.
- Under $H_0$, $\ell$ has the following asymptotic distribution [9]:

$$\ell\sim\chi_q^2\quad\text{as } S\to\infty \tag{19}$$

  where $\sim$ denotes the convergence in distribution, $\chi_q^2$ is the central chi-square distribution with $q$ degrees of freedom.
- Under the alternative hypothesis $H_1$

$$\sqrt{S}\cdot\left(\hat{\bar{c}}_{xx}^{(S)}\cdot\hat{\Sigma}^{-1}\cdot\hat{\bar{c}}_{xx}^{(S)\prime}-\bar{c}_{xx}\cdot\Sigma^{-1}\cdot\bar{c}_{xx}'\right)\sim N\left(0,4\bar{c}_{xx}\cdot\Sigma^{-1}\cdot\bar{c}_{xx}'\right)\quad\text{as } S\to\infty \tag{20}$$

- For a given probability of false alarm $P_{\mathrm{fa}}$ and using the central $\chi_q^2$ tables for $q$ degrees of freedom, find the threshold $\Gamma$ that $P_{\mathrm{fa}}=\Pr\{\chi_q^2\geqslant\Gamma\}$.
- If $\ell\geqslant\Gamma$, declare that $x(n)$is cyclostationary, else decide that $x(n)$ is not cyclostationary over the set $\psi$ of cyclic frequencies.

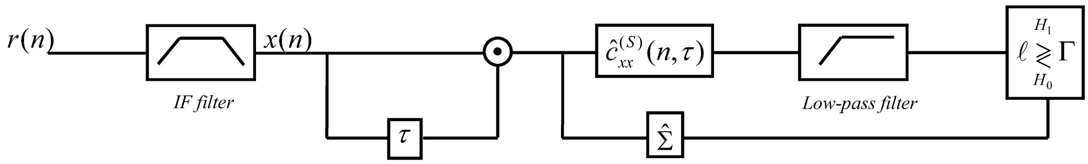


Fig. 4. Block diagram of the *multi-cycles* detector.

In Fig. 4, we depict the block diagram of a possible realization of this method.

## 6. Simulations

Here, we provide some simulation results of the proposed algorithm. As a signal of interest, we take a 4QAM (QPSK) modulated signal sampled at frequency $F_e = 50F_s$ where $F_s$ is the symbol frequency, and filtered by means of a raised cosine filter with parameter $\beta$ (roll-off).

For a theoretical value of $P_{\text{fa}} = 0.05$, Fig. 5 shows $P_d$ versus *RSB* for various values of the parameter $S$ defined in (11). This parameter $S$ corresponds to the number of symbols since the length $N$ of each segment of data is taken equal to the period symbol. Fig. 6 gives the variation of $P_{\text{fa}}$ versus the *detection Threshold* $\Gamma$ for various values of $S$.

From these results, one can observe that for $S = 400$, the obtained $P_d$ can in certain case be satisfactory. However, the corresponding (effective) $P_{\text{fa}}$ remains high compared to the theoretical one, this because of the asymptotic character of the estimator who tends towards its true value (11). For example, with $S = 900$, the obtained $P_{\text{fa}}$ is very close to the theoretical one; in which case, we can use the theoretical curve as a measure of the effective $P_{\text{fa}}$.

Results shown in Figs. 5 and 6 are summarized in Fig. 7 in which we show $P_d$ versus $P_{\text{fa}}$ for different values of the parameter $S$. In Fig. 8, we look to the detector performance for different values of the roll-off coefficient $\beta$. As we know, the larger $\beta$, the more the excess of the band compared to the band of Nyquist $[-\frac{1}{2N_sT_e}; \frac{1}{2N_sT_e}]$ is important. In the Fourier series development of the covariance function, this results in an increasingly clear and intense line at the fundamental cyclic frequency $(\frac{1}{T_s})$ and consequently in an increasingly higher $P_d$. In the special case of no filtering, this Fourier series development contains, besides, lines at the harmonics of the fundamental cyclic frequency.

This is beneficial to *multi-cycles* test especially as it is conceived to detect several cycles at the same time. This situation corresponds well to the case of testing for presence of television (TV) signals as will be specified in the following paragraph.

