

The mystery of the Higgs particle/Le mystère de la particule de Higgs

Spontaneous broken symmetry

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Abstract

The concept of spontaneous broken symmetry is reviewed in the presence of global symmetries both in matter and particle physics. This concept is then taken over to confront local symmetries in relativistic field theory. Emphasis is placed on the basic concepts where, in the former case, the vacuum of spontaneous broken symmetry is degenerate whereas that of local (or gauge) symmetry is gauge invariant. **To cite this article:** R. Brout, F. Englert, C. R. Physique 8 (2007).

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Résumé

Brisure spontanée de symétrie. Le concept de brisure spontanée de symétrie est présenté dans le cadre des symétries globales en physique de la matière condensée et en physique des particules. Ce concept est adapté ensuite aux symétries locales en théorie des champs relativistes. La présentation est centrée sur l'analyse conceptuelle qui conduit d'une part au vide dégénéré de la symétrie brisée globalement mais d'autre part à son invariance de jauge en présence d'une symétrie locale (dite de jauge). **Pour citer cet article :** R. Brout, F. Englert, C. R. Physique 8 (2007).

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1. Introduction

The success of the semi-phenomenological theory of weak and electromagnetic interactions must be counted as one of the triumphs of physics of the latter part of the 20th century [1]. The term 'semi-phenomenological' is used advisedly. Whereas the fundamental framework, spontaneous broken symmetry in the context of gauge theories is well founded in quantum field theory, the choice of the gauge group, its representations, and the mode of its breaking are dictated by observation and have as yet received no theoretical explanation.

Yet, one cannot fail to be impressed by the success of the general framework and, relatively speaking, the paucity of phenomenological input that goes towards the systematic explanation of so much. It is our purpose here to present a synopsis of this framework with special emphasis on its principles.

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We shall begin with the breaking of global symmetry wherein group transformations on the dynamical action are the same everywhere and for all time [2,3]. This is abbreviated SBS (spontaneous breaking of symmetry). Then follows a discussion of local symmetry (also called gauge symmetry) wherein group transformations vary from point to point in space–time, and its asymmetric phase which is the outcome of SBS in gauge theories [4,5].

A major difference in the field theory context between these two chapters of physics is the characterization of the vacuum. Whereas in SBS the vacuum (i.e. the ground state of the field theory) is degenerate and transforms from one state to another of the same energy under group transformations, the vacuum of the asymmetric phase is gauge invariant and unique. The dynamical consequences of this dichotomy are enormous, yet one makes use of the concepts of SBS in the Brout–Englert–Higgs (BEH) construction of the asymmetric phase. In particular, the latter is characterized by gauge invariant masses for some gauge vector fields, hence by the *finite* range of the forces that they transmit when they are exchanged.

The text is organized as follows. For pedagogical reasons, SBS is displayed through the example of ferromagnetism, particularly simple in its conceptual basis. From there we present its field theoretic generalization. In passing, we mention some spectacular successes of the SBS concept applied to physics: superfluidity and superconductivity in matter physics and spontaneous broken chiral symmetry applied to particle physics. We then gauge the symmetry, motivating the introduction of gauge fields. Something of crucial importance turns up. Whereas the degeneracy of the SBS vacuum has as a necessary concomitant the existence of massless collective modes living in the coset G/H where H is the unbroken subgroup of the symmetry group G , these cannot survive the non-degenerate vacuum of the asymmetric phase. Rather they are absorbed in the propagation properties of gauge vectors, providing them with a third (longitudinal) polarization. This is how the gauge vectors coupled to the would-be massless collective modes, that is to those living in G/H , acquire a gauge invariant mass. Finally we outline the consequences of the BEH mechanism and its impact on the quest for unified laws of nature.

2. Global spontaneous symmetry breaking

2.1. Phase transitions

Ferromagnetism is a manifestation of the interaction of an atomic spin with a magnetic field. The interaction is of the form $\mu \mathbf{h} \cdot \mathbf{S}$ where \mathbf{h} is the magnetic field, \mathbf{S} the spin, μ the magnetic moment per spin. From now on μ is absorbed into the definition of \mathbf{h} so \mathbf{h} has the dimension of energy. In a material medium each spin is surrounded by others and interact with them; in good approximation the interaction is among pairs so the total interaction energy is

$$H = H_0 - \sum_{i=1}^N \mathbf{h}_i \cdot \mathbf{S}_i, \quad H_0 = -2 \sum_{i \neq j} v_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \quad (1)$$

We shall take the example of a crystal of spins so that $v_{ij} = v(\mathbf{r}_i - \mathbf{r}_j)$ where \mathbf{r}_i is the position of a crystal site upon which sits spin \mathbf{S}_i . For simplicity, one may think of v_{ij} non vanishing between neighbors. Also its sign is taken positive thereby favoring parallel alignment to minimize the energy. The magnitude of the interaction energy per spin is $O(100 \text{ to } 1000 \text{ K})$, far to great for it to be magnetic energy. The enigma so presented was resolved by Heisenberg in the late 1920s, who showed that such an interaction could arise due to the electrostatic forces when combined with the antisymmetric character of the many electron wave function of quantum mechanics.

For simplicity we shall choose the spin 1/2 case so that $\mathbf{S}_i = \sigma_i^a / 2$, $\sigma_i^a / 2$ ($a = x, y, z$) become the three Pauli matrices. Take $\mathbf{h}_i = h^z \mathbf{1}_z \neq 0$ spatially uniform (independent of i) say in the z direction. The ground state is unique and, as is easily verified, is the symmetric ‘all spin up’ state $|0\rangle = |++++\dots\rangle$ where the normalized spin states of the individual spins $|+\rangle$ are quantized in the z -direction. Upon taking adiabatically the limit $h^z \rightarrow 0$, one remains in the state $|0\rangle$. The $O(3)$ symmetry when $\mathbf{h}_i = 0$ is expressed by $[H_0, S^a] = 0$, where the rotation group generators are $S^a = \sum_{i=1}^N S_i^a$. In particular $[H_0, S^-] = 0$, $S^\pm = S^x \pm iS^y$, and the ground state is $(N+1)$ fold degenerate. The $N+1$ linearly independent ground states are obtained by successive applications of S^- on $|0\rangle$.

