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The mystery of the Higgs particle/Le mystère de la particule de Higgs

The holographic composite Higgs

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Abstract

Theories where the Higgs boson is a composite particle elegantly solve the hierarchy problem. This idea has been recently investigated in the framework of 5-dimensional warped models that, according to the AdS/CFT correspondence, have a 4-dimensional holographic interpretation in terms of strongly coupled field theories. We present a minimal model in which the Higgs arises as a pseudo-Goldstone boson and the electroweak symmetry is dynamically broken. This model can successfully solve the flavor problem and pass all the electroweak precision tests. ***To cite this article: R. Contino, A. Pomarol, C. R. Physique 8 (2007).***

Résumé

Description holographique d'un boson de Higgs composite. Les théories dans lesquelles le boson de Higgs apparaît comme une particule composite apportent une solution élégante au problème de hiérarchie. Ce scénario a récemment connu un regain d'intérêt dans le contexte de théories dans des espaces courbés à cinq dimensions qui sont, selon la correspondence AdS/CFT, une description holographique de théories en couplage fort à quatre dimensions. Nous présentons ici un modèle minimale où le Higgs est un pseudo boson de Goldstone tandis que la symétrie électrofaible est brisée de façon dynamique. Ce modèle est en accord avec tous les tests de précisions et peut également prétendre à une solution du problème de saveur. ***Pour citer cet article : R. Contino, A. Pomarol, C. R. Physique 8 (2007).***

Keywords: Higgs boson; Pseudo-Goldstone boson; 5-dimensional warped model

Mots-clés : Boson de Higgs ; Pseudo boson de Goldstone ; Théorie des espaces courbés à cinq dimensions

1. Introduction

Introducing a Higgs boson represents a simple and economical way to break spontaneously the electroweak gauge symmetry of the Standard Model (SM) and cure the bad high-energy behaviour of its scattering amplitudes, thus allowing one to extrapolate the theory up to very high energies. It is hard to believe that Nature is not using such a simple mechanism to give us a UV-completed theory of electroweak interactions. Nevertheless, naturalness criteria suggest that the Higgs mechanism is unlikely to be just the last ingredient to be incorporated in the SM at the electroweak

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scale. Why is the Higgs mass, which determines the scale of electroweak symmetry breaking, so small compared to the Planck scale?

To answer this question one must postulate new dynamics, hence new particles, around m_W, making the simplicity of the Higgs mechanism for UV completing the SM just a curiosity. At this point, one might think that there is no need for the Higgs particle, and look for other ways to unitarize the SM scattering amplitudes. An example can be found in QCD, where pion–pion scattering is unitarized by the additional resonances that arise from the SU(3) color strong dynamics. This Higgsless approach, however, has to face the present electroweak precision tests (EWPT), and its simple incarnation (Technicolor models [1]) fails to pass them. The reason is that, without a Higgs, we expect the new particles responsible for unitarizing the SM amplitudes to have a mass at around 1 TeV. These same resonances give large (tree-level) contributions to the electroweak observables, in contradiction with the experimental evidence. How can we make the new resonances heavier and safely pass the EWPT?

A possible answer comes again from the Higgs mechanism. This time, however, the Higgs boson will play a less ambitious role, since it will only *partially* unitarize the SM scattering amplitudes. Compared to theories without a Higgs, we will have now that the scale at which new dynamics is needed can be delayed, and therefore the extra resonances that ultimately unitarize the SM amplitudes can be heavier. In this case precision observables will be under control. This is the approach of the composite Higgs models, first considered by Georgi and Kaplan [2]. In these theories a light Higgs arises as a Pseudo-Goldstone boson (PGB) of a strongly interacting theory, in a very similar way as pions in QCD. At low energies the composite Higgs has similar couplings to those of an elementary Higgs, but it behaves differently at high energy. In particular, it does not unitarize completely the SM amplitudes and, consequently, cross sections grow as powers of E/f_π (where f_π is the analog of the pion decay constant). A full unitarization is due to the additional resonances of the model, that we will denote by ρ. From a Naive Dimensional Analysis, and similarly to what happens in QCD, they are expected to have a mass $m_\rho \sim 4\pi f_\pi/\sqrt{N}$, where N is the number of 'colors' of the strongly interacting theory. For large values of f_π, one can have large values of m_ρ and avoid sizeable contributions to the EW precision observables. What determines the value of f_π? The Higgs, being a PGB, has a potential induced at the one-loop level via the SM interactions that do not respect the global symmetry of the new strongly interacting sector. The potential is of the form $V(h/f_\pi)$, since $1/f_\pi$ is the strength of the Higgs coupling (in complete analogy to that of the pions in QCD). We then expect a minimum at $v = \varepsilon f_\pi$, where ε is a model-dependent constant of order one. To have large values of f_π, and therefore large values of m_ρ, one needs $\varepsilon < 1$. This means that composite Higgs models can be potentially realistic if they accommodate small values of ε.

