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Elastic stresses in random flow of a dilute polymer solution and the turbulent drag reduction problem

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Abstract

In this short review I argue that the progress in our understanding the mechanism of turbulent drag reduction is conditioned by obtaining experimental data on dynamics and statistics of polymer stretching and elastic stresses in inertial turbulence at high Reynolds numbers that is a technically challenging task. The suggested way out of the currently unresolved technical problem is to collect the same data in elastic turbulence, which is a smooth random flow similar to that found in inertial turbulence below the dissipation scale. Since the polymer stretching and elastic stresses in inertial turbulence are influenced only by small scales, it is appropriate to use information on the polymer stretching and elastic stresses obtained in elastic turbulence. The experimental data on the statistics of the polymer stretching, the coil–stretch transition, and elastic stresses together with spatial distribution and values of the rms of the velocity gradients were collected in elastic turbulence for the last several years. This information serves a basis for a new hypothesis of turbulent drag reduction. ***To cite this article: V. Steinberg, C. R. Physique 10 (2009).***

Résumé

Efforts élastiques dans l'écoulement aléatoire d'une solution diluée de polyméres : le problème de la réduction de la traînée turbulente. Dans cette brève monographie je suggère que le progrès dans notre compréhension du mécanisme de la réduction de la traînée turbulente est conditionnée par l'obtention de données expérimentales sur la dynamique et la statistique de l'étirement des polymères et des efforts élastiques dans la turbulence inertielle à nombre de Reynolds élevés, ce qui est techniquement un défi. Comme issue au problème technique actuellement non résolu, il est proposé de collecter les mêmes données dans la turbulence élastique, qui est un écoulement aléatoire régulier semblable à celui de la turbulence inertielle en dessous de l'échelle dissipative. Comme l'étirement du polymère et les efforts élastiques dans la turbulence inertielle ne sont influencés que par les petites échelles, il est possible d'utiliser l'information obtenue en turbulence élastique. Dans le régime de turbulence élastique, les données expérimentales sur la statistique de l'étirement, la transition pelote-spaghetti et les efforts élastiques, ainsi que sur la distribution spatiale et la moyenne quadratique des gradients de vitesse, ont été rassemblés pendant les années qui précèdent. Cette information sert de base à une nouvelle hypothèse concernant la réduction de la traînée turbulente. ***Pour citer cet article : V. Steinberg, C. R. Physique 10 (2009).***

Keywords: Elastic stress; Elastic turbulence; Turbulent drag reduction; Stress sensor; Polymer stretching

Mots-clés : Effort élastique ; Turbulence élastique ; Réduction de la traînée turbulente ; Capteur d'effort ; Étirement des polymères

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1. Introduction

For years both theoretical and experimental studies on the turbulent drag reduction problem in a dilute polymer solution were concentrated on extracting relevant information only from a velocity field. In contrast to a Newtonian fluid flow, where the velocity field completely describes its dynamics, the hydrodynamics of a polymer solution is characterized by two fields: velocity and elastic stress. So only knowledge of both fields can lead to understanding of the long standing problem of the turbulent drag reduction [1]. Indeed, hydrodynamics of polymer solution is similar to magneto-hydrodynamics, where both velocity and magnetic fields are measured and are necessary to understand the flow.

Two basic phenomenological theories of turbulent drag reduction differ in their conjectures about dynamics and stretching of a polymer in a random flow and the role of elastic stress in the drag reduction. As Lumley [2] correctly pointed out long time ago, a polymer in a turbulent flow is only influenced by small scales, at which a spatially smooth and random in time flow is realized, since a polymer contour length is, as a rule, smaller than the dissipation length in inertial turbulence. Large scale eddies just transfer a polymer. The reason for a polymer to modify properties of a flow is its flexibility that allows the polymer under an inhomogeneous flow to be stretched from the coiled state. As was qualitatively proved by Lumley [3], the coil–stretch transition of polymer molecules can also occur in a random flow, if the time criterion is fulfilled: $\lambda/\lambda_{\text{turb}} \geqslant 1$, where λ is the longest polymer relaxation time, and λ_{turb} is the appropriate characteristic time in turbulent flow, defined, for example, as $(\partial V_i/\partial x_j)^{-1}_{\text{rms}}$. The ratio $\lambda/\lambda_{\text{turb}}$ is defined as the Weissenberg number *Wi*. These two basic statements were verified experimentally only recently [4]. However, the next assumption, which became the central one for the explanation of the drag reduction in the Lumley's theory [2], stated that the strongly stretched and streamwise oriented polymers mostly in a buffer layer drastically increased viscosity there, similar to the viscosity increase in an elongation flow at large *Wi* [2]. This assumption is doubtful and difficult to verify experimentally. About 20 years later another theory suggested by de Gennes was based on rather opposite assumptions [5]. His model is based on the elasticity arguments of a polymer solution. Contrary to the Lumley's assertion of the occurrence of the coil–stretch transition in a random flow, de Gennes assumed that no coil–stretch transition of a polymer is realized in the random flow, so there is no measurable change in viscosity, but the elastic energy stored by the partially stretched polymers is sufficient to balance the Reynolds stresses in a turbulent flow at some scale and at a polymer concentration above the concentration threshold. The latter looks possible, since the elastic energy increases with decreasing length scale of turbulence, whereas the turbulent energy decreases with scale size. The essential idea is that small scales of turbulence are terminated at some scale larger than the dissipation scale, if the elastic energy is sufficiently large. The larger length scale can lead to increased buffer layer thickness and to drag reduction [5,6]. A more recent theory of the turbulent drag reduction [7] just revived the Lumley's idea by adding a specific dependence of viscosity on the distance from the wall. So the tensorial elastic stress field is replaced by the scalar function with physical arguments that are rather unclear.

