



Observation of black holes and extreme gravitational events/Observation des trous noirs et des événements gravitationnels extrêmes  
**Physics of accretion flows around compact objects**

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**Abstract**

Several physical and astrophysical problems related to accretion onto black holes and neutron stars are briefly reviewed. I discuss the observed differences between these two types of compact objects in quiescent Soft X-ray Transients. Then I review the status of various non-standard objects suggested as an alternative to black holes. Finally, I present new results and a suggestion about the nature of the jet activity in Active Galactic Nuclei. **To cite this article: J.-P. Lasota, C. R. Physique 8 (2007).**

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**Résumé**

**Physique des flots d'accrétion autour de objets compacts.** L'article contient une brève revue de quelques problèmes liés à l'accrétion sur les étoiles à neutrons et les trous noirs. Je discute les différences entre ces deux types d'objets compacts quand ils sont observés dans les Sources X Transitoires quiescentes. Ensuite, j'examine l'intérêt astrophysique, mais aussi fondamental, des divers objets non standard proposés comme alternatives aux trous noirs. La parties finale de l'article contient une présentation de certains résultats récents concernant la nature des jets émanants des Noyaux Actifs de Galaxies. **Pour citer cet article : J.-P. Lasota, C. R. Physique 8 (2007).**

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*Mots-clés:* Trous noirs; Relativité générale; Accrétion; Jets relativistes

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**1. Introduction**

The physics of accretion onto compact objects is interesting for at least two reasons. First, the compact bodies themselves are fascinating objects because of their extreme properties: supernuclear densities and very strong magnetic fields are the prerogative of neutron stars, whereas black holes are a marvel of pure relativistic gravitation. The signatures of neutron stars are usually unmistakable: very regular pulses of electrodynamic radiation or X-ray bursts due to the thermonuclear explosions at their surface. In other cases the presence of neutron stars is deduced from properties analogous to systems in which their presence is well established. Some of the reputed neutron stars could be quark stars. Nobody speaks, however, of 'candidate neutron stars'. The status of black holes is not the same. Although they are a very conservative prediction of Einstein's theory of gravitation, and the calculation showing that a collapsing cloud of dust forms a black hole appeared in 1939 [1], they have a rather louche reputation. One often (although less often than a few years ago) speaks of 'black-hole candidates'. This is not the place to analyze the reasons for

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the status of these fascinating objects. It is certainly due to the fact that they are fascinating, and they are fascinating because they are related to so many fundamental questions of physics. I will come to this later in the article.

From the point view of an astrophysicist, however, they are in a sense rather boring. Their only properties are mass and angular momentum. Maybe a little, but only a little, charge. As an astrophysicist I do not care that when a star collapses to form a black hole, entropy apparently increases by tens of orders of magnitude. I cannot measure this. I can measure the black hole's mass and estimate its angular momentum, but even if I can use these quantities to calculate a quantity called 'black-hole entropy' it has no physical meaning because there is no way I can make the measurement. Black holes have been identified in many binary systems and at the center of many galaxies. They are supposed to be featureless (except for their mass and angular momentum) and in not a single case have they failed to live up to this reputation. More precisely, not a single compact object more massive than  $3 M_{\odot}$  has shown any feature that would allow us to attribute to it a property other than mass and rotation. The ultimate test of their properties will be obtained by measuring gravitational waves emitted during black-hole mergers or black-hole ringing when excited by an orbiting compact body [3].

This said, it is legitimate and interesting to investigate the evidence for the existence of black holes, i.e. whether observations can exclude the existence (or rather presence) of other less orthodox and more exotic bodies. An article I co-authored a few years ago [2] was devoted to this problem. In it, we critically discussed some evidence and arrived at the conclusion that the ultimate test cannot be obtained by electromagnetic observations. Unfortunately, our article was understood by some people as doubting the existence of black holes. This article will try, in part, to dissipate that impression. However, I am not going to review here the whole problem of proving the existence of black holes. For this I refer the reader to the excellent review by Ramesh Narayan [4].

## 2. ADAFs and observations

ADAFs (Advection Dominated Accretion Flows—a name I devised at the 1995 Kyoto conference on the *Physics of accretion disks: advection, radiation and magnetic fields*) are accretion flows in which most of the thermal energy is not radiated but advected onto (in the case of a star) or through (in the case of a black hole) the surface of the accreting body. (Strictly speaking in some types of such flows advection of thermal energy into the central black hole is negligible [5,6].) The idea that such flows have some relevance to astrophysics had been around for some time [7–10]. In 1994–95 it was formalized (see Ref. [11] and references therein) but only the brilliant and successful application of ADAFs to various astrophysical systems by Narayan and collaborators (since I had the privilege of being one of them I am obviously not impartial) showed how useful and powerful the concept is (see Ref. [12] and references therein). The enthusiasm was not universal and the idea has been challenged many times (also by the ADAF authors themselves) but the (slightly bruised) ADAF 'paradigm' is still around and it is unlikely that it will soon disappear (see e.g. Ref. [13]).

Nevertheless, one should keep in mind that the ADAF *model* is just the simplest representation of a radiatively inefficient accretion flow. It cannot be expected to represent faithfully all the complex properties of accretion flows. However, rivals such as ADIOs & Co. or jet models contain more parameters but no more physics. In fact, sometimes they contain less physics because they contradict some of its fundamental laws. As for the geometrically thin disc it is of no use in the inner regions of quiescent accretion flows of transient systems [19]. The situation is less clear in the case of AGN, but also, there, such discs (if present) are very likely to be truncated [31].

