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The mystery of the Higgs particle/Le mystère de la particule de Higgs

Higgs and Wilson lines in field and string theory

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Abstract

This is a short review on basics of the use of the Wilson line to break gauge symmetry in theories with compact extra dimensions. We show how the computation of the one-loop effective field theory leads to a finite result. We then explain the realization of this breaking and the effective potential computation in an open string theory framework with D-branes. ***To cite this article: K. Benakli, C. R. Physique 8 (2007).***

Résumé

Higgs et ligne de Wilson en théories des champs et de cordes. Ceci est une revue courte sur des aspects basiques de l'utilisation des lignes de Wilson pour briser la symétrie de jauge dans les théories avec des dimensions supplémentaires compactes. Nous montrons comment le calcul de la théorie effective de champs à une boucle mène à un résultat fini. Puis, nous illustrons alors la réalisation de cette brisure de symétrie de jauge et le calcul potentiel effectif dans un cadre de théorie de cordes ouvertes se propageant sur des D-branes. ***Pour citer cet article : K. Benakli, C. R. Physique 8 (2007).***

Keywords: Higgs; Wilson line; D-branes

Mots-clés : Higgs ; Lignes de Wilson ; D-branes

1. Introduction

Our universe is full of massive states. Some of them are composite (as protons and neutrons), and most of their mass is explained as a manifestation of binding (confining) interactions between their fundamental constituents. Other massive states are described as elementary particles (electrons, muons or W's and Z bosons). Understanding the origin of their mass is still challenging. Today, there is a strong belief that it is the result of a *Higgs phenomenon* whose experimental discovery may be achieved soon.

The Higgs mechanism arises in theories with a gauge symmetry. The theory has an infinite number of connected degenerate vacua, parametrized by the vacuum expectation value of scalar fields. This spontaneous breaking of symmetry leads to a Higgs mechanism: some 'would be massless' fields, the so-called Nambu–Goldstone particles, that connect the different degenerate vacua, are eaten-up by 'what would have stayed massless' vector bosons. All together, one scalar and one massless vector are traded with one massive vector field.

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This short review presents some very basic facts about the use of Wilson lines as Higgs fields in higher dimensional theories. This study is justified, for instance, as this is a very common ingredient of four dimensional model building in string theory. However, our main motivation here is the possibility of a calculable and predictive model for Higgs physics. This is because the higher dimensional gauge symmetry protects the Higgs potential from sensitivity to details of the UV cut-off. Although the ideas we present, mainly based on [1,2], are simple, all attempts at their implementation in realistic models turned out rapidly to be cumbersome and thus will not be discussed here.

2. Wilson lines and 'gauge-Higgs unification'

The space–time taken to be $D = 4 + d$ dimensional is assumed to factorize as $\mathcal{M}^4 \otimes K$. For our notation, we will use hatted indices $[\hat{\mu}, \hat{\nu}, \ldots = 0, \ldots, 3, 5, 6, \ldots, 4 + d]$, while the Minkowski part $\mathcal{M}^4$ is spanned by the coordinates X^μ, $\mu = 0, \ldots, 3$, and the 'internal space' K is described either by X^M $[M, N, \ldots = 5, 6, \ldots, 4 + d]$ or by y^i, $i = 1, \ldots, d$. The propagation of Yang–Mills gauge fields $A_{\hat{\mu}}$ is described, up to two derivatives, by the Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} \tag{1}$$

with $F_{\hat{\mu}\hat{\nu}} = \sum_a F^{(a)}_{\hat{\mu}\hat{\nu}} t_a$ and $A_{\hat{\mu}} = \sum_a A^{(a)}_{\hat{\mu}} t_a$ and the generators t_a are normalized such that $\mathrm{Tr}(t_a t_b) = \delta_{ab}/2$. With this convention:

$$\begin{aligned} F_{\hat{\mu}\hat{\nu}} &= \partial_{\hat{\mu}} A_{\hat{\nu}} - \partial_{\hat{\nu}} A_{\hat{\mu}} - \mathrm{i}g[A_{\hat{\mu}}, A_{\hat{\nu}}] \\ D_{\hat{\mu}} &= \partial_{\hat{\mu}} + \mathrm{i}g A_{\hat{\mu}} \end{aligned} \tag{2}$$