In the last decade significant progress in dynamics of a single polymer in a random flow and in the hydrodynamics of polymer solutions has been made and three main results were obtained. First, dynamics and statistics of stretching of a polymer molecule in shear, elongation and random flows were studied and a possibility of the coil–stretch transition in these flows was verified experimentally [4,8–11]. Second, a significant progress in numerical simulations of turbulent pipe flow of a dilute polymer solution, described by simplified phenomenological models (Oldroyd-B, FENE-P) was conducted [12,13]. These closely reproduced most of the known features of the drag-reducing polymer flow and dynamics of elastic stresses. Thirdly, direct numerical simulations of the turbulent Kolmogorov flow of a dilute polymer solution reveal the scaling of the friction coefficient, the critical Reynolds number for the onset of drag reduction and the scaling of polymer stresses above the onset with the Reynolds number and the polymer elasticity and its independence of the polymer concentration [14]. The latter is due to the linear character of the viscoelastic model used. Separately I would like to mention the theory of unbounded homogeneous hydrodynamic turbulence with a small addition of polymers [15]. The influence of polymer elastic stresses in particular on velocity power spectra is considered in approximation of linear polymer elasticity and feedback reaction of polymers on a turbulent flow. Since this theory is also applied to elastic turbulence, I will discuss it in detail later on.

From this very short review of the main theories and ideas in turbulent drag reduction, several conceptual ideas and facts are lacking to make further progress in the field:

(i) There are no agreement that just small scale turbulence below the dissipation (or Kolmogorov) scale completely determines the polymer dynamics and statistics of stretching. For example, up to now an influence

of large coherent structures in turbulent flow on turbulent drag is discussed in both theory and experiments.

(ii) Until recently a possibility of the coil–stretch transition in a turbulent flow was discussed for a long time without convincing results in theory and with a lack of the experimental data.

(iii) There is a lack of a theory of turbulent drag reduction in a finite geometry container, where the elastic stresses are employed, and an absence of the experimental data on the value and distribution of the elastic stresses in a turbulent flow.

Thus, one concludes that measurements and characterization of polymer stretching and elastic stresses in a turbulent flow are the key issues to solve the turbulent drag reduction problem.

A novel approach to the turbulent drag reduction problem was taken after a discovery of so-called elastic turbulence, a striking phenomenon of a polymer solution flow [16,17]. It is a chaotic flow at $Re \ll 1$ and large Wi that introduces strong nonlinearity in the flow. In an attempt to understand elastic turbulence, theoretical and experimental investigations of dynamics and statistics of a single polymer in a spatially smooth and random in time flow were undertaken [4,18–27]. Since this random flow is of the same nature as the Batchelor regime [28] below the dissipation scale in inertial turbulence, the investigation of polymer dynamics and statistics in elastic turbulence is also relevant to the dynamics and statistics of polymers in the turbulent drag reduction problem. Thus here the relation between elastic turbulence and turbulent drag reduction problems is disclosed.

2. Elastic turbulence and its main properties

Long polymer molecules added to a fluid make it elastic and capable of storing stresses that depend on the history of deformation, thereby providing the fluid with a memory [29]. Many properties of the polymer solution flows (especially dilute ones) can be understood on the basis of single polymer dynamics, since a polymer there experiences the combined action of the stretching by the flow and the elastic relaxation. The elastic stress generated by the polymer stretching in the flow becomes the main source of nonlinearity in the polymer solution flow at low Re. As the result, an elastic instability shows up, when the elastic energy overcomes the dissipation due to polymer relaxation. The ratio of the nonlinear elastic term to the linear relaxation is defined by the Weissenberg number [29]. In a simple shear flow of a polymer solution the elastic stress is anisotropic and characterized mainly by the difference in the normal stresses along the flow velocity and the velocity gradient directions, called the first normal stress difference (another two normal stress differences are usually much smaller [29]). In a curvilinear shear flow (e.g. Couette flow between rotating cylinders) the first normal stress difference gives rise to a volume force acting on the fluid in the direction of the curvature, the "hoop stress" [29]. The latter becomes the driving force for a rod climbing effect [29] as well as an elastic instability. The mechanism of the elastic instability in the Couette flow was first suggested in Ref. [30] and experimentally verified in Ref. [31]. It was also widely investigated in other flow geometries with curvilinear trajectories, including a curvilinear channel. The streamline curvature is a necessary ingredient to reduce the critical Weissenberg number for the instability onset, Wi_c. On the other hand, the elastic instability and elastic turbulence were also obtained for the first time in a Kolmogorov 2D shear flow of a dilute polymer solution described by the linear Oldroyd-B model with rectilinear streamlines [21,32]. In spite of very simplified geometry without boundaries, this model reproduces most of the features of elastic turbulence. It is particularly surprising in the light of our recent findings discussed below that the elastic stresses produced by linear polymers and their feedback reaction on the flow are not sufficient to stabilize the flow and polymer stretching far above the coil–stretch transition. Another theoretical observation based on stability analysis and numerical simulations of a more realistic shear flow of a viscoelastic fluid in a straight channel also reveals the elastic instability, though its experimental verification is still lacking [33].

Above the purely elastic instability, a path to a chaotic flow in a form of irregular flow patterns at $Wi > Wi_c$ was studied in three flow geometries: Couette flow between cylinders, swirling flow between two disks, and flow in a curvilinear channel [16,17]. It was reasonable to assume that at sufficiently high Wi and vanishingly small Re a random flow, which exhibits continuous velocity spectra in a wide range of temporal and spatial scales similar to a hydrodynamic turbulent flow, can be excited. Indeed, such random flow was observed first in both Couette and swirling flows of dilute polymer solutions and dubbed "elastic turbulence" [16]. In the swirling flow between two disks it was identified due to three main features: a sharp growth in flow resistance, a steep algebraic decay of velocity

power spectra over a wide range of scales, and orders of magnitude more effective way of mixing than in an ordered flow [16,17]. These properties are analogous to those of hydrodynamic turbulence. Elastic turbulence in the swirling flow was observed in dilute polymer solutions of various polymer concentrations down to 7 ppm [16]. Since the elastic energy is proportional to the polymer concentration and should exceed dissipation independent of it, one should expect a minimum threshold value for concentration, below which elastic turbulence cannot be generated.

