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Cloaking by plasmonic resonance among systems of particles: cooperation or combat?

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Abstract

We study quasistatic cloaking by the mechanism of plasmonic resonance, for systems of coated cylinders. Our focus is on the nature of the resonant cloaking interaction: whether systems of particles can be made to cooperate in cloaking a polarizable particle from an applied uniform field. We show that in fact if the cloaking regions of the systems of particles overlap, then they tend to interact in a fashion detrimental to their cloaking of the polarizable particle. If the cloaking regions touch but do not overlap, then the system of particles can cloak a larger region than each would in isolation. ***To cite this article: R.C. McPhedran et al., C. R. Physique 10 (2009).***

Résumé

Cloaking et résonance plasmoniques dans des systèmes de particules : coopération ou combat ? Nous analysons le cloaking en régime quasi-statique à travers le mécanisme des résonances plasmoniques, pour des systèmes de cylindres pelliculés. Nous focalisons notre étude sur la nature de l'interaction résonante du cloaking : à savoir si des systèmes de particules peuvent agir de concert pour cloaker une particule polarisable pour un champ extérieur uniforme. Nous montrons qu'en fait si les régions de cloaking du système de particules se chevauchent, alors elles tendent à interagir d'une manière néfaste pour chaque particule. En revanche, si les régions de cloaking se touchent mais ne se chevauchent pas, alors le système de particules peut cloaker un région plus étendue que chaque particule prise isolément. ***Pour citer cet article : R.C. McPhedran et al., C. R. Physique 10 (2009).***

Keywords: Cloaking; Optical properties; Electromagnetic theory; Resonance

Mots-clés : Cloaking ; Propriétés optiques ; Théorie électromagnétique ; Résonance

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1. Introduction

The topic of cloaking or the hiding of objects from detection by electromagnetic waves is one of the most exciting and active areas in modern physics. The subject builds on pioneering work by Veselago [1], with lesser known but important contributions also being made by Kerker [2]. The field really exploded in interest and activity after seminal contributions by Pendry and co-workers [3–6], Leonhardt and co-workers [7], while Greenleaf and co-workers [8,9] made significant prior contributions. The work we have mentioned so far broadly relied on a mechanism we may describe as *transformation optics*, or *internal cloaking*. The object to be hidden from scrutiny is placed inside a hollow cavity, which is surrounded by a metamaterial shell, carefully crafted to guide electromagnetic waves, of an appropriate frequency, round the cavity, in a fashion such that the detouring of rays is undetectable by an external observer. Similar interesting physics involving the control of rays arises in studies of corner reflectors and checkerboards involving metamaterials [10,11].

There is a second, complementary method of cloaking, which may be described as *cloaking by plasmonic resonance*, or *external cloaking*. In previous papers we have also referred to this mechanism as *cloaking by anomalous resonance* [12–15] or *partial resonance* [16]. Here, the principle is that an object to be cloaked is placed outside, but close to, a body capable of undergoing a plasmonic resonance. This resonance acts to cancel out external fields at the location of the small object. This idea can be traced to papers by some of the present authors [12–16], with interesting contributions also being made by Alu and Engheta [17,18], and Bruno and Lintner [19]. We mention that our previous work [12–15] dealt with full electromagnetic cloaking (not just quasistatics) for the Veselago slab lens. Also we showed fully three-dimensional cloaking of a point dipole in the quasistatic case [12]. We have addressed the problem of two-dimensional cloaking at finite frequencies, in a recent paper [20]. The same effect can be achieved in alternative ways, as discussed by Ramm [21] and Miller [22].

The mechanism of external cloaking is in principle similar to that occurring in more elementary contexts. In chemistry Le Chatelier's Principle and in electromagnetism Lenz's Law both dictate that systems in equilibrium react to external changes in such a way as to oppose or, if possible, cancel them. In the present paper we will explore how effective this reaction to external perturbations can be when a cloaking system is composed of several separated cylindrical particles.

The first two cloaking mechanisms are compared with a third technique (of *unfolded geometries*) [23–25] in a table in Nicorovici et al. [20]. It makes clear that the third technique [23] is intermediate in operation between internal and external cloaking, although in principle it is closely linked to cloaking by plasmonic resonance.