where g is the tree-level gauge coupling. Taking the internal space K to be a d-dimensional torus with radii R_i and assuming periodicity of the wave functions along each compact direction leads to the Fourier expansion:

$$A_{\hat{\mu}}(x_\mu, y_i) = \sum_n A^{(n)}_{\hat{\mu}}(x_\mu) \mathrm{e}^{(\frac{n_i y^i}{R_i^2})} \tag{3}$$

where $A^{(n)}_{\hat{\mu}}(x_\mu)$ are solutions of the four-dimensional equations of motion, for instance plane waves $A^{(n)}_{\hat{\mu}}(x_\mu) = A^{(n)}_{\hat{\mu}} \mathrm{e}^{\mathrm{i}k.x}$. They correspond to the propagation of towers of states with masses:

$$M^2_{KK} \equiv M^2_{\vec{n}} = \left[\frac{n_1}{R_1}\vec{e_1} + \frac{n_2}{R_2}\vec{e_2} + \cdots + \frac{n_d}{R_d}\vec{e_d}\right]^2 \tag{4}$$

where n_i are non-negative integers and $\vec{e_i}$ represent the unitary vectors of the dual lattice. These states with $\sum_i n_i \neq 0$ are denoted as Kaluza–Klein (KK) excitations of the massless mode. In this dimensional reduction process, the higher dimensional gauge fields $A_{\hat{\mu}}$ give birth in four-dimensions to vector fields A^μ and scalar fields A^M transforming in the adjoint representation. The latter have a tree-level scalar potential generated from the reduction to four dimensions of the quartic interactions among gauge bosons:

$$V_{\mathrm{tree}} = \frac{g^2}{2} \sum_{M,N=5}^{d+4} \mathrm{Tr}[A_M, A_N]^2 \tag{5}$$

Note that this interaction is absent in the case of five-dimensional theory ($d = 1$).

We will discuss below some example with $d = 2$ extra dimensions. The two non-contractible cycles of $K \equiv T^2$ have radii R_1 and R_2 and are taken to be along the directions x^5 and x^6 which form an angle θ (see Fig. 1).[1] These parameters appear in the 'internal metric' G_{MN}, $M, N = 5, 6$, the torus area $\sqrt{G}$ and the complex structure modulus U given by:

$$G_{MN} = \begin{pmatrix} R_1^2 & R_1 R_2 c \\ R_1 R_2 c & R_2^2 \end{pmatrix}; \quad \sqrt{G} = R_1 R_2 s; \quad U = \frac{R_2}{R_1}(c + \mathrm{i}s) \tag{6}$$

with the case of orthogonal circles corresponding to $\theta = \pi/2$, thus $c = 0$.

[1] We will use the compact notation $\cos\theta = c$, $\sin\theta = s > 0$.

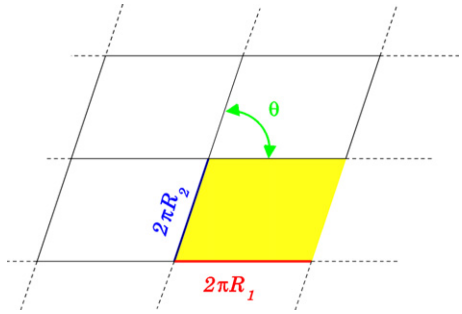

Fig. 1. The two-dimensional torus.

With this metric the squared mass of the KK excitations (4) takes the form:

$$M^2_{\vec{m},I} = \left|\frac{m_2 - (m_1)U}{\sqrt{ImU}G^{1/4}}\right|^2 = \frac{1}{s^2}\left[\frac{(m_1)^2}{R_1^2} + \frac{(m_2)^2}{R_2^2} - 2\frac{m_1 m_2 c}{R_1 R_2}\right] \tag{7}$$

Although our discussion carries over for a generic group G, it is more illuminating to illustrate our discussion with a very simple but explicit example, chosen to be $G = U(2) \equiv U(1) \otimes SU(2)$. The associated gauge fields are

$$A_{\hat{\mu}} = \sum_a A^a_{\hat{\mu}} T^a \tag{8}$$

with

$$T^0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad T^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{9}$$

where T^0 is the generator of the overall $U(1)$ and the $SU(2)$ generators satisfy the commutation relations:

$$[T^3, T^\pm] = \pm T^\pm, \qquad [T^+, T^-] = T^3 \tag{10}$$

Of interest is the use of vacuum expectation values for the scalar field A_M to break the $U(2)$ gauge symmetry. For instance, a simple solution of the equations of motion is given by

$$A_\mu = 0, \qquad A_5 = \frac{1}{g}\frac{a_1}{R_1}T^3 \qquad A_6 = \frac{1}{g}\frac{a_2}{R_2}T^3 \tag{11}$$