In binary systems the ADAF forms only the inner part of the accretion flow, the outer part being a radiatively efficient (geometrically) thin disc. In galactic nuclei pure ADAF models have been proposed, as in the case of Sgr A\* ([14] and references therein). I will limit here myself to the case of binary systems.

### 2.1. Accreting black holes are fainter than accreting neutron stars

If one accepts the ADAF picture for both accreting black holes and neutron stars, one consequence is immediate: neutron stars should be brighter because all the thermal energy that was not radiated in the accretion flow will have to be emitted from the stellar surface. Guided by this idea Narayan, Garcia, and McClintock [15] compared the outburst amplitudes of Black-Hole Soft X-ray Transients (BH SXTs) and Neutrons Star Soft X-ray Transients (NS SXTs) as a function of their maximum luminosities. They found systematically higher amplitudes in BH SXTs implying that in quiescence (when ADAFs are supposed to be present) they are fainter than NS SXTs. Lasota and Hameury [16]

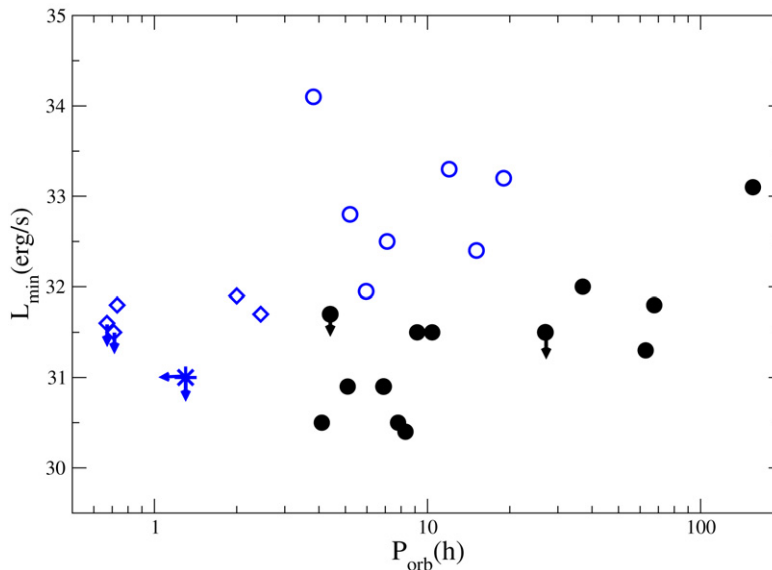


Fig. 1. Quiescent luminosities of black holes (filled circles) and neutron star (open circles and open diamonds) soft X-ray transients. Diamonds correspond to accreting millisecond pulsars. The star represents the system 1H 1905+000 whose orbital period is unknown (see the text). This figure shows that in quiescent transient LMXBs, *at a given orbital period*, neutron stars are brighter than black holes. Data for black holes from Garcia (private communication), for neutron stars from Ref. [27].

pointed out that the ADAF model does not state that accreting black holes are *always* fainter than accreting neutron stars, but that this is the case when both type of objects accrete at the same rate. In order to test the ADAF hypothesis they suggested plotting the quiescent luminosity as a function of the orbital period. This idea was later developed in Ref. [17]. Fig. 1 shows the quiescent luminosity versus orbital period for 13 NS SXTs and 13 BH SXTs. I will discuss some aspects of this figure below, but the separation between the two classes of systems is clear and neat: *at each orbital period* neutron-star systems are brighter than their black-hole counterparts. Doubtless it is a strong argument in favour of ADAFs and black holes. There has been a debate about where the neutron-star quiescent luminosity comes from, what part of it, if any, is due to accretion (see [18] and references therein; [17]), but in BH SXTs the quiescent luminosity is certainly due to accretion and it is unlikely that quiescent discs around neutron stars are not truncated [19] (or truncated but not leaky).

One should stress that the idea of plotting the luminosity as a function of period is *not* based on the assumption that BH SXTs and NS SXTs have the same mass-transfer rates at a given orbital period. It is based on the assumption that the truncation radius where the transition from disc to ADAF occurs, is roughly a constant fraction of the circularization radius [17,20,21]. Another assumption is that the truncated disc is a non-equilibrium disc as described by the disc instability model (see [22] for a review). These two assumptions (the second, although often not understood, should be non-controversial; the first, if one accepts the ADAF model, rather obvious) ensure that quiescent BH and NS SXTs with similar orbital period accrete matter at a similar rate.

Recently Jonker et al. [23] found that the neutron-star soft X-ray transient 1H 1905+000 could be the spoilsport, long-awaited by the ADAF-basher crowd. Its quiescent X-ray luminosity is at most  $1.8 \times 10^{31} \text{ erg s}^{-1}$ , but the upper limit on the 0.5–10 keV luminosity of this source, undetected by *Chandra*, could be as low as  $1.0 \times 10^{31} \text{ erg s}^{-1}$ . Jonker et al. assert that the luminosity of this neutron-star binary is so low that it is similar to the lowest luminosities derived for black-hole SXTs in quiescence and that it “*challenges the hypothesis presented in the literature that black-hole SXTs in quiescence have lower luminosities than neutron star SXTs as a result of the presence of a black-hole event horizon*”. However, looking at Fig. 1, I see no reason to panic. The orbital period of 1H 1905+000 is unknown, but it is certainly very short [23]. I have therefore tentatively assumed a period of 1.3 h, but even a longer period ( $\lesssim 3$  h) would not contradict the claim that black-hole systems are fainter than those harbouring neutron stars.

