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Gamma-ray burst studies in the SVOM era / Étude des sursauts gamma à l'ère de SVOM

Numerical simulations of GRB engines

Simulations numériques des sursauts gamma

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ABSTRACT

Gamma-ray burst central engines may be related with some of the most powerful astrophysical sources of gravitational radiation. Therefore, there are now several numerical models trying to elucidate both the details of the scenarios for the central engine and the information that gravitational waves could provide on the physical processes involved. The models are quite complex and they now consider full general-relativistic simulations, relativistic magneto-hydrodynamics and detailed microphysics for the description of very dense matter and neutrino interactions. This paper analyzes some of these numerical studies, trying to make a link with possible gravitational wave observations for both binary compact star mergers and massive star gravitational collapses.

RÉSUMÉ

Les événements astrophysiques à l'origine des sursauts gamma sont sans doute reliés aux sources les plus puissantes de rayonnement gravitationnel. Il y a ainsi de nombreux modèles numériques qui tentent d'élucider les détails de tels scénarios et les informations que les ondes gravitationnelles pourraient nous apporter sur les processus physiques à l'œuvre dans ces objets. Les modèles numériques sont très complexes et peuvent aujourd'hui prendre en compte aussi bien la théorie de la relativité générale, la magnéto-hydrodynamique relativiste, qu'une description détaillée de physique nucléaire pour la matière dense et son interaction avec les neutrinos. Cet article analyse certaines de ces études numériques, en essayant de faire le lien avec les observations potentielles d'ondes gravitationnelles dans le cas des coalescences de binaires d'astres compacts, comme dans celui des effondrements d'étoiles massives.

1. Introduction

The theoretical studies on gamma-ray burst (GRB) phenomena have most often been devoted to the physics of the relativistic fireball and the formation of the observable properties (gamma-rays, afterglow emission). While these works usually assume the existence of a black hole, acting as a central engine producing the ultra-relativistic outflow, there have been fewer (and more recent) models investigating more in detail the proposed scenario for this central engine. In order to have such a detailed quantitative model of GRB central engines, it is absolutely necessary to perform numerical simulations.

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In the standard picture, mergers of binary compact objects (two neutron stars or a neutron star and a black hole) are supposed to be the sources of short GRBs [1],[1] whereas long GRBs are more likely to be associated with massive stellar collapses [2]. Both scenarios have in common that they would represent the birth of a stellar-mass black hole surrounded by a torus of extremely dense matter. The huge energy release is associated with the high accretion rate, between fractions of a solar mass per second up to many solar masses per second (see e.g. [3]). The huge amount of material accreted by this black hole could explain the energetics of even the most distant GRBs through the release of a significative part of the gravitational binding energy. Possible relevant processes to extract the energy of this black hole accretion disk system and launching a relativistic jet are neutrino–antineutrino ($\nu\bar{\nu}$) annihilations [4]. On the other hand, the relative small scales of stellar-mass black holes (3 km radius per solar mass) can naturally produce rapid variability on millisecond time scales.

Numerical models should therefore incorporate the description of relativistic flows around a black hole, together with a correct description of neutrino interactions and nuclear processes. However, because of computer limitations, there have been two types of approaches that have been used to build approximate numerical models of GRB engines. In the first type of models, the gravitation is modeled using simple Newtonian theory, with possibly some modified potential to catch some of the general-relativistic features appearing near a black hole (e.g. Paczyński and Wiita [5]), in particular the existence of a last stable orbit for the matter flowing around the black hole. These models usually take into account quite detailed microphysics: equations of state (EOS) coming from nuclear model calculations, together with realistic electron capture rates, in order to follow the energy and lepton number changes by neutrino losses. The second and more recent approach comes from the field of *numerical relativity*: the gravitational field is obtained by solving the full Einstein equations of General Relativity, but matter is modeled as a simple polytrope (discarding temperature effects):

$$p = K\rho^{\gamma} \tag{1}$$

or as a simple ideal gas, with a constant adiabatic index:

$$p = (\gamma - 1)\rho\epsilon \tag{2}$$

where p is the pressure, ρ is the density, ϵ is the specific internal energy and K, γ are two constants. The inclusion of General Relativity allows for both a correct description of black hole properties and a clean estimation of gravitational waves emitted by the system.