In the presence of states Φ^q with non-vanishing charge q:

$$[T^3, \Phi^q] = q\Phi^q \tag{12}$$

we define Wilson lines associated with non-contractible cycles C_i by:

$$W^I_{C_i} = \exp\left(\mathrm{i}q^I g \oint_{C_i} A_M \,\mathrm{d}y^M\right) = \mathrm{e}^{\mathrm{i}2\pi a^I_i}, \quad a^I_i \equiv \frac{q^I g}{2\pi}\oint_{C_i} A_M \,\mathrm{d}y^M = q^I a_i \tag{13}$$

Because of the existence of non-contractible cycles C_i along which the associated Wilson lines are non vanishing, A_M can not be cancelled globally, although A_M are locally pure gauge. Equivalent gauge configurations are those related by $qa_i \to qa_i + n_i$ where n_i are integers.

The effect of these non-trivial Wilson lines is the spontaneous breaking of $U(2)$ to $U(1) \otimes U(1)$. In particular, the $W^\pm \equiv A^\pm_\mu$ gauge bosons of the $SU(2)$ associated with the generators $T^\pm$, acquire a mass:

$$M^2_{W^\pm} = M^2_\pm = \frac{1}{s^2}\left[\frac{a_1^2}{R_1^2} + \frac{a_2^2}{R_2^2} - 2\frac{a_1 a_2 c}{R_1 R_2}\right] \tag{14}$$

In the same way, all the KK excitations of the states $A^\pm_\mu$ have their masses shifted to:

$$M^2_{\vec{m},\pm} = \left|\frac{m_2 \pm a_2 - (m_1 \pm a_1)U}{\sqrt{ImU}G^{1/4}}\right|^2 = \frac{1}{s^2}\left[\frac{(m_1 \pm a_1)^2}{R_1^2} + \frac{(m_2 \pm a_2)^2}{R_2^2} - 2\frac{(m_1 \pm a_1)(m_2 \pm a_2)c}{R_1 R_2}\right] \tag{15}$$

As a final remark, note that the mass generation mechanism presented here takes different forms and names as Hosotani [3], Scherk–Schwarz [4], ... but its essence is the Aharonov–Bohm observation [5] that in the presence of a non-contractible loop, a gauge field can not always be gauged away and leads to shifts of momenta with observable effects.

3. The effective potential in field theory

In the example above, A_5 and A_6 have been given vacuum expectations values in commuting directions of the associated gauge groups. This means that at tree level the Wilson lines a_1 and a_2 are flat directions. No symmetry protects them, however, from acquiring a potential through radiative corrections. In fact, we will show now that it can be generated at a one-loop level and it is in some cases finite and calculable.

For this purpose, we start with a generic theory where the bosonic and fermionic fields have field dependent masses M_I. The one-loop contributions are given by:

$$V_{\text{eff}} = \frac{1}{2}\sum_I (-)^{F_I} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \log\left[p^2 + M_I^2\right] \tag{16}$$

where the sum is over all bosonic ($F_I = 0$) and fermionic ($F_I = 1$) degrees of freedom with the field dependence appearing in the masses $M_I(\phi, a_i^I)$. In the Schwinger representation, it takes the form:

$$\begin{aligned} V_{\text{eff}} &= -\frac{1}{2}\sum_I (-)^{F_I} \int_0^\infty \frac{\mathrm{d}t}{t} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{e}^{-t[p^2+M_I^2]} = -\frac{1}{32\pi^2}\sum_I (-)^{F_I} \int_0^\infty \frac{\mathrm{d}t}{t^3} \mathrm{e}^{-tM_I^2} \\ &= -\frac{1}{32\pi^2}\sum_I (-)^{F_I} \int_0^\infty \mathrm{d}l\, l\, \mathrm{e}^{-M_I^2/l} \end{aligned} \tag{17}$$

where we have made the change of variables $t = 1/l$. The integration regions $t \to 0$ ($l \to \infty$) and $t \to \infty$ ($l \to 0$) correspond to the ultraviolet (UV) and infrared (IR) limits, respectively. To proceed further, we need to specify the fields dependence of M_I:

