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C. R. Physique 8 (2007) 1029–1047
Comptes Rendus Physique
http://france.elsevier.com/direct/COMREN/

The mystery of the Higgs particle/Le mystère de la particule de Higgs

The Higgs as a pseudo-Goldstone boson

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Available online 11 June 2007

Abstract

The old idea of the Higgs as a pseudo-Goldstone boson has been revived and re-energized as a possible solution to the little hierarchy puzzle in the Standard Model. Its most natural implementation may be in the context of models with supersymmetry not far above the electroweak breaking scale. ***To cite this article: H. Georgi, C. R. Physique 8 (2007).***

Résumé

Le Higgs comme un pseudo boson de Goldstone. L'idée ancienne selon laquelle le Higgs pourrait correspondre à un pseudo boson de Goldstone a récemment connu un regain d'intérêt du fait de la solution qu'elle apporte au conflit de petite hiérarchie au sein du modèle standard. Elle trouve sans doute sa réalisation la plus naturelle dans le contexte de théories supersymétriques à une échelle proche de l'échelle de brisure de la symétrie électrofaible. ***Pour citer cet article : H. Georgi, C. R. Physique 8 (2007).***

Keywords: Higgs boson; Standard Model; Pseudo-Goldstone boson

Mots-clés : Boson de Higgs ; Modèle standard ; Pseudo boson de Goldstone

1. Introduction

The idea of the Higgs as a Pseudo-Goldstone Boson (PGB) [1–4] was revived in 2001 by the development of 'little higgs' models [5]. One can see this dramatically by looking at the citations to [4] histogrammed in Fig. 1. In the last five years, many new examples of models with a pseudo-Goldstone Higgs (PGH) have been constructed and studied, both within (see [6–80]) and outside (see [81–89]) the little higgs structure. In this review, I hope to introduce the reader to these developments. There are excellent reviews of the little higgs [90–92], and partly for this reason, I do not plan to review little higgs models myself in any comprehensive way. Rather, I want to try to put this class of theories into some context, to discuss the important theoretical ideas, and to discuss how other PGH theories fit in. I will focus particularly on the things that I find particularly beautiful and the things that have surprised me.

2. Accidental symmetry—then and now

The idea of a pseudo-Goldstone boson is in some sense quite old, since the light pseudoscalar mesons of QCD qualify. However, the precise modern formulation is due to Weinberg [1], suggested in the tumultuous times of the early

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1631-0705/$ – see front matter
doi:10.1016/j.crhy.2006.12.003

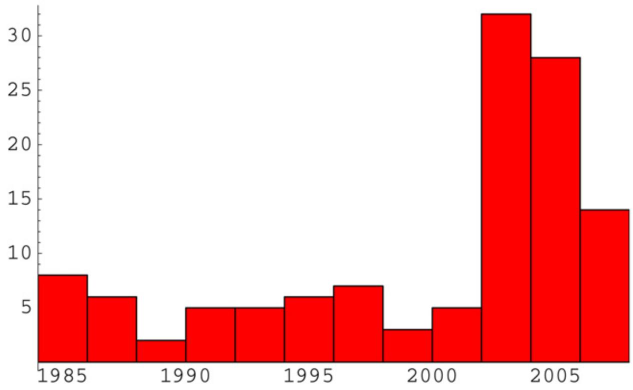

Fig. 1. Citations to [4] by year.

1970s. The issue that Weinberg addressed was the puzzle of approximate symmetries. He showed how they could arise in renormalizable quantum field theories (QFTs) as 'accidental' consequences of the constraints of renormalizability. The issue is always the 'accidental' part. It had long been known how symmetries could be imposed on a QFT, and how, once imposed, they were (in the absence of explicit anomalies) impervious to quantum corrections. But it did not seem to make sense to 'impose' an approximate symmetry when the symmetry breaking corrections were generally subject to renormalization and thus (at least in the prevalent view at the time) completely unknown. This seemed (and still seems to some of us) no better than just assuming the answer you wanted from the beginning. Weinberg noticed that in some QFTs symmetries of *part* of the Lagrangian arose automatically from the constraints of renormalizability without being imposed. If these symmetries were not shared by the rest of the Lagrangian, the symmetry breaking quantum corrections to the symmetric part would be non-zero, but they would be protected by renormalizability and they would get a finite and calculable contribution from the Coleman–Weinberg terms [93,1,94].

If an accidental symmetry is spontaneously broken, the result is a pseudo-Goldstone boson. It's mass and non-derivative interactions are symmetry breaking effects, finite and calculable quantum corrections. Weinberg hoped that this might be related to the approximate Goldstone nature of the pion, as indeed, it was. Such was the rate of progress in those times that the actual connection began to emerge in a matter of months. We now know that the approximate chiral $SU(3) \times SU(3)$ symmetry of low-energy QCD is an automatic consequence of the fact that the light quark masses are small compared to the QCD scale. No symmetry has been imposed by hand.

The reliance of the original definition of accidental symmetry on renormalizability here may seem somewhat quaint, but in fact the idea of accidental symmetry has proven to be extremely robust. But it is important to mention one extension of the original idea. All that is necessary for a PGB is an automatic degeneracy of the surface of minimum potential and this can come about in various ways [3]. In particular, the surface of minimum potential may have a symmetry that is not shared by the full potential. In fact, the only important difference if the symmetry is broken away from the surface of minimum potential is that there are then Coleman–Weinberg contributions to the PGB masses from scalar boson loops. The authors of [3] chose not to introduce a new term for this, and to simply extend the definition of accidental symmetry. Thus the notion of accidental symmetry has broadened somewhat. since its original definition.

The first real examples of PGH theories [4,95–98] made use of composite pseudo-Goldstone bosons built explicitly as pairs of fermions bound by a strong 'ultracolor' gauge interaction. In the simplest ultracolor $SU(3)$ theory, the strong ultracolor dynamics is just a stronger version of QCD with three massless quarks, electroweak $SU(2)$ is the analog of isospin, and the PGH is the analog of the K meson doublet, part of the octet of Goldstone bosons associated with the spontaneous breakdown of chiral $SU(3) \times SU(3)$ to Gell-Mann's flavor $SU(3)$. The point of references [4,95–97] was to explain how additional interactions (and/or masses) of the ultrafermions could break the chiral symmetries that made the Higgs a Goldstone boson and dynamically *build* a Higgs potential that gives rise to electroweak symmetry breaking. In [98], the authors performed a similar construction for an ultracolor theory that spontaneously breaks $SU(5)$ down to $SO(5)$.[1]

[1] This is what we believe happens for an $SO(n)$ color group.

So why is it that the authors of [4,95–98] did not discover little higgs models?

In all these works, fine tuning was required to make the ultracolor scale of Higgs compositeness much larger than the electroweak symmetry breaking scale, which was (and now is much more) required for phenomenological consistency. Note that this is perfectly consistent with the fact that the Higgs itself is a pseudo-Goldstone boson, and therefore has a mass much smaller than the ultracolor scale. The point is that in these models, generically, the Higgs mass is a symmetry breaking effect that is in some sense small because the Higgs is a pseudo-Goldstone boson. But the VEV is large because the VEV is a ratio of the Higgs mass to the square root of the quartic Higgs coupling. The global approximate symmetry that makes the Higgs pseudo-Goldstone boson generically suppresses both the Higgs mass and the quartic Higgs coupling by roughly the same factor, so the VEV remains large in the absence of fine tuning. The problem is to make the Higgs parameter even smaller, higher order in some symmetry breaking parameter, while retaining the leading symmetry breaking correction the quartic coupling. This is what required fine tuning in the original models. In these models, fine tuning is possible because there are two independent sources of breaking of the global symmetry that contribute differently to the Higgs mass and the quartic coupling so that the VEV can be tuned to be small compared to the ultracolor scale. This was the unsatisfactory situation that little higgs models addressed.

The reason that fine tuning was required in the first generation of PGH models is that there was an beautiful and subtle idea missing—the idea of collective symmetry breaking.[2] In interesting little higgs models, as we will see, there are also two sources of symmetry breaking. But each of them preserves a symmetry that leaves the Higgs as an *exact* Goldstone boson. But the symmetries of the two sources are different, so the combination of the two breaks all the symmetries that protect the Higgs. Then generically, without tuning, these terms suppress the Higgs mass more than they suppress the quartic Higgs coupling. I will discuss this in detail in Section 5 below when I talk about the 'littlest higgs', a beautiful model closely related to the model of [98].

However, there is another difference between the current efforts and those of [4,95–98] from the 1980s. The motivations for PGH theories today are somewhat different than they were before the flood of LEP data. In the 1980s, the primary motivation was the desire to do without scalar fields entirely. This motivation has been largely replaced by the so-called 'little hierarchy puzzle'—the surprising agreement of precision electroweak data with the Standard Model with a single light Higgs. The metaphor has changed dramatically in the intervening twenty years. Now one talks about controlling 'large' quadratically divergent contributions to the Higgs mass. This still seems odd to old people like myself. We grew up thinking of all quadratically divergent contributions as 'large'. But the modern view makes perfect sense in the context of Wilsonian effective field theory. Most modern PGH theories are much less ambitious than their ancestors because they are motivated primarily by LEP-energy data and stay at least partly in the effective theory below the scale of beyond-the-standard-model physics.

I will look in varying detail at just three different PGH theories, not attempting to give a complete description, but rather picking out the most interesting features or new theoretical ideas of each. I will not talk about precision tests of any of these theories. While this is an important hurdle for model building, and the creators of these models have learned many interesting things by trying to squeeze their theories into the straight-jacket of precision tests, in some deep sense this is premature. These theories are not crazy enough to be right anyway. I think it is more important to understand the range of theoretical possibilities than to ramify any particular theory. For each I will be particularly interested in the following questions:

(i) What is the accidental symmetry?
(ii) Is it *really* accidental or has some part been imposed by hand?
(iii) Do we get out more than we put in?