The limit $N \rightarrow \infty$ highlights the relation between the degenerate quantum states and the classical geometric notion of a classical magnetization vector pointing in some direction of space. One has

$$\langle 0 | S^x | 0 \rangle = \langle 0 | S^y | 0 \rangle = 0, \quad \langle 0 | S^z | 0 \rangle = NM^z = N|\mathbf{M}| = \frac{N}{2} \quad (2)$$

where we have defined $\mathbf{M} = M^z \mathbf{1}_z = (1/N) \langle 0 | \mathbf{S} | 0 \rangle$. The state $|\theta\rangle$ obtained from $|0\rangle$ by rotating an angle θ about the x -axis is $|\theta\rangle = \exp(iS^x \theta) |0\rangle$. The states $|\theta\rangle$ and $|0\rangle$ are degenerate when $\mathbf{h}_i = 0$ and one gets from the commutation relations

$$\langle \theta | S^x | \theta \rangle = 0, \quad \langle \theta | S^y | \theta \rangle = N |\mathbf{M}| \sin \theta, \quad \langle \theta | S^z | \theta \rangle = N |\mathbf{M}| \cos \theta \quad (3)$$

In this way, the vector \mathbf{M} is the expectation value of the operator \mathbf{S}/N in the different rotated ground states. Consider now the two distinct ground states, $|0\rangle$ and $|\theta\rangle$. One has

$$\begin{aligned} \langle 0 | \theta \rangle &= \langle 0 | e^{iS^x \theta} | 0 \rangle = \langle 0 | \prod_{i=1}^N e^{i(\sigma_i^x/2)\theta} | 0 \rangle \\ &= \prod_{i=1}^N \langle 0 | \cos(\theta/2) + i(\sigma_i^x/2) \sin(\theta/2) | 0 \rangle = [\cos(\theta/2)]^N \longrightarrow_{N \rightarrow \infty} 0 \end{aligned} \quad (4)$$

It is easy to verify that the orthogonality in the limit $N \rightarrow \infty$ still holds between excited states involving finite numbers of ‘wrong spins’ and hence the Hilbert space of the system splits into an infinite number of orthogonal Hilbert subspaces built upon the degenerate ground states labeled by \mathbf{M} . If N is large but finite, the orthogonality condition remains approximatively valid if $\theta > O(1/\sqrt{N})$. This is the expected range of quantum fluctuations around a classical configuration of N aligned spins and \mathbf{M} appears for large N as a *classical* magnetization vector. The characterization of a degenerate vacuum by such *classical* order parameter is the distinctive feature of global SBS, as opposed to the asymmetric phase of gauge theory analyzed in Section 3.

The emergence of the SBS degenerate vacuum can be understood as follows. Fix arbitrarily the orientation of some spin at $\mathbf{h} = 0$. Then its neighbors will orient themselves parallel to it. And the neighbors of the neighbors similarly. And so on, so that the orientation of a *single* degree of freedom fixes the orientation of all N .

A feature related to ground state degeneracy under the rotation group is the onset of zero modes. Namely, there exists a set of normal modes, here called spin waves, such that their excitation energy costs zero energy in the limit of zero wave number. To see this, let us rewrite the Hamiltonian equation (1) in terms of Fourier components. Defining

$$\mathbf{S}(\mathbf{q}) = \frac{1}{\sqrt{N}} \sum_{i=1}^N \mathbf{S}_i e^{i\mathbf{q} \cdot \mathbf{r}_i}, \quad v(\mathbf{q}) = \frac{1}{N} \sum_{i \neq j} v_{ij} e^{-i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \quad (5)$$

Eq. (1) yields, at $h^z = 0$,

$$H_0 = -2 \sum_{\mathbf{q}} v(\mathbf{q}) \mathbf{S}(\mathbf{q}) \cdot \mathbf{S}(-\mathbf{q}) \quad (6)$$

Taking into account the relation $S^z(\mathbf{q})|0\rangle = (\sqrt{N}/2)\delta_{\mathbf{q},0}|0\rangle$ one gets, using the commutation relations of the rotation generators

$$[H_0, S^-(\mathbf{q})]|0\rangle = 2[v(\mathbf{0}) - v(\mathbf{q})]S^-(\mathbf{q})|0\rangle \quad (7)$$

Eq. (7) reveals a spin wave with energy ω related to the wavevector \mathbf{q} by the dispersion relation

$$\omega = 2[v(\mathbf{0}) - v(\mathbf{q})] \quad (8)$$

As stated, its energy vanishes when $\mathbf{q} = 0$. This is a consequence of the ground state degeneracy. The excitation is created by the operator $S^-(\mathbf{q})$ acting on the state $|0\rangle$, which in the limit $\mathbf{q} \rightarrow 0$ is proportional to generators rotating the degenerate ground states, and therefore cannot carry energy. In relativistic field theory, an excitation whose energy vanishes as $\mathbf{q} \rightarrow 0$ characterizes a massless mode and the spin wave may be viewed here as the ‘massless’ mode associated with spontaneous broken rotational invariance. It is the ancestor of the NG boson that will be discussed in the context of field theory. Note that if the external magnetic field h^z is non-zero, Eq. (8) gets an additional term in the RHS linear in h^z , and hence a ‘mass’ term.

We sketch briefly the physics of how one generalizes these considerations to finite temperature. The Helmholtz free energy $F = -T \lim_{N \rightarrow \infty} (1/N) \ln Z$, where Z is the spin partition function, displays a spontaneous magnetization $\mathbf{M} \neq 0$, $\mathbf{h} = 0$ at low temperature. This is understandable thermodynamically from $F = E - TS$. As T is increased

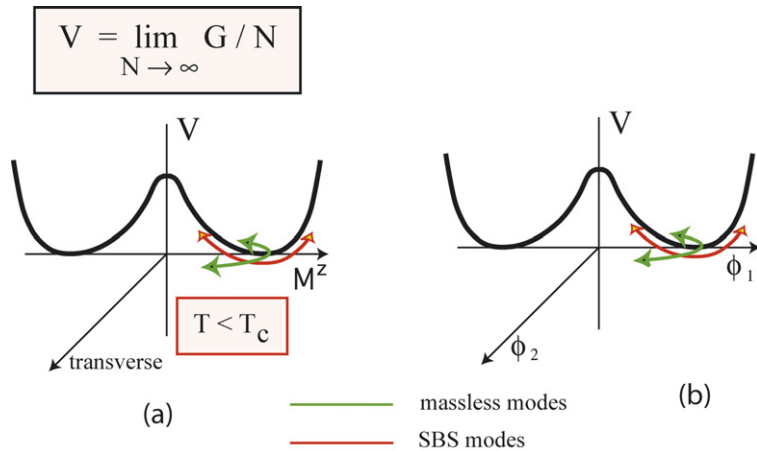


Fig. 1. (a) Effective potential of a Heisenberg ferromagnet. (b) Potential of the Goldstone model.