Even an actual quiescent X-ray luminosity of 1H 1905+000, much lower than the *Chandra* upper limit, would not necessarily be a problem for the black-hole faintness ‘paradigm’; it depends on what kind of system 1H 1905+000 is. The faintness of the secondary implies a short compact binary containing either a hot brown-dwarf companion, similar

to e.g. SAX 1808-365 [24] or an ultra-compact X-ray binary (UCXB), in which case the neutron-star companion would be a low-mass helium or carbon–oxygen white dwarf [25]. When transient (the shortest-period systems are rather persistent [26]), such compact binaries exhibit short ( $\gtrsim 10 - \lesssim 100$  days), exponentially decaying outbursts as expected from small, X-ray irradiated accretion discs [19]. In all these very compact transient systems the neutron star is a millisecond pulsar (MSP). Both their outburst (usually  $\sim$  few percent of the Eddington luminosity) and quiescent X-ray luminosities ( $< 10^{32}$  erg s $^{-1}$ ) are lower than those observed in longer period SXTs [27]. This is very similar to 1H 1905+000 whose outburst luminosity was  $\sim 4 \times 10^{36}$  erg s $^{-1}$ . However, the outburst behaviour of this system is totally different from that observed in other short-period binaries and UCXBs. Instead of short outbursts 1H 1905+000 exhibited one  $\gtrsim 10$  year long outburst that ended the late 1980s or early 1990s. Since then it has been quiet.

It is not clear why 1H 1905+000 is so different. During 11 years, say, it had accreted  $\sim 8 \times 10^{24}$  g. This is a lot, but a hydrogen-dominated accretion disc can contain as much as

$$M_{D,\max} \approx 2.5 \times 10^{24} \left( \frac{\alpha_{\text{cold}}}{0.01} \right)^{-0.83} \left( \frac{M_{\text{ns}}}{1.4 M_{\odot}} \right)^{-0.38} \left( \frac{P_{\text{orb}}}{1.3 \text{ h}} \right)^{2.09} \text{ g} \quad (1)$$

where  $\alpha_{\text{cold}}$  is the cold-disc viscosity parameter. For the disc radius I used

$$\frac{R_d}{a} = \frac{0.60}{1+q} \quad (2)$$

where  $a$  is the binary separation. The critical density of a cold helium disc being  $\sim 50$  times higher, a UCXB disc would contain even more mass (however, standard-disc formulae apply only to mass-ratios  $q \gtrsim 0.02$ , see below).

The maximum outburst luminosity for hydrogen-dominated disc, a  $1.4 M_{\odot}$  neutron star and  $\alpha_{\text{hot}} = 0.2$  can be estimated as

$$L_{\max} \simeq 1.8 \times 10^{36} \left( \frac{P_{\text{orb}}}{1 \text{ h}} \right)^{2.09} \text{ erg s}^{-1} \quad (3)$$

where I crudely re-scaled the formula from [22] to take into account disc irradiation. The maximum luminosity for a helium or carbon–oxygen disc (when it exists) would be a factor  $\sim 2$  higher. Therefore 1H 1905+000 could in principle be a ‘normal’, short-duration X-ray transient source, but it is not. Maybe its long ‘outburst’ was due to irradiations of the secondary.

There is, however, a more fundamental problem. The form of mass-transfer in systems with such very low mass ratios has not been studied and only some general properties of such systems can be conjectured (Dubus, private communication). For values of  $q \lesssim 0.02$  the circularization radius becomes greater than the estimates of the outer radius given by [28] and [29]. Most probably, matter streaming in from the companion circularizes onto unstable orbits. At  $q \approx 0.02$ , matter is added at  $R_{\text{circ}}$  onto orbits that can become eccentric due to the 3:1 resonance. At  $q \approx 0.005$  the circularization radius approaches the 2:1 Lindblad resonance. This might efficiently prevent mass being transferred onto the compact object.

An equivalent system with a black hole instead of a neutron star would have a minuscule mass ratio  $< 0.01$  ( $M_{\text{bh}} > 4 M_{\odot}$ ). It might not be a coincidence that there are no observed black-hole counterparts of neutron-star X-ray binary systems at orbital periods shorter than 2 hours. Such systems might exist [30], but they are not your normal LMXBs.

The 1H 1905+000 challenge is very likely a red herring and a counterexample to the black-hole faintness ‘paradigm’ has yet to be found.

### 3. Demography: the accreting bodies

We wish to know what is accreting because knowing that the object is a black hole would be of fundamental interest, but also the physics of accretion depends on what the matter falls onto. Some aspects of the physics of accretion depend on the nature of the accreting compact objects. No matter can accumulate at the surface of a black hole which prohibits thermonuclear bursts. No boundary layer can exist between an accretion disc and the black-hole surface where the accreted matter must plunge in radially at the speed of light. Since a black hole is strictly axisymmetric no periodic signal can be emitted by a (stationary) black hole. Black holes have no magnetic fields so there is no magnetic disruption of the accretion flow (external currents, however, can create magnetic fields anchored

on black holes—such fields can influence accretion flows; see e.g. [32]). On the other hand rotating black holes are surrounded by a region of absolute no-rest—the ergosphere—which opens possibilities denied to celestial bodies with material surfaces, such as the Blandford–Znajek process [33]. In this section I will mainly (but not only) deal with compact bodies in compact binaries.