Before discussing PGH theories, I will quickly review QCD in Section 3. The chiral symmetries of QCD are interpreted as accidental symmetries and the light pseudoscalar mesons are pseudo-Goldstone boson. Much of our intuition about the properties of pseudo-Goldstone bosons comes from this picture.

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[2] Though I was one of the authors of [5], I really did not understand the beauty and generality of what we had done until I heard from Ann Nelson how it arose from collective symmetry breaking.

We will begin our tour of PGH theories in Section 4 by discussing very briefly the first little higgs model and the 'minimal moose' of Arkani-Hamed, Cohen, Katz, Nelson, Gregoire and Wacker [7] as examples of little higgs models with 'theory spaces' connected by multiple copies of similar pseudo-Goldstone bosons.

In Section 5, I look in much more detail at the 'littlest higgs' theory of [8] as an example of a PGH that arises naturally with a PGB sector different from that of QCD. Again, however, the accidental symmetry arises very naturally in a strongly coupled theory. This model is perhaps the prettiest example of collective symmetry breaking. I will describe it in much more detail than I devote to other little higgs models in order to show how collective symmetry breaking works in detail.

In Section 6 I briefly describe some more general developments in PGB model building that blur the distinctions I have made in the previous two sections.

In Section 7 I discuss what was for me the most surprising set of PGB models, based on the idea of a 'twin higgs'. In this case, the accidental symmetry arises not from strong interactions, but from the imposition of a Z_2 symmetry on a weakly coupled theory, resulting in a doubling of the gauge group and the Higgs structure. That this can accomplish anything at all, I found quite remarkable. But in fact, the simplest models (for example [82]) have some interesting features, and the supersymmetric versions are among the most appealing PGB models I have seen.

I close in Section 8.

3. Pions as pseudo-Goldstone bosons

Before we look at the condensates and Goldstone bosons in PGH theories, let us review how it works in QCD. This is a familiar story. The accidental symmetry of QCD is the chiral $SU(3) \times SU(3)$ symmetry of the light quarks. This symmetry is exact for massless quarks, arising as $SU(n)$ symmetries do so often in quantum mechanics simply because there are several complex fields (the left and right handed fermions in this case) with identical interactions. The quark masses break the chiral symmetry, but to the extent that the quark mass terms can be regarded as a perturbation, some vestige of the chiral symmetry remains. The symmetry is only relevant to the light quarks for which the quark mass can be regarded as small compared to the dimensional scale of Λ_{QCD}. Thus the symmetry requires parameters in the theory (the quark masses) to be in a particular range (small compared to Λ_{QCD}). However, this chiral symmetry requires no fine tuning, because no specific values of the quark masses are required, and the masses can be chosen completely arbitrarily, so long as they are small compared to Λ_{QCD}. In the confined phase of QCD, a quark–antiquark condensate breaks the chiral $SU(3) \times SU(3)$ symmetry of the light quarks spontaneously, preserving Gell-Mann's $SU(3)$, the lightest pseudoscalar mesons are Goldstone bosons, and the light quarks develop dynamical masses related to their couplings to the Goldstone bosons, and nearly independent of their small chiral symmetry breaking masses.

$$\left[\psi_L \gamma^0 \overline{\psi}_R\right]_{\substack{\text{low}\\ \text{energy}}} \propto \Sigma = \exp(\mathrm{i}\Pi/f) = \begin{matrix}\text{unitary}\\ \text{matrix}\end{matrix} \tag{1}$$

There is what is called a vacuum alignment issue here—the vacuum 'direction' of the condensate Σ is determined by the quark mass matrix M. The quark mass matrix produces a contribution to the potential energy that aligns the vacuum direction of Σ to the identity in the basis in which the symmetry breaking mass matrix is diagonal, giving rise to Gell-Mann's approximate $SU(3)$ flavor symmetry [99].

$$-\operatorname{Tr}(M\Sigma) \Rightarrow \langle\Sigma\rangle = I \quad \text{for } M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{2}$$

$$\Leftrightarrow \begin{matrix}\text{Gell-Mann's } SU(3) \text{ approximately}\\ \text{preserved for light quarks}\end{matrix} \tag{3}$$

Mathematically, we say that

$$SU(3) \times SU(3) \to SU(3) \tag{4}$$

Another way of describing this alignment issue is that each flavor does its own thing and condenses, but one does not know exactly what the flavors are until the symmetry is explicitly broken.

The other 'directions' for the condensate are excitations of the Goldstone boson fields

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 - \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 - \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & \sqrt{\frac{2}{3}}\eta \end{pmatrix} \tag{5}$$

In the QCD case, the Goldstone bosons are described by a hermitian matrix because the condensate is unitary.

The Π Goldstone bosons are quark–antiquark bound states, but they are massless in the absence of explicit symmetry breaking because they are bound by the QCD interactions just as much as the vacuum state itself.

In QCD this is very familiar

$$\Pi \propto \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} \tag{6}$$

and simple, except for some funny business for the π^0 and η due to the axial anomaly, which breaks the axial $U(1)$ symmetry and constrains the Goldstone boson fields to the traceless combinations of (5).

The f in

$$\Sigma = \exp(\mathrm{i}\Pi/f) \tag{7}$$

is f_π—the amplitude for a chiral current to create a Goldstone boson out the vacuum. In QCD it is smaller than the typical mass of a non-Goldstone meson state (like the ρ or the a_1 or whatever) ≈ 1 GeV.

The 1 GeV scale is called (confusingly) Λ—the chiral symmetry breaking scale—not to be confused with Λ_{QCD}. The ratio Λ/f plays a large role in the thinking of little higgsers. We believe that this factor is real, important and simple—except for factors having to do with the numbers of colors and flavors, it is a phase space factor of order $4\pi \approx 10$. In the application of these ideas to PGB theories, this factor of 10 is the difference between 1 TeV and 10 TeV. The number of colors and number of flavors affect these scales in ways that we understand in some limits but which in general are rather complicated. Usually, this does not help much.

We have also learned from QCD how to describe the effect of gauge interactions on Goldstone bosons. Because the electromagnetic gauge interactions break the chiral $SU(3) \times SU(3)$ symmetry of the light quark Lagrangian, we would expect electromagnetism to produce a contribution to the Goldstone boson mass. Because of photon exchange, the K^+ is heavier than the K^0 even though the u quark is lighter than the d quark. Electromagnetism gives no mass at all to the K^0 because it does not break the chiral d–s symmetry. Because the d and s quarks have the same charge, chiral U-spin (along with the commuting chiral $U(1)$) is not broken, and so the neutral mesons π^0, K^0, $\overline{K^0}$ and η remain massless.

We can also think of this in terms of the effect of electromagnetism on the attractive force that binds the mesons and produces the condensate. Photon exchange adds to the QCD attraction that forms the condensate, but it adds in the same way in the K^0 bound state, which, therefore, remains an exact Goldstone boson. But the K^+ and π^+ mass squared get a positive contribution from photon exchange because their quark and antiquarks repel one another and they are less bound. Formally, the photon exchange potential is

$$xe^2 f_\pi^4 |Q\Sigma - \Sigma Q|^2 \tag{8}$$

where $x > 0$ and Q is the quark charge matrix

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \tag{9}$$

An interaction of this kinds simply tries to make the condensates neutral to maximize the binding. It produces a contribution to the Goldstone boson mass squared proportional to the square of the charge. Obviously in QCD this gives equal mass squared to the K^+ and π^+ and nothing to any of the neutral states.

In this case, because the nonzero elements of condensate are all neutral, the condensate we already have from the quark masses minimizes this contribution to the potential as well, and the electromagnetic gauge symmetry is not broken by the vacuum.

The description (8) of the effect on the pseudoscalar masses from photon exchange in terms of the condensate Σ is the paradigm for the description of the effects on pseudo-Goldstone bosons of any interactions other than the strong interactions. This is worked out in exquisite detail for QCD [100] and (using [101,102]) in more general theories with spontaneously broken symmetry (see for example [103] and [104]).

4. Theory space models

The first little higgs model was inspired by the idea of deconstructed extra dimensions [5] in which many identical 4-dimensional gauge theories (sites) are linked together to form a 'theory space' that becomes an extra dimension at long distances. The links between different theory space sites are QCD-like Σ fields that spontaneously break the symmetries on the individual sites. Deconstruction allows one to easily translate many extra-dimensional schemes into ordinary 4-dimensional theories. The important thing for PGH theories is that theory space is rife with accidental symmetry. Each link is a potential multiplet of pseudo-Goldstone bosons. Each link could be built like the Σ field of QCD out of fermions transforming under a separate color group and the accidental symmetry could arise as it does in QCD. Indeed, one of the messages of deconstruction is that there is a close link between gauge symmetry and pseudo-Goldstones. The extra-dimensional components of the gauge fields in a deconstructed theory are built out of the pseudo-Goldstone bosons of the links.

It was recognized almost immediately that it was not necessary to have the full structure of a deconstructed extra dimension to produce a PGH. The important thing is to have enough links so that there are PGBs left after the Higgs mechanism has eaten the true Goldstone boson, enough sites so that the gauge interactions do not generate large masses for the PGBs, and enough structure to produce interactions that implement collective symmetry breaking. One of the earliest little higgs models built along these lines is the so-called 'minimal moose' of [7]. In the minimal moose shown in Fig. 2 each link represents an independent $SU(3) \times SU(3)/SU(3)$ Σ field. If these are generated by independent strong interactions, there is $SU(3)^8$ global symmetry spontaneously broken down to $SU(3)^4$ and explicitly broken by the gauge symmetries at the nodes down to $SU(3) \times SU(2) \times U(1)$, as shown in the figure.

In Fig. 2, we have shown only the PGBs on the links. Thus this picture is valid only below the scale at which the PGBs are formed. To see the underlying QCD-like structure diagrammatically, we would replace the solid PGB links by a moose diagram showing fermions and antifermions transforming under a strong ultracolor group. This is shown in Fig. 3, with the independent ultracolor groups indicated by the solid circles. In this form, the dynamical explanation of the global chiral $SU(3) \times SU(3)$ symmetries of the links is obvious.