However, the similarities in the main characteristics of two flows do not imply that the physical mechanism that underlies the two kinds of random motion is the same. Indeed, in contrast to inertial turbulence at high *Re*, where inertial terms are the main source of nonlinearity and large Reynolds stresses, large elastic stresses are a manifestation of nonlinearity and the cause of elastic turbulence in low *Re* flow of a polymer solution. One can suggest that in a random flow driven by elasticity, the elastic stress tensor σ_p should be the object of primary importance and interest, and that it may be appropriate to view elastic turbulence as turbulence of the σ_p-field. It would be then more relevant and instructive to explore the spatial structure, the dynamics and statistics of this field. Currently no experimental technique allows a direct local measurement of the elastic stress in a turbulent flow. On the other hand, properties of the σ_p-field in a boundary layer were inferred from measurements of injected power, whereas its local properties were evaluated from measurements of spatial and temporal distributions of velocity gradients [23,34].

The key early experimental observation in elastic turbulence is the power-law decay of the velocity power spectra in all flow geometries with the exponent $|\delta| > 3$ (between -3.3 and -3.6) [16,17]. Due to the sharp velocity spectrum decay, the velocity and velocity gradient are both determined mostly by the integral scale, i.e., the vessel size. It means that elastic turbulence is essentially a spatially smooth random in time flow, dominated by strong nonlinear interaction of a few large-scale modes. As pointed out above, it is the same random flow that occurs in hydrodynamic turbulence below the dissipation scale and is called the Batchelor flow regime [28]. Further characterization of this random flow in elastic turbulence was conducted by measuring velocity correlation functions via particle image velocimetry (PIV) [35].

3. Statistics of polymer stretching and coil–stretch transition in a random flow, theory of elastic turbulence

3.1. Statistics of polymer stretching and the coil–stretch transition in a random flow

There are two aspects of theory of polymer stretching in a flow. First is statistics of polymer stretching and a coil–stretch transition in a spatially smooth and random in time flow, and second is the properties of elastic turbulence resulting from the polymer stretching.

Two different approaches to the coil–stretch transition of a polymer and to a saturation of polymer stretching in a spatially isotropic random flow (without mean shear) were suggested. In the first approach a polymer is passive [19], i.e. a polymer is just stretched and advected by a flow, whereas in the second approach a polymer is considered to be active [18], i.e. being stretched, it modifies the flow. In the coil state both theories are basically equivalent, and the difference appears only above the transition to the stretched state in regards to a saturation mechanism for the polymer stretching.

Being placed in a inhomogeneous steady flow a polymer is stretched due to a velocity gradient ∇V, if $Wi \equiv \lambda \nabla V \sim 1$. Depending on flow geometry either the sharp coil–stretch transition to the strongly extended state occurs, like in an elongation flow [8,10], or no coil–stretch transition with a continuous increase in stretching takes place in the case of a shear flow [9], which can be presented as a combination of elongation and rotational components. In a contrast to stationary flows, a polymer in a homogeneous smooth random flow at each moment enters regions with a random change of the direction of high stretching. As *Wi* increases, the effect of the stretching becomes more pronounced. The main progress made by the recent theories [18,19] consists in establishing a quantitative relation between statistics of a random velocity field and statistics of polymer extensions in this flow. The rate of the polymer stretching is defined by the principal Lyapunov exponent γ of the flow, which is the average logarithmic divergence rate of nearby Lagrangian trajectories in the flow. According to the theory [18], the tail of probability distribution function (PDF) of the polymer stretching is described by the power law $P(R_i) \propto R_i^{-\alpha-1}$, where $\alpha \sim (\lambda^{-1} - \gamma)$ in the vicinity of the transition, and R_i is the end-to-end distance for a given configuration of a polymer i. Positive α corresponds to the majority of the polymers being in the coil state, whereas $\alpha < 0$ means that the majority of the polymers is strongly stretched. Thus $\alpha = 0$ is interpreted as the criterion of the coil–stretch transition in a random flow [18,19]. It can be rewritten via the *local* Weissenberg number in a random flow defined either by $Wi_{\mathrm{loc}} = \lambda\gamma = 1$

or $Wi_{\rm loc} = \lambda(\nabla V)_{\rm rms} = 1$, since the Lyapunov exponent can be estimated as the characteristic value of rms of the velocity gradient in a random flow. I would like to emphasize that the global Weissenberg number Wi in this case does not have any sense, since the average velocity is zero.

The first attempt to verify experimentally whether the coil–stretch transition occurs in a random flow was made by using rheometrical measurements to estimate elastic stresses in elastic turbulence of a dilute polyacrylamide (PAAm) solution [23]. As the result, the estimates based on these macroscopic measurements showed that the molecules should be stretched about 13 times, i.e. the coil–stretch transition should occur in the random flow [23]. Our later quantitative experiment on the coil–stretch transition verified the existence of the transition, and the critical value of the local Weissenberg number was experimentally confirmed with a reasonable accuracy $Wi_{\rm loc} = 0.77 \pm 0.20$ [4]. The latter was obtained by studying the statistics of both the Lyapunov exponents of the velocity field in a swirling flow between two disks and the polymer stretching in elastic turbulence and by a direct measurement of the polymer relaxation time. To study experimentally polymer dynamics and statistics of stretching in a 3D random flow we used the narrow focal depth high magnification and high NA objective to visualize polymer molecules (fluorescently labeled DNA) moving only in a 2D observation plane with 0.4 μm focus depth. All the rest of the molecules, which partially deviated from the 2D observation plane, were not considered. Of course, in 3D flow such as elastic turbulence a polymer resided in a 2D plane just for a short time interval, about the velocity correlation time on average. But it is sufficient to get up to 50 frames during this time interval. The same approach was used with each couple of particles in a random flow to measure the statistics of the Lyapunov exponents. It means that from the whole ensemble of either molecules or particles only those were selected that resided in the 2D observation plane at any given time interval during the experiment (molecules with an out-of-plane orientation angle different from zero were also not taken into account). If the statistics is sufficiently large, this statistical ensemble is representative to characterize the statistics of the polymer stretching. A somewhat surprising conclusion of these studies is that a generic random flow is, on average, an extensional flow at every point, with the extension rate γ and unlimited Henky strain [4,18].

