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The mystery of the Higgs particle/Le mystère de la particule de Higgs

Higgsless approach to electroweak symmetry breaking

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Abstract

Higgsless models are an attempt to achieve a breaking of the electroweak symmetry via boundary conditions at the end-points of a fifth dimension compactified on an interval, as an alternative to the usual Higgs mechanism. There is no physical Higgs scalar in the spectrum and the perturbative unitarity violation scale is delayed via the exchange of massive spin-1 KK resonances. The correct mass spectrum is reproduced in a model in warped space, which inherits a custodial symmetry from a left–right gauge symmetry in the bulk. Phenomenological challenges as well as collider signatures are presented. From the AdS/CFT perspective, this model appears as a weakly coupled dual to walking technicolour models. ***To cite this article: C. Grojean, C. R. Physique 8 (2007).***

Résumé

Brisure de la symétrie électrofaible sans boson de Higgs. Les modèles « Higgsless » se veulent une alternative au mécanisme de Higgs et se proposent de briser la symétrie électrofaible par l'intermédiaire de conditions aux bords d'une dimension supplémentaire d'espace. Le spectre de masse ne contient aucune excitation scalaire, et l'unitarité perturbative est maintenue jusqu'à des énergies élevées grâce à l'échange de résonances de spin 1 massives. Le spectre du modèle standard est correctement reproduit dans un modèle possédant une symétrie de jauge gauche–droite qui va jouer le rôle de la symétrie custodiale. Les principaux obstacles à la construction d'un modèle complètement réaliste sont présentés, ainsi que les principales signatures expérimentales attendues. Dans le contexte des dualités AdS/CFT, ces modèles s'interprètent comme des versions faiblement couplées des théories de type technicouleur. ***Pour citer cet article : C. Grojean, C. R. Physique 8 (2007).***

Keywords: Electroweak symmetry breaking; Extra dimensions; Higgsless

Mots-clés : Brisure de symétrie électrofaible ; Dimensions supplémentaires ; Higgsless

1. Introduction

The quest to unravel the origin of electroweak symmetry breaking has been at the forefront of particle physics for 25 years, and after a tremendous amount of theoretical effort it is clearer than ever that we will need experiments to answer the question. This has been further emphasized by the explosion, in the last few years, of a plethora of alternative electroweak symmetry breaking scenarios [1], which bear little resemblance to the three traditional solutions:

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the standard model (SM), the minimal supersymmetric standard model, and the technicolour scenario. In addition to large extra dimensions [2], warped extra dimensions [3], gauge component Higgses [4], 'little' Higgses [5], and 'fat' Higgses [6], one of the most recent proposals, and in some ways most radical, is the Higgsless scenario [7,8] (see [1] for a complete list of references on various aspects of Higgsless theories).

The idea behind Higgsless theories is that a momentum along an extra dimension is equivalent to a mass in 4D, so that we can generate a mass by giving a momentum to a particle in an extra dimension. And, as in quantum mechanics, a non-zero momentum along a compact direction can result from non-trivial boundary conditions (BCs). Therefore the work consists in identifying the appropriate BCs and the geometry of the extra dimension to reproduce the spectrum and the couplings of the SM.

How to break a gauge symmetry by orbifold compactification, i.e. by some particular BCs, has been known for a long time. This raises the hope of achieving a breaking of the electroweak (EW) symmetry directly by BCs. However, there are obvious obstacles to the construction of a realistic model [9]:

- *Rank reduction*: In usual (Abelian) orbifold compactifications, the rank of the gauge group cannot be reduced unless the orbifold projection corresponds to an outer automorphism of the gauge symmetry. For a given algebra, the number of automorphisms is limited and, in particular, it is not possible to break $SU(2) \times U(1)$ down to $U(1)$. So more general BCs than those obtained from simple orbifold projection have to be considered.
- *Non-rational mass ratio*: In usual Kaluza–Klein (KK) compactifications, the spectrum is dictated by the geometry of the extra dimensions, and the mass gap between two KK states is given by an integer times the inverse size of the extra dimension. So it seems non-trivial to obtain a mass ratio of the W to the Z that is related to the gauge couplings.
- *Unitarity restoration*: Since the seminal works of [10–14], the Higgs boson has been known to play an essential role in restoring perturbative unitarity in the scattering of longitudinal massive gauge bosons. Thus the question that the 5D theories we want to consider raise, is whether such a breaking of the gauge symmetries via BCs yields a consistent theory or not, or, in other words, whether a momentum along a fifth dimension is UV-safer than a regular 4D gauge boson mass. In order to verify that such a breaking is indeed soft, we need to investigate the issue of unitarity of scattering amplitudes in such 5D gauge theories compactified on an interval, with non-trivial BCs. We will derive the general expression for the amplitude for elastic scattering of longitudinal gauge bosons, and we will write down the necessary conditions for the cancellation of the terms that grow with energy. We will find that all the consistent BCs are unitary in the sense that all terms proportional to E^4 and E^2 vanish. In fact, any theory with only Dirichlet or Neumann BCs is unitary. Surprisingly, this would also include theories where the boundary conditions can be thought of as coming from a very large expectation value of a brane-localized Higgs field. In the limit when the expectation value diverges, there are no scalar degrees of freedom at low energy, thus the name of Higgsless theories.