Obviously, the theory space in Fig. 2 is very simple—consisting of only two sites. But why does it deserve the adjective 'minimal'. The authors of [7] argue that two sites is minimal because one site can never be enough. The point is that any one-site model, such as that shown in Fig. 4, will have contributions to the PGB masses like the electromagnetic contribution to the π^+ mass, (8). In a strongly interacting ultracolor theory, this is of order

$$g^2 f^2 \approx \frac{g^2}{16\pi^2} \Lambda^2 \tag{10}$$

where g is the gauge coupling on the site. This is too big, and we want to suppress contributions of this size. In the two site model, contributions of this order are forbidden by the gauge symmetry.

I will not say any more about these models, and their ilk, but I included them to contrast them with the Littlest Higgs of Section 5. Both make use of accidental symmetries that arise in a very natural way in composite models. But

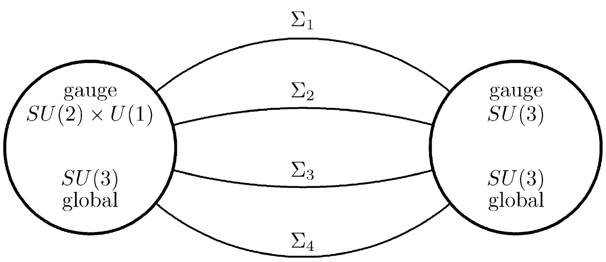

Fig. 2. The minimal moose of [7].

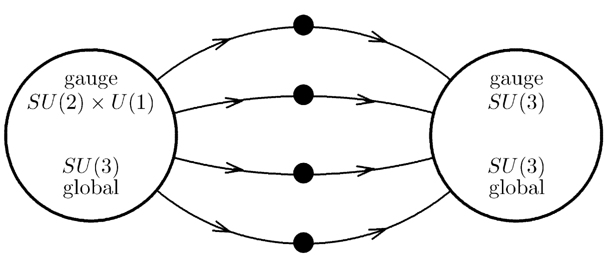

Fig. 3. A UV completion of the PGB sector of the minimal moose of [7].

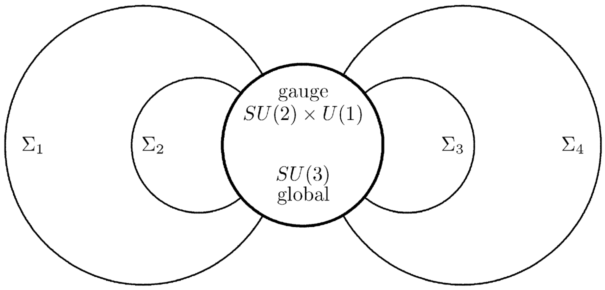

Fig. 4. A 1-site model that inevitably produces quadratically divergent PGB masses.

while in the Littlest Higgs, the pseudo-Goldstone bosons arise from a large unitary symmetry associated with many copies of the matter fields in a single strong gauge interaction, the Minimal Moose is at the other end of the spectrum. The extra symmetry arises precisely because there several independent strong sectors. The theory space models thus have the modular quality of good Rube Goldberg machines.

I will wait and describe collective symmetry breaking in the littlest higgs model in Section 5, because I find this much more beautiful and convincing.

5. The Littlest Higgs

In a way the littlest higgs model is a kind of opposite of the theory space models described in Section 4. In theory space models like the minimal moose, the accidental symmetry has many factors of a small group. The littlest higgs and its ramifications are based on one large accidental symmetry group.

Here I will describe the original littlest higgs model the $SU(5)/SO(5)$ model of Arkani-Hamed, Cohen, Katz and Nelson [8]. In my view this is one of the simplest and most beautiful little higgs models. I shall describe it in some detail because I am so impressed by at least part of it.

One of the things I like about this model is that the higgs structure might emerge from simple specific high energy dynamics different from QCD, so I will start with that. Imagine a high energy theory with an asymptotically free $SO(n)$ gauge group that becomes strongly interacting at a scale of order 10 TeV and that includes, among other things, 5 LH fermions transforming like ns under the $SO(n)$.

In addition there is a much weaker $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ gauge group from which will emerge the electroweak $SU(2) \times U(1)$ low energy gauge symmetry.

The 5 ns transform like $(2,1)+(1,2)+(1,1)$ under the two $SU(2)$s, and it is convenient to talk about this structure in a notation with vectors blocked as follows

$$\psi = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} (2,1) \\ (1,1) \\ (1,2) \end{pmatrix} \tag{11}$$

In this notation, matrices look like

$$\begin{pmatrix} 2\times 2 & 2\times 1 & 2\times 2 \\ 1\times 2 & 1\times 1 & 1\times 2 \\ 2\times 2 & 2\times 1 & 2\times 2 \end{pmatrix} \tag{12}$$

and the weak gauge generators look like this:

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad Q_1' = \begin{pmatrix} q_1 - \frac{1}{2} & 0 & 0 \\ 0 & q_1 & 0 \\ 0 & 0 & q_1 \end{pmatrix} \tag{13}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix}, \qquad Q_2' = \begin{pmatrix} q_2 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_2 + \frac{1}{2} \end{pmatrix} \tag{14}$$

The $U(1)s$ have $SO(n)$ anomalies but who knows what else is happening in the high energy theory—so this may be OK—I will set $q_1 = q_2 = 0$[3]—this keeps the algebra simple and does not change the important parts

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad Q_1' = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{15}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix}, \qquad Q_2' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \tag{16}$$

In the $SO(n)$ theory, the condensate looks like

$$\left[\psi\gamma^0\psi^T\right]_{\substack{\text{low}\\ \text{energy}}} = \Sigma = \begin{matrix}\text{symmetric}\\ \text{matrix}\end{matrix} \tag{17}$$

Unlike the chiral symmetry breaking condensate in QCD, here there is no difference between LH and RH fields. Therefore the condensate is a fermion-fermion condensate—not fermion-anti-fermion $\rightarrow \Sigma = \Sigma^T$.

The fundamental assumption—based on the QCD analog and justified for a large number of colors [105]—is that Σ is also unitary. This is the analog of the statement that each flavor does its own thing. As in QCD, the vacuum 'direction' is determined by symmetry breaking. But if we do not worry about making the small symmetry breaking interactions look complicated, we can always choose a basis for the fermion fields such that

$$\Sigma = I \tag{18}$$

which clearly breaks the $SU(5)$ global symmetry down to $SO(5)$ because under an $SU(5)$ transformation U

$$\Sigma = I \rightarrow U\Sigma U^T = UU^T \tag{19}$$

and only if U is real is $U^T = U^\dagger = U^{-1}$.

But while there is always some basis in which the condensate looks like (18), this may not be the basis in which the weak gauge interactions have the simple form (15), (16). Again there is a vacuum alignment issue. To see what the vacuum looks like in the particular basis defined by (15), (16) we must look at the symmetry breaking effect of the low energy gauge interactions. The result is something I find quite counter-intuitive, but ultimately very beautiful, so I am going to show you how it works in a bit of detail. We will find that in the preferred vacuum, the individual $SU(2) \times U(1)$ break, but the combination of the two is left unbroken—and this becomes the $SU(2) \times U(1)$ of the standard model.

One of the things I find particularly beautiful about the $SU(5)/SO(5)$ model is that the vacuum is picked out largely by the weak gauge interactions. We saw in Section 3 how this works in the generation of the electromagnetic contribution to the $\pi^\pm$ and $\kappa^\pm$ mass in QCD. There are good reasons to believe that this works in a similar way in the $SU(5)/SO(5)$ model—but there are important differences:

(i) Now there are lots of charges—we have to sum over each type of charge, multiplying by the coupling constant. The charges of the Goldstone bosons or the entries in the condensate matrix Σ are just the sums of the charges of the fermion constituents.

(ii) Because the condensate is symmetric, the potential from a charge Q looks like

$$xe^2f^4\left|Q\Sigma + \Sigma Q^T\right|^2 \tag{20}$$

(iii) This time, we do not already know the form of the vacuum from something like the quark masses that we had in QCD. These terms determine the vacuum structure.

(iv) Finally, we will not be able to find elements of the condensate matrix that preserves all the gauge symmetries—some will get spontaneously broken.

So, for example, the $U(1)$ charge in (15) gives a charge squared of the form

[3] The charges of the bosons do not depend on the qs.

$$Q_1' = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix} \tag{21}$$

$$= \begin{pmatrix} (\frac{1}{2}+\frac{1}{2})^2 & (\frac{1}{2}+0)^2 & (\frac{1}{2}+0)^2 \\ (0+\frac{1}{2})^2 & 0 & 0 \\ (0+\frac{1}{2})^2 & 0 & 0 \end{pmatrix} \tag{22}$$

Similarly the $U(1)$ charge in (16) gives a charge squared of the form

$$Q_2' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix} \tag{23}$$

For the $SU(2)$ gauge groups, we want the sum of the squares of the components, which gives $i(i+1)$ for the representation. Under $SU(2)_1 \times SU(2)_2$, the various parts of the condensate matrix have isospin

$$\begin{pmatrix} (1,0) & (1/2,0) & (1/2,1/2) \\ (1/2,0) & (0,0) & (0,1/2) \\ (1/2,1/2) & (0,1/2) & (0,1) \end{pmatrix} \tag{24}$$

Notice that in these entries

$$\begin{pmatrix} \boxed{(1,0)} & (1/2,0) & (1/2,1/2) \\ (1/2,0) & (0,0) & (0,1/2) \\ (1/2,1/2) & (0,1/2) & \boxed{(0,1)} \end{pmatrix} \tag{25}$$

there is no $(0,0)$ component because of the symmetry of the matrix.