Above the coil–stretch transition, two theories differ by the mechanism limiting the polymer stretching in a spatially homogeneous random flow. The first mechanism is based on nonlinear elasticity of a polymer [19,24]. If this mechanism dominates, the polymer can be stretched up to $R_{\max}$, where $R_{\max}$ is the polymer contour length. Then $R/R_{\max}$ as well as σ_p increases nonlinearly with Wi, and at each Wi a stationary state of a stretched polymer as well as its stationary probability distribution exists [19,24]. The second mechanism is due to the feedback reaction of the stretched polymers on the flow [18]. In this case a polymer of linear elasticity is considered. If a feedback reaction of stretched polymers on the flow is disregarded and the critical value of Wi_c for the coil–stretch transition is exceeded, both a stretched polymer and its PDF of the extension do not reach a stationary state, i.e. PDF becomes no longer normalizable. So all the moments of the PDF of polymer extension are diverged. Thus the linear model is not suited to describe the stretched state in the passive regime. When the feedback reaction of the stretched polymers is taken into account, i.e. polymers are considered to be active, both polymer stretching and the PDF of polymer extension can reach the stationary state. Indeed, according to the theoretical assumption [18] and numerical simulations [20, 22,37], the feedback mechanism turns out to be sufficient to stabilize the polymer stretching at some intermediate value $R_{\rm back} \ll R_{\max}$, where stationary PDFs are found [4]. Moreover, peaks of the PDFs at various $Wi > Wi_c$ were found in the experiment at $R_p \leqslant R_{\max}$ and the value of R_p increases with Wi that is definitely in odds with one of the main assumptions of the second theory: $R_{\rm back} \ll R_{\max}$ [18] (see e.g. Fig. 1, where PDFs of T4DNA stretching in a buffer solution with 25.16 μg/mL polymer concentration in a wide range from the coil to the stretched state are shown). First, the transition from the coil to stretched state is obvious. Second, the PDFs are saturated at the highest 2–3 values of Wi. The PDFs are taken at the radial location $r_1 = 300$ μm, close to the center. It was agreed that the first theory works for very low polymer concentrations, where the feedback reaction on the flow is negligible and polymer is considered to be passive [19], whereas the second is applicable at sufficiently high polymer concentrations [18]. In fact, both theories were considered for a spatially homogeneous smooth random flow, which occurs in the Batchelor regime below the dissipation scale in inertial turbulence and in elastic turbulence without shear.

One significant difference between the random flow used in our experiment and considered by the theory was a non-zero shear contribution into an experimental realization of the random flow, though the shear rate was smaller or of the order of rms of the velocity gradient [4]. Later on, the influence of the shear component in a random flow on the polymer stretching was further investigated theoretically [38] and just recently by us experimentally [36]. The theory suggests that at strong shear and $Wi \gg 1$ PDF of the polymer stretching above the coil–stretch transition has a well-defined sharp peak at $R \leqslant R_{\max}$ and a wide plateau of rather small probability at smaller values of R.

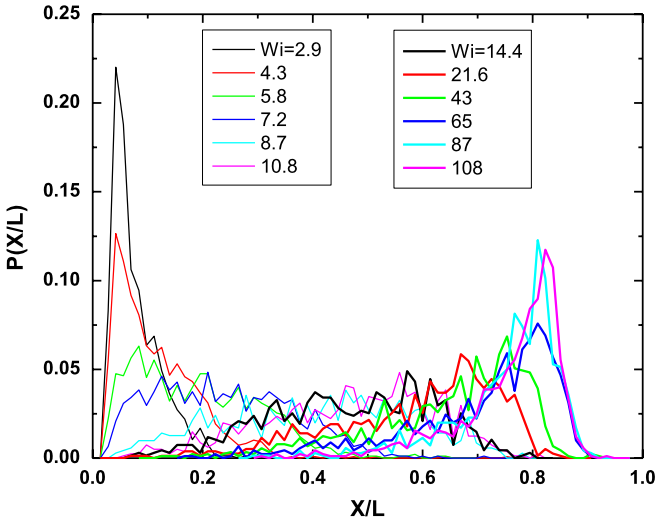

Fig. 1. PDF of x/L in elastic turbulence at $r_1 = 300$ μm ($r_1/\Re = 0.13$) and various values of Wi from the coil state via transition to the stretched state. The experiment was conducted in a swirling flow with flexible polymer T4DNA with concentration $c = 25.16$ μg/mL that is about 30% higher than c^* [36].

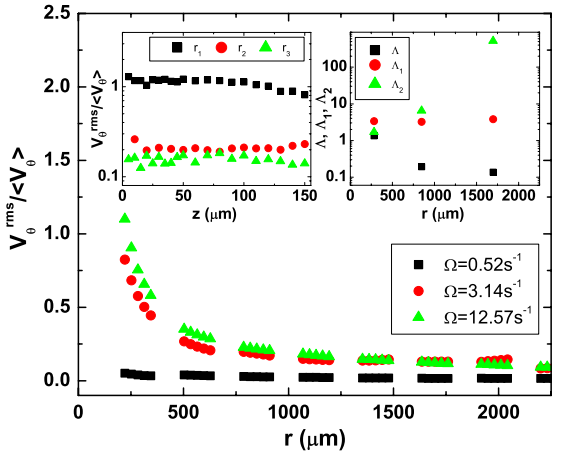

Fig. 2. Ratio $v_\theta^{\rm rms}/\langle v_\theta\rangle$ as a function of r. Insert: Ratio $v_\theta^{\rm rms}/\langle v_\theta\rangle$ as a function of z. Inset: Ratios $\Lambda \equiv (\partial v_\theta/\partial z)^{\rm rms}/\langle \partial v_\theta/\partial z\rangle$, $\Lambda_1 \equiv (\partial v_\theta/\partial z)^{\rm rms}/(\partial v_\theta/\partial r)^{\rm rms}$, $\Lambda_2 \equiv \langle \partial v_\theta/\partial z\rangle/\langle \partial v_\theta/\partial r\rangle$ at three radial locations.