at $\mathbf{h} = 0$, the spin-spin energy E increases from its ground state value as spin disorient relative to each other and the entropy increases due to this disordering. For T less than some critical Curie temperature T_c one finds a minimum at $\mathbf{M} \neq 0$ and for $T > T_c$ one finds $\mathbf{M} = 0$; for $T > T_c$ the entropic disordering dominates and order at $\mathbf{h} = 0$ is no longer sustainable. In the statistical theory, this can be shown by computing a ‘restricted trace’ contribution to Z . This means one limits spin configurations in the complete trace to a subset restricted to some fixed magnetization \mathbf{m} . Call this restricted trace $Z_R(\mathbf{m})$. For a given magnetic field \mathbf{h} , $Z_R(\mathbf{m})$ is stationary at $\mathbf{m} = \mathbf{M}^*$ and it is easily shown that in the thermodynamic limit $N \rightarrow \infty$ one has $\mathbf{M}^* = \mathbf{M}$, i.e. $\partial \ln Z / \partial \mathbf{h} = \mathbf{M} / T$. Equivalently, one may compute the effective potential V , that is the Gibbs potential per spin $G/N = (E - TS)/N + \mathbf{M} \cdot \mathbf{h}$. Its behavior is characteristic of second order phase transitions with spontaneous broken symmetry. Above the Curie point, V has a single minimum at $\mathbf{M} = 0$. This minimum flattens at $T = T_c$ and two symmetric minima appear for $T < T_c$ in the VM^z -plane. This would be the whole story for a system with discrete symmetry, such as the Ising model obtained from the Hamiltonian equation (1) by retaining only the z -component of the spin. The discrete Z_2 symmetry of the action would be spontaneously broken below the Curie point when, as $h^z \rightarrow 0$, the system ends in one of the equivalent minima in the VM^z -plane exhibited in Fig. 1(a). But, when $\mathbf{h} = 0$, the Hamiltonian H_0 is invariant under the full rotation group $SO(3)$. This continuous symmetry implies that the thermodynamics of the ferromagnetic phase does not depend on the orientation of the magnetization. The effective potential $V(T, |\mathbf{M}|)$ only depends on the norm of the magnetization vector \mathbf{M} . Hence the equivalent minima are not only doubly degenerate but span the full 2-sphere, that is the coset space of $SO(3)/U(1)$. By selecting an orientation at a given minimum, the system in the ferromagnetic phase spontaneously breaks the $SO(3)$ symmetry down to $U(1)$.

The effective potential below the Curie point, depicted in Fig. 1(a), summarizes the essential features of SBS. At a given minimum $\mathbf{h} = 0$. Taking, say, $\mathbf{M} = M^z \mathbf{1}_z$, the curvature of the effective potential measures the inverse susceptibility which determines the energy for infinite wavelength fluctuations, in other words, the ‘mass’. The inverse susceptibility is zero in directions transverse to the order parameter and positive in the longitudinal direction. One recovers, even at non-zero temperature, the massless transverse mode characteristic of broken continuous symmetry¹ and we learn that there is also a (possibly unstable) ‘massive’ longitudinal mode which corresponds to fluctuations of the order parameter and which is present in any spontaneous broken symmetry, continuous or even discrete. Such generically massive mode characterize any ordered structure, be it the broken symmetry phase in statistical physics, the vacuum of the global SBS in field theory presented in Section 2.2 or of the Yang–Mills asymmetric phase discussed in Section 3. The ‘SBS mass’ of the longitudinal mode measures the rigidity of the ordered structure. In material systems one often refers to this as to the ‘stretching mode’.

To conclude this section we evoke two other second order phase transitions.

¹ The fact that there is only one massless mode while the space of constant $|\mathbf{M}|$ (the coset space $SO(3)/U(1)$) is two-dimensional is due to the axial vector character of the magnetization vector which selects only one linear combination of S^x and S^y , namely S^- to describe classically the precession of the spins around the z -axis.

Superfluidity in He 4 occurs when below a critical temperature a condensate forms out of zero momentum states of the bosonic atoms. This phenomenon is related to the Bose–Einstein condensation of a free boson gas and can be characterized by the breaking of the $U(1)$ symmetry of the quantum phase. This results in a degeneracy of the ground state and in the existence of a concomitant massless mode which here are sound waves. A $U(1)$ broken symmetry also occurs in superconductivity through condensation of Cooper pairs bound states of zero momentum spin singlets formed by an attractive force in the vicinity of the electron Fermi surface induced by phonon exchange [6]. Cooper pair condensation leads to a gap at the Fermi surface. This gap would host a massless mode were it not for the presence of the long-range Coulomb interactions. The massless mode disappears by being absorbed in the electron density oscillations induced by the Coulomb field, namely, in the longitudinal massive plasma mode [7,8]. These features and the occurrence of a penetration depth for the magnetic field are precursors of the asymmetric Yang–Mills phase where the broken symmetry concept is adapted to local gauge symmetries in relativistic field theories. This will be presented in Section 3.