The first proposals for a composite Higgs [2], presented in the eighties, lacked several ingredients. First, they did not incorporate a heavy top (since its mass was not known at that time), and the largest SM contribution to the Higgs potential was that of the SM weak bosons. Since the latter always leads to an electroweak-preserving vacuum, the authors of Ref. [2] had to enlarge the SM gauge group in order to trigger EWSB. Second, the corrections to the EW precision observables were not calculated,[1] and flavor was also not successfully incorporated.

More recently, a new approach to building realistic composite Higgs theories was proposed in Refs. [3–6]. The idea was that of studying the composite Higgs in the framework of 5-dimensional (5D) field theories defined on a slice of Anti-de-Sitter (AdS) spacetime [7]. This approach was inspired by the AdS/CFT correspondence [8], which states that weakly coupled theories in 5D AdS have a 4D holographic description in terms of strongly coupled conformal field theories (CFT) with a large number of 'colors' N. Such correspondence gives a definite prescription on how to construct 5D theories that have the same physical behaviour and symmetries of the desired strongly coupled 4D theory.

One can proceed in the following way [4]. First, one can define the properties of the 4-dimensional CFT theory based on symmetry principles only. This is done in Section 2, following the criterium of minimality. Such a qualitative description, however, does not allow one to make definite, quantitative predictions for the various physical observables, mainly due to two reasons: first, we do not know if a CFT exists which fulfills our requirements; second, even if we knew the CFT in its Lagrangian formulation, we would not be able to make perturbative calculations, due to the strong regime of the theory. To overcome these problems, we take the following second step. Using the dictionary of the AdS/CFT correspondence, we construct the 5D AdS theory that leads to the same effective Lagrangian as the 4D CFT model described before. This is done in Section 3. Since the 5D theory is weakly coupled, we are able to

[1] LEP and SLD precision data were not yet available at the time, and, on the theory side, the strongly interacting regime of the theory would have prevented any kind of perturbative calculation.

perturbatively compute all the physical quantities of central interest. The model obtained in this way can address successfully the flavor problem, and also pass all the electroweak precision tests.

This particular example shows that theories of this kind can be considered as serious alternatives to supersymmetry. Most importantly, they give well defined predictions for the LHC: their spectrum contains a light Higgs, a tower of new states with the quantum numbers of each SM particle, plus other towers of massive states with exotic quantum numbers. In the specific case of the model of Section 3, the lightest resonances are color-triplet weak doublets of hypercharge $Y = 1/6$ and $7/6$, with a mass in the range 500–1500 GeV. They are therefore accessible at the LHC. We will briefly discuss their most promising production processes and decay channels in Section 4.

2. Higgs as a PGB of a strongly interacting sector

Let us consider a 4D theory that contains a strongly interacting sector with the following properties. It has a large number of 'colors' N, a mass gap at the infrared scale $\mu_{\rm IR} \sim$ TeV, and it is conformal at energies much higher than $\mu_{\rm IR}$. The mass gap is responsible for the formation of a tower of bound states with lowest mass of order $m_\rho \sim \mu_{\rm IR}$. These resonances interact among themselves with a coupling of order $4\pi/\sqrt{N}$ [9]. We assume that the global symmetry of the CFT is $G = {\rm SU}(3)_c \times {\rm SO}(5) \times {\rm U}(1)_X$, spontaneously broken down to $H = {\rm SU}(3)_c \times {\rm O}(4) \times {\rm U}(1)_X$ (with ${\rm O}(4) \supset {\rm SO}(4) \sim {\rm SU}(2)_L \times {\rm SU}(2)_R$) at a scale f_π (the analog of the pion decay constant). The operator responsible for this spontaneous breaking is assumed to have a large dimension. The group G is the minimal one that (i) contains the SM group, (ii) has a custodial symmetry, and (iii) delivers the Higgs doublet as a Goldstone boson of the breaking $G \to H$. Other groups have been proposed in Ref. [2] based on different motivations. We will assume that the SM gauge bosons and fermions are elementary fields external to the strongly interacting CFT. The top quark constitutes an exception and will be mostly composite, as we will see later. The SM gauge bosons couple to the CFT through its conserved currents, gauging the subgroup ${\rm SU}(3)_c \times {\rm SU}(2)_L \times {\rm U}(1)_Y$ contained in G ($Y = T_{3_R} + X$). In the following we will neglect the ${\rm SU}(3)_c$ color group, since it plays no role in the mechanism of EWSB. The Goldstone theorem tells us that the CFT must contain a massless scalar transforming as a **4** of SO(4), a real bidoublet of ${\rm SU}(2)_L \times {\rm SU}(2)_R$. We will identify this field with the Higgs boson.