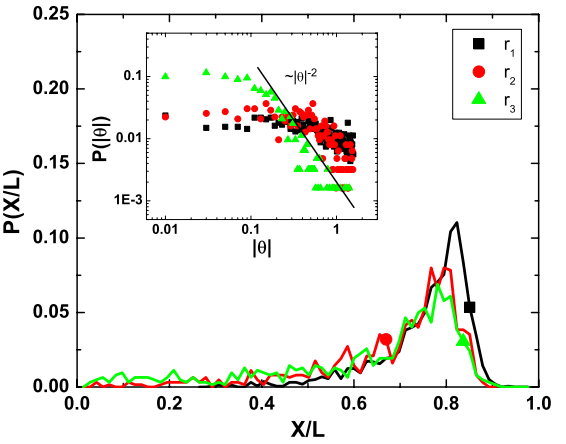

Fig. 3. $P(X/L)$ at r_1, r_2, r_3 and $Wi = 87$ & 108 for r_1, $Wi = 260$ at r_2, and $Wi = 491$ at r_3. Insert: $P(|\theta|)$ at the same locations and Wi.

The latter is resulted from a deterministic contribution of polymer rotational dynamics in the shear flow [38–40]. The experimental verification of these predictions and the role of the shear component on the statistics of polymer stretching were investigated in our recent paper [36]. In this paper we study the polymer stretching in elastic turbulence of a swirling flow at three radial locations: $r_i = 300, 900, 1700$ μm with the rotating rod of the radius $\Re = 2250$ μm. Three components of the velocity field and the main components of the velocity gradient tensor were measured via PIV. At a large global $Wi = \lambda\Omega r/d$ in the developed elastic turbulence regime both $V_\theta^{\rm rms}$ and $(\partial V_i/\partial x_j)_{\rm rms}$ change insignificantly with radial position but the ratios of these parameters to the corresponding average values $V_\theta^{\rm rms}/\langle V_\theta\rangle$ and $\Lambda \equiv (\partial V_\theta/\partial z)_{\rm rms}/\langle \partial V_\theta/\partial z\rangle$ vary over orders of magnitude (Fig. 2). Thus near the center of the swirling flow, a uniform random and isotropic in azimuthal direction flow at a small shear contribution is found, whereas near the edge of the rotating rod the average values of both parameters are about an order of magnitude larger than their corresponding rms values (Fig. 2). Despite of the orders of magnitude change in the shear component along the radius,

PDFs of the polymer stretching are rather close for all three radial locations at the highest Wi with small differences in the PDF peak locations (less stretched and longer tails towards the coil state at the higher shear contribution) (Fig. 3). For smaller values of Wi, a large difference in the polymer stretching is observed, so for smaller values of Wi (smaller shear component contribution), the coil–stretch transition occurs earlier. Thus, the smaller Wi, the larger polymer stretching. For this reason the global Weissenberg number Wi is irrelevant to characterize elastic turbulence, since does not define unambiguously the degree of the polymer stretching and so the elastic stresses. On the other hand, instead of the uniform distribution in an inclination angle in the 2D observation plane close to the center, one finds $|\theta|^{-2}$ tail in the PDF at large inclination angles far away from the center (insert in Fig. 3), similar to the PDF tail of $|\theta|$ in a simple shear flow [38–40].

3.2. Theory of elastic turbulence

The coil–stretch transition has remarkable macroscopic consequences on a flow: properties of the polymer solution flow become essentially non-Newtonian at $Wi_{\mathrm{loc}} > 1$ and at sufficiently large Wi_{loc} the flow is also chaotic. The theory of both elastic and inertial turbulence of a polymer solution, based on polymers with linear elasticity and the feedback reaction on the flow, was developed by Lebedev et al. [15,41]. The mechanism of elastic turbulence based on polymers with nonlinear elasticity [19] was never brought into completion. The theory of elastic turbulence uses the set of equations for the elastic stress tensor and velocity fields. The hydrodynamic description of a polymer solution flow and of the dynamics of elastic stresses for linear polymers is analogous to that of a small-scale fast dynamo in magneto-hydrodynamics (MHD) and also to that of turbulent advection of a passive scalar in the Batchelor regime [15,28], although some significant differences exist. The stretching of the magnetic lines is similar to the polymer stretching, and the difference with MHD lies in the relaxation term that replaces the diffusion term in MHD description, whereas in the passive scalar advection problem the dynamo effect is absent. In all three cases the basic physics is the same, rather general and directly related to the classical Batchelor regime of mixing: stretching and folding of the stress field by a random advecting flow.

Theory of elastic turbulence in an unbounded flow of a polymer solution is based on the following assumptions:

(i) A statistically stationary state occurs due to the feedback reaction of stretched polymers (or the elastic stress) on the velocity field that leads to a saturation of σ_p and $(\partial V_i/\partial x_j)_{\mathrm{rms}}$ (and so $Wi_{\mathrm{loc}} = (\partial V_i/\partial x_j)_{\mathrm{rms}}\lambda$) even for a polymer with a linear elasticity [15,41]. The saturation value in a bulk of elastic turbulence is $Wi_{\mathrm{loc}} \simeq 1$ and is constant at all Wi above the coil–stretch transition. It is the key theoretical prediction [15,41].

(ii) Both dissipative terms due to viscosity and polymer relaxation, which appear in the equation for the dissipation of elastic energy [41], are of the same order, i.e. $\sigma_p/\lambda \sim \eta(\nabla V)^2$ or otherwise $\sigma_p\lambda/\eta \sim Wi_{\mathrm{loc}}^2$.

Then both assumptions lead to the following result: the normalized elastic stress $\sigma_p\lambda/\eta \simeq 1$ also saturates. These two important theoretical predictions deserved an experimental test.

The further theoretical analysis leads to a power-like decaying spectrum for the elastic stresses and for the velocity field fluctuations with the exponent $|\delta| > 3$ (about -3.3) in a good accord with the experimental results [16,17]. Close value of the exponent in the velocity power spectra decay was also obtained in the numerical simulations of elastic turbulence based on the Kolmogorov shear flow of a dilute polymer solution described by the Oldroyd-B model [32].