2. Gauge symmetry breaking by boundary conditions

2.1. Boundary conditions for a gauge field

From now on, we will consider a single extra dimension that consists of an interval, $y \in [0, \pi R]$. We need to specify boundary conditions at the end of this interval for the various fields that propagate in the bulk. The appropriate BCs can be derived from a variational principle. A gauge field in 5D, A_M, contains a 4D gauge field, A_μ, and a 4D scalar, A_5. The 4D vector will contain a whole KK tower of massive gauge bosons; however, the KK tower of the A_5 will be eaten by the massive gauge fields and (except for a possible zero mode) will be non-physical. We can guess that this is what happens from the fact that the Lagrangian contains a mixing term between the gauge fields and the scalar, reminiscent of the usual 4D Higgs mechanism. Including appropriate gauge-fixing terms in the bulk and on the brane, the full Lagrangian can be written:

$$\mathcal{S} = \int \mathrm{d}^5 x \left(-\frac{1}{4} F^a_{MN} F^{MNa} - \frac{1}{2\xi} (\partial_\mu A^{\mu a} - \xi \partial_5 A^a_5)^2 - \frac{1}{2\xi_{bd}} (\partial_\mu A^{\mu a} \pm \xi_{bd} A^a_5)^2_{|0,\pi R} \delta(y - y_{bd}) \right) \tag{1}$$

where the field strength is given by the usual expression, $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_5 f^{abc} A^b_M A^c_N$, g_5 is the 5D gauge coupling, which has mass dimension $-1/2$, and ξ and ξ_{bd} are some gauge-fixing parameters (the $-$ sign is for $y = 0$, the + for $y = \pi R$). The theory is non-renormalizable, so it has to be considered as a low-energy effective theory valid below a cutoff scale, that we will be calculating later on.

We can see that the A_5^a field has a term $\xi \partial_5^2 A_5^a$ in its equation of motion. This will imply that if the wave function is not flat (e.g. the KK mode is not massless), then the field is not physical (since in the unitary gauge $\xi \to \infty$, this field will have an infinite effective 4D mass and decouples). This shows that the scalar KK tower of A_5^a will be completely unphysical, owing to the 5D Higgs mechanism, except perhaps for a zero mode for A_5^a. Whether or not there is a zero mode depends on the BCs for the A_5 field. In Higgsless models, there would not be any A_5 zero mode. Requiring that the variation of the action vanish at the boundaries leads to the BCs obeyed by the gauge fields

$$\partial_5 A^{\mu a} \pm \frac{1}{\xi_{bd}} \partial_\mu \partial^\mu A^{\mu a} = 0 \quad \text{and} \quad \xi \partial_5 A_5^a \pm \xi_{bd} A_5^a = 0 \tag{2}$$

This simplifies quite a bit if we go to the unitary gauge on the boundary given by $\xi_{bd} \to \infty$. In this case we are left with the simple set of boundary conditions

$$\partial_5 A^{\mu a} = 0 \quad \text{and} \quad A_5^a = 0 \tag{3}$$

These are the BCs that one usually imposed for gauge fields in the absence of any boundary terms. With these BCs, the full gauge group would remain unbroken. To reduce the gauge symmetry, one needs to introduce some dynamics at the boundaries, as for example some scalar fields acquiring vacuum expectation values (*vev*'s).

2.2. *Higgs mechanism localized on a boundary: scalar decoupling limit*

Let us now consider the case when scalar fields that develop *vev*'s are added on the boundary. Instead of repeating a full and general analysis (that can be found in [15]), we will present a concrete example. We consider (see Fig. 1) a $SU(2)$ gauge group with Newmann BCs for the A_μ components at both ends of the interval. We then assume that at $y = \pi R$, $SU(2)$ is fully broken by the *vev* of a Higgs doublet. As for the scalar case, the boundary mass generated by the Higgs *vev* induces a mixed BC of the form