2.2. Field theory

Spontaneous symmetry breaking was introduced in relativistic quantum field theory by Nambu in analogy with the BCS theory of superconductivity. The problem studied by Nambu [3] and Nambu and Jona-Lasinio [9] is the spontaneous breaking of chiral symmetry induced by a fermion condensate $\langle \bar{\psi}\psi \rangle \neq 0$. They consider massless fermions interacting through the four Fermi interaction $g[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$ that is invariant under the $U(1)$ chiral group transformation $\psi \rightarrow \exp(i\gamma_5\alpha)\psi$. This is a global symmetry as α is constant in space–time. Although no fermion mass may arise perturbatively, summing up an infinity of diagrams allow the self-energy to acquire self-consistently a non-zero contribution from $\langle \bar{\psi}\psi \rangle$. This yields a fermion mass m and an eigenvalue equation $g = f(m/\Lambda)$ where Λ is a ultraviolet cut-off. The eigenvalue equation in turn implies the existence of a massless pseudoscalar mode coupled to the axiovector current. This is a consequence of the chiral Ward identity relating the axial vector vertex $\Gamma_{\mu 5}$ to the self energy $\Sigma = A(p^2)\gamma^\mu p_\mu + M(p^2)$ in a chiral invariant theory,

$$q^\mu \Gamma_{\mu 5} = \gamma_5 \Sigma \left(p + \frac{q}{2} \right) + \Sigma \left(p - \frac{q}{2} \right) \gamma_5 \quad (9)$$

As $q_\mu \rightarrow 0$, the form factor $A(p^2)$ drops out of Eq. (9) and

$$\lim_{q_\mu \rightarrow 0} q_\mu \Gamma_{\mu 5} = 2M(p^2)\gamma_5, \quad \Gamma_{\mu 5} \rightarrow \frac{2M(p)\gamma_5 q_\mu}{q^2} \quad (10)$$

The pole at $q^2 = 0$ in Eq. (10) signals the appearance of the pseudoscalar boson.

The model is not intended to be realistic but sets the scene for more general considerations. The pion is identified with the massless mode of spontaneous broken chiral invariance. It gets its tiny mass (on the hadron scale) from a small explicit breaking of the symmetry, just as a small external magnetic field h^z imparts a small gap in the spin wave spectrum. This interpretation of the pion mass constituted a breakthrough in our understanding of strong interaction physics. It resulted in a number of interesting insights into the so called soft pion sector of strong interaction physics, in particular the theoretical understanding of the Goldberger–Treiman [10] relation which relates β decay, charged pion decay and the pion–nucleon interaction.

General features of SBS in relativistic quantum field theory were further analyzed by Goldstone, Salam and Weinberg [11,12]. Here, symmetry is broken by non vanishing vacuum expectation values of scalar fields. The method is designed to exhibit the appearance of a massless mode out of the degenerate vacuum and does not really depend on the significance of the scalar fields. The latter could be elementary or represent collective variables of more fundamental fields, as would be the case in the original Nambu model. Compositeness affects details of the model considered, such as the behavior at high momentum transfer and the stability of the SBS massive scalar, but not the existence of the massless excitations encoded in the degeneracy of the vacuum.

Let us illustrate the occurrence of this massless Nambu–Goldstone (NG) boson in a simple model of a complex scalar field with $U(1)$ symmetry [11]. The Lagrangian density,

$$L = \partial^\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \quad \text{with } V(\phi^* \phi) = -\mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2, \lambda > 0 \quad (11)$$

is invariant under the $U(1)$ group $\phi \rightarrow e^{i\alpha}\phi$. The global $U(1)$ symmetry is broken by a vacuum expectation value of the ϕ -field given, at the classical level, by the minimum of $V(\phi^*\phi)$. Writing $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$, one may choose $\langle\phi_2\rangle = 0$. Hence $\langle\phi_1\rangle^2 = \mu^2/\lambda$ and we select, say, the vacuum with $\langle\phi_1\rangle$ positive. The potential $V(\phi^*\phi)$ depicted in Fig. 1(b) is similar to the effective potential below the ferromagnetic Curie point shown in Fig. 1(a) and leads to similar consequences. In the unbroken vacuum the field ϕ_1 has negative mass and acquires a positive mass $2\mu^2$ in the broken vacuum where the field ϕ_2 is massless. The latter is the NG boson of broken $U(1)$ symmetry and is the analog of the massless spin wave mode in ferromagnetism, corresponding to the vanishing of the inverse transverse susceptibility. The massive scalar describes the fluctuations of the order parameter $\langle\phi_1\rangle$ and is the analog of the SBS massive mode in the ordered phase of a many-body system, encoded in a non-vanishing inverse longitudinal susceptibility.

The origin of the massless NG boson is, as in the ferromagnetism phase, a consequence of the vacuum degeneracy. The vacuum characterized by the order parameter $\langle\phi_1\rangle$ is rotated into an equivalent vacuum by an operator proportional to the field ϕ_2 at zero space momentum. Such rotation costs no energy and thus the field ϕ_2 at space momenta $\vec{q} = 0$ has $q_0 = 0$ in the equations of motion, and hence zero mass.

This Goldstone theorem can be formalized and extended to spontaneous global symmetry breaking of a semi-simple Lie group G to H . Let ϕ^A be scalar fields spanning a representation of G generated by the (antihermitian) matrices T^{aAB} . If the G -invariant potential has minima for nonvanishing ϕ^A , the global symmetry is broken and the vacuum is degenerate under G -rotations. The conserved charges are $Q^a = \int \partial_\mu \phi^B T^{aBA} \phi^A d^3x$. The propagators of the fields ϕ^B such that $\langle [Q^a, \phi^B] \rangle = T^{aBA} \langle \phi^A \rangle \neq 0$ have a NG pole at $q^2 = 0$ and the NG bosons live in the coset G/H .

3. The asymmetric Yang–Mills phase

3.1. Global to local symmetry

The global $U(1)$ symmetry in Eq. (11) is extended to a local one $\phi(x) \rightarrow e^{i\alpha(x)}\phi(x)$ by introducing a vector field $A_\mu(x)$ transforming as $A_\mu(x) \rightarrow A_\mu(x) + (1/e)\partial_\mu\alpha(x)$. The corresponding Lagrangian density is

$$L = D^\mu \phi^* D_\mu \phi - V(\phi^* \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (12)$$

with covariant derivative $D_\mu \phi = \partial_\mu \phi - ie A_\mu \phi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Local invariance under a semi-simple Lie group G is realized by extending the Lagrangian equation (12) to incorporate non-Abelian Yang–Mills vector fields A_μ^a

$$L_G = (D^\mu \phi)^{*A} (D_\mu \phi)^A - V - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \quad (13)$$

$$(D_\mu \phi)^A = \partial_\mu \phi^A - e A_\mu^a T^{aAB} \phi^B, \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - ef^{abc} A_\mu^b A_\nu^c \quad (14)$$

Here, ϕ^A belongs to the representation of G generated by T^{aAB} and the potential V is invariant under G .