Indeed, in most of the proposed scenarios for the GRB central engine there should be a strong emission of gravitational radiation. This can be seen from the following approximate formula, giving the gravitational luminosity L_{grav} of a system, as a function of its mass M, typical scale R, and velocity v (see also Misner, Thorne and Wheeler [6], Section 36.3):

$$L_{\text{grav}} \sim \frac{c^5}{G} s^2 \left(\frac{R_S}{R}\right)^2 \left(\frac{v}{c}\right)^6 \tag{3}$$

s is a geometrical parameter measuring how much the system deviates from sphericity ($s = 0$ for a spherically symmetric setting and $s \leqslant 1$), G is the gravitational constant and c is the speed of light. R_S is the *Schwarzschild radius* of the system defined as:

$$R_S = \frac{2GM}{Rc^2} \tag{4}$$

it can also be seen as the radius of a black hole having the same mass M as the system. From these arguments, GRB sources with a black hole and relativistic motion are expected to be good sources for gravitational wave detectors too. The typical time-scales for GRBs and the progenitor models imply that the resulting gravitational radiation would be in the high-frequency range (10 Hz–10 kHz), which is probed by current ground-based detectors, such as Virgo or LIGO.

In principle, it is necessary to solve the full set of Einstein equations of General Relativity to obtain reliable waveforms for gravitational wave data analysis. Therefore, one would expect simulations using only Newtonian gravity to be strongly inaccurate. But it is possible to use an approximate formula, called the *quadrupole formula*, to extract gravitational waveforms out of a simulation (see e.g. [7]). This formula is not gauge invariant and only valid in the Newtonian slow-motion limit. However, it was shown [8] that the method seems to be sufficiently accurate to compare to more elaborate techniques and to work reasonably well in the case of stellar collapse, before the appearance of the black hole. It seems thus possible to derive rather precise gravitational waveforms from non-general-relativistic simulations of mergers of neutron stars, or of massive star gravitational collapse, but one must keep in mind that these waveforms become inaccurate once the black hole is formed.

More details about numerical models of GRB central engines and implications for gravitational waves shall be given in the rest of this paper, which is organized as follows. Section 2 deals with the simulations of mergers of two compact objects: neutron star and black hole, or two neutron stars. These are proposed as progenitors of short-duration GRBs. In Section 3 some recent studies on the possible imprints of the different physical scenarios for short GRBs on the gravitational wave spectrum are reviewed, before discussing the case of GRB 070201. Section 4 addresses the models of the collapse of a

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[1] A binary system of two black holes does not contain a priori any matter, so it cannot form an accretion disk and a jet to emit gamma-rays.

massive rotating star to a black hole, so-called *collapsar*, as well as the resulting gravitational radiation. Finally, Section 5 gives some concluding remarks and possible perspectives.

2. Mergers of compact star binaries

In the study of the central engine for short GRBs, standard models invoke the merger of a binary system of compact objects. On the one hand, binary neutron stars are known to exist [9] and, for example, the time until the final merging of the binary pulsar J0737-3039 is less than the Hubble time. One can thus reasonably assume that binary neutron star mergers are rather frequent and give rise to both electromagnetic and gravitational wave signals. On the other hand, no neutron star–black hole binary is known, although theoretical models (population synthesis [10]) suggest that they do exist.

The binary coalescence process can be divided into three phases [11], both for binary neutron stars and neutron star–black hole systems:

(i) **In-spiral**: during this phase the binary loses energy and angular momentum by emission of gravitational radiation and the two objects gradually spiral toward each other. The gravitational radiation reaction timescale is much longer than the orbital period and the system can be considered as quasi-adiabatic during an orbital period. This evolution can be well described, considering the compact stars as point-masses, and using a perturbative approach for the gravitational field (e.g. post-Newtonian expansion, see [12]).

(ii) **Merger**: when the timescale of change of the orbital period becomes comparable to the orbital period itself, or when the two stars come into contact with each other, the system goes through a violent merger phase that usually leads to the rapid formation of a black hole, or a hypermassive neutron star sustained by rapid differential rotation, surrounded by a torus of dense matter. This stellar material possesses too much angular momentum to be directly accreted by the black hole.

(iii) **Ring-down/collapse**: the black hole is formed in a strongly perturbed and deformed state, with respect to stationary configurations. In this phase, it rapidly radiates away gravitational waves and settles into a stationary rotating state (Kerr geometry). In the case of the hypermassive neutron star, gravitational wave emission, magneto-rotational instability [13] or neutrino radiation [14] can lead the star to collapse to a black hole.