$$\partial_5 A_\mu^a(\pi R) = -\frac{1}{4} g_{5D}^2 v^2 A_\mu^a(\pi R) \tag{4}$$

The canonically normalized KK modes are given by

$$A_\mu^a(x, y) = \sum_{k=1}^{\infty} f_k(y) A_\mu^{(k)}(x) \tag{5}$$

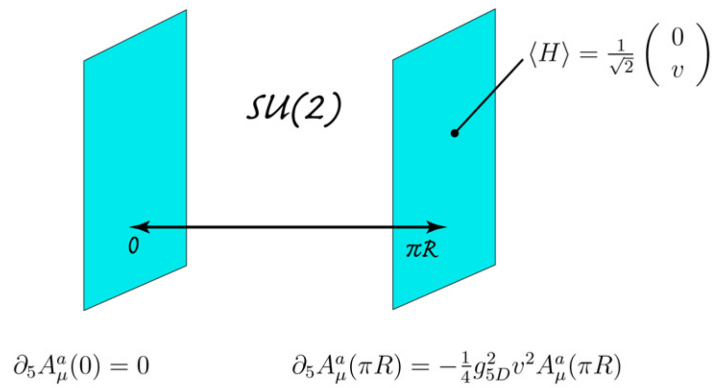


Fig. 1. Example of a Higgs mechanism localized on a boundary. For a finite Higgs *vev*, we obtain a mixed BC which, in the infinite *vev* limit, simply becomes a Dirichlet BC: all the gauge bosons that couple to the Higgs have a wave-function that vanishes at the point where the Higgs is localized. In that limit, there is no scalar degree of freedom in the low energy effective action and the gauge symmetry is entirely broken by the BCs and the mass of the lightest KK state is simply inversely proportional to the size of the extra dimension.

with

$$f_k(y) = \frac{\sqrt{2}}{\sqrt{\pi R(1 + 16M_k^2/(g_{5D}^4 v^4)) + 4/(g_{5D}^2 v^2)}} \frac{\cos(M_k y)}{\sin(M_k \pi R)} \tag{6}$$

The BC at the origin, $y = 0$, is trivially satisfied while the condition at $y = \pi R$ determines the mass spectrum through the equation:

$$M_k \tan(M_k \pi R) = \frac{1}{4} g_{5D}^2 v^2 \tag{7}$$

In the large *vev* limit, we obtain that the wave-functions at the $y = \pi R$ boundary vanish like $1/v^2$

$$f_k(\pi R) \sim 2\sqrt{\frac{2}{\pi R}} \frac{2k+1}{g_{5D}^2 R v^2} \tag{8}$$

while the KK masses remain finite

$$M_k \sim \frac{2k+1}{2R}\left(1 - \frac{4}{g_{5D}^2 \pi R v^2}\right) \tag{9}$$

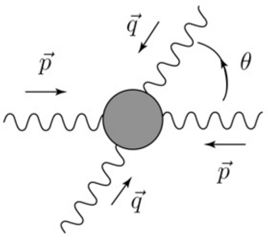
This limit exactly corresponds to a Dirichlet BC: in the large *vev* limit, the wave-functions of the gauge bosons that couple to the Higgs vanish. It can also be checked that, in that limit, A_5 actually obeys a Neumann BC. Though, in our example, because of the other Dirichlet BC at $y = 0$, there is still no physical massless mode for A_5, while the would-be massive ones are eaten to give the longitudinal polarizations of the massive A_μ. What allows us to decouple the Higgs degree of freedom from the low energy action is that, contrary to 4D, the masses of the gauge bosons are not proportional to the Higgs *vev*.

3. Unitarity restoration by KK modes. Sum rules of Higgsless theories

Our aim is to build a Higgsless model of electroweak symmetry breaking using BC breaking in extra dimensions. However, there is a problem in theories with massive gauge bosons without a Higgs scalar: the scattering amplitude of longitudinal gauge bosons will grow with the energy and violate unitarity at a low scale [10–14]. What we would like to first understand is what happens to this unitarity bound in a theory with extra dimensions [7] (for a complete list of references, see [1]). For simplicity we will be focusing on the elastic scattering of the longitudinal modes of the nth KK mode (see Fig. 2). The energy dependence can be estimated from $\epsilon \sim E$, $p_\mu \sim E$ and a propagator $\sim E^{-2}$. This way we find that the amplitude could grow as quickly as E^4, and then for $E \gg M_W$ can expand the amplitude in decreasing powers of E as

$$\mathcal{A} = \mathcal{A}^{(4)} \frac{E^4}{M_n^4} + \mathcal{A}^{(2)} \frac{E^2}{M_n^2} + \mathcal{A}^{(0)} + \mathcal{O}\left(\frac{M_n^2}{E^2}\right) \tag{10}$$

In the SM (and any theory where the gauge kinetic terms form the gauge-invariant combination $F_{\mu\nu}^2$) the $\mathcal{A}^{(4)}$ term automatically vanishes, while $\mathcal{A}^{(2)}$ is only cancelled after taking into account the Higgs exchange diagrams.



$$\epsilon_\mu = \left(\frac{|\vec{p}|}{M_n}, \frac{E}{M_n}\frac{\vec{p}}{p}\right)$$
$$p_\mu^{in} = \left(E, 0, 0, \pm\sqrt{E^2 - M_n^2}\right)$$
$$q_\mu^{out} = \left(E, \pm\sqrt{E^2 - M_n^2}\sin\theta, 0, \pm\sqrt{E^2 - M_n^2}\cos\theta\right)$$

Fig. 2. Elastic scattering of longitudinal modes of KK gauge bosons, $n + n \to n + n$, with the gauge index structure $a + b \to c + d$. The E-dependence can be estimated from $\epsilon \sim E$, $p_\mu \sim E$ and a propagator $\sim E^{-2}$.