It is the object of this paper to present our first results on an investigation of a widely recognized limitation of all three cloaking methods: their narrow-band effectiveness. This topic is of great importance to any effective use of cloaking, since electromagnetic detection will almost inevitably involve use of a range of frequencies, and current cloaking designs either rely on geometric structures optimized for a narrow band of frequencies, or on optimal material properties which again are narrow-band. This problem has recently been discussed by Farhat et al. [26], and by Alu and Engheta [27]. The difference between the work here and that of Alu and Engheta is that we deal with the two-dimensional case of electrostatics and seek materials offering optimal dielectric permittivity, while Alu and Engheta deal with coated spheres and electromagnetic scattering, and combine both the dielectric permittivity and the magnetic permeability as design parameters. Also, our focus will be on whether cloaking by resonance can be made to work in a cooperative fashion, with individual particles "looking after" cloaking in a narrow range of frequencies, thereby in aggregate delivering wider-bandwidth invisibility. Alu and Engheta concentrate on non-resonant designs, and achieve promising results in this way.

In fact rather than discussing here the frequency overlap of plasmonic resonances in several coated cylinders, we will discuss the spatial overlap of their cloaking regions. The reinforcement of cloaking by the spatial overlap would be useful in practice, given that the physical choices of the cloaking material will not be ideal, and spatial overlap could deliver aggregate performance closer to that of the ideal materials. Again, the idea of spatial overlap is easier to investigate since it involves the use of only one material. A full investigation of spectral overlap would require designs for a range of cloaking materials with plasmonic resonance frequencies closely spaced.

In the next section, we briefly describe the numerical methods we have used to obtain the results of later sections. We continue in Section 3 with a discussion of systems of two and three coated cylinders, and show that in general resonant cloaking *cannot* be made to operate in a cooperative fashion: the interaction between cloaking cylinders is combative, in that, loosely speaking, each tries to cloak the others, rather than the external particle it is desired

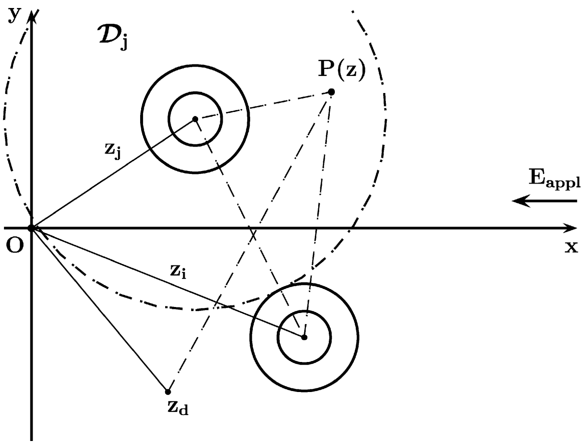

Fig. 1. Two coated cylinders, placed at the points z_i and z_j, and a polarizable line dipole at z_d. $\mathbf{E}_{\mathrm{appl}}$ is an electric field with sources at infinity. The annular domain $\mathcal{D}_j$, between the shell of the cylinder j and the dashed circle, is chosen so that it contains no other cylinder or field sources. P(z) is an arbitrary point in the plane where the total electric potential is evaluated.

be cloaked. We show that this is true not only for highly-idealized choices of permittivity, which exhibit extremely strong resonances, but also for the permittivity of silver, for which the resonances are damped by ohmic dissipation. In Section 4, we discuss a means by which cooperative resonant cloaking can be achieved, with the cloaking regions of successive coated cylinders touching rather than overlapping, so delivering a cloaking region larger in space but not widened in frequency. We summarize our results in Section 5.

2. Formulation

Here, we only outline the formulation. Detailed theoretical considerations and the corresponding numerical method are given in the electronic supplementary material.

We consider the problem of a finite cluster of identical coated cylinders with the core radius r_c and the shell radius r_s, having the core permittivity ε_c and the shell permittivity ε_s, embedded in a matrix (background) of permittivity ε_b. The cylinders are subjected to a uniform electric field sourced at infinity and the field of a polarizable line dipole added to the system so that it becomes a source of electric field. Fig. 1 shows a pair of nearest neighbors in the cluster.

For a physical system comprising a cluster of N coated cylinders, a dipole of magnitude $\mathbf{d} = (k^{(e)}, k^{(o)})$ located at $z_d \neq 0$, and a uniform field $\mathbf{E}$, the analytic potential in the matrix is given by the Wijngaard type series [28]

$$f_b^{(p)}(z) = C^{(p)} + E^{(p)}z + q\frac{k^{(p)}}{z - z_d} + q\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\frac{B_\ell^{(p,i)}}{(z - z_i)^\ell} \tag{1}$$

Here, the cylinders are represented by multipole series, $p = e, o$, and $q = 1$ if $p = e$, and $q = -1$ if $p = o$ [12,13]. The complex potential is obtained from the formula [12,13]

$$V_b(z) = \left[f_b^{(e)}(z) + f_b^{(e)}(\bar{z})\right]/2 + \left[f_b^{(o)}(z) - f_b^{(o)}(\bar{z})\right]/(2\mathrm{i}) \tag{2}$$

where the superposed bar denotes complex conjugation and the subscript b stands for background (matrix). Note that $f_b^{(p)}(\bar{z})$ means to take the complex conjugates of all complex coordinates z, z_d, and z_i.