### 3.1. Neutron and quark stars

Compact objects in close binary systems show mass segregation (see Fig. 2). They are clearly divided into two mass ranges:  $M < 3 M_{\odot}$  and  $M > 3 M_{\odot}$ . All members of the lower mass set are confirmed neutron stars, i.e. they are either (radio or X-ray) pulsars or X-ray bursters.<sup>1</sup> The members of the higher mass set are certainly not neutron stars, since the maximum mass of these celestial bodies satisfies the inequality [35]:

$$M_{\max} \leq M_{\max}^{\text{CL}} = 3.0 \left( \frac{5 \times 10^{14} \text{ g cm}^{-3}}{\rho_u} \right)^{1/2} M_{\odot} \quad (4)$$

where ‘CL’ stands for ‘causality limit’;  $\rho_u \lesssim 2\rho_0$  ( $\rho_0 = 2.7 \times 10^{14} \text{ g cm}^{-3}$  is the normal nuclear density) is the fiducial density above which the equation of state (EOS) of (super)nuclear matter EOS is unknown, i.e. there exist a whole bunch of EOS that could describe matter at these supernuclear densities. If  $\rho_u \lesssim 2\rho_0$  the contribution of the outer layers ( $\rho < \rho_u$ ) of the neutron star to the maximum mass is negligible (for this and the following see the excellent lectures by Haensel [35]). The maximum mass of neutron stars with causal ( $c_s \leq c$ , where  $c_s$  is the adiabatic speed of sound) EOSs is only slightly increased by rotation:  $M_{\max}^{\text{rot}} \simeq 1.18 M_{\max}^{\text{stat}}$  [36]. The upper bound  $M_{\max}^{\text{CL}}$  is increased by rotation up to  $\sim 30\%$ .

$M_{\max}^{\text{CL}}$  can be considered to be the maximum mass of a star with surface density  $\rho_s = \rho_u$  and a pure causality-limit EOS ( $c_s = c$ ). In other words, the maximum mass of  $M_{\max} \sim (\rho_s)^{-1/2}$ . Quark stars cannot be very massive because their surface density is still in the nuclear regime, but by lowering the allowed surface density, i.e.  $\rho_u$  one can easily obtain maximum masses in the black-hole ‘range’.

In any case, we know that hypothetical objects more massive than  $3 M_{\odot}$  cannot be made of normal matter, or matter in a normal state (this includes also quark matter in various states). Therefore normal matter accreting onto

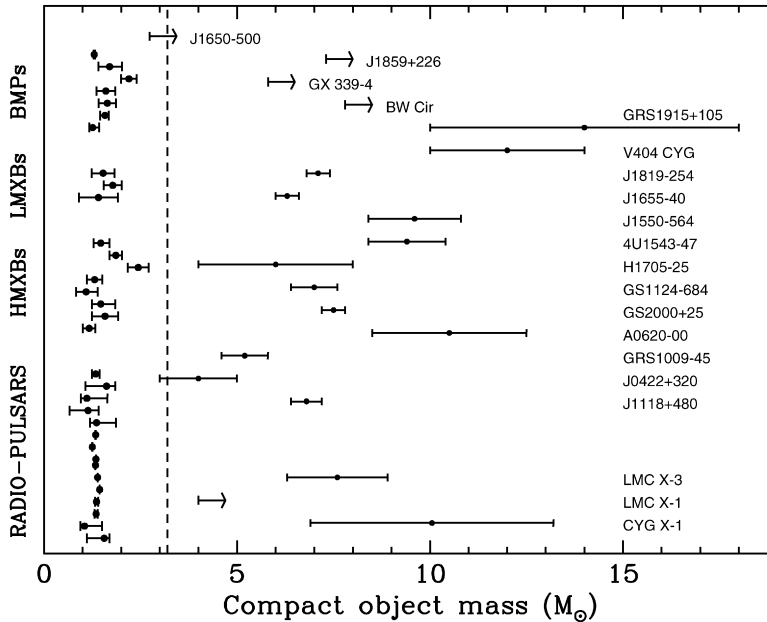


Fig. 2. Masses of neutron stars and black holes. From Casares [34].

<sup>1</sup> Some of them could be quark (‘strange’) stars.

the surface of such bodies might as well be undergoing a phase transition transmuting itself into whatever forms the accreting object. Therefore calculating models of thermonuclear explosions at the surface of ‘stars’ more massive than  $3 M_{\odot}$  [37,38] is rather pointless. There may be no nuclei to burn [2]. However, of course, the absence of X-ray bursts from systems believed to host black holes is a strong argument in favour of their real presence there.

### 3.2. Exotic interlopers

There exist several exotic alternatives to black holes. The oldest are boson stars, which were not invented as black-hole competitors (e.g. [39]). Other compact exotic bodies were explicitly introduced as black-hole challengers.

#### 3.2.1. Q-stars

This is the story of Q-stars [40]. They are objects made of a hypothetical state of matter which is a macroscopic self-bound superdense scalar-field condensate with a well defined electric and baryonic charge. They generated some astrophysical interest because the maximum mass of Q-stars could be as large as  $\sim 10 M_{\odot}$ . However, in fact, such high masses were obtained by allowing sufficiently low values of the fiducial density  $\rho_u$  [35]. Of course we do not know what it could be in reality. No experimental evidence for the existence of Q-matter has been found until now, but the same is true of cosmological dark matter.