Thus the $SU(2)$ contributions look like

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 2 & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{pmatrix} \tag{26}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^T}{2} \end{pmatrix} \to \begin{pmatrix} 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} & 2 \end{pmatrix} \tag{27}$$

Putting this together gives the contribution to (20) proportional to

$$\begin{aligned} \mathrm{Tr}\Bigg\{ \Bigg[& g_1^2 \begin{pmatrix} 2 & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{pmatrix} + g_2^2 \begin{pmatrix} 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} & 2 \end{pmatrix} + {g_1'}^2 \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix} + {g_2'}^2 \begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix} \Bigg] \\ & \times \begin{pmatrix} |\Sigma_{(1\mathrm{or}2)(1\mathrm{or}2)}|^2 & |\Sigma_{(1\mathrm{or}2)3}|^2 & |\Sigma_{(1\mathrm{or}2)(4\mathrm{or}5)}|^2 \\ |\Sigma_{3(1\mathrm{or}2)}|^2 & |\Sigma_{33}|^2 & |\Sigma_{3(4\mathrm{or}5)}|^2 \\ |\Sigma_{(4\mathrm{or}5)(1\mathrm{or}2)}|^2 & |\Sigma_{(4\mathrm{or}5)3}|^2 & |\Sigma_{(4\mathrm{or}5)(4\mathrm{or}5)}|^2 \end{pmatrix} \Bigg\} \end{aligned} \tag{28}$$

Because of the zeros in the 33 spot in (28), the potential (20) is minimized by a condensate of the form

$$\begin{pmatrix} |\Sigma_{(1\mathrm{or}2)(1\mathrm{or}2)}|^2 & |\Sigma_{(1\mathrm{or}2)3}|^2 & |\Sigma_{(1\mathrm{or}2)(4\mathrm{or}5)}|^2 \\ |\Sigma_{3(1\mathrm{or}2)}|^2 & |\Sigma_{33}|^2 & |\Sigma_{3(4\mathrm{or}5)}|^2 \\ |\Sigma_{(4\mathrm{or}5)(1\mathrm{or}2)}|^2 & |\Sigma_{(4\mathrm{or}5)3}|^2 & |\Sigma_{(4\mathrm{or}5)(4\mathrm{or}5)}|^2 \end{pmatrix} = \begin{pmatrix} ? & 0 & ? \\ 0 & 1 & 0 \\ ? & 0 & ? \end{pmatrix} \tag{29}$$

And you can see explicitly that a condensate of the form

$$\begin{pmatrix} I & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{pmatrix} \tag{30}$$

has higher energy than one of the form

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & I \\ 0 & 1 & 0 \\ I & 0 & 0 \end{pmatrix} \tag{31}$$

Both the $SU(2)$ and the $U(1)$ contributions tend to stabilize the vacuum (31). Looking at (24) you can see that the off-diagonal components are like $SU(2) \times SU(2)$ sigma models:

$$\begin{pmatrix} (1,0) & (1/2,0) & \boxed{(1/2,1/2)} \\ (1/2,0) & (0,0) & (0,1/2) \\ \boxed{(1/2,1/2)} & (0,1/2) & (0,1) \end{pmatrix} \tag{32}$$

so (31) spontaneously breaks the gauge $SU(2) \times SU(2)$ down to a single $SU(2)$ which is identified as the electroweak $SU(2)$ gauge symmetry.

Notice that this completely determines the vacuum up to gauge transformations, so we expect that there are no exact Goldstone bosons left over. But something interesting happens when you look at the pseudo-Goldstone bosons. We will first do this explicitly, and then go back and understand qualitatively what is going on.

The pseudo-Goldstone fields are small deformations around the vacuum (31) that we can parametrize in terms of a field-dependent symmetric condensate Σ as follows:

$$\Sigma = \mathrm{e}^{2\mathrm{i}\Pi/f}\,\Sigma_0 \tag{33}$$

where Π is a Hermitian matrix field satisfying

$$\Pi = \Pi^\dagger = \Sigma_0 \Pi^T \Sigma_0 \quad \text{and} \quad \mathrm{Tr}(\Pi\Sigma_0) = 0 \tag{34}$$

which guarantees the symmetry of Σ. This may look a little funny because it is Π that is Hermitian to ensure the unitarity of Σ, but it is $\Pi\Sigma_0$ that is symmetric and traceless.

$$\Pi = \begin{pmatrix} \xi + \eta/(2\sqrt{5}) & h/\sqrt{2} & \phi \\ h^\dagger/\sqrt{2} & -2\eta/\sqrt{5} & h^T/\sqrt{2} \\ \phi^* & h^*/\sqrt{2} & \xi^* + \eta/(2\sqrt{5}) \end{pmatrix} \tag{35}$$

where ξ is a traceless, Hermitian 2×2 matrix field, η is a real singlet field, ϕ is a complex symmetric matrix, and h is the doublet putative Higgs field. The electric charges are

$$\begin{pmatrix} 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & -1 & -1 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 2 & 1 & 1 & 0 \end{pmatrix} \tag{36}$$

With a gauge transformation, we can choose

$$h = \begin{pmatrix} h_0/\sqrt{2} \\ 0 \end{pmatrix} \tag{37}$$

where h_0 is a real field. In this gauge, the other critical field is the imaginary part of Π_{14}, which I will call $\phi_2/\sqrt{2}$. As a function of h_0 and ϕ_2 with the other fields zero, the contributions to (20) from Q_1^a and Q_1' are proportional to

$$3f^4 + \phi_2^2 f^2 + \frac{h_0^2 \phi_2 f}{\sqrt{2}} + \frac{h_0^4}{8} - \frac{2\phi_2^4}{3} - \frac{2h_0^2\phi_2^2}{3} + \mathrm{O}(1/f) \tag{38}$$

For $\phi_2/f, h_0^2/f^2 \ll 1$, this is approximately

$$3f^4 + \left(\phi_2 f + h_0^2/\sqrt{8}\right)^2 \tag{39}$$

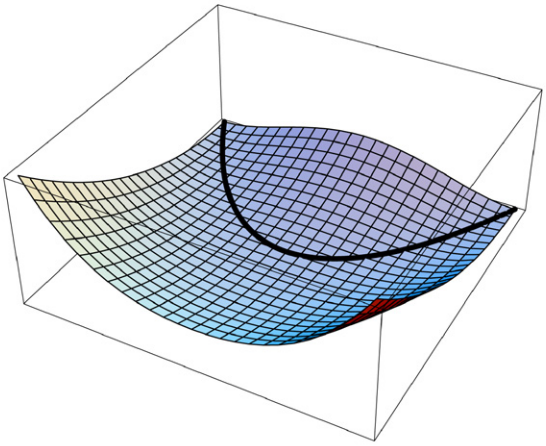

Fig. 5. The potential from Q_1. The potential is minimized and constant along the thick black curve.

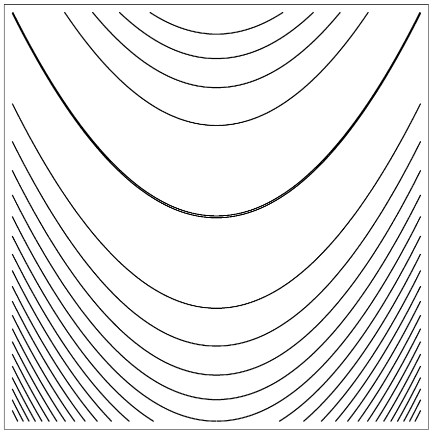

Fig. 6. Contour plot of the potential from Q_1. The potential is minimized and constant along the thick black curve.

which is constant on the curve $h_0^2 = \sqrt{8}\phi_2 h$. While the specific form of (39) is valid only near the origin, the fact that it is constant on a curve in field space is an exact consequence of the Goldstone theorem.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & x & x \\ 0 & x & x \end{pmatrix} \quad \begin{matrix}\text{commutes}\\ \text{with}\end{matrix} \quad Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad Q_1' = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{40}$$

and therefore the weak gauge couplings associated with these charges preserve an $SU(3)$ global symmetry acting on indices 3–5 in addition to the $SU(2)$ gauge symmetry acting on the 1–2 indices. This symmetry is spontaneously broken giving rise to an uneaten Goldstone boson. The constancy of (39) on a curve is a consequence of this spontaneously broken symmetry.

The potential (39) as a function of h_0 and ϕ_2 is plotted in Fig. 5. Superimposed on the graph is the curve in field space along which the potential is constant. A contour plot of (39) is shown in Fig. 6.

Similarly, as a function of h_0 and ϕ_2 with the other fields zero, the contributions to (20) from Q_2^a and Q_2' are proportional to

$$3f^4 + \phi_2^2 f^2 - \frac{h_0^2\phi_2 f}{\sqrt{2}} + \frac{h_0^4}{8} - \frac{2\phi_2^4}{3} - \frac{2h_0^2\phi_2^2}{3} + \mathrm{O}(1/f) \tag{41}$$

and for $\phi_2/f, h_0^2/f^2 \ll 1$, this is approximately

$$3f^4 + \left(\phi_2 f - h_0^2/\sqrt{8}\right)^2 \tag{42}$$

which is constant on the curve $h_0^2 = -\sqrt{8}\phi_2 h$. Again the constancy arises from the Goldstone theorem, while

$$\begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{matrix}\text{commutes}\\ \text{with}\end{matrix} \quad Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix} \quad \text{and} \quad Q_2' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \tag{43}$$

and therefore these weak gauge couplings associated with these charges preserve an $SU(3)$ global symmetry acting on indices 1–3 in addition to the $SU(2)$ gauge symmetry acting on the 4–5 indices. This global symmetry is spontaneously broken resulting in an uneaten Goldstone boson. The constancy of (42) on a curve is a consequence of this spontaneously broken symmetry.

The potential (42) as a function of h_0 and ϕ_2 is plotted in Fig. 7. Superimposed on the graph is the curve in field space along which the potential is constant. A contour plot of (42) is shown in Fig. 8.