The difficulty of simulating these phenomena stems from the fact that the merger is an intrinsically three-dimensional event and it must be addressed with accurate, high-resolution dynamical simulations. The strong gravitational fields in the vicinity of black holes play an important role in the dynamics of the merger. Very detailed pseudo-Newtonian calculations have been made e.g. by Janka [15], who compared black hole–neutron star to binary neutron star mergers, using a Eulerian piecewise-parabolic code which took into account the emission and back-reaction of gravitational waves. This was the first such simulation done with the realistic nuclear EOS by Lattimer and Swesty [16], thus providing important information about thermodynamic evolution and neutrino emission. This study has shown that the neutrinos and anti-neutrinos emitted by the torus could deposit up to 10^{51} ergs above the poles, which could in turn allow for a relativistic expansion with Lorentz factors around 100 and explain short GRB luminosities up to $10^{53}\ \mathrm{erg\,s^{-1}}$.

Nevertheless, the final answer clearly requires a fully-relativistic calculation and, as of now several groups are able to perform numerical simulations of binary neutron star mergers in General Relativity, starting with the pioneering work of Shibata and Uryū [17]. A nice and more recent example is given by the work of Baiotti et al. [18], who present a systematic study of the dynamics of a binary neutron star system. They use high-resolution shock-capturing methods for the solution of the relativistic hydrodynamics equations and high-order finite difference methods for the Einstein equations, with adaptative mesh-refinement techniques. However, the use of polytropic EOS of the form (1), prevented them from studying the full thermodynamics of the system. They show in particular that higher-mass binaries lead to the prompt formation of the black hole–torus system, lower-mass binaries give rise to the hypermassive and differentially rotating neutron star, which eventually undergoes a delayed collapse to a black hole. One of their high-mass binary simulations is displayed in Fig. 1, with the two neutron stars undergoing strong tidal effects (left panel), merging (central panel), and the appearance of a black hole with a torus (right panel). Note that most of these simulations in General Relativity rely on initial data obtained from a quasi-equilibrium hypothesis, i.e. an approximation of the inspiral phase discarding gravitational waves from the computational domain defining the initial data [19].

The final result of the neutron star–black hole system mostly depends on the black hole mass. When it is small enough, the neutron star will be tidally disrupted before the merger, thus forming an accretion disk. For larger black hole masses, the neutron star can be swallowed by the black hole, without leaving enough material to power a GRB. In the case of the neutron star–black hole systems, an additional difficulty in the numerical simulations comes from the fact that the mathematical model of a black hole implies the presence of a singularity in the computational grid. Because of this singularity, many physical fields are diverging and essentially two types of techniques have been devised for the modeling of black holes in numerical relativity:

- **Punctures:** black holes are described in the initial data in coordinates that do not reach the singularity, but follow a *wormhole* through another copy of the asymptotically flat exterior spacetime. This one is compactified, so that infinity is represented by a single point, called *puncture* [20].

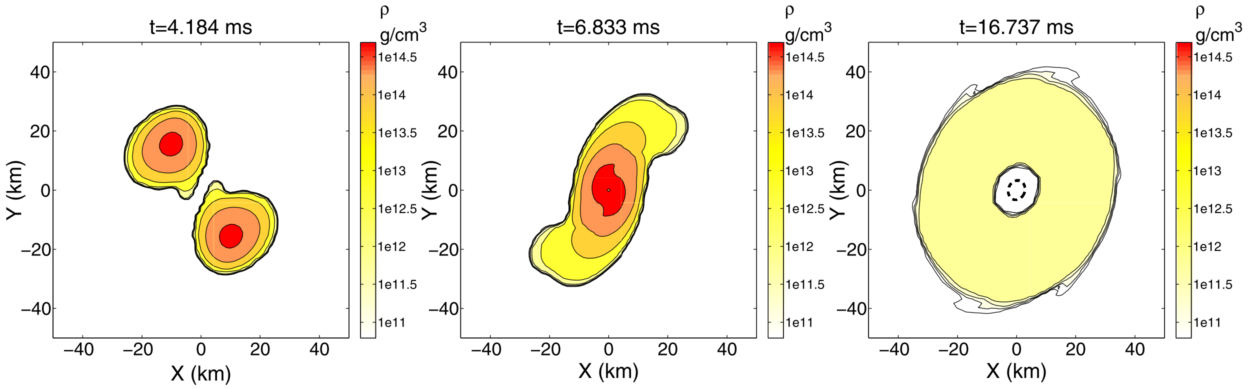

Fig. 1. Snapshots of a fully-relativistic simulation of a binary neutron star merger. Shown are the density isocontours in the orbital plane, for a high-mass binary, collapsing to a black hole. The thick dashed line represents the black hole apparent horizon.
Reprinted figure with permission from Baiotti, Giacomazzo, Rezzolla, Physical Review D 78 (2008) 084033 [18]. Copyright (2008) by the American Physical Society.