#### 3.2.2. Boson stars

The typical mass of a star made of non-interacting bosons is  $\sim m_{\text{Pl}}^2/m_b$ , where  $m_{\text{Pl}} = (\hbar c/G)^{1/2}$  is the Planck mass and  $m_b$  is the boson mass, but the maximum mass of a self-interacting boson star is  $\sim \lambda^{1/2} m_{\text{Pl}}^3/m_b^2$ , where  $\lambda \sim 1$  is the scalar field self-coupling. This is similar to the mass of a baryon star ( $\sim$ Chandrasekhar mass)  $\sim m_{\text{Pl}}^3/m_p^2 = 1.9 M_{\odot}$ , where  $m_p$  is the proton mass, so that for  $m_b \sim m_p$ , a boson star would have a mass comparable to that of a neutron star. Since, by construction, a boson star is a macroscopic quantum state ‘supported’ against gravity by the uncertainty principle, its radius  $\sim 1/m_b$  is (for relativistic bosons) comparable to the radius of a neutron star. Although for the sake of completeness Yuan, Narayan and Rees [41] calculated models of accreting boson stars in close binaries (and came to the conclusion that they would have Type I X-ray bursts) the presence of these exotic bodies in such systems is rather doubtful, because it would require a fermion star evolving into a boson star. I have no doubt that the imagination (and skills) of some of my colleagues would find an easy way around this objection, but if one wants to consider a boson star as a serious substitution for a black hole, its place is the Galactic Center. Indeed, for a boson mass  $m_b \sim 1$  MeV the mass of a boson ‘star’ would be  $\sim 10^6 M_{\odot}$  and its radius a bit larger than that of the Sun (see e.g. [42,43]). Such an object, and except for black holes only such an object, would fit nicely into the constraints on the nature of the observed dark mass in the Galactic Center [44]. The ultimate test of the black-hole’s presence will be brought by the gravitational-wave observatory *LISA* [3,45,65].

#### 3.2.3. Dark energy stars aka gravastars

Dark-energy stars (DES) (also called gravastars) have been proposed as an alternative to black holes [46,47]. In these objects, the event horizon of a black hole is replaced by a quantum phase transition of the vacuum of space–time, analogous to the liquid–vapor critical point of a Bose fluid. Since outside such objects General Relativity is supposed to be valid, they are described by the Schwarzschild solution down to a distance  $\varepsilon$

$$\varepsilon \sim \left(\frac{M_{\text{Pl}}}{M}\right)^{2/3} \sim 10^{-25} \left(\frac{M_{\odot}}{M}\right)^{2/3} \quad (5)$$

from the Schwarzschild radius. The inner negative-pressure gravitational vacuum condensate is protected by a very thin shell, which effectively forms the DES surface [47]. Since this surface is at  $R_{g^*} = R_S(1 + \varepsilon)$  gravastar sizes are not restricted by the Buchdahl–Bondi [48,49] bound<sup>2</sup> ( $R_* > 9/8 R_S$ , or the redshift  $z < 0.33$ —for comparison, the maximum redshift of a neutron star is 0.851 [79]) their ridiculously large redshifts make them apparently indistinguishable from black holes, hence their potential astrophysical interest [2]. Such compactness is achieved at a price: to avoid the presence of naked singularities gravastars must have anisotropic pressures and a very peculiar equation of

<sup>2</sup> This bound was first noticed by Karl Schwarzschild [50].

state [51]. Of course, if one accepts negative central pressures and violation of the dominant energy condition (which requires  $\rho \geq 0$ ;  $|p| \leq \rho$ ) this cannot be the reason to dismiss dark energy stars (incidentally, but not accidentally, ‘normal’ boson stars are subject to anisotropic stress and cannot be described by an equation of state [39]).

The problem with dark energy stars is, indeed, more fundamental: their existence does not emerge from a (new) theory. They are constructed by analogy with superfluidity, liquid helium, optical fibers etc. and rather belong to the category of ‘Analog Gravity’ models [52] and should be treated as such. Although DES are advertised as panacea for the inconsistencies between quantum mechanics and General Relativity, Einstein’s theory of gravitation deserves better than to be replaced (if and when necessary) just by an analogy—especially when that analogy is far from perfect. In the latest installment of the DES saga [53] it has been proposed that the answer to the “*long-standing puzzle of astrophysics; namely, how [ . . . ] during the gravitational collapse of a massive stellar core the baryon number of the core disappears in  $\sim 10^{-5}$  sec*” be that the nucleons undergoing gravitational collapse disappear, being converted to vacuum energy, when according to General Relativity a trapped surface forms. If true, this would be in violent contradiction with the equivalence principle (to an observer in a free falling frame everything appears normal when crossing the horizon) and requires something better than just the affirmation that (in some reference frames) space–time behaves like a superfluid. Incidentally, the ‘disappearance’ of baryons during collapse has never been an astrophysical puzzle.

DES try to find their place in the dark, multidimensional landscape where tachyons chase phantoms<sup>3</sup> in their quest to couple to various types of (presumably) supersymmetric matter, so they do not deserve the indifference with which they were met. They are no more eccentric than most ‘theories’ appearing everyday on arXiv.org and they are more interesting than most. Their astrophysical interest, however, is rather limited. No solution for a rotating gravastar has been found (the Gödel-like metric of [54] is not such a solution, as it is not matched to an external vacuum solution). Since DES properties are only vaguely defined, the suggested astrophysical ‘tests’ cannot be taken seriously. Of course, the same can be said about many models elaborated by astrophysicists, but at least these do not claim to result from a revision of the fundamental laws of physics.