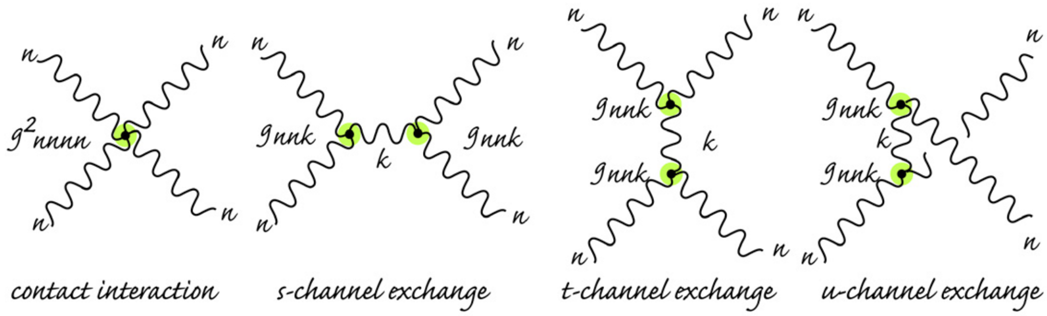


Fig. 3. The four diagrams contributing at tree level to the elastic scattering amplitude of the nth KK mode.

In the case of a theory with an extra dimension with BC breaking of the gauge symmetry, there are no Higgs exchange diagrams; however, one needs to sum up the exchanges of all KK modes, as in Fig. 3. As a result we will find the following expression for the terms in the amplitudes that grow with energy:

$$\mathcal{A}^{(4)} = \mathrm{i}\left(g_{nnnn}^2 - \sum_k g_{nnk}^2\right) a^{(4)}(\theta) \tag{11}$$

with

$$a^{(4)}(\theta) = \left(f^{abe} f^{cde}\left(3 + 6\cos\theta - \cos^2\theta\right) + 2\left(3 - \cos^2\theta\right) f^{ace} f^{bde}\right) \tag{12}$$

In order for the term $\mathcal{A}^{(4)}$ to vanish it is sufficient to ensure that the following sum rule between the couplings of the various KK modes is satisfied [7]:

$$E^4 \text{ sum rule:} \quad g_{nnnn}^2 = \sum_k g_{nnk}^2 \tag{13}$$

Assuming $\mathcal{A}^{(4)} = 0$ we get

$$\mathcal{A}^{(2)} = \frac{\mathrm{i}}{M_n^2}\left(4 g_{nnnn} M_n^2 - 3\sum_k g_{nnk}^2 M_k^2\right) a^{(2)}(\theta) \tag{14}$$

with

$$a^{(2)}(\theta) = \left(f^{ace} f^{bde} - \sin^2\frac{\theta}{2}\, f^{abe} f^{cde}\right) \tag{15}$$

Assuming that relation (13) holds, we can find a sum rule that ensures the vanishing of the $\mathcal{A}^{(2)}$ term [7]:

$$E^2 \text{ sum rule:} \quad g_{nnnn} M_n^2 = \frac{3}{4}\sum_k g_{nnk}^2 M_k^2 \tag{16}$$

Here g_{nnnn}^2 is the quartic self-coupling of the nth massive gauge field, while g_{nnk} is the cubic coupling between the KK modes. In theories with extra dimensions these are of course related to the extra dimensional wave-functions, $f_n(y)$, of the various modes as[1]

$$g_{mnk} = g_5 \int \mathrm{d}y\, f_m(y) f_n(y) f_k(y) \quad \text{and} \quad g_{mnkl}^2 = g_5^2 \int \mathrm{d}y\, f_m(y) f_n(y) f_k(y) f_l(y) \tag{17}$$

[1] These expressions of the effective cubic and quartic couplings are valid in flat space. When the fifth dimension is curved, appropriate powers of the warp factor appear. The scalar product used in the completeness relation has to be modified accordingly. At the end of the day, the same sum rules still hold.