4. Experimental test of theory of elastic turbulence

Further experimental studies of elastic turbulence first for a single polymer concentration of 80 ppm PAAm [34] in swirling and curvilinear channel flows [42] and then in a wide range of PAAm concentrations up to 3000 ppm [43] show that first, the experimental values of Wi_{loc} grow with Wi and then saturate at values at least of the order of 10 and higher than theoretically predicted in the well-developed elastic turbulence regime. The saturation level reduces with increasing concentration (see Figs. 4 and 5). So the experiment, on the one hand, confirms theoretical predictions about the saturation of Wi_{loc} in a bulk random flow, although, on the other hand, both the growth with Wi and the saturation level are very different from the theoretical predictions. Thus the assumption of the theory that the saturation of Wi_{loc}, as well as the polymer stretching, occur only due to the feedback reaction of polymers on the flow [15,41] does not hold, except in the vicinity of the coil–stretch transition. As we mentioned above, this assumption is a consequence

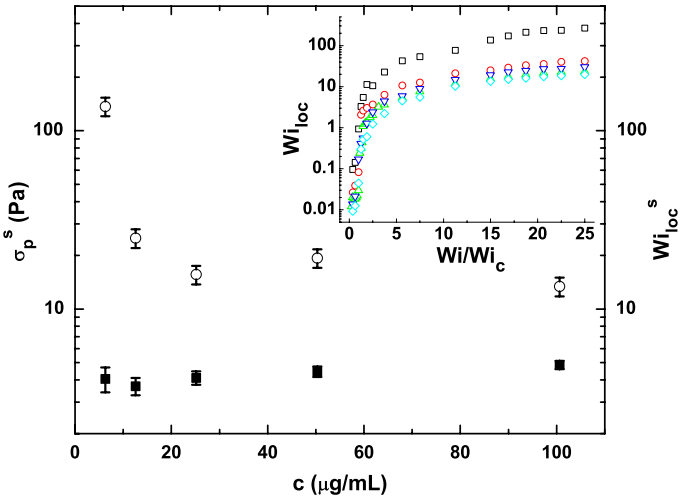

Fig. 4. σ_p^s and $Wi_{\rm loc}^s$ versus c: (■) σ_p^s, (○) $Wi_{\rm loc}^s$. Inset: $Wi_{\rm loc} \equiv \lambda(\partial V_\theta/\partial r)^{\rm rms}$ versus Wi/Wi_c for 5 values of polymer concentration $c = 6.29$ (black squares), 12.58 (red circles), 25.16 (green upper triangles), 50.32 (black down triangles), 100.64 μg/mL (blue rhombs).

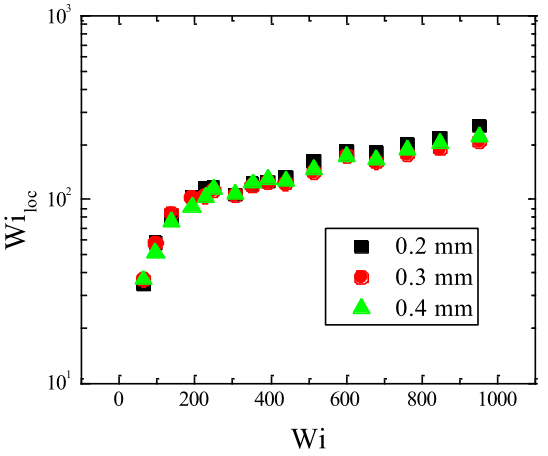

Fig. 5. $Wi_{\rm loc}$ as a function of Wi in a curvilinear channel flow at 3 different position across the channel: 0.2 (squares), 0.3 (circles), and 0.4 mm (triangles), respectively.

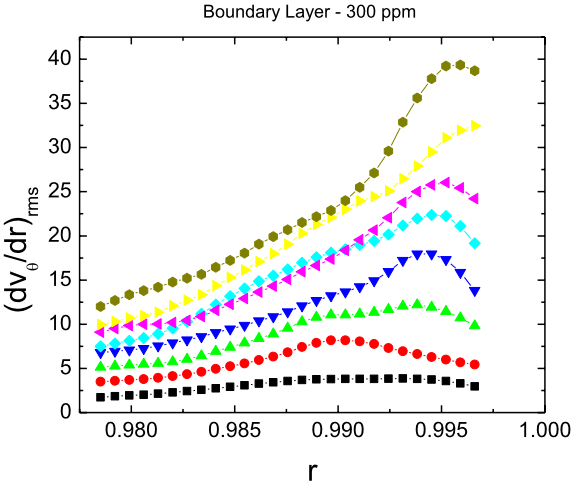

Fig. 6. The rms of the velocity gradient $(\partial V_{\rm theta}/\partial r)_{\rm rms}$ for 300 ppm polymer concentration in a swirling flow near the wall for 8 values of Wi from 40 up to 110 [42].

of the linear elasticity model for a polymer and of the initial assumption of rather small polymer extension in elastic turbulence $R_{\rm back} \ll R_{\rm max}$. The latter also disagrees with the experiment on the polymer extension in a random flow above the coil–stretch transition, as we already pointed out. One of the way out of this discrepancy is the nonlinear elasticity of a stretched polymer, which can lead to the saturation of the polymer stretching at each value of $Wi_{\rm loc}$ [19,24]. In this case $Wi_{\rm loc}$ grows with Wi and is not limited by the value of the order of unity.

The experiments also reveal a non-uniform distribution of the rms of the velocity gradients across the cell in elastic turbulence due to presence of boundaries in both swirling flow between two disks and curvilinear channel flow. The following features of elastic turbulence are found experimentally [34,42,43]:

(i) Contrary to the slow growth and the saturation in the bulk, the rms of the velocity gradients (and thus $Wi_{\rm loc}$) grows linearly with Wi near the driving disk (Fig. 6). Thus, a new length scale, i.e. the thickness of the boundary layer measured from the profile of the rms of the velocity gradients, which is probably to be relevant to the boundary layer of the elastic stresses, is discovered and found to be much smaller than the vessel size (Fig. 7).