The local Abelian or non-Abelian gauge invariance of Yang–Mills theory hinges *apparently* upon the massless character of the gauge fields A_μ , hence on the long-range character of the forces they transmit, as the addition of a mass term for A_μ in the Lagrangian equation (12) or (13) destroys gauge invariance. But short-range forces such as the weak interaction forces seem to be as fundamental as the electromagnetic ones. A natural hypothesis would be that these forces be mediated by the exchange of vector and axial vector fields coupled to currents, in strong analogy to the electromagnetic current, but which apparently seem to be non-conserved. One is therefore tempted to account for these facts by appeal to spontaneous broken symmetry. However, the problem of SBS is different for global and for local symmetries.

In perturbation theory, gauge invariant quantities are evaluated by choosing a particular gauge by adding to the action a gauge fixing term. Gauge invariance is ensured by including the corresponding Fadeev–Popov ghosts and by summing over subsets of graphs satisfying Ward Identities.

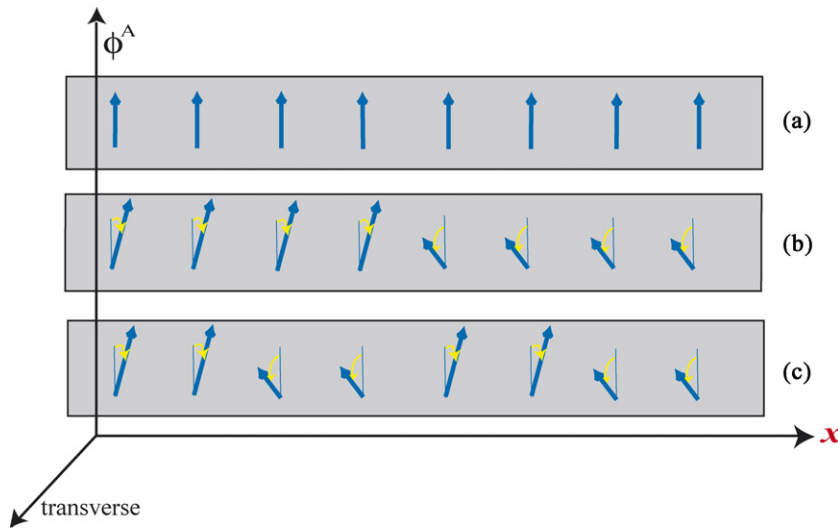


Fig. 2. The disappearance of the massless Nambu–Goldstone boson in gauge theories.

Consider the Yang–Mills theory defined by the Lagrangian² equation (13). To exhibit the similarities and the differences between spontaneous breaking of a global symmetry and its local symmetry counterpart, it is convenient to choose a gauge which preserves Lorentz invariance and a residual global G symmetry. This can be achieved by adding to the Lagrangian a gauge fixing term $(2\eta)^{-1} \partial_\mu A^{a\mu} \partial_\nu A^{a\nu}$. The gauge parameter η is arbitrary and is not observable.

In such gauges, the global symmetry can be spontaneously broken for suitable potential V , by non zero expectation values $\langle \phi^A \rangle$ of scalar fields. In Fig. 2 we have represented motions of this field in the ‘transverse’ coset space G/H (H is the unbroken subgroup) propagating in the spatial x -direction. Fig. 2(a) pictures the spontaneously broken vacuum of the gauge fixed Lagrangian. Figs. 2(b) and (c) mimic motions with decreasing wavelength λ . Clearly, as $\lambda \rightarrow \infty$, such motions can only induce global rotations in the internal space. In absence of gauge fields, they would give rise, as in spontaneously broken global continuous symmetries, to massless NG modes. In a gauge theory, transverse fluctuations of $\langle \phi^A \rangle$ are just local rotations in the internal space and are unobservable gauge motions. Hence the would-be NG bosons induce only gauge transformations and their excitations disappear from the physical spectrum. A formal proof of the failure of the Goldstone theorem in presence of gauge fields, in relativistic quantum field theory, was given by Higgs [13]. As a consequence, the vacuum is generically non degenerate and points in no particular direction in group space.³ In this sense, local gauge symmetry cannot be spontaneously broken.⁴ The disappearance of the NG bosons and the concomitant removal of the vacuum degeneracy stems from the fact that no energy is needed to change the *relative* orientation of neighboring ‘spins’, that is of neighboring configurations of the scalar fields in group space. No classical configuration is available to protect a degenerate vacuum against quantum fluctuations. Recalling that the explicit presence of a gauge vector mass breaks gauge invariance, we are thus faced with a dilemma. How can gauge fields acquire mass without breaking the local symmetry?

What makes local internal space rotations unobservable in a gauge theory is precisely the fact that they can be absorbed by the Yang–Mills fields. The absorption of the NG fields renders massive the gauge fields living in the coset G/H by transferring to them their degrees of freedom which become longitudinal polarizations.

We shall see in the next sections how these considerations are realized in relativistic quantum field theory and give rise to vector masses in the coset G/H , leaving long-range forces only in a subgroup H of G . Despite the unbroken local symmetry, the group G appears broken to its subgroup H in the asymptotic state description of field theory by

² A similar reasoning for the Abelian case Eq. (12) leads to a mass for the gauge field A_μ when the scalar field gets expectation value, say $\langle \phi_1 \rangle \neq 0$.

³ Note that for global symmetry breaking, one can always choose a linear combination of degenerate vacua which is invariant under, say, the $U(1)$ symmetry. This choice has no observable consequences because of the splitting into orthogonal Hilbert spaces.

⁴ For a detailed proof, see Ref. [14].

the *choice* of the orientation of the vacuum expectation value $\langle \phi^A \rangle$. This is why we often term SBS or asymmetric such a Yang–Mills phase. The onset of SBS will be described in detail mostly in lowest order perturbation theory around the self-consistent vacuum, both in the field-theoretic [4] and in the equation of motion [5] formulations. This contains already the basic ingredients of the phenomenon and comparison between the two methods gives some insight on the renormalization issue.

3.2. The field theoretic approach

Let us first examine the Abelian case as realized by the complex scalar field ϕ exemplified in Eq. (12).

In the covariant gauges, the free propagator of the field A_μ is

$$D_{\mu\nu}^0 = \frac{g_{\mu\nu} - q_\mu q_\nu / q^2}{q^2} + \eta \frac{q_\mu q_\nu / q^2}{q^2} \quad (15)$$

where η is the gauge parameter.