Let j be a cylinder in the cluster and $\mathcal{D}_j$ an annular region free of any field sources or cylinders/inclusions (see Fig. 1). For $z \in \mathcal{D}_j$ and inside the cylinder j, we may write the complex potential in terms of the analytic potentials [12,13,15]

$$f_{\mathrm{out}}^{(p,j)} = \sum_{\ell=1}^{\infty} A_\ell^{(p,j)}(z - z_j)^\ell, \quad f_{\mathrm{in}}^{(p,j)} = q\sum_{\ell=1}^{\infty}\frac{B_\ell^{(p,j)}}{(z - z_j)^\ell}$$

$$\underline{f}_{\text{out}}^{(p,j)} = \sum_{\ell=1}^{\infty} C_\ell^{(p,j)} (z - z_j)^\ell, \quad \underline{f}_{\text{in}}^{(p,j)} = q \sum_{\ell=1}^{\infty} \frac{D_\ell^{(p,j)}}{(z - z_j)^\ell}$$
$$\underline{f}_{\text{c}}^{(p,j)} = \sum_{\ell=1}^{\infty} E_\ell^{(p,j)} (z - z_j)^\ell \tag{3}$$

Consequently, for $z \in \mathcal{D}_j$ we construct the complex potentials

$$V_{\text{out}}^{(j)} = \big[f_{\text{out}}^{(e)}(z) + f_{\text{out}}^{(e)}(\bar{z})\big]/2 + \big[f_{\text{out}}^{(o)}(z) - f_{\text{out}}^{(o)}(\bar{z})\big]/(2\text{i}) \tag{4}$$
$$V_{\text{in}}^{(j)} = \big[f_{\text{in}}^{(e)}(z) + f_{\text{in}}^{(e)}(\bar{z})\big]/2 + \big[f_{\text{in}}^{(o)}(z) - f_{\text{in}}^{(o)}(\bar{z})\big]/(2\text{i}) \tag{5}$$

the total complex potential for $z \in \mathcal{D}_j$ being

$$V_{\mathcal{D}_j}(z) = \widetilde{A}_0^{(j)} + V_{\text{out}}^{(j)}(z) + V_{\text{in}}^{(j)}(z) \tag{6}$$

where $\widetilde{A}_0^{(j)}$ is a constant to be determined. Also, in terms of analytic potentials (3) we have

$$f_{\mathcal{D}_j}^{(p)}(z) = A_0^{(p,j)} + f_{\text{out}}^{(p,j)}(z) + f_{\text{in}}^{(p,j)}(z) \tag{7}$$

which has to be identical to $f_b^{(p)}(z)$ for $z \in \mathcal{D}_j$. Hence, $f_b^{(p)}(z)$ is the analytical continuation of $f_{\mathcal{D}_j}^{(p)}(z)$ in the matrix. Consequently, for $z \in \mathcal{D}_j$ we have

$$A_0^{(p,j)} + \sum_{\ell=1}^{\infty} A_\ell^{(p,j)} (z - z_j)^\ell = C^{(p)} + E^{(p)} z + q \frac{k^{(p)}}{z - z_d} + q \sum_{i \neq j}^{N} \sum_{\ell=1}^{\infty} \frac{B_\ell^{(p,i)}}{(z - z_i)^\ell} \tag{8}$$

The constant $C^{(p)}$ can be set so that the potential at the origin of coordinates is zero ($f_b^{(p)}(0) = 0$):

$$C^{(p)} = q \frac{k^{(p)}}{z_d} - q \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \frac{B_\ell^{(p,i)}}{(-z_i)^\ell} \tag{9}$$

To show the symmetries of the cylinder cluster one has to set the origin of coordinates at the centroid of the cluster, and the direction of axes according to the direction of the cluster symmetry axes.

Now, introduce the notations $\zeta_j = z - z_j$, $z_{jd} = z_d - z_j$, and $z_{ji} = z_i - z_j$, and note that for $z \in \mathcal{D}_j$ we have $|\zeta_j| < |z_{jd}|$ and $|\zeta_j| < |z_{ji}|$, $\forall i \neq j$ (see Fig. 1), so that we may use the series expansions of $1/(\zeta_j - z_{jd})$ and $1/(\zeta_j - z_{ji})^\ell$ to rewrite (8) in the form

$$A_0^{(p,j)} + \sum_{\ell=1}^{\infty} A_\ell^{(p,j)} \zeta_j^\ell = C^{(p)} + E^{(p)}(\zeta_j + z_j) - q \frac{k^{(p)}}{z_{jd}} \sum_{\ell=0}^{\infty} \left(\frac{\zeta_j}{z_{jd}}\right)^\ell$$
$$+ q \sum_{i \neq j}^{N} \sum_{m=1}^{\infty} \sum_{s=m-1}^{\infty} B_m^{(p,i)} \frac{(-1)^m}{z_{ji}^m} \binom{s}{m-1} \left(\frac{\zeta_j}{z_{ji}}\right)^{s-m+1} \tag{10}$$