The SM fermions are assumed to couple linearly to the strong sector through operators $\mathcal{O}$ made of CFT fields: $\mathcal{L} = \lambda\bar{\psi}\mathcal{O} + {\rm h.c.}$ The running coupling $\lambda(\mu)$ obeys the RG equation

$$\mu\frac{{\rm d}\lambda}{{\rm d}\mu} = \gamma\lambda + a\frac{N}{16\pi^2}\lambda^3 + \cdots \tag{1}$$

where the dots stand for terms subleading in the large-N limit, and a is an $\mathcal{O}(1)$ positive coefficient. The first term in Eq. (1) drives the energy scaling of λ as dictated by the anomalous dimension $\gamma = {\rm Dim}[\mathcal{O}] - 5/2$, ${\rm Dim}[\mathcal{O}]$ being the conformal dimension of the operator $\mathcal{O}$. The second term originates instead from the CFT contribution to the fermion wave-function renormalization. The low-energy value of λ is determined by γ. For $\gamma > 0$, the coupling of the elementary fermion to the CFT is irrelevant, and λ decreases with the energy scale μ. Below $\mu_{\rm IR}$, we have

$$\lambda \sim \left(\frac{\mu_{\rm IR}}{\Lambda}\right)^\gamma \tag{2}$$

where $\Lambda \sim M_{\rm Pl}$ is the UV cutoff of the CFT. Therefore, fermions with $\gamma > 0$ will have a small mixing with the CFT bound states, and thus small Yukawa couplings. For $\gamma < 0$, the coupling is relevant and λ flows at low energy to the fixed-point value $\lambda = (4\pi/\sqrt{N})\sqrt{-\gamma/a}$. In this case the mixing between the fermion and the CFT is large, and sizable Yukawa couplings can be generated for moderate values of N.

The model is then described by the Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\rm CFT} + \mathcal{L}_{\rm SM} + J^{a_L\mu}W_\mu^{a_L} + J_Y^\mu B_\mu + \sum_r \lambda_r\bar{\psi}_r\mathcal{O}_r + {\rm h.c.} \tag{3}$$

where the index r runs over the SM fermions (a family index is understood), and $W_\mu^{a_L}$ ($a_L = 1,2,3$) and B_μ stand respectively for the ${\rm SU}(2)_L$ and ${\rm U}(1)_Y$ gauge bosons. At the tree level the massless spectrum of the theory is that of the SM. The Higgs is the Goldstone boson and can be parametrized by the fluctuations along the SO(5)/SO(4) broken generators $T^{\hat{a}}$, $\hat{a} = 1,2,3,4$:

$$\Sigma = \Sigma_0 {\rm e}^{\Pi/f_\pi} = \frac{s_h}{h}\left(h^1, h^2, h^3, h^4, \frac{c_h}{s_h}h\right), \quad \Sigma_0 = (0,0,0,0,1), \Pi = -{\rm i}T^{\hat{a}}h^{\hat{a}}\sqrt{2} \tag{4}$$

where $h = \sqrt{(h^{\hat{a}})^2}$, and $c_h \equiv \cos(h/f_\pi)$, $s_h \equiv \sin(h/f_\pi)$. By integrating out the CFT dynamics, one can write an effective Lagrangian for the external fields. It is convenient to express this Lagrangian in an SO(5)-symmetric way. To do so, we promote the elementary SM fields to fill complete representations of SO(5). For the gauge bosons we introduce extra non-dynamical vectors, i.e. spurions, to form complete adjoint representations A_μ, B_μ of $\mathrm{SO}(5) \times \mathrm{U}(1)_X$. For the fermions we must choose which SO(5) representations they are embedded in. We consider the case in which the SM fermions are embedded in fundamentals (**5**) of SO(5) [10,6]. A slightly minimal possibility is to embed the fermions in spinorial representations (**4**) of SO(5) [4], though such choice leads to generally large corrections to the $Zb\bar{b}$ coupling [5,10]. Another alternative is to use antisymmetric representations (**10**) of SO(5) [6], which leads to a model similar to the one described here, see [6]. We will furthermore assume that each SM fermion is embedded in a different SO(5)-multiplet, and that the three SM families have all the same embedding. For the up-quark sector, we find that the minimal realistic choice is

$$\Psi_q = \begin{bmatrix} \begin{pmatrix} q'_L \\ q_L \end{pmatrix} \\ u'_L \end{bmatrix}, \qquad \Psi_u = \begin{bmatrix} \begin{pmatrix} q^u_R \\ q'^u_R \end{pmatrix} \\ u_R \end{bmatrix} \tag{5}$$

The multiplets Ψ_q and Ψ_u transform as $\mathbf{5_{2/3}}$ representations of $\mathrm{SO}(5) \times \mathrm{U}(1)_X$, and the components q'_L, u'_L and q^u_R, q'^u_R are the non-dynamical spurion fields. For the embedding of the down-quark sector and leptons, see Ref. [6].

We can now write the effective Lagrangian using an SO(5)-invariant notation. We integrate out all the massive CFT states at tree level, including the fluctuations of the Higgs field around a constant classical background Σ. The most general effective Lagrangian for the external gauge bosons is, in momentum space and at the quadratic level,

$$\mathcal{L}^g_{\text{eff}} = \frac{1}{2} P_{\mu\nu} \big[\Pi^B_0(p) B^\mu B^\nu + \Pi_0(p) \mathrm{Tr}\big[A^\mu A^\nu\big] + \Pi_1(p) \Sigma A^\mu A^\nu \Sigma^T \big] \tag{6}$$

where $P_{\mu\nu} = \eta_{\mu\nu} - p_\mu p_\nu / p^2$. For the up-quark sector we have