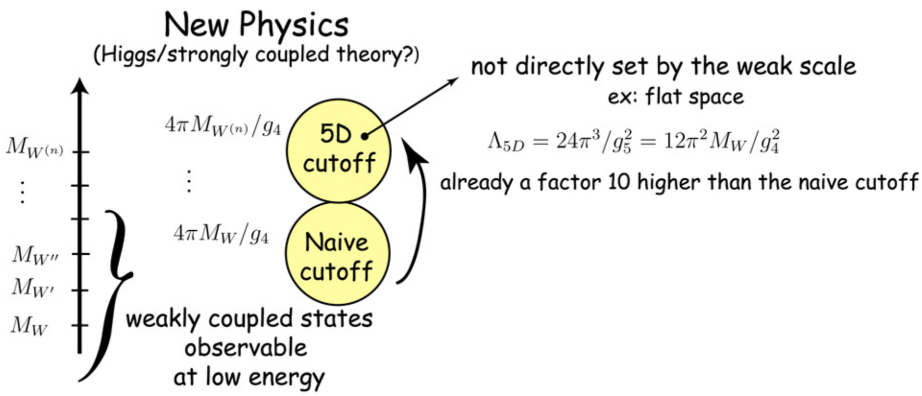


Fig. 4. The scattering amplitude of the longitudinal components of the lightest massive KK gauge boson would naively become non-perturbative at an energy scale $4\pi M_W/g_4$. However, before reaching that scale, the exchange of the KK excitations starts cancelling the scattering amplitude. The story repeats itself until reaching the heaviest KK mode below the 5D cutoff, for which no heavier excitations can intervene to smoothen its scattering amplitude. Thus the perturbative unitarity breakdown has been delayed and pushed to a scale that is not directly related to the mass of the lightest massive gauge bosons. A detailed analysis [16] of inelastic channels confirms the loss of perturbative unitarity at an energy scale related to the 5D cutoff.

Amazingly, higher-dimensional gauge invariance will ensure that both of the sum rules are satisfied as long as the breaking of the gauge symmetry is spontaneous. For example, it is easy to show the first sum rule via the completeness of the wave functions $f_n(y)$:

$$\int_0^{\pi R} \mathrm{d}y\, f_n^4(y) = \sum_k \int_0^{\pi R} \mathrm{d}y \int_0^{\pi R} \mathrm{d}z\, f_n^2(y) f_n^2(z) f_k(y) f_k(z) \tag{18}$$

and using the completeness relation $\sum_k f_k(y) f_k(z) = \delta(y-z)$, we can see that the two sides of Eq. (13) will indeed agree. One can similarly show [7] that the second sum rule will also be satisfied if the boundary conditions are natural and all terms in the Lagrangian (including boundary terms) are gauge-invariant. Let us insist on the particular case of a Higgs mechanism localized at the boundary: for finite Higgs *vev*, the cancellation of the E^2 term requires the exchange of the brane Higgs scalar degree of freedom; however, in the infinite *vev* limit, the contribution of the Higgs exchange to the scattering amplitude actually cancels out and we are left with simple Dirichlet BCs for which the scattering amplitude is unitarized by the sole exchange of spin-1 KK excitations.

At this point, it should be noted that the two sum rules cannot be satisfied with a *finite* number of KK modes. This is in full agreement with the old theorem by Cornwall et al. [12], who established that the only way to restore perturbative unitarity in the scattering of massive spin-1 particles is through the exchange of a scalar Higgs boson. Our 5D theory is non-renormalizable anyway, so it is valid up to a finite cutoff. What our result really shows is that, through the exchange of the KK gauge bosons, the perturbative unitarity breakdown is postponed from an energy scale of the order of the mass of the lightest KK state to the true 5D cutoff of the order of the mass of the heaviest KK state; see Fig. 4.

What we see from the above analysis is that in any gauge-invariant extra-dimensional theory the terms in the amplitude that grow with the energy will cancel. However, this will not automatically mean that the theory itself is unitary. The reason is that there are two additional worries: even if $\mathcal{A}^{(4)}$ and $\mathcal{A}^{(2)}$ vanish, $\mathcal{A}^{(0)}$ could be too large and spoil unitarity. This is what happens in the SM if the Higgs mass is too large. In the extra-dimensional case what this would mean is that the extra KK modes would make the scattering amplitude flatten out to a constant value. However, if the KK modes themselves are too heavy then this flattening out will happen too late, when the amplitude already violates unitarity. The other issue is that, in a theory with extra dimensions, there are infinitely many KK modes and thus as the scattering energy grows one should not only worry about the elastic channel, but the ever growing number of possible inelastic final states. The full analysis taking into account both effects has been performed in [16], where it was shown that, after taking into account the opening up of the inelastic channels, the scattering amplitude will grow linearly with energy and will always violate unitarity at some energy scale. This is a consequence of the intrinsic non-renormalizability of the higher-dimensional gauge theory. It was found in [16] that the unitarity violation scale due to the linear growth of the scattering amplitude is equal (up to a small numerical factor of order 2–4) to the cutoff scale of the 5D theory obtained from naive dimensional analysis (NDA). This cutoff scale can be estimated in the following way. The one-loop amplitude in 5D is proportional to the 5D loop factor $g_5^2/(24\pi^3)$. The dimensionless quantity obtained from this loop factor is $g_5^2 E/(24\pi^3)$, where E is the scattering energy. The cutoff scale can be obtained by calculating the energy scale at which this loop factor will become of order 1 (that is the scale at which the loop and tree-level contributions become comparable). From this we get $\Lambda_{\mathrm{NDA}} = 24\pi^3/g_5^2$. We can express this scale by using the matching of the higher-dimensional and the lower-dimensional gauge couplings. In the simplest theories this is usually given by $g_5^2 = \pi R g_4^2$, where πR is the length of the interval, and g_4 is the effective 4D gauge coupling. So the final expression of the cutoff scale can be given as