The zeroth order term is

$$A_0^{(p,j)}(z_j, z_{jd}, z_{ji}) = C^{(p)} + E^{(p)} z_j - q \frac{k^{(p)}}{z_{jd}} + q \sum_{i \neq j}^{N} \sum_{\ell=1}^{\infty} B_\ell^{(p,i)} \frac{(-1)^\ell}{{z_{ji}}^\ell}$$
$$= E^{(p)} z_j + q k^{(p)} \left[\frac{1}{z_d} - \frac{1}{z_{jd}}\right] + q \sum_{i \neq j}^{N} \sum_{\ell=1}^{\infty} B_\ell^{(p,i)} \frac{(-1)^\ell}{{z_{ji}}^\ell} - q \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \frac{B_\ell^{(p,i)}}{(-z_i)^\ell} \tag{11}$$

Taking into account that

$$V_{\mathcal{D}_j}(z) = \frac{1}{2}\big[f_{\mathcal{D}_j}^{(e)}(z) + f_{\mathcal{D}_j}^{(e)}(\bar{z})\big] + \frac{1}{2\text{i}}\big[f_{\mathcal{D}_j}^{(o)}(z) - f_{\mathcal{D}_j}^{(o)}(\bar{z})\big]$$

we may express the constant $\widetilde{A}_0^{(j)}$ from (6) in the form

$$\widetilde{A}_0^{(j)} = \frac{1}{2}\big[A_0^{(e,j)}(z_j, z_{jd}, z_{ji}) + A_0^{(e,j)}(\overline{z_j}, \overline{z_{jd}}, \overline{z_{ji}})\big] + \frac{1}{2\mathrm{i}}\big[A_0^{(o,j)}(z_j, z_{jd}, z_{ji}) - A_0^{(o,j)}(\overline{z_j}, \overline{z_{jd}}, \overline{z_{ji}})\big] \tag{12}$$

For $m \geqslant 1$

$$A_m^{(p,j)} - q\sum_{i\neq j}^{N}\sum_{\ell=1}^{\infty} B_\ell^{(p,i)} \frac{(-1)^\ell}{{z_{ji}}^{\ell+m}}\binom{m+\ell-1}{\ell-1} = E^{(p)}\delta_{m,1} - q\frac{k^{(p)}}{{z_{jd}}^{m+1}} \tag{13}$$

and from the boundary conditions at the shell–matrix interface we obtain a second relation

$$B_m^{(p,i)} = \beta_m^{(i)} A_m^{(p,i)} \tag{14}$$

where

$$\beta_m^{(i)} = \frac{{r_c^{(i)}}^{2m}(\varepsilon_s^{(i)} - \varepsilon_c^{(i)})(\varepsilon_b + \varepsilon_s^{(i)}) + {r_s^{(i)}}^{2m}(\varepsilon_s^{(i)} + \varepsilon_c^{(i)})(\varepsilon_b - \varepsilon_s^{(i)})}{{r_c^{(i)}}^{2m}(\varepsilon_s^{(i)} - \varepsilon_c^{(i)})(\varepsilon_b - \varepsilon_s^{(i)}) + {r_s^{(i)}}^{2m}(\varepsilon_s^{(i)} + \varepsilon_c^{(i)})(\varepsilon_b + \varepsilon_s^{(i)})}{r_s^{(i)}}^{2m} \tag{15}$$

Here, $r_c^{(i)}$ and $r_s^{(i)}$ denote the core and shell radii of the cylinder i. Also, $\varepsilon_c^{(i)}$ and $\varepsilon_s^{(i)}$ are respectively the core and shell permittivities, while ε_b represents the permittivity of the matrix (background). Finally, by substituting Eq. (14) into Eq. (13) we obtain the linear system

$$A_m^{(p,j)} - q\sum_{i\neq j}^{N}\sum_{\ell=1}^{\infty} \frac{(-1)^\ell}{{z_{ji}}^{\ell+m}}\binom{m+\ell-1}{\ell-1}\beta_\ell^{(i)} A_\ell^{(p,i)} = E^{(p)}\delta_{m,1} - q\frac{k^{(p)}}{{z_{jd}}^{m+1}} \tag{16}$$