$$\begin{aligned} \mathcal{L}^f_{\text{eff}} = {} & \bar{\Psi}^i_q \not{p} \big[\delta^{ij} \Pi^q_0(p) + \Sigma^i \Sigma^j \Pi^q_1(p)\big] \Psi^j_q + \bar{\Psi}^i_u \not{p} \big[\delta^{ij} \Pi^u_0(p) + \Sigma^i \Sigma^j \Pi^u_1(p)\big] \Psi^j_u \\ & + \bar{\Psi}^i_q \big[\delta^{ij} M^u_0(p) + \Sigma^i \Sigma^j M^u_1(p)\big] \Psi^j_u + \text{h.c.} \end{aligned} \tag{7}$$

where the indices i, j run over the SO(5) components. The form factors $\Pi(p)$, $M(p)$ encode the effects of the strong dynamics, and cannot be determined perturbatively in the 4D theory. Their poles match with the CFT spectrum. We have not written down possible bare kinetic terms and gauge-fixing terms for the external fields, i.e. terms not induced by the strong dynamics. They can be included in a straightforward way. We are only interested in the two-point form factors since, as we will see below, these are the only ones needed for the calculation of the Peskin–Takeuchi S parameter and of the Higgs potential.

From Eq. (6) one can derive the low-energy effective theory. This is the theory of the light states, the SM fields and the Higgs, obtained by performing an expansion in derivatives and light fields over m_ρ (the equivalent of the chiral Lagrangian in QCD):

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} - V(\Sigma) + \Delta\mathcal{L} \tag{8}$$

The term $\mathcal{L}_{\text{kin}}$ contains the kinetic terms of the dynamical fields

$$\mathcal{L}_{\text{kin}} = \frac{f^2_\pi}{2} (D_\mu \Sigma)\big(D^\mu \Sigma\big)^T + \sum_r Z_r \bar{\psi}_r \not{D} \psi_r - \frac{1}{4g^2} W^{a_L}_{\mu\nu} W^{a_L \mu\nu} - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} \tag{9}$$

where

$$\begin{aligned} & Z_{u_L} = \Pi^q_0(0) + s^2_h \Pi^q_1(0)/2, \quad Z_{u_R} = \Pi^u_0(0) + c^2_h \Pi^u_1(0) \\ & f^2_\pi = \Pi_1(0), \quad 1/g^2 = -\Pi'_0(0), \quad 1/g'^2 = -\Pi^{B\prime}_0(0) \end{aligned}$$

The Higgs potential $V(\Sigma)$ is generated by SM gauge and fermion loops. We will show below that the top contribution can trigger the EWSB and the Higgs field h acquires a VEV, breaking O(4) down to the custodial O(3) group. From the kinetic term of Σ we obtain $M^2_W = g^2 (s_h f_\pi)^2/4$, which implies

$$v \equiv \varepsilon f_\pi = s_h f_\pi = 246\ \text{GeV} \tag{10}$$

The value of ε can vary between 0 (no EWSB) and 1 (maximal EWSB). The term $\mathcal{L}_{\rm yuk}$ of Eq. (8) contains the Yukawa couplings between the Higgs and the elementary fermions, and comes from expanding the last term of Eq. (7). When the Higgs acquires a VEV, this term will give mass to the fermions

$$m_u = \frac{s_h c_h}{\sqrt{2}} \frac{M_1^u(0)}{\sqrt{Z_{u_L} Z_{u_R}}} \equiv y_u v \tag{11}$$

By choosing $\gamma_{q,u} > 0$, we have, according to Eq. (2), that $\lambda_{q,u}$ are strongly suppressed at low energies, and the fermions are weakly coupled to the CFT. This can be used to explain in a natural way the smallness and the hierarchical structure of the masses of the light fermions [11,12]:

$$m_u \sim \frac{\sqrt{N}}{4\pi} \left(\frac{\mu_{\rm IR}}{\Lambda}\right)^{\gamma_q+\gamma_u} v \tag{12}$$

It is interesting to notice that this theory has a (sort of) GIM mechanism, since flavour changing neutral current (FCNC) effects involving light fermions are also suppressed by the couplings $\lambda_{q,u}$ (see for example Refs. [12,13]). For the third family we need $\gamma_{q,u} < 0$, in order to have a large top quark mass. Finally, there is a very important condition to be fulfilled if we want to generate non-zero fermion masses: as it is clear from Eq. (11), we must require $0 < s_h c_h < 1$, i.e. $0 < \varepsilon < 1$. Therefore, maximal EWSB $\varepsilon = 1$ is not allowed.

The last term $\Delta\mathcal{L}$ in the effective Lagrangian (8) contains all higher-order operators in the chiral expansion. The only one that is relevant for us here is that responsible for the Peskin–Takeuchi S parameter that originates from the third term of Eq. (6):

$$\Delta\mathcal{L} \supset \frac{1}{2} \Pi_1'(0) W_{\mu\nu}^{a_L} B^{\mu\nu} \Sigma T^{a_L} Y \Sigma^T \tag{13}$$

where T^{a_L}, Y are respectively the generators of $SU(2)_L$ and hypercharge.