In absence of symmetry breaking, the lowest order contribution to the self-energy, arising from the covariant derivative terms in Eq. (12), is given by the one-loop diagrams of Fig. 3. The self-energy (suitably regularized) takes the form of a polarization tensor

$$\Pi_{\mu\nu} = (g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi(q^2) \quad (16)$$

where the scalar polarization $\Pi(q^2)$ is regular at $q^2 = 0$, leading to the gauge field propagator

$$D_{\mu\nu} = \frac{g_{\mu\nu} - q_\mu q_\nu / q^2}{q^2 [1 - \Pi(q^2)]} + \eta \frac{q_\mu q_\nu / q^2}{q^2} \quad (17)$$

The polarization tensor in Eq. (16) is transverse and hence does not affect the gauge parameter η . The transversality of the polarization tensor reflects the gauge invariance of the theory⁵ and, as we shall see below, the regularity of the polarization scalar signals the absence of symmetry breaking. This guarantees that the A_μ -field remains massless.

Symmetry breaking adds tadpole diagrams to the previous ones. To see this write

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \quad \langle \phi_1 \rangle \neq 0 \quad (18)$$

The scalar field ϕ_1 , whose expectation value plays the role of an order parameter in the gauge considered and whose fluctuations have a gauge invariant SBS mass, is often called the Higgs field and its fluctuations the Higgs boson. This massive mode is not a specific property of the BEH mechanism but is a necessary concomitant of *any* SBS structured vacuum, as pointed out in Section 2.1. The would-be NG-field is ϕ_2 . The additional diagrams are depicted in Fig. 3.

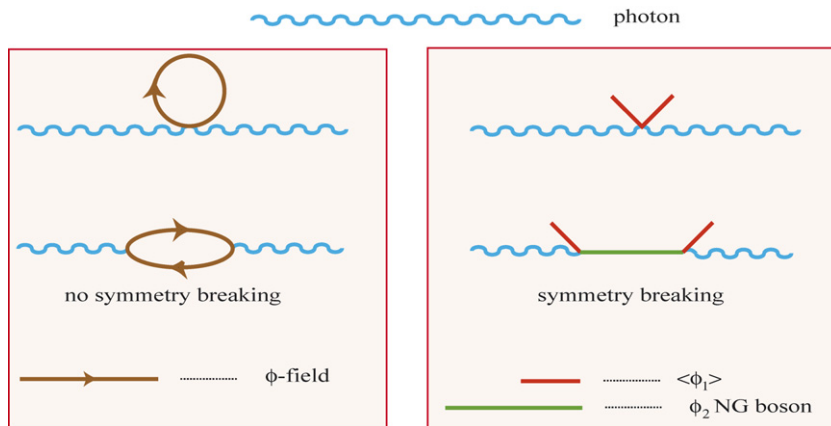


Fig. 3. The Yang–Mills mass.

⁵ The transversality of polarization tensors is a consequence of the Ward Identities alluded to in the preceding section.

The polarization scalar $\Pi(q^2)$ in Eq. (16) acquires a pole from the tadpole contribution

$$\Pi(q^2) = \frac{e^2 \langle \phi_1 \rangle^2}{q^2} \quad (19)$$

and, in lowest order perturbation theory, the gauge field propagator becomes

$$D_{\mu\nu} = \frac{g_{\mu\nu} - q_\mu q_\nu / q^2}{q^2 - \mu^2} + \eta \frac{q_\mu q_\nu / q^2}{q^2} \quad (20)$$

which shows that the A_μ -field gets a mass

$$\mu^2 = e^2 \langle \phi_1 \rangle^2 \quad (21)$$

The generalization of Eqs. (16) and (19) to the non Abelian case described by the action equation (13) is straightforward. One gets

$$\Pi_{\mu\nu}^{ab} = (g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi^{ab}(q^2) \quad (22)$$

$$\Pi^{ab}(q^2) = \frac{e^2 \langle \phi^{*B} \rangle T^{*aBC} T^{bCA} \langle \phi^A \rangle}{q^2} \quad (23)$$

Eq. (23) defines the mass matrix

$$(\mu^2)^{ab} = e^2 \langle \phi^{*B} \rangle T^{*aBC} T^{bCA} \langle \phi^A \rangle \quad (24)$$

In terms of its non-zero eigenvalues $(\mu^2)^a$, the propagators of the massive gauge vectors take the same form as Eq. (20),

$$D_{\mu\nu}^a = \frac{g_{\mu\nu} - q_\mu q_\nu / q^2}{q^2 - (\mu^2)^a} + \eta \frac{q_\mu q_\nu / q^2}{q^2} \quad (25)$$

The gauge invariance is expressed, as it was in absence of symmetry breaking, through the transversality of the polarization tensors equations (16) and (22). The singular $1/q^2$ contributions to the polarization scalars equations (19) and (23) preserve transversality and yield gauge invariant masses for the gauge bosons. They stem from the long-range NG boson fields. The latter are, as such, unobservable gauge terms but their absorption in the gauge field propagators transfers the degrees of freedom of the would-be NG bosons to the third degree of polarization of the massive vectors. Indeed, on the mass shell $q^2 = (\mu^2)^a$, one easily verifies that the numerator in the transverse propagator in Eq. (25) is

$$g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} = \sum_{\lambda=1}^3 e_\mu^{(\lambda)} \cdot e_\nu^{(\lambda)}, \quad q^2 = (\mu^2)^a \quad (26)$$

where the $e_\mu^{(\lambda)}$ are three polarization vectors orthonormal in the rest frame of the particle.

In this way, the would-be NG bosons generate massive propagators for the gauge fields in G/H . Long-range forces only survive in the subgroup H of G which leaves invariant the non vanishing expectation values $\langle \phi^A \rangle$.