Eq. (16) can be used when we know the magnitude of the dipole. In the case of a polarizable dipole, the magnitude of the dipole is determined by the total electric field at the point where the dipole is located. Thus, the total electric field at an arbitrary point z is $\mathcal{E} = -\nabla V_b(z)$ [29], which is singular at $z = z_d$. Hence, we have to remove the dipole term from $V_b(z)$ or, equally well, from $f_b^{(p)}(z)$ defined in Eq. (1), and use

$$\widetilde{f}_b^{(p)}(z) = E^{(p)}z + q\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\frac{B_\ell^{(p,i)}}{(z-z_i)^\ell} \tag{17}$$

Note that we have also removed the constant which vanishes after differentiation. From (17) we derive the expression of the complex potential

$$\widetilde{V}_b(z) = \frac{1}{2}\big[\widetilde{f}_b^{(e)}(z) + \widetilde{f}_b^{(e)}(\bar{z})\big] + \frac{1}{2\mathrm{i}}\big[\widetilde{f}_b^{(o)}(z) - \widetilde{f}_b^{(o)}(\bar{z})\big] \tag{18}$$

and the total electric field at z_d

$$\mathcal{E} = -\nabla\widetilde{V}_b(z)|_{z=z_d} \tag{19}$$

Thus, we obtain

$$k^{(e)} = -\alpha E^{(e)} + \alpha\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\ell\left[B_\ell^{(e,i)}\,\mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right) - B_\ell^{(o,i)}\,\mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right] \tag{20}$$

$$k^{(o)} = -\alpha E^{(o)} - \alpha\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\ell\left[B_\ell^{(e,i)}\,\mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right) + B_\ell^{(o,i)}\,\mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right] \tag{21}$$

Finally, by substituting $k^{(e)}$ and $k^{(o)}$ into (16) we obtain a linear system for the coefficients $A_m^{p,j}$. Note that, in the case $E^{(e)} = E^{(o)} = 0$ this linear system becomes homogeneous and the solution is the trivial one, except the resonant states. Therefore, all the multipole coefficients vanish and there is no effect. Actually, with no external source the whole system of polarizable dipoles and cylinders is "in darkness" with all the components being then invisible.

The resonant states, if they exist, are interesting because the only parameters which can be varied to achieve the resonance effect are the polarizability of the dipole and the distances between the dipole and cylinders. The arrangement of cylinders could be important too.