2.1. Higgs potential and vacuum misalignment

The dominant contribution to the Higgs potential comes at the one-loop level from the virtual exchange of the elementary $SU(2)_L$ gauge bosons and top quark. It is given by the Coleman–Weinberg potential

$$V(h) = -\frac{9}{2} \int \frac{d^4 p}{(2\pi)^4} \log \Pi_W + (2N_c) \int \frac{d^4 p}{(2\pi)^4} \left[\log \Pi_{b_L} + \log\left(p^2 \Pi_{t_L} \Pi_{t_R} - \Pi_{t_L t_R}^2\right)\right] \tag{14}$$

where $\Pi_i(p)$ are the self-energies of the corresponding SM fields in the background of h. These can be written as functions of the form factors of Eqs. (6) and (7):

$$\begin{aligned} \Pi_W &= \Pi_0 + \frac{s_h^2}{4}\Pi_1, & \Pi_{b_L} &= \Pi_0^q \\ \Pi_{t_L t_R} &= \frac{s_h c_h}{\sqrt{2}} M_1^u, & \Pi_{t_L} &= \Pi_0^q + \frac{s_h^2}{2}\Pi_1^q \\ & & \Pi_{t_R} &= \Pi_0^u + c_h^2 \Pi_1^u \end{aligned} \tag{15}$$

Apart from a constant piece, the potential of Eq. (14) is finite, since the form factors Π_1 and M_1 drop with the momentum as $|\langle\Phi\rangle|^2/p^{2d}$, where Φ is the CFT operator of dimension $d \gg 1$ responsible for the SO(5) spontaneous breaking. This fast decrease with the momentum allows us to expand the logarithms in Eq. (14) and write the approximate formula

$$V(h) \simeq \alpha s_h^2 - \beta s_h^2 c_h^2 \tag{16}$$

where α and β are integral functions of the form factors. For $\alpha < \beta$ and $\beta \geqslant 0$ we have that the electroweak symmetry is broken: $\varepsilon \neq 0$. If $\beta > |\alpha|$, the minimum of the potential is at

$$s_h = \varepsilon = \sqrt{\frac{\beta - \alpha}{2\beta}} \tag{17}$$

while for $\beta < |\alpha|$ the minimum corresponds to $c_h = 0$, and the EWSB is maximal: $\varepsilon = 1$. As we said before, this latter case leads to zero fermion masses and must be discarded. The gauge contribution gives $\alpha > 0$ and tends to align the vacuum along the $(\mathrm{SU}(2)_L)$-preserving direction. A misalignment of the vacuum, however, can come from the top loops, which can give $\alpha < 0$ and $\beta > 0$. The physical Higgs mass is given by

$$m^2_{\mathrm{Higgs}} \simeq \frac{8\beta s_h^2 c_h^2}{f_\pi^2} \sim \frac{8N_c}{N} y_t^2 v^2 \tag{18}$$

and, for moderate values of N, it can be above the experimental bound $m_{\mathrm{Higgs}} > 114$ GeV.

Remarkably, this model can give a realistic account of electroweak symmetry breaking. To make this statement more quantitative, we need to compute the precise value of α, β and check if the model passes the EWPT. This can be done by resorting to the 5D theory, which we define in the next section.

3. The 5D model

The 4D theory presented above can be obtained as the holographic description of a 5-dimensional, weakly coupled model. Such model is defined as follows. The 5D spacetime metric is given by [7]

$$\mathrm{d}s^2 = \frac{1}{(kz)^2}\left(\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \mathrm{d}z^2\right) \equiv g_{MN}\,\mathrm{d}x^M\,\mathrm{d}x^N \tag{19}$$

where the 5D coordinates are labeled by capital Latin letters, $M = (\mu, 5)$, with $\mu = 0, \ldots, 3$, $z = x^5$ represents the coordinate for the fifth dimension and $1/k$ is the AdS curvature radius. This spacetime has two boundaries at $z = L_0 \equiv 1/k \sim 1/M_{\mathrm{Pl}}$ (UV brane) and at $z = L_1 \sim \mu_{\mathrm{IR}} \sim 1/\mathrm{TeV}$ (IR brane). The theory is thus defined on the line segment $L_0 \leqslant z \leqslant L_1$. The gauge symmetry in the 5D bulk is taken to be $\mathrm{SU}(3)_c \times \mathrm{SO}(5) \times \mathrm{U}(1)_X$, reduced to $\mathrm{SU}(3)_c \times \mathrm{O}(4) \times \mathrm{U}(1)_X$ on the IR brane and to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ on the UV brane. The Higgs field is identified with the fifth component of the SO(5)/SO(4) gauge bosons, as it occurs in the Hosotani mechanism for symmetry breaking [14].