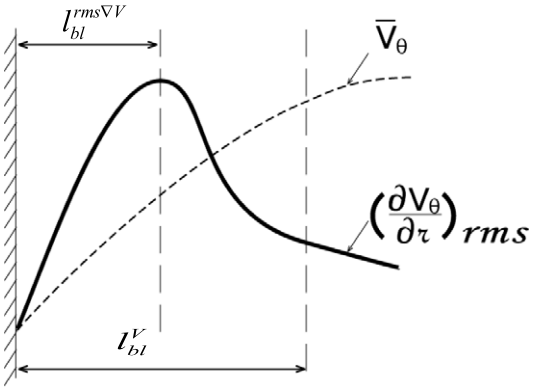

Fig. 7. Schematic of a new characteristic length scale–boundary layer width in elastic turbulence.

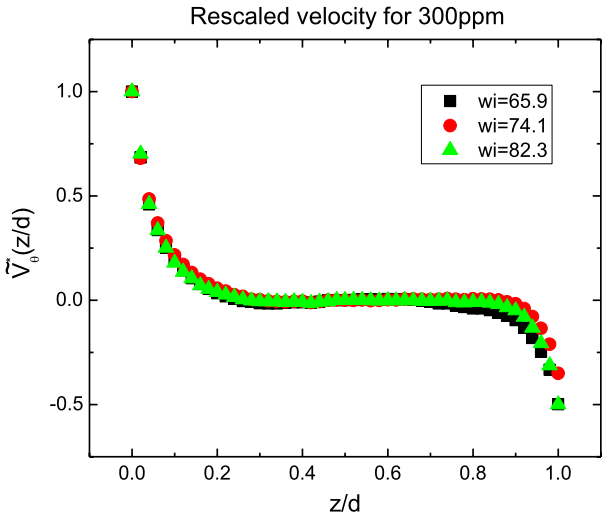

Fig. 8. The rescaled mean velocity profiles for different *Wi* [42].

A similar Wi_{loc} spatial distribution is observed in a curvilinear channel flow in the vicinity of walls [42]. The velocity boundary layer in elastic turbulence just reflects some of the features of the boundary layer of the rms of the velocity gradients or the elastic stresses (Figs. 7 and 8).

(ii) The rms of the velocity gradients in the boundary layer is one to two orders of magnitude larger than in the bulk, suggesting that the elastic stresses are accumulated near the wall and are intermittently injected into the bulk (Fig. 6).

(iii) The PDFs of the injected power P and pressure p fluctuations show skewness and exponential tails, which both indicate intermittent statistical behavior over a wide range of polymer concentrations. Striking similarity in statistical properties of P and p fluctuations in high Re and elastic turbulence and their independence on concentration and Wi, despite of different underlying mechanisms, suggests universality in statistical properties of P and p in a wide class of non-equilibrium hydrodynamic systems [43].

(iv) One of the main messages of the experimental studies is a surprising similarity in scaling, statistics, and spatial distribution of the rms of the velocity gradients (or the elastic stresses) and of passive scalar mixing in elastic turbulence in a finite size vessel, in spite of the important difference in the dynamo effect [44,45]. The latter occurs due to the feedback reaction of stretched polymers (or the elastic stress) on the velocity field. The scaling of the structure functions of the velocity gradients, injected power and pressure fluctuations is found to be close to that of a passive scalar advected by the velocity field in elastic turbulence [44,45]. These properties provide a basis for a model of elastic turbulence, in which elastic stress is introduced into the fluid by the driving boundary,

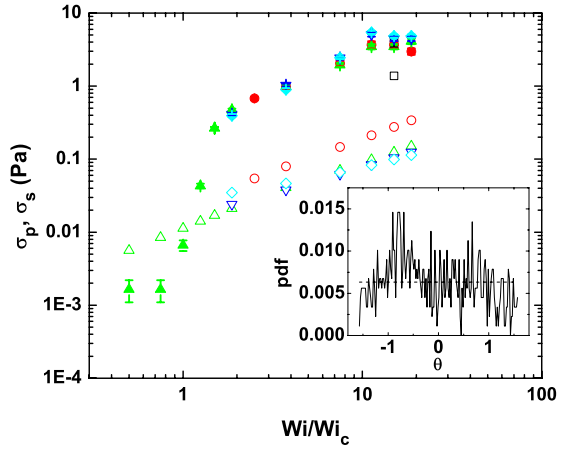

Fig. 9. Elastic σ_p and viscous σ_s stresses versus Wi/Wi_c for 5 values of c. Symbols: Full – σ_p, open – σ_s. Insert: PDF of inclination angle θ at $Wi/Wi_c = 7.5$ and $c = 25.16$ μg/mL. The dash line is the average uniform PDF.

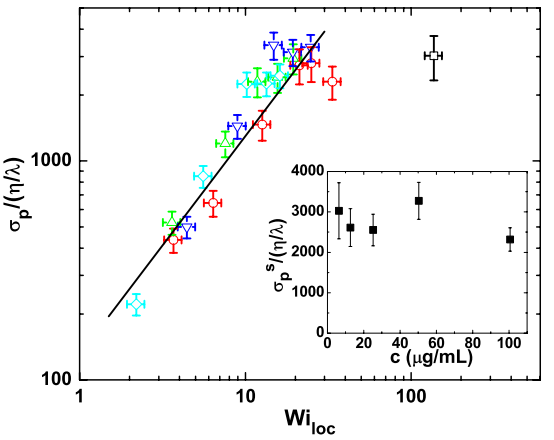

Fig. 10. Normalized elastic stress versus $Wi_{\rm loc}$ for 5 values of c. Insert: Saturated normalized elastic stress σ_p^s versus c. The error bars include error in x_p of ±0.5 μm, 5% in η, and 2% in λ.

accumulates in the boundary layer and is intermittently injected into the bulk of the flow. The situation is entirely similar to the trapping of a passive scalar in the mixing boundary layer in a finite size vessel, and to its intermittent injection into the bulk [44,45]. On the other hand, elastic turbulence can be produced in unbounded domain, as was shown in Ref. [32], although its detailed statistical properties can differ from the bounded flow, as occurred for passive scalar mixing [42,46].