$$M_{\vec{m},I}^2 = M_{0I}^2 + \sum_{i=1}^d \left[\frac{m_i + a_i^I}{R_i}\right]^2 \tag{18}$$

where $\vec{m} = \{m_1, \ldots, m_d\}$ with m_i integers. These are towers of KK states as defined in (4), with a lightest mode of mass M_{0I} More precisely, this is the $(4+d)$-dimensional mass which remains in the limit $R_i \to \infty$. The shifts a_i^I for internal momenta are associated to the Wilson lines, $a_i^I = q^I \oint \frac{\mathrm{d}y^i}{2\pi} g A_i$, where A_i is the internal component of a gauge field with gauge coupling g and q^I is the charge of the I field with respect to the appropriate generator.

Cases with constant M_{0I}, i.e. all field dependence is through the Wilson lines are those of interest. We will restrict for simplicity to $M_{0I}^2 = 0$, as a non-vanishing finite value would otherwise play the role of an infrared cut-off but does not introduce new UV divergences. The effective potential obtained from (17) is then given by:

$$V_{\text{eff}}\left(a_i^I\right) = -\sum_I \sum_{\vec{m}} (-)^{F_I} \frac{1}{32\pi^2} \int_0^\infty \mathrm{d}l\, l\, \mathrm{e}^{-\sum_i \frac{(m_i + a_i^I)^2}{R_i^2 l}} \tag{19}$$

Commuting the integral with the sum over the KK states, and performing a Poisson re-summation, allows one to write the effective potential as:

$$V_{\text{eff}}\left(a_i^I\right) = -\sum_I (-)^{F_I} \frac{\prod_{i=1}^d R_i}{32\pi^{\frac{4-d}{2}}} \sum_{\vec{n}} \mathrm{e}^{2\pi\mathrm{i}\sum_i n_i a_i^I} \int_0^\infty \mathrm{d}l\, l^{\frac{2+d}{2}} \mathrm{e}^{-\pi^2 l \sum_i n_i^2 R_i^2} \tag{20}$$

The term with $\vec{n} = \vec{0}$ gives rise to a (divergent) field-independent contribution that needs to be dealt with in the framework of a full quantum gravity theory. This is irrelevant for our discussion and can be forgotten. For all other

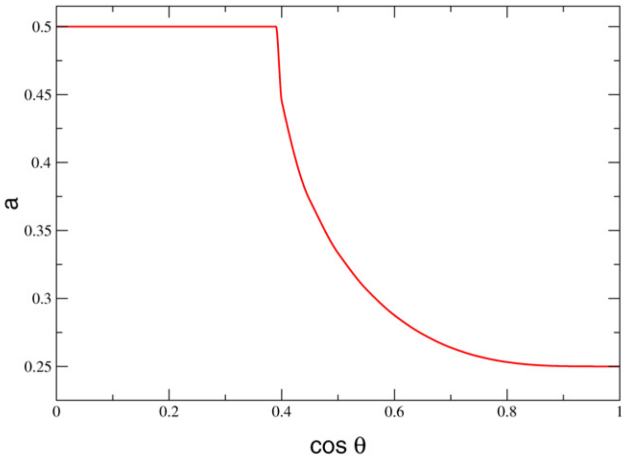

Fig. 2. The minima of the effective potential (22), for $R_1 = R_2$, as a function of $\cos\theta$.

(non-vanishing) vectors $\vec{n} \neq \vec{0}$ in (20), we make the change of variables: $l' = \pi^2 l \sum_i n_i^2 R_i^2$ and perform the integration over l' explicitly. This leads to a *finite* result for the field-dependent part of the effective potential:

$$V_{\text{eff}}\big(a_i^I\big) = -\sum_I (-)^{F_I} \frac{\Gamma(\frac{4+d}{2})}{32\pi^{\frac{12+d}{2}}} \prod_{i=1}^{d} R_i \sum_{\vec{n}\neq\vec{0}} \frac{\mathrm{e}^{2\pi\mathrm{i}\sum_i n_i a_i^I}}{[\sum_i n_i^2 R_i^2]^{\frac{4+d}{2}}} \tag{21}$$