In the fermion sector, each SM generation is identified with the zero modes of a set of 5D bulk multiplets transforming as fundamentals of SO(5). For the up-quark sector, these 5D fields are

$$\xi_q = \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{q}}_{\mathbf{L}} = \begin{bmatrix} q'_L(-+) \\ q_L(++) \end{bmatrix} & (\mathbf{2},\mathbf{2})^{\mathbf{q}}_{\mathbf{R}} = \begin{bmatrix} q'_R(+-) \\ q_R(--) \end{bmatrix} \\ (\mathbf{1},\mathbf{1})^{\mathbf{q}}_{\mathbf{L}}(--) & (\mathbf{1},\mathbf{1})^{\mathbf{q}}_{\mathbf{R}}(++) \end{bmatrix}, \qquad \xi_u = \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{L}}(+-) & (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}}(-+) \\ (\mathbf{1},\mathbf{1})^{\mathbf{u}}_{\mathbf{L}}(-+) & (\mathbf{1},\mathbf{1})^{\mathbf{u}}_{\mathbf{R}}(+-) \end{bmatrix} \tag{20}$$

where $\xi_{q,u}$ transform as $\mathbf{5}_{2/3}$ of $\mathrm{SO}(5) \times \mathrm{U}(1)_X$. Chiralities under the 4D Lorentz group have been denoted by L, R, and $(\pm, \pm)$ is a shorthand notation to denote Neumann $(+)$ or Dirichlet $(-)$ boundary conditions on the two boundaries. In Eq. (20) we have grouped the fields of each multiplet $\xi_{q,u}$ in representations of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, and used the fact that a fundamental of SO(5) decomposes as $\mathbf{5} = (\mathbf{2},\mathbf{2}) \oplus (\mathbf{1},\mathbf{1})$. The bulk masses in units of k of the 5D fields $\xi_{q,u}$ will be denoted by $c_{q,u}$. Localized on the IR boundary, we consider the most general set of mass terms invariant under $\mathrm{O}(4) \times \mathrm{U}(1)_X$:

$$\widetilde{m}_u \overline{(\mathbf{2},\mathbf{2})}^{\mathbf{q}}_{\mathbf{L}} (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}} + \widetilde{M}_u \overline{(\mathbf{1},\mathbf{1})}^{\mathbf{q}}_{\mathbf{R}} (\mathbf{1},\mathbf{1})^{\mathbf{u}}_{\mathbf{L}} + \mathrm{h.c.} \tag{21}$$

The 5D model described above has exactly the same properties as the 4D CFT theory described in Section 2. In particular, it leads to the same effective Lagrangian of Eqs. (6), (7), and to the same low-energy chiral theory of Eq. (8). The anomalous dimensions γ of the operators of the 4D CFT theory are related to the 5D fermion masses c_i according to [15]

$$\gamma_q = \left| c_q + \frac{1}{2} \right| - 1, \qquad \gamma_u = \left| c_u - \frac{1}{2} \right| - 1 \tag{22}$$

Therefore the requirement $\gamma_{q,u} > 0$ for the light fermions (see Eq. (12) above) implies $c_q > 1/2$ and $c_u < -1/2$, while for the top $\gamma_{q,u} \lesssim 0$ implies $|c_{q,u}| \lesssim 1/2$.

The Higgs potential and the corrections to the EW precision observables are largely controlled by the relevant parameters of the top quark and gauge sector:

$$N, c_q, c_u, \widetilde{m}_u, \widetilde{M}_u \tag{23}$$

We have defined $N \equiv 16\pi^2/(g_5^2 k)$, where g_5 is the SO(5) bulk gauge coupling. Here, and from now on, $\widetilde{m}_u$, $\widetilde{M}_u$, c_q and c_u denote the mass parameters of the top quark, $q = (t_L, b_L)$ and $u = t_R$. The scale L_1 has been traded for v. The five parameters of Eq. (23) cannot be completely determined by the present experimental data. Two constraints come from fixing the top quark mass to its experimental value, $m_t = 173$ GeV, and by requiring $0 < \varepsilon < 1$. Moreover, a portion of the parameter space will be excluded by the precision tests. How extended is this portion gives us a measure of the 'degree of tuning' required in our model. This is the subject of the next section.

3.1. Electroweak precision tests

There are two types of corrections to the electroweak observables that any composite Higgs model must address, since they are usually sizable: non-universal corrections to the $Zb\bar{b}$ coupling, and universal corrections to the gauge boson self-energies. The results of Ref. [10] show that our choice of bulk fermionic representations guarantees that non-universal corrections to $Zb\bar{b}$ are small, due to the O(3) custodial symmetry of the bulk and IR boundary. Therefore, we need to consider only universal effects, which can be parametrized in terms of four quantities: S, T, W and Y [16]. The last two parameters are suppressed by a factor $\sim(g^2N/16\pi^2)$ compared to S and T, and can be neglected [4]. The parameter T is zero at tree-level due to the custodial symmetry. Loop effects can be estimated to be small ($T \lesssim 0.3$), and explicit calculations in similar 5D models confirm this expectation [5,17].

The Peskin–Takeuchi S parameter gives the most robust and model-independent constraint. Neglecting a small correction from boundary kinetics terms, one has $S = 3N\varepsilon^2/(8\pi)$ [4]. The 99% CL experimental bound $S \lesssim 0.3$ [16] then translates into

$$\varepsilon^2 \lesssim \frac{1}{4}\left(\frac{10}{N}\right) \tag{24}$$

For $N = 10$ this rules out the values $1/4 \lesssim \varepsilon^2 < 1$, which we naively expect to correspond to $\sim 3/4$ of the allowed region. A detailed numerical analysis confirms this expectation [10], as the contour plots in Fig. 1 show. This means that a sizable portion of the parameter space is still allowed, and that no large fine tuning is required to pass the electroweak tests.