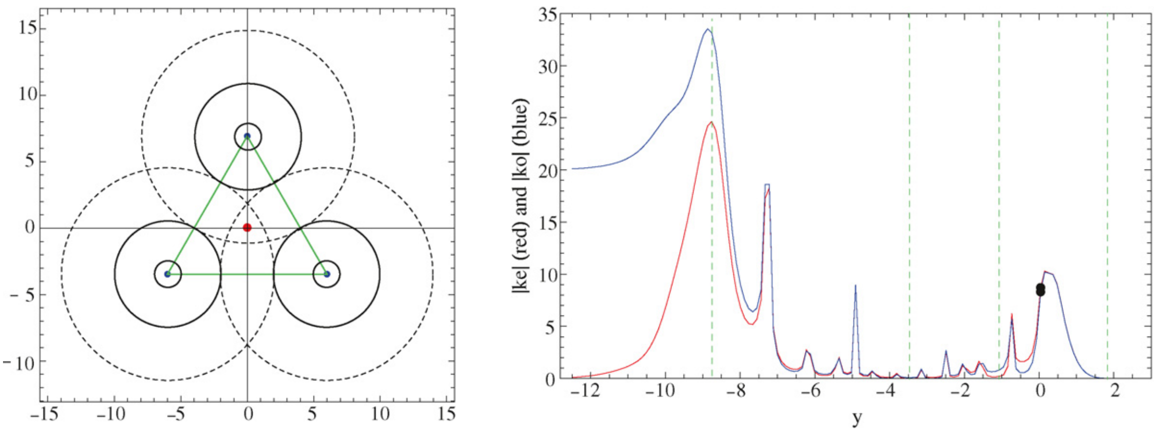

Fig. 2. Numerical computations for the magnitude of a polarizable dipole moving along the y-axis through the cloaking regions of a system of three cylinders. Left: the physical structure containing coated cylinders with $r_c = 1$, $r_s = 4$, $\varepsilon_c = \varepsilon_m = 1$, $\varepsilon_s = -1 + 10^{-6}\mathrm{i}$, and $\mu_c = \mu_s = \mu_m = \mu_0$. The applied external electric field is $\mathbf{E} = (0, -1)$ and the polarizability of the dipole is $\alpha = 20$. The dashed line denotes the cloaking radius $r_\# = \sqrt{r_s^3/r_c} = 8$ (see [12,13]), and the centers of the cylinders are located at the points $z = d\{1 - \mathrm{i}\sqrt{3}/3, 2\mathrm{i}\sqrt{3}/3, -1 - \mathrm{i}\sqrt{3}/3\}$, with $d = 16$. Right: the values of $|k_e|$ and $|k_o|$ corresponding to the position of the polarizable dipole in the left panel (the black dots in the right panel show the x-coordinate of the red dot on the k_e- and k_o-curve in the left panel). Here, the black dots correspond to $(x = 0, y = 0.04)$, where $k_e = 1.7205 + 8.54009\mathrm{i}$ and $k_o = -4.6295 + 6.89136\mathrm{i}$.

3. Combative cloaking

We now consider a polarizable particle placed in a uniform external field, and approaching a system of three cloaking cylinders with centers arranged on the vertices of an equilateral triangle. The inner and outer radii of the coated cylinders are chosen so that the boundaries of their cloaking regions (indicated by dashed lines in Fig. 2, left panel) just touch the outer surfaces of the other cylinders. The three cloaking regions overlap in a small, almost triangular region. It was our expectation that the cloaking action of the first pair of coated cylinders would set in as the particle penetrated their overlapping cloaking region, and the cloaking would be enhanced when the particle entered the region where the third cylinder could enhance the cloaking.

We show results for two cases: one where the cloaking cylinders have a coating with a dielectric permittivity with a very small imaginary part (Fig. 2), and one where its imaginary part is that of silver at just that wavelength where the real part of the dielectric permittivity passes through -1 (Fig. 3).

Considering first the idealized case, we see that cloaking tends to be best in the region of two-cylinder interaction, but is bad in the region where three cylinders interact. However, even in the region of pair interaction, the cloaking interaction is inferior to that of a single coated cylinder acting in isolation. Note the small regions of local maxima in Fig. 2 (right panel), where cloaking is less effective, and the very evident peak at the center of the three-cylinder cloaking region. The best cloaking interaction is evident at the right of the curve in Fig. 2, where only the top cylinder is involved.

Consider next the case of an actual metal as the cloaking material (Fig. 3). Naturally enough, the cloaking action is a much smoother function of position, as we would expect given the lower quality factor of the plasmon resonance associated with the damping effect of the imaginary part of the dielectric permittivity. Roughly speaking, this is the major difference between the results in Fig. 2 and Fig. 3: there is a good cloaking action at the center of the pair interaction region, but this is lost in the three cylinder interaction region.

We can see no evidence in either Fig. 2 and Fig. 3 of the cylinders acting to help each other in diminishing the applied field acting on the polarizable external particle. This conclusion also relates to other numerical experiments on different geometrical configurations of cloaking cylinders. For example, in Fig. 4 we show the interaction of the polarizable dipole with a pair of cloaking cylinders: the similarity of the results here with those for three cloaking cylinders is evident. In the case of highly resonant cloaking, if there are interactions then the cloaking is fragile and spatially oscillatory, and rarely helped by cloaking particle interaction. If the cloaking corresponds to realistic material parameters, and is thereby only moderately resonant, then the most important factor is how deeply the particle to be cloaked penetrates into the cloaking region, rather than whether there are multiple cloaking interactions.