The main motivation behind DES and similar enterprises is, however, not astrophysical. Some physicists are depressed by the presence of the singularity hidden behind the event horizon, and some are unhappy both with the singularity and the event horizon. This motivates them to find an alternative to black holes. At a more fundamental level, the worry is the apparent incompatibility between General Relativity and quantum mechanics. The unitary character of quantum mechanics does not agree well with the presence of event horizons and, especially, black-hole evaporation. A fashionable response to this is that (super)string theory has the answer to these problems. It seems, however, that for the moment this is only a hope and that “(understanding) *how string theory prevents quantum information from being destroyed by black holes*” and “(understanding) *when and how string theory resolves space-time singularities*” are still ‘remaining problems’ of string theory [55]. There are also attempts to suitably generalize quantum mechanics so that black-hole evaporation would not be in conflict with its principles [56].

Obviously, the source of all these difficulties is the absence of a quantum theory of gravitation. Both Einstein’s theory of gravitation and quantum mechanics are extremely well tested experimentally in their respective domains of application. One does not need General Relativity to describe an atom, and quantum mechanics is not good in describing planetary orbits. In fact, there is not a single observed phenomenon or experimental fact that requires a quantum gravity explanation. Hawking radiation, although generally treated as fact, has never been observed. Gravitational waves produced by inflation have yet to be detected.

It is possible that we will have to live with two distinct theories describing the Universe at different scales and that their unification is physically meaningless [57,58]. If this is true, black holes are purely classical objects.

#### 4. Accretion, jets and spin

Rotating accretion flows often show evidence of the presence of more or less collimated outflows. One speaks of *jets* when the observed outflow is very collimated, but no precise definition of jet exists; sometimes the jet could be just the collimated outflow’s core that managed to bore through the surrounding medium, or simply the central part of a more extended outflow (see e.g. [59]). Accreting compact objects produce relativistic jets, i.e. well-collimated outflows with

<sup>3</sup> Phantoms are states with negative free kinetic energy.

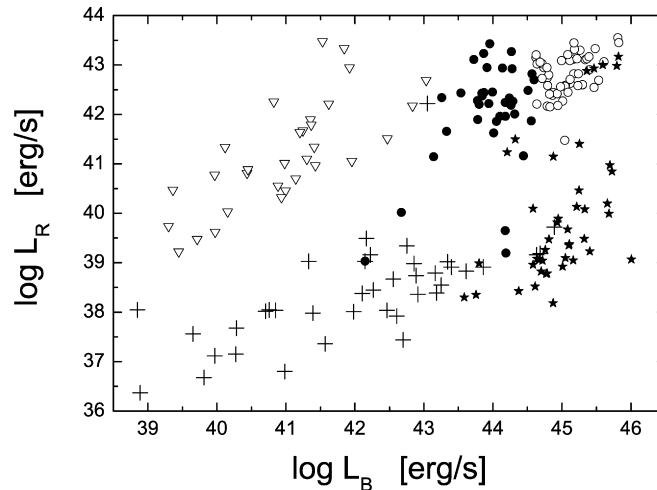


Fig. 3. Total 5 GHz luminosity versus  $B$ -band nuclear luminosity. BLRGs are marked by filled circles, radio-loud quasars by open circles, Seyfert galaxies and LINERS by crosses, FR I radio galaxies by open triangles, and PG Quasars by filled stars. From [60].

bulk motion corresponding to  $\Gamma = \sqrt{1/(1 - (v_{\text{bulk}}/c)^2)} > 1$ . Contrary to naive expectations, the  $\Gamma$  of black-hole jets does not seem to be larger than the  $\Gamma$  of ejecta from neutron stars; there is even evidence to the contrary, but one should keep in mind that determination of the flow's relativistic factors is rather a deduction than a direct measurement. The jet launching mechanism is unknown. This is rather embarrassing and some well-intentioned authors prefer to write that it is the *details* of this mechanism that are unknown, but this is a rather huge understatement. In most models of jet launching, the accretion-flow anchored magnetic field plays a crucial role.

Not all accreting compact objects show jet activity. In fact most quasars are radio-quiet (i.e. do not eject powerful jets). It is not clear if the distribution of radio-loudness is bi-modal, as had been believed for a long time (see [60] and references therein), but there are certainly quasars with very similar optical properties, whose radio luminosity differs by several orders of magnitude (see Fig. 3). Also, compact binaries do not always show jets. In their case, jet activity seems to correlate with accretion luminosity and with the spectral states of the accretion flow [61]. The advantage of LMXBs is their short timescales of variations, which makes it possible to keep track of their various states in real time. The drawback is their scarcity (e.g. the correlation found in [62] is based on observations of two systems). The advantage of quasars and Active Galactic Nuclei (AGN) is their large numbers and variety of types, but their relevant timescales are much too long to be of any use in direct observations. Therefore, LMXBs in which one can observe jets appearing and vanishing exert an irresistible attraction and tempt one to sometimes hasty generalizations. For example, Gallo, Fender, and Pooley [62] found that radio luminosities  $L_X$  of black-hole binaries at low accretion rates correlate with X-ray luminosities,  $L_X$ . In two systems, GX 339-4 and V404 Cyg, the relation  $L_R \propto L_X^{0.7}$  holds for more than three orders of magnitude in  $L_X$  with the same normalization within a factor of 2.5. This discovery led to speculations that jet activity in LMXBs is entirely determined by accretion, at least at low accretion rates, or more exactly during so called hard/low states. At higher rates, in the so called high/soft state, jet activity is suppressed. This conclusion was extended to AGN and quasars whose luminosities were supposed to follow the same type of single relation. However, as found recently by Sikora, Stawarz and Lasota [60] in the case of AGN there are *two* such relations (Fig. 3). They have similar slopes, and at higher luminosities one notices in both classes of objects signs of saturation and intermittency.