Now when we add these two contributions, we get something wonderful. The result is shown in Figs. 9 and 10 in the special case in which the two contributions have the same coefficients. Now, as expected, the potential is minimized at the origin. However, the two directions in field space are very different. In the ϕ_2 direction (vertical in Fig. 10)

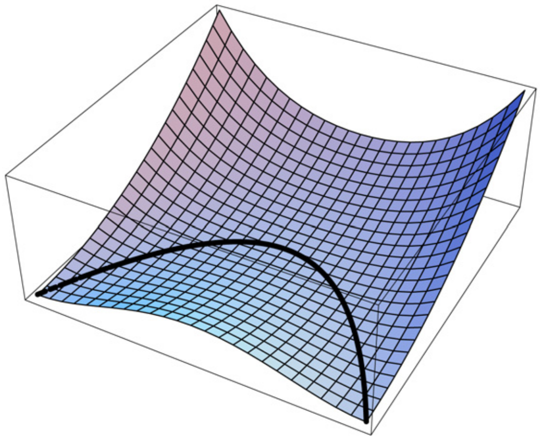

Fig. 7. The potential from Q_2. The potential is minimized and constant along the thick black curve.

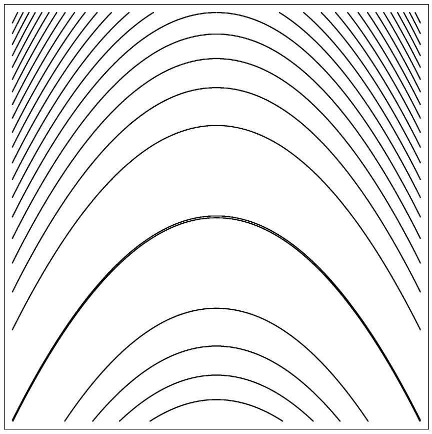

Fig. 8. Contour plot of the potential from Q_2. The potential is minimized and constant along the thick black curve.

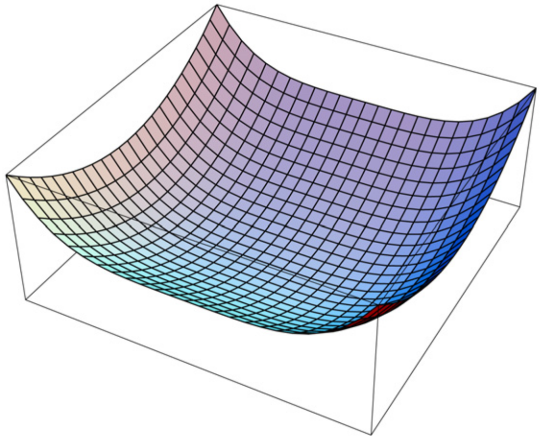

Fig. 9. The sum of the potentials from Q_1 and $Q_2 Q_s$ with equal coefficients.

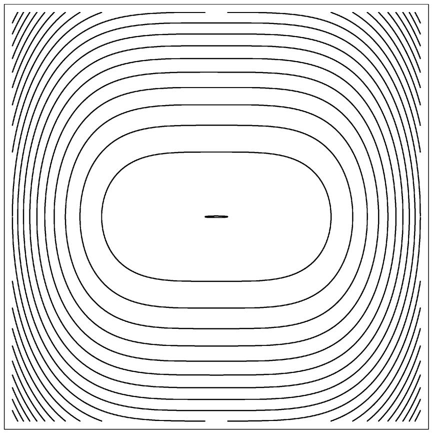

Fig. 10. Contour plot of the sum of the potentials from Q_1 and Q_2 with equal coefficients.

the potential is quadratic so the ϕ_2 field has a mass. But in the h direction, the potential is much flatter because both contributions are moving along the Goldstone boson direction. But it is not perfectly flat because the Goldstone boson directions from the two contributions diverge as we move away from the origin. This is the source of the λh^4 interaction that is crucial to stabilize spontaneous symmetry breaking.

The general case gives similar results as shown in Figs. 11 and 12. These look different far away from the origin, but are qualitatively similar near the origin.

Thus collective symmetry breaking seems to very naturally produce a λh^4 coupling without an h mass. There is one question that could still be asked. For this mechanism to work the Goldstone boson directions must coincide at the origin. If the Goldstone boson directions were different both fields would get mass. Of course, the directions did line up in this case by explicit calculation. But did we secretly sneak in a tuning to enforce this? Or it is an automatic consequence of some principle. In this case, there are various ways to see that no tuning has been done. One simple one is to note that there are symmetries between the 1 and 2 gauge structures in the limit that the gauge couplings are equal. This symmetry, along with the fact that h_0 is part of a $SU(2)$ doublet, requires the Goldstone boson direction to line up with the h_0 direction when $\phi_2 \to 0$. In discussions of collective breaking in the literature, this issue is seldom discussed, but it is implicit in the assumption that it is the same Goldstone field that is protected by the different components of the collective symmetry breaking.

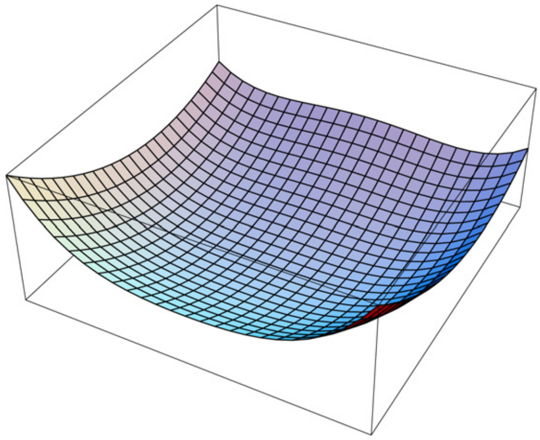

Fig. 11. The sum of the potentials from Q_1 and $Q_2 Q_s$ with different coefficients.

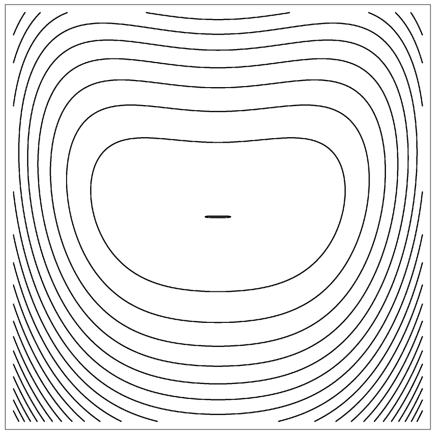

Fig. 12. Contour plot of the sum of the potentials from Q_1 and Q_2 with different coefficients.

The collective symmetry breaking in the littlest higgs model is clever and beautiful. To my mind, this is the most gorgeous example of the phenomenon because the quartic interactions of the little higgs boson are generated just by the gauge interactions with no additional structure required. The rest of the model is merely clever. As in all composite little higgs models, one of the interesting issues is how to generate a large t quark mass without spoiling the structure of collective symmetry breaking.

I will discuss the mechanism for generating the t mass because it demonstrates dramatically both the cleverness and the lack of beauty. The authors of [8] include the left-handed third family quarks t_3 and b_3 with an electroweak singlet left-handed quark $\tilde{t}$ in a triplet under the global $SU(3)$ that acts on the indices $1, 2, 3$ in (43):

$$\chi = \begin{pmatrix} t_3 \\ b_3 \\ -\mathrm{i}\tilde{t} \end{pmatrix} \tag{44}$$

Then the following terms in the Lagrangian generate a Yukawa coupling for the t:

$$\mathrm{i}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\chi_i \Sigma_{jx}\Sigma_{ky}{u'_3}^c + \lambda_2 f \tilde{t}\tilde{t}^c \tag{45}$$

where the sums over repeated indices in (45) run over $i, j, k = 1$ to 3 and $x, y = 4, 5$. The first term preserves the global $SU(3)$ acting on indices $1, 2, 3$ of Σ and the second term breaks that symmetry because $\tilde{t}$ is included not as part of the triplet (44), but because it does not involve Σ at all, it does not break the global $SU(3)$ acting on $3, 4, 5$ under which the quarks are singlets. Thus we expect that contributions to the higgs mass will require both couplings to be nonzero.

Expanding the first term to zeroth order in $1/f$ using (33), (35) and (37), (45) becomes

$$2\lambda_1 f \tilde{t}{u'_3}^c + 2\lambda_1 h t_3 {u'_3}^c + \lambda_2 f \tilde{t}\tilde{t}^c + \cdots \tag{46}$$

Now when the heavy quark $\tilde{t}$ is integrated out, the linear combination

$$t_3^c = \frac{\lambda_2 {u'_3}^c - \lambda_1 \tilde{t}^c}{\sqrt{\lambda_1^2 + \lambda_2^2}} \tag{47}$$

remains in the low energy theory and (46) generates a Yukawa coupling of the higgs to the t_3 fermion field

$$\frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \tag{48}$$

This is in accord with what we expect from collective symmetry breaking. t-quark loops contribute to the higgs mass (more precisely to the negative mass-squared term that drives electroweak symmetry breaking) but the mass of the t

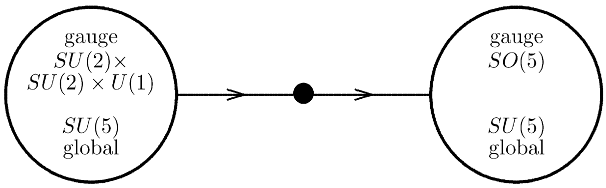

Fig. 13. A QCD-like UV completion of the PGB sector of the littlest higgs model of [8]. The black dot in the center represents an $SU(n)$ gauge group.

is proportional to the product of λ_1 and λ_2 so the contribution is under control. The $\tilde{t}$ mass is larger and does not go to zero as $\lambda_1 \to 0$ or as $\lambda_2 \to 0$, but $\tilde{t}$ loops do not contribute to the h mass.