Note that the explicit form of the scalar potential V does not enter the computation of gauge field propagators which depend only on the expectation values at its minimum. This is because trilinear terms arising from covariant derivatives can only couple the tadpoles to other scalar fields through group rotations and hence couple them only to the would-be NG bosons. These are the eigenvectors with zero eigenvalue of the scalar mass matrix given by the quadratic term in the expansion of the potential V around its minimum. Hence the massive scalars decouple from the tadpoles at the tree level considered above. To visualize how the asymmetric phase arises in a simple non-Abelian example, consider a $SO(3)$ Yang–Mills theory broken to $SU(1)$ by say a scalar field in the adjoint representation. In the coset space $SO(3)/U(1)$, there are two would-be NG bosons hence two massive vectors associated to the rotations depicted in red in Fig. 4. The rotation around the remaining symmetry axis is drawn in green and characterizes the third gauge vector which remains massless.

The symmetry breaking giving mass to gauge vector bosons need not be due to fundamental scalar fields. For example as in global SBS, it could arise from a fermion condensate such as that which breaks chiral symmetry. This is illustrated by the following chiral invariant Lagrangian

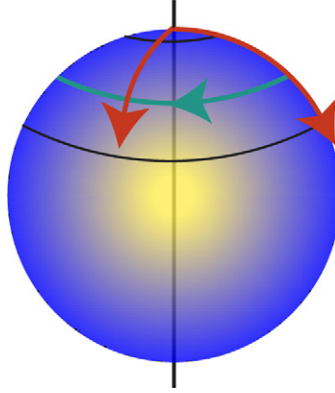


Fig. 4. Massive and massless gauge vectors in a $SO(3)/U(1)$ asymmetric phase model.

$$L = L_0^F - e_V \bar{\psi} \gamma_\mu \psi V_\mu - e_A \bar{\psi} \gamma_\mu \gamma_5 \psi A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}(V) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}(A) \quad (27)$$

Here $F_{\mu\nu}(V)$ and $F_{\mu\nu}(A)$ are Abelian field strength for $U(1) \times U(1)$ symmetry. Chiral anomalies are eventually canceled by adding in the required additional fermions.

As in global SBS, the Ward identity for the chiral current equation (9) shows that if the fermion self-energy $\gamma^\mu p_\mu A(p^2) - M(p^2)$ acquires a non vanishing $M(p^2)$ term, thus a dynamical mass, the axial vertex $\Gamma_{\mu 5}$ develops a pole at $q^2 = 0$. In leading order in q , we get as in Eq. (10)

$$\Gamma_{\mu 5} \rightarrow 2M(p) \gamma_5 \frac{q_\mu}{q^2} \quad (28)$$

The pole in the vertex function induces a pole in the suitably regularized gauge invariant polarization tensor $\Pi_{\mu\nu}^{(A)}$ of the axial vector field A_μ

$$\Pi_{\mu\nu}^{(A)} = e_A^2 (g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi^{(A)}(q^2) \quad (29)$$

with

$$\lim_{q^2 \rightarrow 0} q^2 \Pi^{(A)}(q^2) = \mu^2 \neq 0 \quad (30)$$

The field A_μ acquires in this approximation⁶ a gauge invariant mass μ .

This example illustrates the fact that the transversality of the polarization tensor used in the quantum field theoretic approach to mass generation is a consequence of a Ward identity. This is true whether vector masses arise through fundamental scalar or through fermion condensate. The generation of gauge invariant masses is therefore not contingent upon the ‘tree approximation’ used to get the propagators equations (20) and (25). It is a consequence of the $1/q^2$ singularity in the vacuum polarization scalars equations (19), (22) or (30) which comes from the would-be NG boson contribution.

3.3. The equation of motion formulation

The BEH mechanism can be understood in terms of equations of motions which illustrate nicely the fate of the NG bosons. This is shown below for the Abelian case described by the action equation (12).

Taking as in Eq. (18), the expectation value of the scalar field to be $\langle \phi_1 \rangle$, and expanding the NG field ϕ_2 to first order, one gets from the action equation (12) the classical equations of motion

⁶ The validity of the approximation, and in fact of the dynamical approach, rests on the high momentum behavior of the fermion self energy, but this problem will not be discussed here.

$$\partial^\mu \{ \partial_\mu \phi_2 - e \langle \phi_1 \rangle A_\mu \} = 0 \quad (31)$$

$$\partial_\nu F^{\mu\nu} = e \langle \phi_1 \rangle \{ \partial^\mu \phi_2 - e \langle \phi_1 \rangle A^\mu \} \quad (32)$$

Defining

$$B_\mu = A_\mu - \frac{1}{e \langle \phi_1 \rangle} \partial_\mu \phi_2 \quad \text{and} \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu} \quad (33)$$

one gets

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2 \langle \phi_1 \rangle^2 B^\mu = 0 \quad (34)$$

Eq. (34) shows that B_μ is a massive vector field with mass squared $e^2 \langle \phi_1 \rangle^2$ in accordance with Eq. (21). As pointed out in the previous section, the vector boson mass does not depend explicitly on the scalar potential, but only on the value of $\langle \phi_1 \rangle$ at its minimum.

The value of $\langle \phi_1 \rangle$ and the mass of the massive scalar boson are determined by the potential and are of course not affected by the gauging. For the potential equation (11) one recovers from the equation of motion for the massive scalar,

$$\{ \partial^2 - V''(\langle \phi_1 \rangle) \} \delta \phi_1 = 0 \quad (35)$$

the mass $2\mu^2$ using $\langle \phi_1 \rangle^2 = \mu^2/\lambda$.

In this formulation, we see clearly from Eq. (33) how the NG boson is absorbed into a redefined massive vector field. The disappearance of the NG boson was further analyzed in Ref. [15]. In the gauge defined by Eq. (34), the field B_μ , which contains only the physical degrees of freedom of the massive vector, does appear. This is the ‘unitary gauge’ of the theory. In contradistinction, the field theoretic approach introduces a spurious $1/q^2$ pole in the polarization equation (19), which is not observable. The comparison between these two different approaches to massive gauge vector boson masses contains the germ of the renormalizability of the BEH mechanism, as will now be discussed.