3.2. Fermionic resonances and the Higgs mass

A crucial prediction of our model is that the requirement of a large top quark mass always forces some of the fermionic Kaluza–Klein (KK) resonances to be lighter than their gauge counterpart. The reason is the following. The

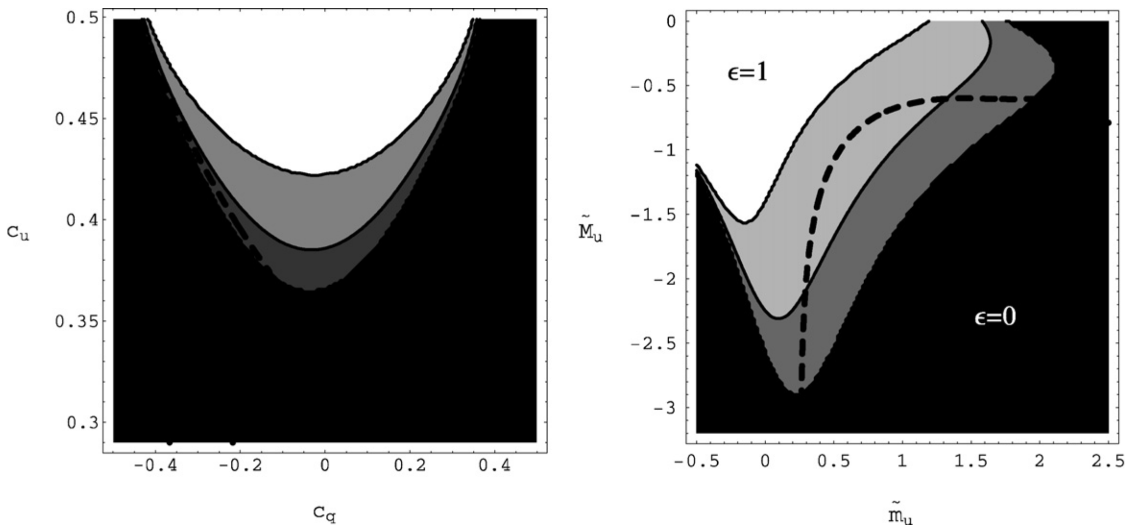

Fig. 1. Contour plots in the plane (c_q, c_u) with $\widetilde{m}_u = 0$, $\widetilde{M}_u = -3.5$, $N = 8$ (left), and in the plane $(\widetilde{m}_u, \widetilde{M}_u)$ with $c_q = 0.35$, $c_u = 0.45$, $N = 8$ (right). The black (white) area denotes the region with $\varepsilon = 0$ ($\varepsilon = 1$). The gray intermediate area with $0 < \varepsilon < 1$ is the region with EWSB and non-zero fermion masses. Its lighter gray portion is excluded by the bound $S \lesssim 0.3$ for $N = 8$, see Eq. (24). The dashed black line represents the curve with $m_t^{\overline{\mathrm{MS}}}(2\ \mathrm{TeV}) = 150$ GeV, equivalent to $m_t^{\mathrm{pole}} = 173$ GeV.

embedding of t_L and t_R into SO(5) bulk multiplets implies that some of their SO(5) partners have $(\pm,\mp)$ boundary conditions, an assignment that is necessary to avoid extra massless states (see Eqs. (20)). Consider for example the case in which the left-handed chirality of the 5D field is $(+,-)$ (hence the right-handed one is $(-,+)$): for values of the 5D mass $c_i > 1/2$, the lightest Kaluza–Klein mode – we will denote it by q^* – has its left-handed chirality exponentially peaked on the UV brane, while its right-handed is localized on the IR brane. This implies that the mass of q^* is exponentially suppressed. In the opposite limit $c_i < -1/2$, both chiralities are localized on the IR brane and the mass of q^* is of the same order as that of the other KKs: $m_{q^*} \simeq m_\rho$. In the intermediate region $-1/2 < c_i < 1/2$, the one we chose for top quark bulk fields, one finds that m_{q^*} is given by [5]

$$m_{q^*} \simeq \frac{2}{L_1}\sqrt{\frac{1}{2} - c_i} \tag{25}$$

which means that it is still parametrically smaller than m_ρ by a factor $\sqrt{1/2 - c_i}$. Analogous results hold in the case of a $(+,-)$ right-handed chirality, but for $c_i \to -c_i$.

Let us concentrate on the region $-1/2 < c_u < 1/2$ and $c_q > 0$. From the argument above and by inspecting Eqs. (20), one finds that the lightest KK modes are those arising from the $(\mathbf{2},\mathbf{2})^{\mathbf{u}}$ component of the bulk multiplet ξ_u. This field contains two $SU(2)_L$ doublets of hypercharge $Y = 7/6$ and $Y = 1/6$. Fig. 2 shows the spectrum of the lowest fermionic KK states. The lightest states are those predicted. Their mass is around 500–1500 GeV for $\varepsilon = 1/2$ and $N = 8$, much smaller than that of the lightest gauge KK, $m_\rho \simeq 2.6$ TeV, and of other fermionic excitations.