So why do I feel that (45) and all that goes with it is less compelling than (40) and (43) and the rest of the analysis of the higgs potential. The difference gets to the heart of what it means for the Higgs to be a pseudo-Goldstone boson. The difference is that it is easy for me to imagine a high energy theory that generates a Σ field with an $SU(5)$ accidental symmetry. In fact, we do not have to imagine. I discussed an example—5 fermions transforming like ns of a strong $SO(n)$ gauge group. And I will discuss more possibilities later. This $SU(5)$ accidental symmetry is all we need to make collective symmetry breaking that gives rise to the higgs potential completely natural. But the options for a high energy theory from which the fermion couplings (45) emerges in any natural way are much more limited, and this is a potential problem.

In particular, if the $SU(3)$ symmetry of the λ_1 term is imposed by hand, rather than arising naturally as an accidental symmetry, then one is actually doing a tuning to suppress the quadratically divergent contributions to the higgs mass term. If this is the situation, it is not clear how much we have gained with all superstructure of a little higgs model.

This difficultly is part of a more general problem. We have less flexible tools for generating fermions with accidental symmetries. This may ultimately be just a failure of imagination, but it is not a new problem. From the very beginning of composite models of electroweak symmetry breaking, it has been difficult to get the fermions right. Little higgs models may be easier than technicolor models in this regard because we are not trying to do without scalars entirely.

In fact, one could regard this issue as an argument for some supersymmetry in the UV completion of little higgs models—at least those in which the accidental symmetry arises in the strongly coupled sector of the theory. A nice explicit example of how SUSY can generate an appropriate UV completion of the littlest higgs is the composite little Higgs model of [27]. We will discuss the advantages of SUSY in PGH models further in Section 7.

6. Hidden local symmetry

One of the surprises (at least to me) that came out of the spate of activity surrounding little higgs models was the realization that models like the littlest higgs models which apparently have a very different structure from the QCD-like theory space models are in fact not so different after all. As shown in [106], you can do anything with QCD-like theories. Before discussing the general situation, let us look at the littlest higgs. As discussed in Section 5, we can obtain the PGB sector of the theory by having five identical ultrafermions transforming like ns of an $SO(n)$ gauge group with a large gauge coupling. The theory has an $SU(5)$ global symmetry acting on the identical multiplets. When the $SO(n)$ gauge theory confines, the chiral $SU(5)$ symmetry breaks down to $SO(n)$, producing the 14 PGBs of the littlest higgs model. Thaler in [106] noticed that the same set of PBGs can be produced in the model shown in Fig. 13.

The details of how this works depends on the relative strength of the $SU(n)$ gauge coupling and the $SO(5)$ gauge coupling (I will assume that both are stronger than the $SU(2) \times SU(2) \times U(1)$ couplings). If the $SO(5)$ coupling is much stronger, then when the $SO(5)$ theory confines, the $SU(n)$ gauge symmetry is spontaneously broken down to an $SO(n)$ gauge symmetry. All the Goldstone bosons are eaten in the Higgs mechanism by the $SU(n)$ gauge bosons that not associated with the unbroken $SO(n)$ subgroup. The fermions on the left of Fig. 13 are unaffected by the confinement of the $SO(5)$ fields. Thus in this limit, at energies below the $SO(5)$ confinement scale, the theory actually looks like the original version with five fermions transforming under an $SO(n)$ gauge group, which in turn on confinement, produces the 14 PGBs of $SU(5)/SO(5)$.

However, if the $SU(n)$ is much more strongly coupled, then the $SU(n)$ gauge theory confines. The chiral $SU(5) \times SU(5)$ global symmetry spontaneously breaks down to $SU(5)$, producing 24 Goldstone bosons. However 10 of these are eaten by the $SO(5)$ gauge bosons leaving 14 PGBs with the same $SU(5)/SO(5)$ structure. There seems to be no dramatic transition required to go from one of these limits to the other. Thus it appears that there is a kind of duality between the strong $SO(n)$ models and the QCD-like models for these PGB sectors.

In [106], Thaler also makes connections between little higgs theories and other apparently different schemes with PBHs, such as holographic theories.

7. Twin Higgs

To me, one of the most surprising developments in PGH theories is the 'Twin Higgs'. I find it surprising because when I first encountered the idea, I did not think it could possibly go anywhere. Briefly the idea is this. Suppose that there are two copies of the standard model with independent $SU(2)$s, one the ordinary electroweak $SU(2)$ and all that goes with it—fermions and a scalar doublet H and another $SU(2)'$ with a corresponding set of fermion fields and a scalar doublet H'. Now suppose there is a Z_2 symmetry that interchanges these two sectors. This Z_2 restricts the form of the scalar potential, and in particular, the H mass terms must be proportional to

$$H^\dagger H + H'^\dagger H' \tag{49}$$

But this term has an $SU(4)$ symmetry[4] under which the two Hs form a 4 of $SU(4)$,

$$\mathcal{H} = \begin{pmatrix} H \\ H' \end{pmatrix} \tag{50}$$

and the symmetry is

$$\mathcal{H} \to U\mathcal{H} \tag{51}$$

where U is a unitary 4×4 matrix. This $SU(4)$ symmetry is, of course, broken by the gauge interactions. But if the full potential had this enlarged symmetry, then when any linear combination of the H and H' developed a VEV, all the other components would be either true Goldstone bosons or PGBs (depending on the vacuum alignment). In particular, if H' develops a large VEV, then the particles in the 2-world are heavy, and the H doublet would be a PGB.

This all sounds very interesting. The trouble is that (51) generically is not a symmetry of the full potential, and tuning is required to make it so. The authors of [82] argue that by extending the Higgs structure and the symmetry still further, they can naturally obtain something like the collective symmetry breaking that one finds in little higgs models. They refer to the resulting Higgs boson as a 'Partially Goldstone Twin Higgs'.

In my view, a much more promising class of twin higgs models arises when one combines the twin higgs idea with SUSY, as in [64,71,107,86,108,109]. SUSY does at least three good things for you in this context.

(i) The constraints imposed Z_2 symmetry on the superpotential are much more effective in producing accidental symmetries than the corresponding constraints on the potential in a theory without SUSY. This is simply because the renormalizable superpotential is at most cubic in the fields, and the troubles in the twin higgs potential come from quartic terms. In particular, consider a term in the superpotential like

$$\lambda N \mathcal{H}_u \mathcal{H}_d \tag{52}$$

where N is a singlet field and the other fields are multiples of the u and d Higgs field for $SU(2)$ and $SU(2)'$:

$$\mathcal{H}_u = \begin{pmatrix} H_u & H_u' \end{pmatrix} \quad \text{and} \quad \mathcal{H}_d = \begin{pmatrix} H_d \\ H_d' \end{pmatrix} \tag{53}$$

Here we have imposed the Z_2 symmetry and this *automatically* implies that (52) has an $SU(4)$ symmetry under which $H_u^\dagger$ and H_d transform like 4s.

(ii) The second important feature of SUSY theories in this context is the incredibly special role of the SUSY superpotential. The important point is that not all terms in the superpotential are relevant to the structure and symmetry of the surface of minimum potential. The surface of minimum potential in the SUSY limit is determined by the vanishing of the derivatives of the superpotential. Thus the only terms in the superpotential that matter for the

[4] Actually the full symmetry is $SO(8)$, but $SU(4)$ is easier to talk about and is enough to guarantee the PGB character of the light Higgs.

surface of minimum potential in the SUSY limit in some region of parameter space are those that contain at most one field with zero expectation value. If (52) and perhaps a term linear in N are the only such terms, then the superpotential contribution to the potential will have an $SU(4)$ symmetric surface of minimum potential in the relevant region even if the other term in the superpotential break $SU(4)$ (which they almost certainly do). So, for example, it does not matter that Z_2 invariant terms like

$$f(u^c H_u \psi + u'^c H'_u \psi') \tag{54}$$

which might be responsible for a charged $2/3$ quark mass have no vestige of the $SU(4)$ symmetry. The Higgs is still a PGB by the criterion of [3]. No tuning is required!

(iii) Finally, one of the most exciting things about SUSY twin higgs models is that it appears quite natural to construct successful models of this kind that preserve the unification of couplings in a SUSY GUT theory [110]. This exciting prospect is discussed in [109].

8. Chindōgu

Not all reviews have any sort of conclusion, but I confess that I came away from my study of recent progress in PGH theories with a very different viewpoint than what I expected when I began. From several points of view, for both strongly coupled models like the littlest higgs and weakly coupled models like the twin higgs, the most natural home for these ideas seems to be in SUSY theories. Indeed, the ideas of SUSY and the PGH seem to complement one another very well. SUSY enables the accidental symmetry that drives the pseudo-Goldstone phenomenon. The PGH can ameliorate the little hierarchy in SUSY models and enable models with little or no tuning.