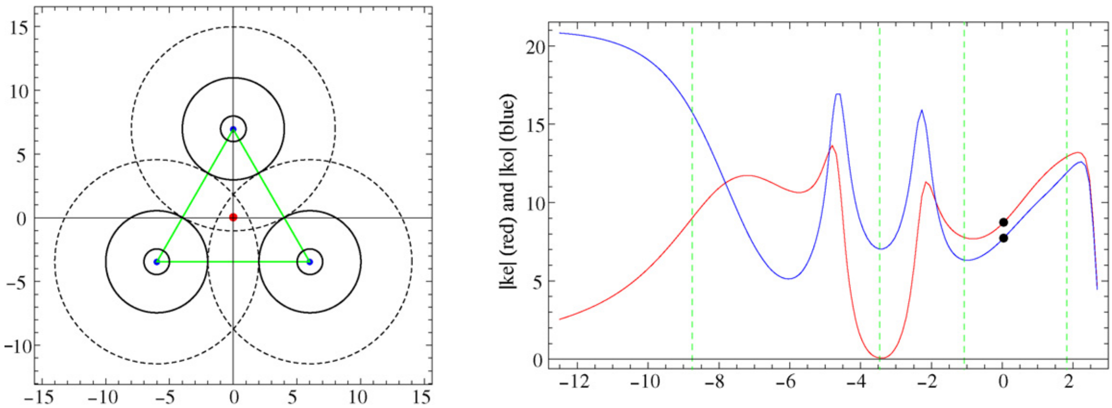

Fig. 3. Numerical computations for the magnitude of a polarizable dipole moving along the y-axis through the cloaking regions of a system of three coated cylinders with silver shell. The parameters are the same as in Fig. 2 except shell permittivity which is $\varepsilon_s = -1 + 0.585385\text{i}$ for an external radiation of wavelength $\lambda = 336.9681$ μm (r_c, r_s and d are also given in μm). For the position of the dipole shown in the left panel ($x = 0$, $y = 0.04$) we have $k_e = 4.67281 + 7.38056\text{i}$ and $k_o = -7.71414 - 0.49838\text{i}$.

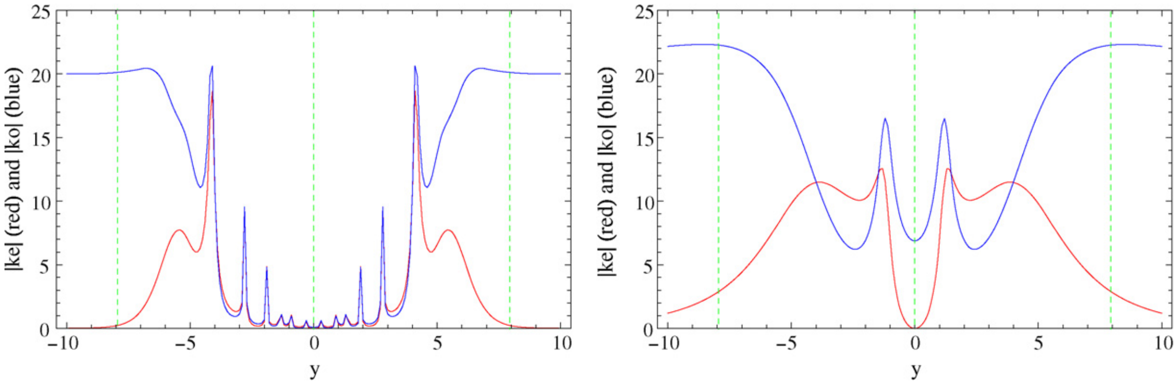

Fig. 4. The same as Fig. 2 (left panel) and Fig. 3 (right panel) but with the top cylinder removed. The vertical dashed lines mark the lower intersection of the cloaking areas, the position of the cylinder centers, and the upper intersection of the cloaking areas.

4. Cooperative cloaking

We now consider a design where the coated cylinders have cloaking regions which touch, but do not overlap (see Fig. 5, left panel). The result is that we can follow the polarizable particle along a trajectory where it is only interacting strongly with one cloaking cylinder at a time. In Fig. 5 (right panel) we see that this gives us quite a good cloaking effect over a trajectory whose length can be increased indefinitely. There are small “blips”, where the particle moves from one cloaking region into the next, but overall this design works well. The best cloaking occurs as we would expect, at the points where the particle is deepest in the cloaking zone of a single cylinder.

For the next example (Fig. 6), we show a two-layer arrangement of cloaking cylinders, again with an idealized imaginary part of the dielectric permittivity. The cloaking behavior is similar to that in Fig. 5, except that the “blips” in cloaking where overlap regions touch is larger, and there is an interesting alternance in their size. The reasons for the differences between the behavior in Figs. 5 and 6 are not fully understood, and need to be explored if this mechanism of collaborative cloaking is to be further developed.

5. Conclusions

We have explored phenomena which arise when quasistatic cloaking due to plasmonic resonance is attempted with systems of coated cylinders. The hoped-for strengthening of cloaking effects due to overlapping resonances has not been in evidence, at least in the cases we have investigated. Other geometries may well be required, perhaps with