5. Measurements of elastic stresses in elastic turbulence and further test of theory

Recently, the non-uniform distribution of the elastic stresses across the cell, which is caused by non-uniform distribution of the polymer stretching, is indeed verified experimentally by studying the statistics of a single polymer stretching in elastic turbulence generated by the same polymers [47,48]. A single fluorescently stained T4DNA molecule with known elastic properties is used as a stress sensor in a random flow [49]. Statistics of the elastic stresses and the average elastic stress value in elastic turbulence as a function of Wi and polymer concentration are obtained from the measured statistics of stretching of the fluorescently labeled polymers and their known elastic properties. These results present for the first time direct elastic stress measurements in a flow (Fig. 9) and confirm the saturation of the elastic stress in a bulk flow at the highest values of Wi (Fig. 4) and the existence of boundary layer near the wall [36,49]. The most remarkable result of these studies is the linear relation $\sigma_p/(\eta/\lambda) \approx 130 Wi_{\rm loc}$ in the entire elastic turbulence regime for a wide range of polymer concentrations and its saturation at the highest value of $Wi_{\rm loc}$ for all polymer concentrations (Fig. 10). Thus one concludes that $Wi_{\rm loc}$ is the only relevant control parameter in elastic turbulence and determines uniquely the elastic stress in a flow. We should point out that both the estimated value of the elastic stresses averaged over the measured molecule ensemble and the inclination angle PDF and the linear dependence of σ_p on $Wi_{\rm loc}$ are both not consistent with the theory of elastic turbulence [15,41]. These results allow us to disregard the theory of stretching of polymers with linear elasticity in a random flow based on the feedback reaction to the flow [18] as well as theory of elastic turbulence based on the assumptions of balance of viscous and elastic stresses and the saturation of elastic stresses due to their feedback reaction on the flow. I believe that the theory of elastic turbulence based on polymers with nonlinear elasticity [19,24] is better suited to explain the observations and I hope that the new results will initiate further theoretical development in this direction.

Finally, I would like to suggest a possible resolution of the theory. There exists an exact relation between pressure and elastic stresses, which follows from the Navier–Stokes equation for a visco-elastic fluid. It is $\Delta p = \nabla_i \nabla_j \sigma_{ij}$, which allows us to compare microscopic measurements of elastic stress variations with macroscopic measurements of pressure variations. On the other hand, time average of the pressure and elastic stress gradients are also equal in the approximation of very small viscous stresses that follows from our experiment. Since our measurements show that

$p_{\rm rms}$ are much smaller than $\sigma_{\rm rms}$, one concludes that only the non-divergent part of $\sigma_{\rm rms}$ participates in this relation, while the divergent part is zero. Then the theory of elastic turbulence based on a polymer with nonlinear elasticity should use the equation for elastic stresses to find out the solution and then using it to get scaling for the velocity field.

6. Conclusion

There are several important messages presented in this review that can be relevant to the turbulent drag reduction problem:

(i) Elastic turbulence is a spatially smooth and random in time flow, which is the same as existing in high *Re* inertial turbulence below the dissipation scale [16,17,35].
(ii) Study of the statistics of the polymer stretching as a function of *Wi* in a random flow shows that a polymer can be stretched considerably, the coil–stretch transition occurs in such a flow and the threshold is defined by $Wi_{\rm loc} \simeq 1$ as predicted theoretically [4,36,49].
(iii) Above the coil–stretch transition in the developed elastic turbulence regime the polymers are considerably stretched (and can be even overstretched like T4DNA molecules [49]) but the degree of their stretching is determined by the rms of the velocity gradient fluctuations [49]. With an increasing contribution of shear into the random flow, the polymer stretching even reduces, contrary to conventional expectations [36].
(iv) Elastic stresses exceed viscous stresses up to two orders of magnitude in elastic turbulence [49].
(v) Elastic stresses are proportional to the local Weissenberg number $Wi_{\rm loc}$ in elastic turbulence [49].
(vi) Contrary to the predictions, $Wi_{\rm loc}$ as well as σ_p slowly grow with *Wi* in elastic turbulence up to the saturation at the values that exceed the theoretical ones by more than two orders of magnitude [49].
(vii) $Wi_{\rm loc}$ (and probably σ_p) is non-uniform across the cell and has a peak near the wall, where it reaches up to two orders of magnitude compared to the bulk values [34,47,48].

The peak is defined as the edge of the new boundary layer in both $Wi_{\rm loc}$ and σ_p. The thickness of the boundary layer is the new length scale, which can be relevant also to inertial turbulence of a polymer solution [34,47]. Thus elastic turbulence is an appropriate flow to study statistics of the polymer stretching, and statistics, values and spatial distribution of the elastic stresses in a random flow that are relevant to the turbulent drag reduction problem.

In this short review, which presents the results of the measurements of the velocity field and the elastic stresses as well as polymer dynamics and statistics of stretching in elastic turbulence of various polymer solutions in various macro- and micro-flows, I convey my point of view on the relations between statistics of polymer stretching, the coil–stretch transition and elastic stresses in a smooth random flow such as elastic turbulence and statistical and scaling properties of its velocity gradient field. By using early idea by Lumley [2,3] and its further development by Lebedev et al. [15,41] that polymer dynamics and statistics of polymer stretching are determined just by small scales in high *Re* inertial flow below the dissipation scale, I generalize further our results obtained in elastic turbulence. I assert that the value of the elastic stresses and their non-uniform spatial distribution near the walls with the new boundary layer should be observed in the case of inertial turbulence in a polymer solution. Then the peak of the elastic stresses at the edge of the elastic stress boundary layer and large elastic stresses in the bulk indicate that most of the momentum, pumped into the system to drive the flow, is stored in the stretched molecules than in Reynolds stresses and not transfer to the wall, in such a way reducing the turbulent drag. Thus, measurements and characterization of polymer stretching and elastic stresses in inertial turbulence are the key issues to resolve the turbulent drag reduction puzzle.

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