Take, for instance, the case $d = 2$, then plugging the form (15) in the effective potential and performing a Poisson re-summation, one can extract the part of the effective potential dependent on a_1 and/or on a_2 that takes the form:

$$V_{\text{eff}}\big(a_i^I\big) = -\sum_I (-)^{F_I} \frac{R_1 R_2 s}{16\pi^7} \sum_{\vec{n}\neq\vec{0}} \frac{\cos[2\pi(n_1 a_1^I + n_2 a_2^I)]}{[n_1^2 R_1^2 + n_2^2 R_2^2 + 2c n_1 R_1 n_2 R_2]^3} \tag{22}$$

In realistic models, a full treatment requires to consider minimization of the potential with respect to the radii R_i as they are given by vacuum expectation of scalar fields (radions), but for our concern here, we will suppose that they are fixed parameters and discuss the extrema of the potential (22) as function of $a_i^I = a_i$ supposed to independent of present species. In the case of one extra dimension, the vacuum expectation value $d = 1$, the minimum of the potential is at $a = 1/2$. We will discuss in more details the case of $d = 2$, restricting ourselves to the case of equal radii, i.e. $R_1 = R_2 \equiv R$ and leaving the torus angle as a free parameter. Using torus periodicity, we can restrict the potential to the region $-1/2 \leqslant a_1, a_2 \leqslant 1/2$. The potential (22) being symmetric with respect to $|a_2| \leftrightarrow |a_1|$, implies that at the minimum $|a_2| = |a_1| \equiv a$. Fig. 2 shows the minimum as a function of $\cos\theta$. We can see that for $\cos\theta < 0.4$ the minimum is at $a = 1/2$, while, for $\cos\theta > 0.4$ it goes from $a = 1/2$ to $a = 1/4$.

A generic $(4+d)$-dimensional gauge theory is not expected to be consistent and its UV completion (the embedding in a consistent higher dimensional theory, such as string theory) is needed. However, we found here that some one-loop effective potentials can be finite, computable in the field theory limit and insensitive to most of the details of the UV completion. Before commenting on this, we wish to show how this picture is embedded in such a UV complete framework.

4. Gauge symmetry breaking using D-branes

In the previous sections we described the use of Wilson lines to spontaneously break gauge symmetry in field theories with compact extra dimensions. We will give, in the following, a corresponding picture in the case of string theory [6]. More precisely, we consider the case of open strings propagating on D-branes, themselves living in a ten-dimensional target space–time.

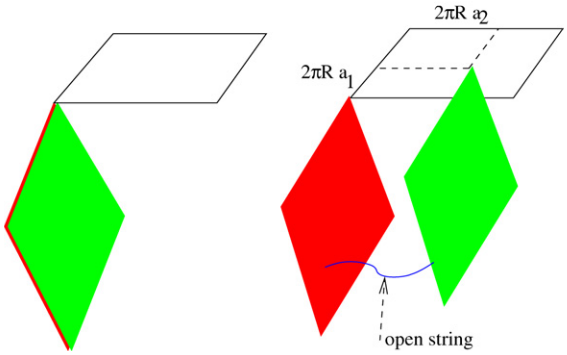

Fig. 3. The effect of Wilson lines is moving the branes away from each other.

A (supersymmetric) $U(2)$ brane version of the scenario described above can now be realized as a low energy effective field theory of the world volume of two D5-branes warping the same compact torus. On each brane lives a $U(1)$ gauge field whose generators, Y_1 and Y_2 respectively, are embedded inside the $U(2)$ as:

$$Y_1 = T^0 + T^3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad Y_2 = T^0 - T^3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad T^3 = \frac{1}{2}(Y_1 - Y2) \tag{23}$$

We can classify the open strings into two sets:

– The first have both ends on the same brane, the second set is made of strings with one end on the first brane and the other end on the second brane. The first set of strings are insensitive to the presence of a Wilson line along T^3. This is because the two ends of the string carry opposite $U(1)$ charges, thus they have vanishing total charge under one of the Y_i's, and none under the other. Their mass formula is given by:

$$M^2 = \sum_i \frac{n_i^2}{R_i^2} + (N-1) \tag{24}$$

in string units. N is integer and $N = 1$ reproduces the field theory spectrum of KK states.