$$\Lambda_{\mathrm{NDA}} = \frac{24\pi^2}{g_4^2 R} \tag{19}$$

We will see that in the Higgsless models $1/R$ will be replaced by M_W^2/M_{KK}, where M_W is the physical W mass, and M_{KK} is the mass of the first KK mode beyond the W. Thus the cutoff scale will indeed be lower if the mass of the KK mode used for unitarization is higher. However, this Λ_{NDA} could be significantly higher than the cutoff scale in the SM without a Higgs, which is around 1.2 TeV.

4. Warped Higgsless model with custodial symmetry

It is clear that in order to find a Higgsless model with the correct W/Z mass ratio one needs to find an extra-dimensional model that has the custodial $SU(2)$ symmetry incorporated [17]. Therefore we need to somehow involve $SU(2)_R$ in the construction. The simplest possibility is to put an entire $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group in the bulk of an extra dimension [7]. In order to mimic the symmetry-breaking pattern in the SM most closely, we assume that on one of the branes the symmetry breaking is $SU(2)_L \times SU(2)_R \to SU(2)_D$, with $U(1)_{B-L}$ unbroken. On the other boundary, the bulk gauge symmetry must be reduced to that of the SM, and thus have a symmetry-breaking pattern $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$, which is illustrated in Fig. 5. The custodial symmetry is broken on one boundary. To reduce the effect of this breaking on the KK modes, we need to engineer a setup such that all the KK wave-functions are localized away from the point where the custodial symmetry is broken. This is automatically achieved if the space is curved by a negative vacuum energy to an anti-de Sitter (AdS) background.

The appropriate BCs are

$$\text{at } z = R: \quad \partial_z\big(g_{5R} B_\mu + \tilde{g}_5 A_\mu^{R3}\big) = 0, \quad \partial_z A_\mu^{La} = 0, \quad A_\mu^{R1,2} = 0, \quad \tilde{g}_5 B_\mu - g_{5R} A_\mu^{R3} = 0 \tag{20}$$

$$\text{at } z = R': \quad \partial_z\big(g_{5R} A_\mu^{La} + g_{5L} A_\mu^{Ra}\big) = 0, \quad \partial_z B_\mu = 0, \quad g_{5L} A_\mu^{La} - g_{5R} A_\mu^{Ra} = 0 \tag{21}$$

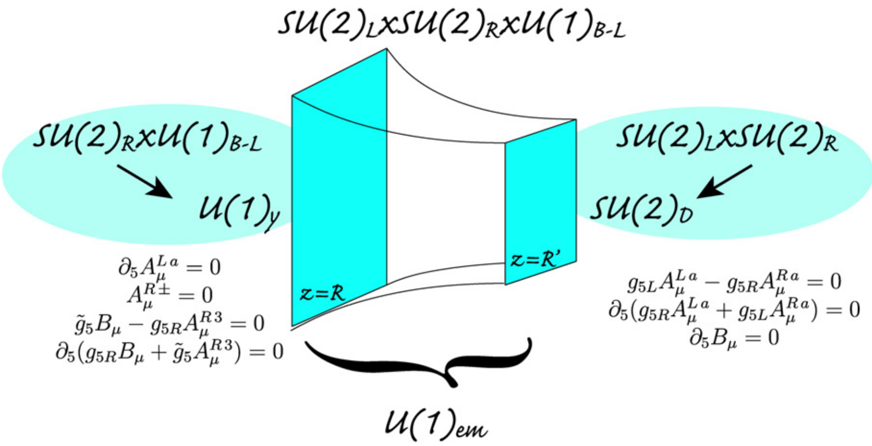


Fig. 5. The symmetry-breaking structure of the warped Higgsless model [8]. We will be considering a 5D gauge theory in the fixed gravitational anti-de Sitter (AdS) background. The UV brane (sometimes called the Planck brane) is located at $z = R$ and the IR brane (also called the TeV brane) is located at $z = R'$. R is the AdS curvature scale. In conformal coordinates, the AdS metric is given by $\mathrm{d}s^2 = (R/z)^2(\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \mathrm{d}z^2)$.