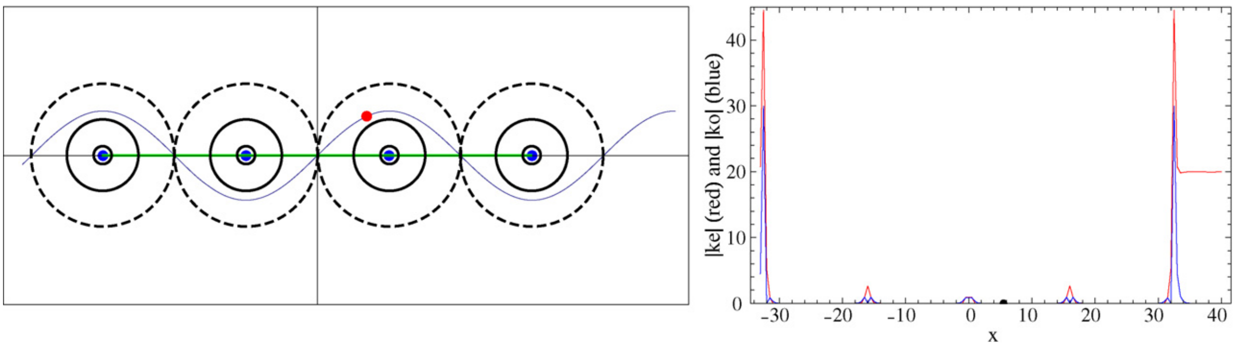

Fig. 5. Numerical computations for the magnitude of polarizable dipole moving along a sin-type trajectory ($y = 5\sin(\pi x/d)$) through the cloaking regions of a chain of coated cylinders. Left: the physical structure containing coated cylinders with $r_c = 1$, $r_s = 4$, $\varepsilon_c = \varepsilon_m = 1$, $\varepsilon_s = -1 + 10^{-6}\mathrm{i}$, and $\mu_c = \mu_s = \mu_m = \mu_0$. The applied external electric field is $\mathbf{E} = (-1, 0)$ and the polarizability of the dipole is $\alpha = 20$. The dashed line denotes the cloaking radius $r_\# = 8$ and the distance between the centers of the cylinders is $d = 2r_\# = 16$. Right: the values of $|k_e|$ and $|k_o|$ corresponding to the position of the polarizable dipole in the left panel (the black dots in the right panel show the x-coordinate of the red dot in the left panel). The black dots correspond to $k_e = -0.0000991 - 0.0004102\mathrm{i}$ and $k_o = 0.0003424 - 0.0000991\mathrm{i}$.

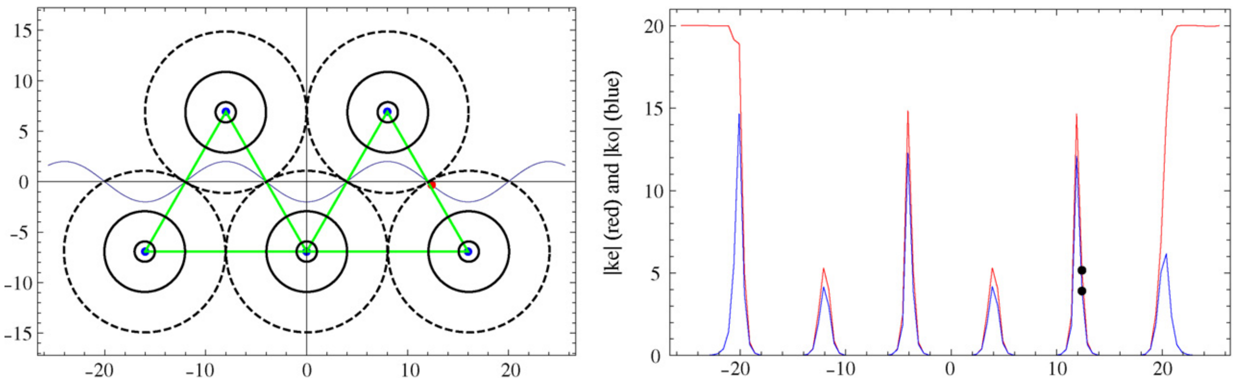

Fig. 6. Numerical computations for the magnitude of polarizable dipole moving along a sin-type trajectory ($y = 2\sin(2\pi x/d - \pi/2)$) through the cloaking regions of a cluster of coated cylinders. Left: the physical structure containing coated cylinders with $r_c = 1$, $r_s = 4$, $\varepsilon_c = \varepsilon_m = 1$, $\varepsilon_s = -1 + 10^{-6}\mathrm{i}$, and $\mu_c = \mu_s = \mu_m = \mu_0$. The applied external electric field is $\mathbf{E} = (-1, 0)$ and the polarizability of the dipole is $\alpha = 20$. The dashed line denotes the cloaking radius $r_\# = 8$ and the distance between the centers of the cylinders is $d = 2r_\# = 16$. Right: the values of $|k_e|$ and $|k_o|$ corresponding to the position of the polarizable dipole in the left panel (the black dots in the right panel show the x-coordinate of the red dot in the left panel). The black dots correspond to $k_e = -0.461442 - 5.14197\mathrm{i}$ and $k_o = 3.88616 - 0.439134\mathrm{i}$.

multiple coatings or with sectored coating. to achieve the sought-after effects of better cloaking bandwidth, or stronger cloaking with non-optimal materials. Of course, the investigations reported here need also to be undertaken in the case of the full Maxwell equations, and not just in the quasistatic limit.

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Supplementary information

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