The two AGN sequences have rather interesting properties. The plotted sample consists of radio-loud broad-line AGNs (Broad-Line Radio Galaxies and radio-loud quasars); Seyfert galaxies and LINERS; FR I radio galaxies; and optically selected quasars, and contains active nuclei hosted by both elliptical and disc galaxies. The sample *does not* include blazars, i.e., OVV-quasars, HP-quasars, and BL Lac objects, because their observed emission is significantly Doppler boosted, a property too often forgotten in attempts to find general correlations extending from binaries to quasars. The two sequences are separated by  $\sim 3$  orders of magnitude in radio luminosity. Manifestly, in AGN an additional parameter is at work in jet production.



All AGN hosted by disc galaxies (i.e. Seyfert galaxies and LINERS), including those that according to some criteria are ‘radio-loud’, are found only on the lower (‘radio-quiet’) branch. AGN hosted by elliptical galaxies (quasars and radio-galaxies), however, are found on both sequences, i.e. they can be radio-loud or radio-quiet. It had been believed that all AGNs in giant elliptical galaxies are radio-loud and only recently, using the HST, luminous, radio-quiet quasars have been found to be hosted by giant elliptical galaxies. Clearly all this supports the idea that an extra parameter must play a role in explaining why the upper, radio-loud sequence is reachable only by AGNs hosted by early type galaxies.

#### 4.1. Spin paradigm revisited

There is no much choice in additional parameters. The most natural one is obviously the black-hole’s spin. The idea of the ‘spin paradigm’ (like all ideas in about black-hole accretion and ejection) is not new [63] and it went through various phases of popularity. Wilson and Colbert [66] assumed that black-hole spin evolution is determined by mergers and argued that this produces a broad, ‘heavy-bottom’ distribution of the spin, consistent with a distribution of radio-loudness. On the other hand Moderski and Sikora ([67], see also [68]) showed that spin-up by accretion (neglected in [66]) is so efficient, that to match the distribution of radio-loudness to observed spin distribution (i.e. to obtain a large proportion of radio-quiet quasars corresponding to low spins), one has to assume that accretion events take the form of both co-rotating and counter-rotating discs. Both these conclusions have been strongly challenged, however. On the one hand, it was found [69] that, unless the merging binary’s more massive member already spins rapidly and the merger with the smaller hole is consistently near prograde, or if the binary’s mass ratio approaches unity, mergers typically spin black holes down. On the other, Volonteri et al. [70] argued that the angular-momentum coupling between black holes and accretion discs is so strong, that the innermost parts of a disc are always forced to co-rotate with the black hole, and therefore that all AGN black holes should have large spins. In a sense the situation is now inverted: it is apparently impossible to spin up black holes by mergers but, nonetheless, nothing can stop them spinning very rapidly. If true, the implications are important: if all AGN black holes have very large spins, then their masses did not grow through mergers and, maybe less importantly, jet production has nothing to do with the spin of the central black hole. Of course, the last point seems to be strengthened by the presence of relativistic jets in neutron-star LMXBs.

However, the fact that radio-loudness of galactic centers depends on the host-galaxy morphology makes it worth trying to revive the spin paradigm. In its framework the much lower radio-loudness of AGNs hosted by disc galaxies implies (very) low spins of their central black holes. One has therefore to elucidate how in an accretion-dominated evolution, black holes in disc galaxies are protected against spinning up, whereas those in elliptical galaxies are not. The physics of the angular-momentum coupling between black holes and accretion discs is notoriously complex (see e.g. [72] and references therein) but one can expect that if nuclei of disc galaxies evolve through a sequence of randomly oriented *short* accretion events will result in small values of black-hole spins [67,68]. The required shortness of the accretion events can be expressed (Sikora et al., in preparation) in terms of the accreted-mass increments  $m_{\text{acc}}$ :

$$\frac{m_{\text{acc}}}{M_{\text{BH}}} \ll a \sqrt{\frac{R_S}{R_w}} \quad (6)$$

where  $a = J_c / GM_{\text{BH}}^2$  is the dimensionless black-hole spin and  $R_w$  the warp radius [71].

However, if in contrast to disc galaxies, elliptical galaxies undergo at least one major merger during their evolution (see, e.g. Ref. [73]) the following mass-accretion event is too massive to satisfy the condition equation (6). Then regardless of the initial relative disc-hole orientation the alignment processes [71] will finally produce co-rotating configurations and, provided that  $m_{\text{acc}}/M_{\text{BH}} \sim 1$ , this will result in  $a \sim 1$ .

In black-hole LMXBs the situation is simpler: since to reach the maximum spin a black hole has at least to double its mass (see below), black-hole spins in low-mass binaries do not evolve during the lifetime of the systems. Therefore, if black holes in LMXBs are born with roughly the same spin (or at least with no bimodal distribution), one should not expect in this case the presence of two radio-loudness sequences as observed [62].