We denoted by A_μ^{Ra}, A_μ^{La} and B_μ the gauge bosons of $SU(2)_R$, $SU(2)_L$ and $U(1)_{B-L}$ respectively; g_{5L} and g_{5R} are the gauge couplings of the two $SU(2)$'s, and $\tilde{g}_5$ the gauge coupling of the $U(1)_{B-L}$. The corresponding KK decomposition is given by

$$\begin{cases} B_\mu = g_5 a_0 \gamma_\mu(x) + \sum_{k=1}^\infty \psi_k^{(B)}(z) Z_\mu^{(k)}(x) \\ A_\mu^{L3} = \tilde{g}_5 a_0 \gamma_\mu(x) + \sum_{k=1}^\infty \psi_k^{(L3)}(z) Z_\mu^{(k)}(x) \\ A_\mu^{R3} = \tilde{g}_5 a_0 \gamma_\mu(x) + \sum_{k=1}^\infty \psi_k^{(R3)}(z) Z_\mu^{(k)}(x) \end{cases} \qquad \begin{cases} A_\mu^{L\pm} = \sum_{k=1}^\infty \psi_k^{(L\pm)}(z) W_\mu^{(k)\pm}(x) \\ A_\mu^{R\pm} = \sum_{k=1}^\infty \psi_k^{(R\pm)}(z) W_\mu^{(k)\pm}(x) \end{cases} \tag{22}$$

The wavefunctions, solutions of the bulk equation of motion in AdS space, involve some Bessel functions of order 1

$$\psi_k^{(A)}(z) = z\big(a_k^{(A)} J_1(q_k z) + b_k^{(A)} Y_1(q_k z)\big) \tag{23}$$

On top of a flat massless mode, corresponding to the photon of the unbroken $U(1)_{\text{em}}$, the spectrum involves two light gauge bosons, naturally identified as the SM W and Z gauge bosons, with masses suppressed by $\log R'/R$ compared with the rest of the KK towers ($M_{\text{KK}} \sim \mathcal{O}(1)/R'$):

$$M_W^2 \sim \frac{2g_{5L}^2}{g_{5L}^2 + g_{5R}^2} \frac{1}{R'^2 \log R'/R} + \cdots, \qquad M_Z^2 \sim \frac{2g_{5L}^2}{g_{5L}^2 + g_{5R}^2} \frac{g_{5R}^2 + \tilde{g}_5^2(1 + g_{5R}^2/g_{5L}^2)}{g_{5R}^2 + \tilde{g}_5^2} \frac{1}{R'^2 \log R'/R} + \cdots \tag{24}$$

where $\cdots$ denote corrections in $1/\log^2(R'/R)$. The coupling of the photon allows us to identify the 4D SM gauge couplings as functions of the 5D parameters:

$$\frac{1}{g^2} = \frac{R \log R'/R}{g_{5L}^2} \quad \text{and} \quad \frac{1}{g'^2} = R \log R'/R \left(\frac{1}{g_{5R}^2} + \frac{1}{\tilde{g}_5^2} \right) \tag{25}$$

The ρ parameter is thus equal to 1, as announced earlier. This equality would not occur if the extra dimension were flat, it is a consequence of the localization property of the KK wave-functions, which ensure that the bulk gauge $SU(2)_R$ symmetry acts as a custodial symmetry. The presence of this approximate global symmetry can also be easily understood from the AdS/CFT duality. From that perspective, our 5D warped Higgsless model appears as a weakly coupled dual of walking technicolour models [8].

Finally, after redshift due to the warping of the space, the NDA cutoff scale is estimated to be

$$\Lambda_{\text{NDA}} \sim \frac{24\pi^3}{g_5^2} \frac{R}{R'} \sim \frac{24\pi^3}{g^2} \frac{1}{R' \log R'/R} \sim \frac{50\pi^3}{g^2} \frac{M_W^2}{M_{KK}} \tag{26}$$

As dictated by intuition, the smaller M_{KK}, the higher the scale where perturbative control is lost. Phenomenologically, the preferred range of M_{KK} will be around 500 GeV to 1 TeV.

5. Fermion masses

In the Standard Model, quarks and leptons acquire a mass after electroweak symmetry breaking through their Yukawa couplings to the Higgs. In the absence of a Higgs, one cannot write any Yukawa coupling and one should expect the fermions to remain massless. However, as for the gauge fields, appropriate BCs will force the fermions to acquire a momentum along the extra dimension and this is how they will become massive from the 4D point of view.