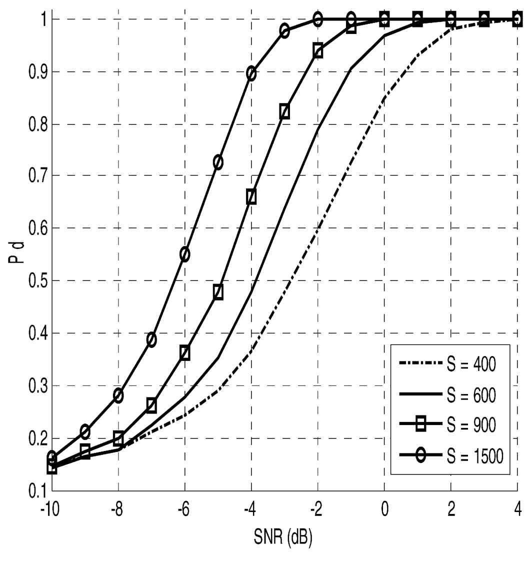


Fig. 5. $P_d$ vs. *SNR* for $\beta = 0.5$ and $\tau = 0$.

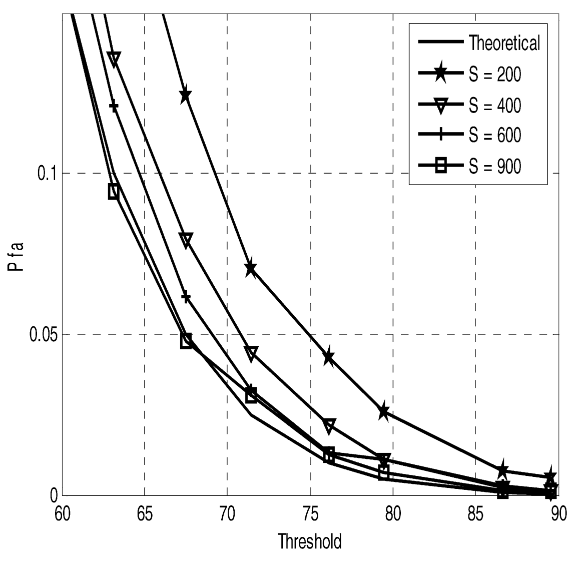


Fig. 6. $P_f$ vs. *Threshold*.

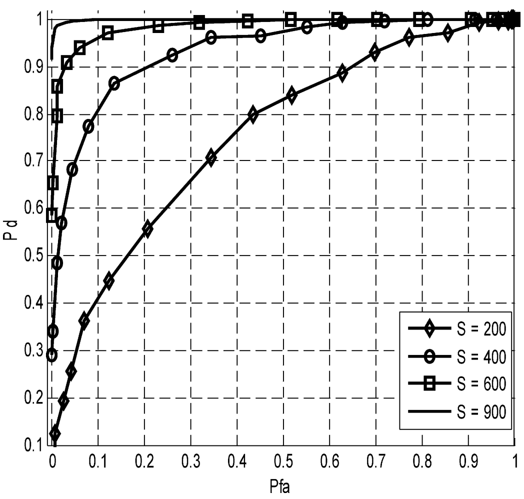


Fig. 7. $P_d$ versus $P_f$ for $SNR = 0$ dB, $\beta = 0.5$ and $\tau = 0$.

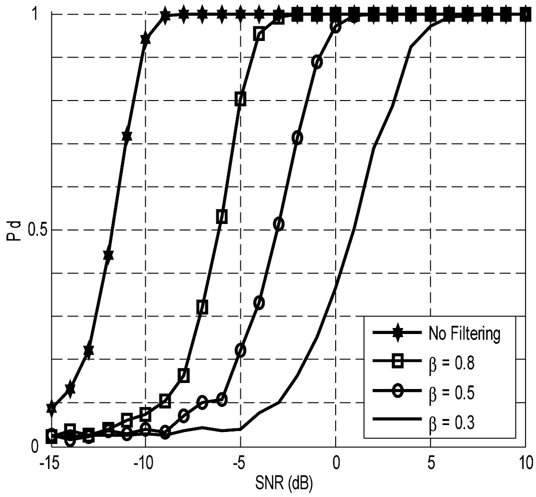


Fig. 8. $P_d$ versus $SNR$ for $S = 512$ and $\tau = 0$.

## 7. Vacant channel detection within TV bands

Detection of free channels over TV bands has taken on much interest since the FCC has authorized to cognitive radio devices the right to operate within these bands [16]. In the following we will be interested in SECAM, PAL, NTSC and DVBT TV systems. For each one of them, we determine the characteristic cyclic frequency.

In the analog TV systems [17]; the luminance information waveform is random and it exhibits (Fig. 9) synchronizing pulses at the rate of one pulse every $T_L = 64$ µs (63.5 µs for system NTSC). This leads to a cyclostationary video signal although its lower sideband is partially removed (see Fig. 10). Moreover, in the Fourier series development of the covariance function, we obtain the fundamental cyclic frequency (15 625 Hz $= 1/64$) and its harmonics.