## 4.2. Jets from neutron-star X-ray binaries

Relativistic jets are also observed in NS LMXBs [61,64]. Neutron stars have no ergospheres so the ejection mechanism cannot be the exactly the same as in BH systems. However, the condition for launching a Poynting-flux dominated outflow, which later becomes converted to the matter dominated relativistic jet, is to supply a high magnetic-to-rest-mass energy ratio ( $\gg 1$ ) at the base of the outflow. This is obviously satisfied in the case of the magnetic field anchored on a (rotating) neutron star (see [60] for more details).

However, it is rather amusing to notice that when a jet-like structure is observed in an X-ray binary it is immediately assumed that the compact component is a black hole. This allows one to call the system a ‘micro-quasar’, which sounds good in a press release. Of course, this is possible only if there is no direct evidence that the compact body is a neutron star (incidentally there is no *direct* evidence that the compact body in Sco X-1 and Cir X-1, the two ‘radio-loud’ neutron-star binaries is indeed a neutron star but somehow nobody, as far as I know, tried to claim that they could be micro-quasars.) So when very high energy  $\gamma$ -rays were observed with HESS [74] from the X-ray binary LS 5039 which exhibits a jet-like structure, it seemed inevitable that this was due to particle acceleration in a microquasar jet. However, as showed by Dubus [75], the compact object in this *high-mass binary* is a young pulsar. In a similar system the object PSR B1259-63 is a pulsar and a third system LSI +61.303 also belongs to this category of gamma-ray binaries. The recent criticism [76] of this model is not very thoughtful. It is true that the pulsar model underestimates the  $\gamma$ -ray fluxes but this can be explained by the simplifying assumptions (isotropy, no pair cascades) and in any case the micro-quasar model is not doing better. The argument that the ‘jets’ in LS 5039 seem to have relativistic bulk motions, as in micro-quasars, is irrelevant because it assumes that these jet-like features *are* jets and their speeds then deduced from their asymmetry. In the pulsar model, however, these features are not jets but radio-tails that just mimic (micro-quasar) jets. Finally the absence of major radio outbursts in LS 5039 is not an argument against the pulsar model but simply a consequence of the fact that in this system the stellar companion is an *O* star so that the pulsar has no circumstellar disc to plunge through, whereas its cousins in PSR B1259-63 and LSI +61.303 enjoy the presence of *Be* companions and can therefore produce radio splashes.

## 4.3. Black hole spin-up

It is (too) often forgotten that although to spin a black hole up to the maximum rotation-rate through accretion from a Keplerian disc is, in principle, easy, it is *impossible* in LMXBs. The reason is simple and has been known for 36 years: to spin-up to maximum rate a black hole must accrete more than twice its mass [77]. Since apparently this is not universally known and also the difference, in this respect, between black holes and neutron-stars is not clear to everybody, it is worth showing it once more. Bardeen’s solution can be found in [70]—e.g. an initially non-rotating black hole gets spun up to the maximum rate after accreting  $\sqrt{6}$  of the initial mass  $M_0$ . It is indeed a ‘modest’ [70] amount by extragalactic standards, but it is at least several times more than the mass of the black-hole companion in LMXBs. Here some simple reasoning will show the difference between spinning up a neutron star and a black hole.

In the case of a neutron star, assuming the disc extends to the star’s surface, the mass  $\Delta M$  accreted during the spin-up to an angular frequency  $\Omega_*$  can be expressed as

$$\Delta M \approx \frac{I_* \Omega_*}{(GM_* R_*)^{1/2}} \quad (7)$$

where  $I_*$ ,  $M_*$  and  $R_*$  are the neutron star’s moment of inertia, mass and radius respectively. It is therefore enough to accrete  $0.1 M_\odot$  to spin up a  $1.4 M_\odot$  neutron star to millisecond periods. More generally, to spin up a neutron star to break-up, one needs to accrete:

$$\Delta M \approx \alpha(x) M_* \quad (8)$$

where I have used  $I_* \approx \alpha(x) M_* R_*^2$  [78], where  $x = (M_*/M_\odot)(\text{km}/R_*) < 0.24$  [79] is the compactness parameter. For the most compact neutron star  $\alpha(x) \lesssim 0.489$  [78].

One can ‘adapt’ Eq. (7) to black holes, but it is preferable to use the inequality expressing the black-hole surface area theorem:

$$\Delta M > \Omega_H \Delta J \quad (9)$$

Here

$$\Omega_{\text{H}} = \frac{a}{2Mr_+} \quad (10)$$

is the black-hole angular velocity and  $r_+ = M + \sqrt{M^2 - a^2}$  the black hole (outer horizon) radius. For a maximally rotating black hole  $a = M$ ,  $r_+ = M$  and one finds that

$$\Delta M > M_1 - M_0 \approx \frac{1}{2} M_1 \quad (11)$$

A more refined treatment of spin-up of black holes in LMXBs can be found in Ref. [80]. (A black hole initially counterrotating with respect to the accretion disc must triple its mass to achieve maximum spin [72].)

Therefore black holes in LMXBs keep their inborn spin. Observations seem to suggest that it is not very close to maximal ( $a/M \sim 0.1\text{--}0.8$  [81,82], but one should remember that such conclusions are strongly model-dependent. Simulations suggest formation of stellar-mass black holes with  $j = J/M \sim 0.7$  [83].

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