The SM fermions cannot be completely localized on the UV boundary: since the unbroken gauge group on that boundary coincides with the SM $SU(2)_L \times U(1)_Y$ symmetry, the theory on that brane would be chiral and there is no way for the chiral zero-mode fermions to acquire a mass. The SM fermions cannot live on the IR brane either since the unbroken $SU(2)_D$ gauge symmetry will impose an isospin-invariant spectrum and the up-type and down-type quarks will be degenerate, as well as the electron and the electron neutrino. The only possibility is thus to embed the SM fermions into 5D fields living in the bulk and feeling the gauge symmetry breakings on both boundaries. Bulk fermions are generically Dirac fermions; however, on an interval in warped space only one of the chiralities will have a zero mode. The location of the zero mode in warped space depends on the bulk mass term [18], and can be localized close to the UV brane for all the fermions of the first two generations and the leptons of the third generation. For the right-handed top quark, one can localize the wave function of the zero mode closer to the IR brane. Since the theory on the IR brane is vector-like (only $SU(2)_D$ is unbroken there), a mass for the zero modes can be added on the IR brane (which corresponds to a dynamical isospin symmetric fermion mass in the CFT language). The size of the physical mass will then depend on the location of the zero mode and the value of the mass term on the IR brane. However, because of the unbroken $SU(2)_D$ symmetry on the IR brane, these masses must be isospin-symmetric, that is the mass for the up and down type quarks are equal at this point. Isospin splitting can be introduced by adding operators on the UV brane. For instance one can introduce different brane-localized kinetic terms for the up and down right-handed quarks. The full spectrum of quarks and leptons can be easily reproduced this way [19].

6. Electroweak precision constraints, collider signatures and conclusions

Waiting for the LHC to reveal any signs of new physics, the major stumbling block for any theory beyond the SM is the level of corrections to electroweak precision measurements. And sharing so many resemblance with technicolour models, it is not a surprise that generically a large contribution to the S parameter of order unity is found [20]. This contribution can be lowered by introducing a brane kinetic term on the IR brane for the B–L gauge group, albeit at the price of lowering the mass of one of the Z' to phenomenologically unacceptable levels. In Ref. [21], it was pointed out that one can in fact easily eliminate the large contributions to the S parameter by changing the position of the light fermions. The reason behind this is simple: the oblique correction parameters on their own are meaningless until the normalization of the couplings between the fermions and the gauge bosons is fixed. An overall shift in the fermion–gauge-boson couplings can be reabsorbed in the oblique correction parameters and thus effectively change the predicted values of S, T [22]. This is exactly what happens when one changes the localization parameters of the light fermions. When the fermions are strictly localized on the UV brane, one obtains a positive S parameter. However, it has been known that if fermions are localized on the TeV brane then the S parameter in the Randall–Sundrum model is in fact negative. Therefore it should be expected that there should be an intermediate position where S exactly vanishes [23]. This actually happens when the fermion wave-functions are 'flat'. This is just a simple consequence of the orthogonality of the KK mode wave functions of the gauge bosons: when the fermion wave-functions are flat, the coupling of the KK gauge bosons to the fermions vanishes, eliminating any possible additional LEP or Tevatron constraints on this setup. This way, with an appropriate tuning of the localization of the fermions in the bulk, the model can pass the electroweak precision constraints.

A reason for localizing the light generations near the UV brane was that corrections to Flavour Changing Neutral Currents, coming from higher-order operators, should be suppressed by a large scale, of order $1/R$ rather than the strong coupling scale estimated earlier [24]. If we delocalize the light fermions, such scale is red-shifted to a dangerously low energy. In order to escape experimental bounds, we need to implement a flavour symmetry in the bulk and on the IR brane. Moreover, the mechanism that generates masses for the fermions themselves will induce some distortions in the wave functions, thus modifying in a non-universal way the couplings with the SM gauge bosons.

A more serious problem arises when one tries to introduce the third family [25]: there is a tension between the heaviness of the top and the coupling of the left-handed b_L to the Z gauge boson. It has recently been argued that this problem can be alleviated by a suitable embedding [26] of the SM third generation into non-standard representations of $SU(2)_L \times SU(2)_R$.

Many different realizations of Higgsless models have been proposed, differing in the way the SM fermions are introduced or even in the number of extra dimensions. All these models will have different particular signatures. However, the fundamental mechanism by which Λ is raised is a common feature to all these models: new massive spin-1 particles, with the same quantum numbers as the SM gauge bosons, appear at the TeV scale and their couplings to the W, Z and γ obey unitarity sum rules like (13) and (16), which enforce the cancellation of the energy-growing contributions to the scattering amplitudes of the longitudinal W, Z. Vector boson fusion processes will thus provide a model-independent test of the Higgsless scenario.

The non-observation of a physical scalar Higgs would be the first indication for a Higgsless scenario. Yet, *the absence of proof is not the proof of the absence* and some other models exist in which the Higgs is unobservable at the LHC and we need to look for other distinctive features of Higgsless models, such as the presence of spin-1 KK resonances with the W, Z quantum numbers, some slight deviations in the universality of the light fermion couplings to the SM gauge bosons, or some deviations in the gauge boson self-interactions compared with the SM. More than ever, experimental data are eagerly awaited to disentangle what may be the most pressing question faced by particle physics today: How is electroweak symmetry broken?

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