In the case of DVBT standard [18], we employ a multi-carrier modulation (OFDM). Cyclostationarity in the transmitted video signal arises, due to the guard interval insertion, at cyclic period equal the OFDM symbol period, i.e., in 2K mode and for a TV channel of 8 MHz of large, we obtain [8] the data of Table 1.

As for analog TV systems, the Fourier series development of the covariance function of the OFDM signal exhibits lines at the fundamental cyclic frequency (inverse of the cyclo-period) and at its harmonics.

Fig. 11 shows the detection results of the multi-cycles detection method when it is applied to detect a noisy OFDM signal. These results are compared with the radiometer detector under the assumption of uncertain noise power indicated by $U$. From theses curves, it is clear that increasing the number of OFDM symbols used in the calculation of the

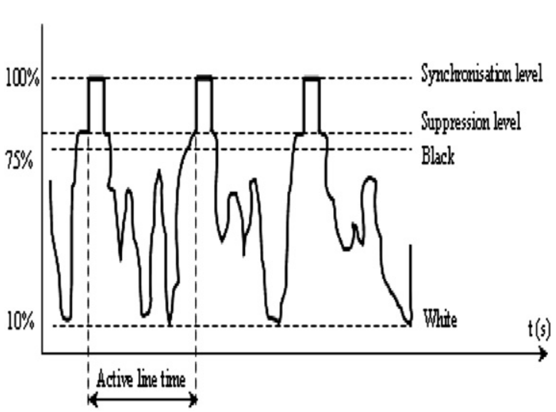


Fig. 9. Video waveform [17].

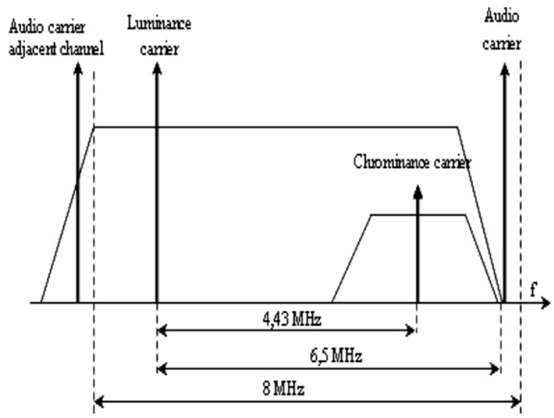


Fig. 10. Analog TV spectrum [17].

Table 1
Data for a TV channel of 8 MHz

| | | | | |
|---|---|---|---|---|
| Duration of symbol part $T_u$ (μs) | 224 | | | |
| Guard interval $\Delta/T_u$ | 1/4 | 1/8 | 1/16 | 1/32 |
| Cyclic period (μs) | 280 | 252 | 238 | 231 |

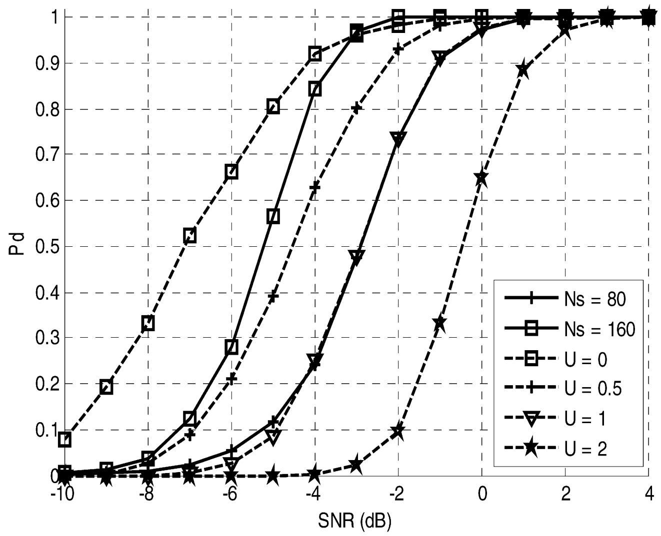


Fig. 11. $P_D = F$ (*SNR*). Detection of the numerical signal of TV by the radiometer and the detector *multi-cycles*.

statistical test leads to an enhanced detection performances for the multi-cycles method. Moreover, this performance remains insensitive, compared to the radiometer, when the noise power uncertainty is varying.

## 8. Conclusions

This article has dealt with an opportunistic detection of vacant bands, which is suitable for emerging cognitive radios. Specifically, we have proposed a cyclostationary based detection method. This method, referred to as a *multi-cycles detector*, is based on the estimation of the time varying covariance function of the received signal. It takes advantage of the fact that it tests several cyclic frequencies at the same time. A second advantage can be profited from when the harmonics of the fundamental cyclic frequency exist and in this case the detection performances are enhanced compared to the test of one cyclic frequency. Moreover, this method facilitates the detection of signals the cyclic frequencies of which are unknown, by simply increasing the duration of the segment of data used in the calculation of the correlation function. First simulations of this method for detection of free channels in TV bands prove to be encouraging compared to the simple energy detection.