Light fermionic resonances are a generic property of natural models of EWSB with a light Higgs. In the class of composite Higgs models that we are considering, the Higgs mass can be expressed as [6]

$$m^2_{\text{Higgs}} \simeq \frac{N_c}{\pi^2}\frac{m_t^2}{v^2}\varepsilon^2\Lambda^2, \quad \text{where } \Lambda^2 = a_1 m^2_{q^*} + a_2 m_{q^*} M + a_3 M^2 \tag{26}$$

Here a_i are numerical coefficients and $M \equiv m_\rho = 3\pi/(4L_1)$ parametrizes the scale of heavier color resonances. A numerical fit to the set of points of Fig. 2 gives $a_i = (-0.1, 0.3, 0.006)$. The dispersion of the points around the fitted curve can be explained as follows. In Fig. 2 we have fixed $N = 8$, $\varepsilon = 0.5$ and $m_t = 173$ GeV, which leaves two of the five parameters of Eq. (23) free to vary. If c_u is traded for m_{q^*}, by means of Eq. (25), we are left with one free parameter, for example, c_q. The coefficients a_i of Eq. (26) will thus depend on c_q, and since we have scanned over the values $0.2 < c_q < 0.38$ to generate the points in Fig. 2, this explains their dispersion.

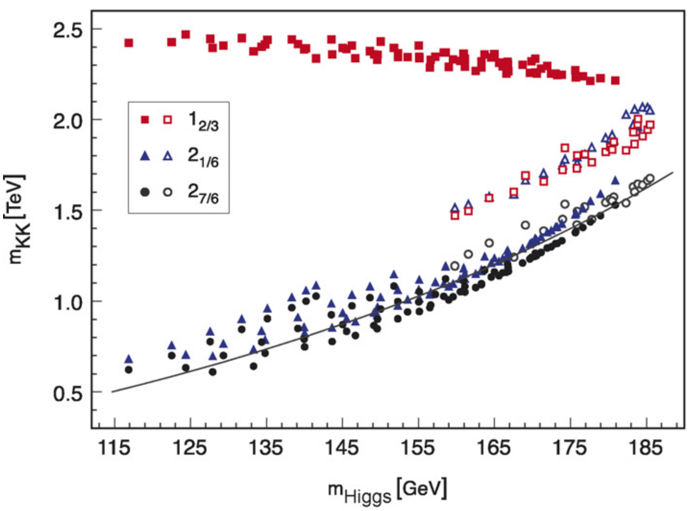

Fig. 2. Masses of the lightest colored KK fermions. Different symbols denote KKs with different quantum numbers under $SU(2)_L \times U(1)_Y$. We have fixed $\varepsilon = 0.5$, $N = 8$, and varied $0.28 < c_q < 0.38$, $0 < c_u < 0.41$, $0.32 < \widetilde{m}_u < 0.42$, $-3.5 < \widetilde{M}_u < -2.2$ (filled points), or $0.2 < c_q < 0.35$, $-0.25 < c_u < -0.42$, $-1.3 < \widetilde{m}_u < 0.2$, $0.1 < \widetilde{M}_u < 2.3$ (empty points). The black continuous line is the fit to the mass of the lightest resonance of Eq. (26).

4. LHC phenomenology

Detecting the light KK resonances that transform as $SU(2)_L$ doublets of hypercharge $Y = 7/6$ and $Y = 1/6$ would be a smoking gun of our model. In particular, discovering the 'exotic' state with electromagnetic charge $Q_{\text{em}} = 5/3$ from the first doublet would clearly distinguish our model from other scenarios that predict heavy excitations of the top and bottom quarks. This exotic KK, as well as the other resonances, can be pair produced at the LHC via its QCD interactions. Single production via $t_R W$ fusion might also be important, since the small top quark content of the proton can be compensated by the large coupling [4] (for t_R composite, i.e. $c_u \to 1/2$, such coupling is expected to be much larger than the top Yukawa coupling). Once produced, the $Q_{\text{em}} = 5/3$ resonance will decay to a top quark and a longitudinally polarized W boson. In the case of pair production, this leads to a final state with two tops and two W's, which should be possible to isolate over the SM background. A dedicated analysis is required to establish the full reach at the LCH. Similar considerations apply to the production of the other KKs inside the two weak doublets, which have electromagnetic charge $Q_{\text{em}} = -1/3, 2/3$, with the difference that these states can also be produced in association with a top or bottom quark via the exchange of a gluon KK. They will decay mostly to a top plus a longitudinal W or Z or a Higgs boson H.

Finally, the gauge KK resonances will be most easily produced in a $q\bar{q}$ Drell–Yan scattering. Heavy excitations of the W and Z can also be produced via weak boson fusion, in analogy to the case of technicolor models, though the cross section for this process is expected to be small for KK mass of order 2–3 TeV or larger. The gauge KKs will decay mostly to pairs of longitudinally polarized weak bosons (or alternatively a weak boson plus a Higgs), and to pairs of tops and bottoms. As for the fermions, a detailed study of all these processes will be required to fully explore the reach at the LHC and at future colliders.

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