– The second set contains strings with charges ± 1 under Y_1 and ∓ 1 under Y_2, which correspond respectively to the states T^+ and T^- of $SU(2)$. These have charges $+1$ and -1 respectively under T^3 generator and feel the Wilson line a_i. Their mass formula is given by:

$$M_+^2 = M_-^2 = \sum_i \frac{(n_i + a_i)^2}{R_i^2} + (N-1) \tag{25}$$

This reproduces the field theory discussion above with the $W^\pm$ getting a mass through a Higgs mechanism.

A nice geometrical picture of the Wilson line breaking mechanism is obtained by performing a T-duality. This transformation transforms the Neumann to Dirichlet boundary conditions and as a result the two D5-branes are transformed into two D3-branes localized in the T-dual torus. The value of the Wilson lines along Y_i parametrize, as an angular variable, the location of the D3-brane along the corresponding direction y^i. The Wilson line along T^3 parametrize then the separation between the two D-branes in the compact space (see Fig. 3). The strings stretching between these two branes have masses proportional to their length:

$$M^2 = M_+^2 = M_-^2 = \sum_i (n_i + a_i)^2 R_i^2 + (N-1) \tag{26}$$

An important remark is that the breaking presented here does not reduce the rank. For that purpose, we can resort to the case where the Higgs is identified with the lightest mode of strings localized at brane intersections. The condensation of the tachyonic modes of such strings lead to the recombination of two branes into a single one as illustrated in Fig. 4.

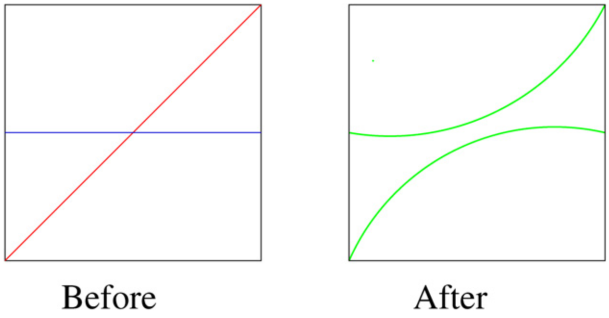

Fig. 4. Recombination of two branes into a single one.

5. The effective potential in string models

We wish to address here the fate of the computation of the effective potential presented above when embedded in a string theory. In order to obtain a non-vanishing result, the configuration needs to be non-supersymmetric. The important ingredient is to put on top of each other type of (anti-)orientifolds and (anti-)branes that preserve different parts of the original supersymmetry. In this case, the brane world-volume theory massless fields no longer form supersymmetric multiplets. Such models have been constructed, for example, in [7–9].

For instance, starting with N D-branes, the world volume theory contains N^2 bosons and N^2 fermions. The orientifold projection is then chosen to act in opposite ways for bosons and fermions. This is achieved if the Ramond–Ramond (RR) charge cancels locally while the Neuveu–Schwarz–Neuveu–Schwarz (NS–NS) charge does not (case (1)). In this case, the massless states contain $n_B^1 = N(N+1)/2$ bosons and $n_F^1 = N(N-1)/2$ fermions and the gauge group is $USp(N)$. Or instead (case (2)) the NS–NS charge is locally cancelled while RR charge is not which leads to $n_B^2 = N(N-1)/2$ bosons and $n_F^2 = N(N+1)/2$ fermions with a gauge group $SO(N)$.

One-loop contributions from world-sheet diagrams with topologies of torus, Klein-bottle can be forgotten here. They do give contributions to the part we are interested in, which is the Wilson line dependent part, because closed strings are not charged under the gauge symmetry. We are left with two contributions: amplitudes with topologies of an annulus and of a Mœbius strip.

The annulus contribution represents interactions between branes and is relevant in our case only if both brane charges (i.e. branes and anti-branes) are present. We will restrict our analysis to the Mœbius strip amplitude which is on the other side always present in the models of interest to us. Moreover, without loss of generality, we will restrict to a Wilson line a acting on all the N branes along one compact dimension of radius R. The others are all treated on the same footing, forming, for example, a torus T^5 with a common radius r. The amplitude in the transverse, due to the exchange of closed strings can be written, in each case, as:

$$
\begin{aligned}
V_{\mathrm{eff}}(a) &= \frac{8N}{32\pi^4}\int_0^\infty \mathrm{d}l\, \frac{\theta_2^4}{16\eta^{12}}\left(\mathrm{i}l+\frac{1}{2}\right)\frac{R}{r^5}\sum_{\vec m} \mathrm{e}^{-2\pi\frac{\vec m^2}{r^2}l}\sum_n \mathrm{e}^{-4\mathrm{i}\pi na}\mathrm{e}^{-2\pi n^2R^2l} \\
&= \frac{n_F^i-n_B^i}{32\pi^4}\int_0^\infty \mathrm{d}l\, \frac{\theta_2^4}{16\eta^{12}}\left(\mathrm{i}l+\frac{1}{2}\right)\frac{R}{r^5}\sum_{\vec m} \mathrm{e}^{-2\pi\frac{\vec m^2}{r^2}l}\left(1+2\sum_{n>0}\cos(4\pi na)\mathrm{e}^{-2\pi n^2R^2l}\right) \qquad (27)
\end{aligned}
$$

where dimensionful quantities are defined in units of α'.

The canonically normalized scalar field h associated to the Wilson line a is $h = a/gR$, where g is the gauge coupling (11). Expanding the effective potential in powers of h allows one to extract its quadratic (squared mass) term $\mu^2h^2/2$. The result is:

$$
\mu^2 = -g^2\frac{n_F^i-n_B^i}{2\pi^2\alpha'}\int_0^\infty \mathrm{d}l\, \frac{\theta_2^4}{16\eta^{12}}\left(\mathrm{i}l+\frac{1}{2}\right)\frac{R^3}{r^5}\sum_{\vec m} \mathrm{e}^{-2\pi\frac{\vec m^2}{r^2}l}\sum_n n^2\mathrm{e}^{-2\pi n^2R^2l} \qquad (28)
$$

In order to see that the integral converges, note that, in the limit $l \to \infty$ the integrand falls off exponentially, while for $l \to 0$ one can use the Poisson re-summations

$$\sum_m \mathrm{e}^{-2\pi \frac{m^2}{r^2} l} = \frac{r}{\sqrt{2l}} \sum_p \mathrm{e}^{-\pi \frac{r^2}{2l} p^2} \tag{29}$$

$$\sum_n n^2 \mathrm{e}^{-2\pi n^2 R^2 l} = \frac{1}{R\sqrt{2l}} \sum_n \left(\frac{1}{4\pi R^2 l} - \frac{n^2}{4R^4 l^2}\right) \mathrm{e}^{-\frac{\pi}{2R^2 l} n^2} \tag{30}$$

and the identity

$$\frac{\theta_2^4}{\eta^{12}}\left(\mathrm{i}l + \frac{1}{2}\right) = (2l)^4 \frac{\theta_2^4}{\eta^{12}}\left(\frac{\mathrm{i}}{4l} + \frac{1}{2}\right) \tag{31}$$

to show that the integrand goes to a constant.

At the origin, μ^2 is negative or positive depending on the sign of $n_F^i - n_B^i$. As is expected the sign is such that the μ^2 is negative in the case (2) where the brane and orientifold have the same RR charge and so repulse each other, while it is attractive if the RR charges are of opposite sign (attraction). Even if a is a periodic variable of period 1, V_{eff} is periodic under the shift $a \to a + 1/2$ (since it originates from the Möbius amplitude) and it has a global minimum at $a = 1/4$ where its second derivative gives

$$V''_{\text{eff}}|_{a=1/4} = \frac{n_F^i - n_B^i}{2\pi^2} \int_0^\infty \mathrm{d}l\, \frac{\theta_2^4}{16\eta^{12}}\left(\mathrm{i}l + \frac{1}{2}\right) \frac{R}{r^5} \sum_{\vec{m}} \mathrm{e}^{-2\pi \frac{\vec{m}^2}{r^2} l} \sum_n (-)^{n+1} n^2 \mathrm{e}^{-2\pi n^2 R^2 l} \tag{32}$$

The sign is given again by the sign of $n_F^i - n_B^i$. Positivity of the last sum can be seen if it is rewritten as $\partial_\tau \theta_4(\tau)/2\mathrm{i}\pi$ with $\tau = 2\mathrm{i}R^2 l$.

In the T-dual picture, the VEV $a = 1/4$ corresponds to separating a brane at a distance from the origin equal to half the compact interval πR. However, the string framework, presented up to this point, is not complete. As the internal space is compact the RR charge needs to be globally cancelled, so that the case $n_F^i - n_B^i > 0$ implies existence of some similar objects with RR charge opposite to those of the branes.[2] These anti-branes or anti-orientifolds are localized somewhere else in the internal space, that can be put far away enough to avoid the presence of tachyonic states at tree-level. These new objects will attract the branes and destabilize the potential presented here, and whose only purpose was to compare with the field theory result.