## Appendix A. Bias

$$E\{\hat{c}_{xx}^{(S)}(n,\tau)\} = \frac{1}{S}\sum_{s=0}^{S-1} E\{x(n+sN)x(n+sN+\tau)\} = \frac{1}{S}\sum_{s=0}^{S-1} c_{xx}(n+sN,\tau) \tag{A.1}$$

Using assumption (A1), we obtain $E\{\hat{c}_{xx}^{(S)}(n,\tau)\} = c_{xx}(n,\tau)$, then the estimator $\hat{c}_{xx}^{(S)}(n,\tau)$ is unbiased.

## Appendix B. Consistence

We have

$$\mathrm{Cov}\left\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\right\}=\frac{1}{S^2}\sum_{s,s'=0}^{S-1}\mathrm{Cov}\left\{x(n+sN)x(n+sN+\tau),x(m+s'N)x(m+s'N+\gamma)\right\} \tag{B.1}$$

or

$$\begin{aligned}&\mathrm{Cov}\left\{x(n+sN)x(n+sN+\tau),r(m+s'N)x(m+s'N+\gamma)\right\}\\&\quad=\mathrm{Cum}\left\{x(n+sN),x(n+sN+\tau),x(m+s'N),x(m+s'N+\gamma)\right\}\\&\qquad+\mathrm{Cov}\left\{x(n+sN),x(m+s'N)\right\}.\mathrm{Cov}\left\{x(n+sN+\tau),x(m+s'N+\gamma)\right\}\\&\qquad+\mathrm{Cov}\left\{x(n+sN),x(m+s'N+\gamma)\right\}.\mathrm{Cov}\left\{x(n+sN+\tau),x(m+s'N)\right\}\end{aligned} \tag{B.2}$$

Then, with use of assumption (A3) and cumulant properties, it comes that each term of the right hand of (14) will be zero equal if $s\neq s'$. Next with use (A1), we obtain:

$$\begin{aligned}&\mathrm{Cov}\left\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\right\}\\&\quad=\frac{1}{S}\Big(\mathrm{Cum}\left\{x(n),x(n+\tau),x(m),x(m+\gamma)\right\}+\mathrm{Cov}\left\{x(n),x(m)\right\}.\mathrm{Cov}\left\{x(n+\tau),x(m+\gamma)\right\}\\&\qquad+\mathrm{Cov}\left\{x(n),x(m+\gamma)\right\}.\mathrm{Cov}\left\{x(n+\tau),x(m)\right\}\Big)\end{aligned} \tag{B.3}$$

Finally, from assumption (A2) we infer that $\mathrm{Cov}\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\}=\mathrm{O}(S^{-1})$ and $\lim_{S\to\infty}\mathrm{Cov}\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\}=0$ showing the consistency of the estimator.

## Appendix C. Normality

Let $x_1^n(s)=x(n+sN)$ and $x_2^{n+\tau}(s)=x(n+sN+\tau)$. Can be seen as a stationary time series and being jointly stationary. Thus, $\hat{c}_{xx}^{(S)}(t,\tau)$ is being a sample average of stationary discrete-time random signals. Under the mixing condition given by assumption (A2), Asymptotic Theory of Mixed Time Averages has been studied in [15], from which we conclude that $\sqrt{S}\,[\hat{c}_{xx}^{(S)}(n,\tau)-c_{xx}(n,\tau)]$ is asymptotically normal.

## Appendix D. Covariance

Eq. (14) can be simplified as $\mathrm{Cov}\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\}=\frac{1}{S}\mathrm{Cov}\{x(n)x(n+\tau),x(m)x(m+\gamma)\}$. Intuitively, for fixed $n$, $m$, $\tau$ and $\gamma$, $z^{n,\tau}(s)=x(n+sN)x(n+sN+\tau)$ and $z^{m,\gamma}(s)=x(m+sN)x(m+sN+\gamma)$ each one of them is the product of two jointly stationary time series; hence, they are also jointly stationary. Consequently, the $\mathrm{Cov}\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\}=\frac{1}{S}\mathrm{Cov}\{z^{n,\tau}(s),z^{m,\gamma}(s)\}$ can be estimated by

$$\mathrm{Cov}\left\{\hat{c}_{xx}^{(S)}(n,\tau),\hat{c}_{xx}^{(S)}(m,\gamma)\right\}=\frac{1}{S^2}\sum_{s=0}^{S-1}z^{n,\tau}(s)z^{m,\gamma}(s) \tag{D.1}$$

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