The models, however, are still models. While thinking about all the models I looked at in preparing this note, I came across the concept of *chindōgu*, a kind of Japanese version of the Rube Goldberg machine, examples of which are shown in Fig. 14. PGH models have something of this feeling. To go beyond model building and really construct a plausible theory, it is necessary but not sufficient to find clever solutions to all of the puzzles that nature throws at us. We begin to be convinced that our model is more than a model only when our clever solutions start to fit together, to do more than the one job they were invented for and to produce a model that feels like more than the sum of its parts. This has not really happened for PGH models. That does not mean that Nature does not make use of the PGH. But it probably means that if it is Nature's way, we have not found quite the right way of thinking about it. Of course, none of the alternatives to PGH really satisfy this criterion either. We are just going to have to wait and see what happens at the LHC! Let me close with a telling quote from [64]:

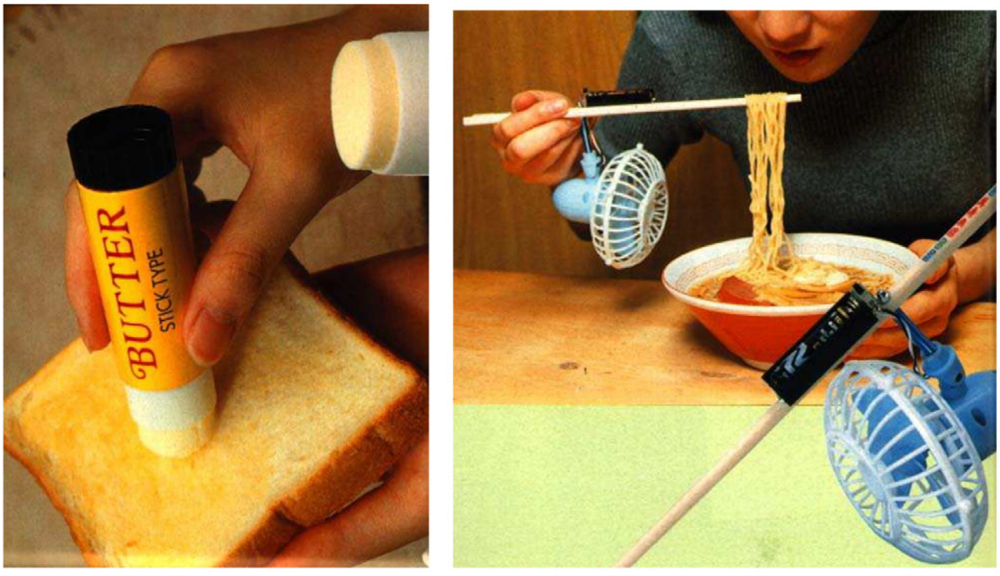

Fig. 14. Examples of chindōgu from http://www.chindogu.com/.

"*When using the lack of positive experimental results as a guideline for exploring new directions one should pay attention in not inventing a medicine that is worse than the illness: a complicated model might look less plausible than the MSSM fine-tuned at a few % level.*"

This caveat is repeated in some form in a number of the papers on the subject of the Higgs as a pseudo-Goldstone boson. And in the papers in which it does not appear—*it ought to*!

References

[1] S. Weinberg, Approximate symmetries and pseudogoldstone bosons, Phys. Rev. Lett. 29 (1972) 1698–1701.
[2] H. Georgi, A. Pais, Calculability and naturalness in gauge theories, Phys. Rev. D 10 (1974) 539.
[3] H. Georgi, A. Pais, Vacuum symmetry and the pseudogoldstone phenomenon, Phys. Rev. D 12 (1975) 508.
[4] D.B. Kaplan, H. Georgi, SU(2) × U(1) breaking by vacuum misalignment, Phys. Lett. B 136 (1984) 183.
[5] N. Arkani-Hamed, A.G. Cohen, H. Georgi, Electroweak symmetry breaking from dimensional deconstruction, Phys. Lett. B 513 (2001) 232–240, hep-ph/0105239.
[6] N. Arkani-Hamed, A.G. Cohen, T. Gregoire, J.G. Wacker, Phenomenology of electroweak symmetry breaking from theory space, JHEP 0208 (2002) 020, hep-ph/0202089.
[7] N. Arkani-Hamed, et al., The minimal moose for a little higgs, JHEP 0208 (2002) 021, hep-ph/0206020.
[8] N. Arkani-Hamed, A.G. Cohen, E. Katz, A.E. Nelson, The littlest higgs, JHEP 0207 (2002) 034, hep-ph/0206021.
[9] T. Gregoire, J.G. Wacker, Mooses, topology and higgs, JHEP 0208 (2002) 019, hep-ph/0206023.
[10] I. Low, W. Skiba, D. Smith, Little higgses from an antisymmetric condensate, Phys. Rev. D 66 (2002) 072001, hep-ph/0207243.
[11] J.G. Wacker, Little higgs models: New approaches to the hierarchy problem, hep-ph/0208235.
[12] C. Csaki, J. Hubisz, G.D. Kribs, P. Meade, J. Terning, Big corrections from a little higgs, Phys. Rev. D 67 (2003) 115002, hep-ph/0211124.
[13] J.L. Hewett, F.J. Petriello, T.G. Rizzo, Constraining the littlest higgs. ((u)), JHEP 0310 (2003) 062, hep-ph/0211218.
[14] T. Han, H.E. Logan, B. McElrath, L.-T. Wang, Phenomenology of the little higgs model, Phys. Rev. D 67 (2003) 095004, hep-ph/0301040.
[15] D.E. Kaplan, M. Schmaltz, The little higgs from a simple group, JHEP 0310 (2003) 039, hep-ph/0302049.
[16] A.E. Nelson, Dynamical electroweak superconductivity from a composite little higgs, hep-ph/0304036.
[17] W. Skiba, J. Terning, A simple model of two little higgses, Phys. Rev. D 68 (2003) 075001, hep-ph/0305302.
[18] S.C. Park, J.-h. Song, Muon anomalous magnetic moment and the heavy photon in a little higgs model, hep-ph/0306112.
[19] H.E. Logan, Phenomenology of the littlest higgs model, hep-ph/0307340.
[20] H.-C. Cheng, I. Low, TeV symmetry and the little hierarchy problem, JHEP 0309 (2003) 051, hep-ph/0308199.
[21] M. Perelstein, M.E. Peskin, A. Pierce, Top quarks and electroweak symmetry breaking in little higgs models, Phys. Rev. D 69 (2004) 075002, hep-ph/0310039.
[22] N. Mahajan, Littlest higgs model and unitarity constraints, hep-ph/0310098.
[23] H.E. Logan, Little higgs phenomenology, Eur. Phys. J. C 33 (2004) s729–s731, hep-ph/0310151.
[24] M.-C. Chen, S. Dawson, One-loop radiative corrections to the rho parameter in the littlest higgs model, Phys. Rev. D 70 (2004) 015003, hep-ph/0311032.
[25] R. Casalbuoni, A. Deandrea, M. Oertel, Little higgs models and precision electroweak data, JHEP 0402 (2004) 032, hep-ph/0311038.
[26] S. Chang, H.-J. He, Unitarity of little higgs models signals new physics of UV completion, Phys. Lett. B 586 (2004) 95–105, hep-ph/0311177.
[27] E. Katz, J.-y. Lee, A.E. Nelson, D.G.E. Walker, A composite little higgs model, JHEP 0510 (2005) 088, hep-ph/0312287.
[28] C. Dib, R. Rosenfeld, A. Zerwekh, Higgs production and decay in the little higgs model, hep-ph/0302068.
[29] T. Han, H.E. Logan, B. McElrath, L.-T. Wang, Loop induced decays of the little higgs: $H \to gg, \gamma\gamma$, Phys. Lett. B 563 (2003) 191–202, hep-ph/0302188.
[30] S. Chang, J.G. Wacker, Little higgs and custodial su(2), Phys. Rev. D 69 (2004) 035002, hep-ph/0303001.
[31] G.D. Kribs, Electroweak precision tests of little higgs theories, hep-ph/0305157.
[32] T. Gregoire, D.R. Smith, J.G. Wacker, What precision electroweak physics says about the SU(6)/Sp(6) little higgs, Phys. Rev. D 69 (2004) 115008, hep-ph/0305275.
[33] W.-j. Huo, S.-h. Zhu, $b \to s\gamma$ in littlest higgs model, Phys. Rev. D 68 (2003) 097301, hep-ph/0306029.
[34] S. Chang, A 'littlest higgs' model with custodial SU(2) symmetry, JHEP 0312 (2003) 057, hep-ph/0306034.
[35] Z. Sullivan, How to rule out little higgs (and constrain many other models) at the LHC, hep-ph/0306266.
[36] O.C.W. Kong, A completed chiral fermionic sector model with little higgs, hep-ph/0307250.
[37] O.C.W. Kong, Anomaly free gauged $SU(4)_L \times U(1)$ models with little higgs, hep-ph/0308148.
[38] C. Csaki, J. Hubisz, G.D. Kribs, P. Meade, J. Terning, Variations of little higgs models and their electroweak constraints, Phys. Rev. D 68 (2003) 035009, hep-ph/0303236.
[39] C.-x. Yue, S.-z. Wang, D.-q. Yu, Littlest higgs model and associated ZH production at high energy e^+e^- collider, Phys. Rev. D 68 (2003) 115004, hep-ph/0309113.
[40] O.C.W. Kong, Flavor and little higgs, J. Korean Phys. Soc. 45 (2004) S404–S409, hep-ph/0312060.
[41] C. Kilic, R. Mahbubani, Precision electroweak observables in the minimal moose little higgs model, JHEP 0407 (2004) 013, hep-ph/0312053.
[42] C.-x. Yue, W. Wang, The branching ratio $R(b)$ in the littlest higgs model, Nucl. Phys. B 683 (2004) 48–66, hep-ph/0401214.
[43] O.C.W. Kong, Little higgs model completed with a chiral fermionic sector, Phys. Rev. D 70 (2004) 075021, hep-ph/0409238.