To compare our result with the field theory one we start by taking the limit $r \to \infty$, using Eq. (29) for each of the five dimensions, we see that only $p = 0$ contributes to the sum. The expression (27) becomes then:

$$V_{\text{eff}}^i(a) = \frac{n_F^i - n_B^i}{32\pi^4} \int_0^\infty \mathrm{d}l\, l^{3/2} f_s(l) R \sum_{\vec{m}} \mathrm{e}^{-2\pi \frac{\vec{m}^2}{r^2} l} \sum_n \mathrm{e}^{-4\mathrm{i}\pi n a} \mathrm{e}^{-2\pi n^2 R^2 l} \tag{33}$$

In the infrared limit $l \to 0$, the effects of the string oscillators drop as:

$$f_s(l) = \left[\frac{1}{2l}\frac{\theta_2}{\eta^3}\left(\mathrm{i}l + \frac{1}{2}\right)\right]^4 \to 1 \quad \text{for } l \to 0 \tag{34}$$

and one recovers the result (20) with $d = 1$ and after two change of variables $l \to 2l/\pi$ and $a \to 2a$ have been performed.

Finally, let us comment on the form of the mass term at the origin. In the limit $r \to \infty$ and for arbitrary R, it can be written as:

$$\mu^2(R) = -\varepsilon^2(R) g^2 M_s^2 \tag{35}$$

with

$$\varepsilon^2(R) = \frac{n_F^i - n_B^i}{2^{7/2}\pi^2} \int_0^\infty \mathrm{d}l\, l^{3/2} f_s(l) R^3 \sum_n n^2 \mathrm{e}^{-2\pi n^2 R^2 l} \tag{36}$$

[2] Such complete models have been constructed, for example, in [8,9].

For R of order one, the whole string spectrum contributes, but in the limit $R \to \infty$ only the region $l \to 0$ dominates and the effective field theory result $\varepsilon(R) \sim 1/(RM_s)$ is recovered. The mass is *finite and calculable* in the effective field theory.

6. Discussions and conclusions

We have discussed simple models where the Higgs field is identified with the internal component of a gauge field. We have derived one-loop effective potentials which remain finite, computable in the effective field theory limit, insensitive to details of the UV completion. A string computation showed that the results hold as long as the scale of compactification is well separated from the UV cut-off (the string scale). These results call for a few remarks:

– The fact that the effective potential for the fields a_i are finite is due to their protection from quadratic divergences by the higher dimensional gauge symmetry. This symmetry is spontaneously broken by the compactification, remains non-linearly realized among the KK excitations as could be very easily seen by writing the corresponding Lagrangian. It is recovered by taking the compactification radius to infinity. This is a smooth limit that constrains the presence of any cut-off dependences.
– The property of the UV theory we made use of, is to allow us to sum over the *whole infinite* tower of KK modes and to commute sum and integration. This was necessary in order to perform the Poisson re-summation in (20). Our string theory example legitimates such a procedure.
– In the presence of a $(4+d)$-dimensional field dependent mass $M_I^2(\phi)$, not related to a gauge symmetry, the effective potential contains a divergent contribution:

$$V^{(\infty)} = \frac{1}{2}\sum_I (-)^{F_I} \int \frac{\mathrm{d}^{4+d}p}{(2\pi)^{4+d}} \log\bigl[p^2 + M_I^2(\phi)\bigr] \tag{37}$$

which signals sensitivity to the UV physics introduced to regularize it. This part identically cancels in the presence of supersymmetry.
– Another issue is related with compactification on a space with boundaries. These compactifications are useful in order to provide chiral fermions. They can be obtained from the above toroidal compactification by dividing by a discrete symmetry group. The orbifolding procedure introduces singular points, fixed under the action of the discrete symmetry, where new localized (twisted) matter can appear. These new states have no KK excitations along the directions where they are localized. The higher dimensional symmetry does not allow their direct coupling to the internal component of gauge field. Engineering such couplings should be done with parsimony as they could introduce sensitivity to cut-off physics in radiative corrections.

After these remarks, the next step is obviously to implement this mechanism in realistic model buildings. We refer the reader interested in the subject to more recent literature [10].

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