3.4. The renormalization issue

The massive vector propagator equation (25) differs from a conventional free massive propagator in two respects. First the presence of the unobservable longitudinal term reflects the arbitrariness of the gauge parameter η . Second the NG pole at $q^2 = 0$ in the transverse projector $g_{\mu\nu} - q_\mu q_\nu / q^2$ is unconventional. Its significance is made clear by expressing the propagator of the A_μ field in Eq. (25) as (putting η to zero)

$$D_{\mu\nu}^a \equiv \frac{g_{\mu\nu} - q_\mu q_\nu / q^2}{q^2 - (\mu^2)^a} = \frac{g_{\mu\nu} - q_\mu q_\nu / (\mu^2)^a}{q^2 - (\mu^2)^a} + \frac{1}{(\mu^2)^a} \frac{q_\mu q_\nu}{q^2} \quad (36)$$

The first term in the right-hand side of Eq. (36) is the conventional massive vector propagator. It may be viewed as the (non-Abelian generalization of the) free propagator of the B_μ -field defined in Eq. (33) while the second term is a pure gauge propagator due to the NG boson ($[1/e \langle \phi_1 \rangle] \partial_\mu \phi_2$ in Eq. (33)) which converts the gauge field A_μ into the massive vector field B_μ .

The propagator equation (25) which appeared in the field theoretic approach contains thus, in the covariant gauges, the transverse projector $g_{\mu\nu} - q_\mu q_\nu / q^2$ in the numerator of the massive gauge field A_μ^a propagator. This is in sharp contradistinction to the numerator $g_{\mu\nu} - q_\mu q_\nu / (\mu^2)^a$ characteristic of the conventional massive vector field B_μ propagator. It is the transversality of the polarization tensor in covariant gauges, which led in the tree approximation to the transverse projector in Eq. (25). As mentioned above, the transversality of the polarization tensor is a consequence of a Ward identity and therefore does not rely on the tree approximation. This fact is already clear from the dynamical example equation (29) but was proven in more general terms in a subsequent publication⁷ [16]. The importance of this fact is that transversality in covariant gauges determines the power counting of irreducible diagrams. It is then straightforward to verify that the quantum field theory formulation has the required power counting for a renormalizable field theory. On this basis it was suggested that it indeed was renormalizable [16].

⁷ The proof given in Ref. [16] was not complete because closed Yang–Mills loops, which would have required the introduction of Fadeev–Popov ghosts were not included.

However power counting is not enough to prove the renormalizability of a theory with local gauge invariance. In addition, to be consistent, the theory must also be unitary, a fact which is not apparent in ‘renormalizable’ covariant gauges but is manifest in the ‘unitary gauge’ defined in the free theory by the B_μ -field introduced in Eq. (33). In the unitary gauge however, power counting requirements fail. The equivalence between the A_μ and B_μ free propagators, *which is only true in a gauge invariant theory* where their difference is the unobservable NG propagator appearing in Eq. (36), is a clue of the consistency of the BEH theory. It is of course a much harder and subtler affair to prove that the full interacting theory is both renormalizable and unitary. This was achieved in the work of ’t Hooft and Veltman, which thereby established the consistency of the BEH mechanism [17].

4. Consequences and perspectives

In the electroweak theory [1], the gauge group is taken to be $SU(2) \times U(1)$ with corresponding generators and coupling constants $g A_\mu^a T^a$ and $g' B_\mu Y'$. The $SU(2)$ acts on left-handed fermions only. The scalar field ϕ is a doublet of $SU(2)$ and its $U(1)$ charge is $Y' = 1/2$. Breaking is characterized by $\langle \phi \rangle = 1/\sqrt{2} \{0, v\}$ and $Q = T^3 + Y'$ generates the unbroken subgroup. Q is identified with the electromagnetic charge operator. The only residual massless gauge boson is the photon and the electric charge e is usually expressed in terms of the mixing angle θ as $g = e/\sin \theta$, $g' = e/\cos \theta$.

Using Eqs. (21) and (24) one gets the mass matrix

$$|\mu^2| = \frac{v^2}{4} \begin{vmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g'^2 & -gg' \\ 0 & 0 & -gg' & g^2 \end{vmatrix}$$

whose diagonalization yields the eigenvalues

$$M_{W^+}^2 = \frac{v^2}{4} g^2, \quad M_{W^-}^2 = \frac{v^2}{4} g^2, \quad M_Z^2 = \frac{v^2}{4} (g'^2 + g^2), \quad M_A^2 = 0 \quad (37)$$

This permits to relate v to the four Fermi coupling G , namely $v^2 = (\sqrt{2}G)^{-1}$.

Although the electroweak theory has been amply verified by experiment, the existence of the massive scalar boson has, as yet, not been confirmed. It should be noted that its physics is, as previously discussed, more sensitive to the dynamical assumptions of the model than the massive vectors W^\pm and Z , be it a genuine elementary field or a manifestation of a composite due to a more elaborate mechanism. Observation of its mass and width is of particular interest for further understanding of the mechanism at work.

The discovery that confinement could be found in the strong coupling limit of quantum chromodynamics based on the ‘color’ gauge group $SU(3)$ led to tentative Grand Unification schemes where electroweak and strong interaction could be unified in a simple gauge group G containing $SU(2) \times U(1) \times SU(3)$ [18]. Breaking occurs through vacuum expectation values of scalar fields and unification is apparent at high energies because, while the renormalization group makes the small gauge coupling of $U(1)$ increase logarithmically with the energy scale, the converse is true for the asymptotically free non Abelian gauge groups.

Also, the BEH mechanism has put into evidence concepts which may have a profound impact on further research. One of the richest sources of such concepts is the discovery by ’t Hooft and Polyakov of regular monopoles in non Abelian gauge theories [19]. Also of interest is the geometrical interpretation of the mechanism in the context of the string theory approach. Discussion of these developments are outside the scope of this paper and are reviewed for instance in Ref. [20].

Originally the BEH mechanism was conceived to unify the theoretical description of long-range and short-range forces. The success of the electroweak theory made the mechanism a candidate for further unification. Grand unification schemes, where the scale of unification is pushed close to the scale of quantum gravity effects, strengthen the believe in a still larger unification that would include gravity. This trend towards unification received a further impulse from the developments of string theory and from its connection with eleven-dimensional supergravity. The latter is often viewed as a classical limit of a hypothetical M-theory into which all perturbative string theories would merge to yield a comprehensive theory of ‘all’ interactions.

Such vision may be premature. Quite apart from obvious philosophical questions raised by a ‘theory of everything’ formulated in the present framework of theoretical physics, the transition from perturbative string theory to its

M-theory generalization hitherto stumbles on the treatment of nonperturbative gravity. This might well be a hint that new conceptual elements have to be found to cope with the relation between gravity and quantum theory and which might not be directly related to the unification program.

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