[44] J.-J. Liu, W.-G. Ma, G. Li, R.-Y. Zhang, H.-S. Hou, Higgs boson pair production in the little higgs model at hadron collider, Phys. Rev. D 70 (2004) 015001, hep-ph/0404171.
[45] C. Quigg, Beyond the standard model in many directions, hep-ph/0404228.
[46] A. Deandrea, Little higgs and precision electroweak tests, hep-ph/0405120.
[47] S.C. Park, J. Song, Phenomenology of the heavy B_H in a littlest higgs model, Phys. Rev. D 69 (2004) 115010.
[48] W. Kilian, D. Rainwater, J. Reuter, Pseudo-axions in little higgs models, Phys. Rev. D 71 (2005) 015008, hep-ph/0411213.
[49] J. Hubisz, P. Meade, Phenomenology of the littlest higgs with t-parity, Phys. Rev. D 71 (2005) 035016, hep-ph/0411264.
[50] P. Batra, D.E. Kaplan, Perturbative, non-supersymmetric completions of the little higgs, JHEP 0503 (2005) 028, hep-ph/0412267.
[51] M. Schmaltz, The simplest little higgs, JHEP 0408 (2004) 056, hep-ph/0407143.
[52] M. Piai, A. Pierce, J.G. Wacker, Composite vector mesons from QCD to the little higgs, hep-ph/0405242.
[53] D.E. Kaplan, M. Schmaltz, W. Skiba, Little higgses and turtles, Phys. Rev. D 70 (2004) 075009, hep-ph/0405257.
[54] S.R. Choudhury, N. Gaur, A. Goyal, N. Mahajan, B/d–anti-b/d mass difference in little higgs model, Phys. Lett. B 601 (2004) 164–170, hep-ph/0407050.
[55] S.R. Choudhury, N. Gaur, G.C. Joshi, B.H.J. McKellar, $K_L \to \pi^0 \nu \bar{\nu}$ in little higgs model, hep-ph/0408125.
[56] J.y. Lee, A vector-like heavy quark in the littlest higgs model, JHEP 0412 (2004) 065, hep-ph/0408362.
[57] C.-x. Yue, W. Wang, F. Zhang, Probing the gauge bosons Z' and B' from the littlest higgs model in the high-energy linear e^+e^- colliders, Nucl. Phys. B 716 (2005) 199–214, hep-ph/0409066.
[58] J.R. Espinosa, M. Losada, A. Riotto, Symmetry nonrestoration at high temperature in little higgs models, Phys. Rev. D 72 (2005) 043520, hep-ph/0409070.
[59] A.J. Buras, A. Poschenrieder, S. Uhlig, Particle antiparticle mixing, ϵ_K and the unitarity triangle in the littlest higgs model, Nucl. Phys. B 716 (2005) 173–198, hep-ph/0410309.
[60] D.G. Sutherland, A not so little higgs? Phys. Lett. B 607 (2005) 139–143, hep-ph/0411115.
[61] C.-x. Yue, W. Wang, Single production of new gauge bosons from the littlest higgs model at the TeV energy e^- gamma colliders, Phys. Rev. D 71 (2005) 015002, hep-ph/0411266.
[62] A. Datta, X. Zhang, Vacuum stability in split SUSY and little higgs models, Int. J. Mod. Phys. A 21 (2006) 2431, hep-ph/0412255.
[63] J. Thaler, I. Yavin, The littlest higgs in anti-de Sitter space, JHEP 0508 (2005) 022, hep-ph/0501036.
[64] C. Csaki, G. Marandella, Y. Shirman, A. Strumia, The super-little higgs, Phys. Rev. D 73 (2006) 035006, hep-ph/0510294.
[65] J. Hubisz, S.J. Lee, G. Paz, The flavor of a little higgs with t-parity, JHEP 0606 (2006) 041, hep-ph/0512169.
[66] J.y. Lee, Neutrino masses, lepton flavor violations, and flavor changing neutral currents in the composite little higgs model, JHEP 0506 (2005) 060, hep-ph/0501118.
[67] C.-x. Yue, L. Wang, J.-X. Chen, New gauge bosons from the littlest higgs model and the process $e^+e^- \to$ t anti-t, Eur. Phys. J. C 40 (2005) 251–258, hep-ph/0501186.
[68] M. Lechowski, Test of the 'little higgs' model in atlas at LHC, and simulation of the digitization of the electromagnetic calorimeter (in French), CERN-THESIS-2005-042.
[69] F. Bazzocchi, M. Fabbrichesi, M. Piai, The littlest higgs is a cruiserweight, Phys. Rev. D 72 (2005) 095019, hep-ph/0506175.
[70] Z. Han, W. Skiba, Little higgs models and electroweak measurements, Phys. Rev. D 72 (2005) 035005, hep-ph/0506206.
[71] T. Roy, M. Schmaltz, Naturally heavy superpartners and a little higgs, JHEP 0601 (2006) 149, hep-ph/0509357.
[72] S. Fajfer, S. Prelovsek, Effects of littlest higgs model in rare d meson decays, Phys. Rev. D 73 (2006) 054026, hep-ph/0511048.
[73] M.-C. Chen, Models of little higgs and electroweak precision tests, Mod. Phys. Lett. A 21 (2006) 621–638, hep-ph/0601126.
[74] A. Dobado, L. Tabares, S. Penaranda, On the electroweak symmetry breaking in the littlest higgs model, hep-ph/0606031.
[75] C. Cheung, J. Thaler, (Reverse) engineering vacuum alignment, hep-ph/0604259.
[76] K. Agashe, R. Contino, L. Da Rold, A. Pomarol, A custodial symmetry for $Zb\bar{b}$, hep-ph/0605341.
[77] H.-C. Cheng, J. Thaler, L.-T. Wang, Little m-theory, hep-ph/0607205.
[78] X. Wang, J. Chen, Y. Liu, S. Liu, H. Yang, New gauge boson B_H production associated with W boson pair via gamma gamma collision in the littlest higgs model, Phys. Rev. D 74 (2006) 015006, hep-ph/0606093.
[79] X.-l. Wang, Y.-b. Liu, J.-h. Chen, H. Yang, The correction of the littlest higgs model to the higgs production process $e^+e^- \to e^+e^-h$ at the ILC, hep-ph/0607131.
[80] Y. Liu, L. Du, X. Wang, The correction of the littlest higgs model to the higgs production process $e^-\gamma \to \nu_e w^- h$ in $e^-\gamma$ collisions, hep-ph/0608289.
[81] Z. Chacko, H.-S. Goh, R. Harnik, The twin higgs: Natural electroweak breaking from mirror symmetry, Phys. Rev. Lett. 96 (2006) 231802, hep-ph/0506256.
[82] Z. Chacko, Y. Nomura, M. Papucci, G. Perez, Natural little hierarchy from a partially Goldstone twin higgs, JHEP 0601 (2006) 126, hep-ph/0510273.
[83] Z. Chacko, H.-S. Goh, R. Harnik, A twin higgs model from left-right symmetry, JHEP 0601 (2006) 108, hep-ph/0512088.
[84] K. Agashe, R. Contino, A. Pomarol, The minimal composite higgs model, Nucl. Phys. B 719 (2005) 165–187, hep-ph/0412089.
[85] K. Agashe, R. Contino, The minimal composite higgs model and electroweak precision tests, Nucl. Phys. B 742 (2006) 59–85, hep-ph/0510164.
[86] A. Birkedal, Z. Chacko, M.K. Gaillard, Little supersymmetry and the supersymmetric little hierarchy problem, JHEP 0410 (2004) 036, hep-ph/0404197.
[87] K. Lane, A. Martin, Accidental Goldstone bosons, Phys. Rev. D 71 (2005) 076007, hep-ph/0501204.
[88] K. Lane, A. Martin, A new mechanism for light composite higgs bosons, Phys. Lett. B 635 (2006) 118–122, hep-ph/0511002.
[89] E. Katz, A.E. Nelson, D.G.E. Walker, The intermediate higgs, JHEP 0508 (2005) 074, hep-ph/0504252.

[90] M. Schmaltz, Physics beyond the standard model (theory): Introducing the little higgs, Nucl. Phys. Proc. Suppl. 117 (2003) 40–49, hep-ph/0210415.
[91] M. Schmaltz, D. Tucker-Smith, Little higgs review, Ann. Rev. Nucl. Part. Sci. 55 (2005) 229–270, hep-ph/0502182.
[92] M. Perelstein, Little higgs models and their phenomenology, hep-ph/0512128.
[93] S.R. Coleman, E. Weinberg, Radiative corrections as the origin of spontaneous symmetry breaking, Phys. Rev. D 7 (1973) 1888–1910.
[94] H. Georgi, S.L. Glashow, Spontaneously broken gauge symmetry and elementary particle masses, Phys. Rev. D 6 (1972) 2977–2982.
[95] D.B. Kaplan, H. Georgi, S. Dimopoulos, Composite higgs scalars, Phys. Lett. B 136 (1984) 187.
[96] H. Georgi, D.B. Kaplan, Composite higgs and custodial su(2), Phys. Lett. B 145 (1984) 216.
[97] H. Georgi, D.B. Kaplan, P. Galison, Calculation of the composite higgs mass, Phys. Lett. B 143 (1984) 152.
[98] M.J. Dugan, H. Georgi, D.B. Kaplan, Anatomy of a composite higgs model, Nucl. Phys. B 254 (1985) 299.
[99] R.F. Dashen, Some features of chiral symmetry breaking, Phys. Rev. D 3 (1971) 1879–1889.
[100] J. Gasser, H. Leutwyler, Chiral perturbation theory to one loop, Ann. Phys. 158 (1984) 142.
[101] S.R. Coleman, J. Wess, B. Zumino, Structure of phenomenological lagrangians. 1, Phys. Rev. 177 (1969) 2239–2247.
[102] J. Callan, G. Curtis, S.R. Coleman, J. Wess, B. Zumino, Structure of phenomenological lagrangians. 2, Phys. Rev. 177 (1969) 2247–2250.
[103] H. Georgi, A tool kit for builders of composite models, Nucl. Phys. B 266 (1986) 274.
[104] H. Leutwyler, On the foundations of chiral perturbation theory, Ann. Phys. 235 (1994) 165–203, hep-ph/9311274.
[105] S.R. Coleman, E. Witten, Chiral symmetry breakdown in large *n* chromodynamics, Phys. Rev. Lett. 45 (1980) 100.
[106] J. Thaler, Little technicolor, JHEP 0507 (2005) 024, hep-ph/0502175.
[107] Z. Berezhiani, P.H. Chankowski, A. Falkowski, S. Pokorski, Double protection of the higgs potential, Phys. Rev. Lett. 96 (2006) 031801, hep-ph/0509311.
[108] A. Falkowski, S. Pokorski, M. Schmaltz, Twin SUSY, Phys. Rev. D 74 (2006) 035003, hep-ph/0604066.
[109] S. Chang, L.J. Hall, N. Weiner, A supersymmetric twin higgs, hep-ph/0604076.
[110] S. Dimopoulos, H. Georgi, Softly broken supersymmetry and SU(5), Nucl. Phys. B 193 (1981) 150.