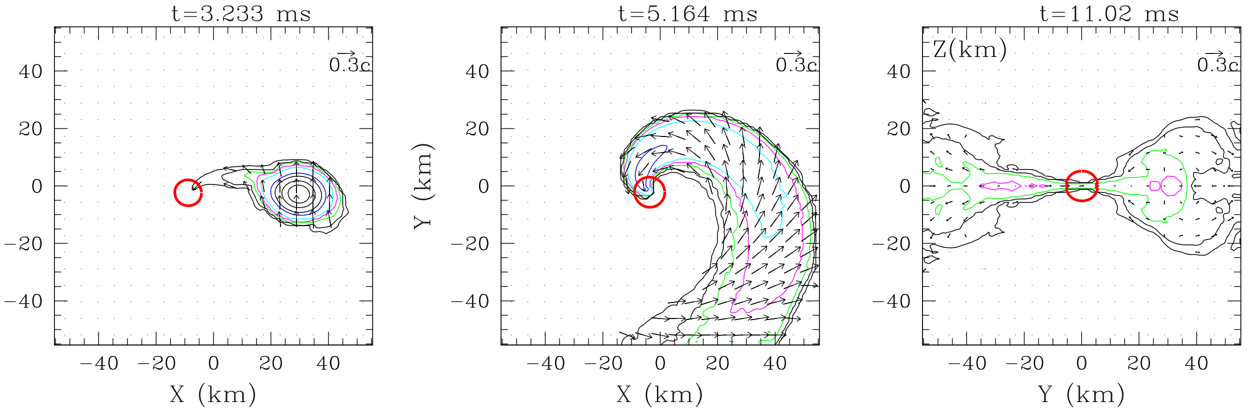

Fig. 2. Snapshots of a fully-relativistic simulation of a neutron star–black hole merger. Shown are the density isocontours in the orbital plane (left and central panels), and in the $x = 0$ place (right panel), together with the velocity field. The blue, cyan, magenta and green curves denote 10^{14}, 10^{13}, 10^{12} and 10^{11} g/cm^3 respectively. The thick red line represents the black hole apparent horizon.
Reprinted figure with permission from Shibata, Taniguchi, Physical Review D 77 (2008) 084015 [22]. Copyright (2008) by the American Physical Society.

– **Excision:** a neighborhood of the central singularity is removed from the computational domain and replaced with boundary conditions. The boundary is chosen in order to have well-defined physical and/or geometrical properties: most of the studies using this approach have chosen the *apparent horizon* as the excision surface [21].

For example, the study by Shibata and Taniguchi [22] uses the moving puncture approach to model the black hole, and a γ-law EOS of the form (2) for the neutron star matter. This work is interesting because it makes the link with the short GRB central engine: the authors find that the merger between a low-mass black hole and a neutron star may form a central engine of short GRBs of total energy $\sim 10^{49}$ ergs if the compactness ($R_S/2R$, see Eq. (4)) is rather small, $\leqslant 0.145$. However, for more standard features of the neutron star ($M = 1.35M_\odot$ and $R = 12$ km), the merger results in a small torus mass and hence, it can only be a candidate for the low-energy short GRBs of total energy of the order 10^{48} ergs. In Fig. 2 are given some snapshots of the merger between a $4M_\odot$ black hole and a $1.3M_\odot$ neutron star. The neutron star is tidally disrupted and a large fraction of its mass goes into the black hole, while a smaller fraction ($\sim 0.1M_\odot$) forms a torus around the black hole (right panel of Fig. 2).

Finally, one can mention here the black hole–white dwarf merger, too, which has been discussed as a mechanism for producing long GRBs. When the merger is occurring, the black hole spirals inward through the envelope of the white dwarf, making the white dwarf tidally disrupted and most of its matter going into an accretion disk around the black hole. Fryer et al. [23] have performed three-dimensional hydrodynamic simulations of this phenomenon, using a smooth particle hydrodynamics (SPH–Lagrangian) code, with the Lattimer and Swesty EOS [16]. They find that nuclear reactions induced by this merger give a negligible energy release, while the peak accretion rates are $\sim 0.05M_\odot$ s^{-1} and last for approximately a minute, possibly producing long-duration GRBs.

Numerical studies have also been devoted to the dynamical evolution of the torus around the black hole. Indeed, a detailed treatment of the equation of state and neutrino physics is a very important ingredient to understand the evolution and

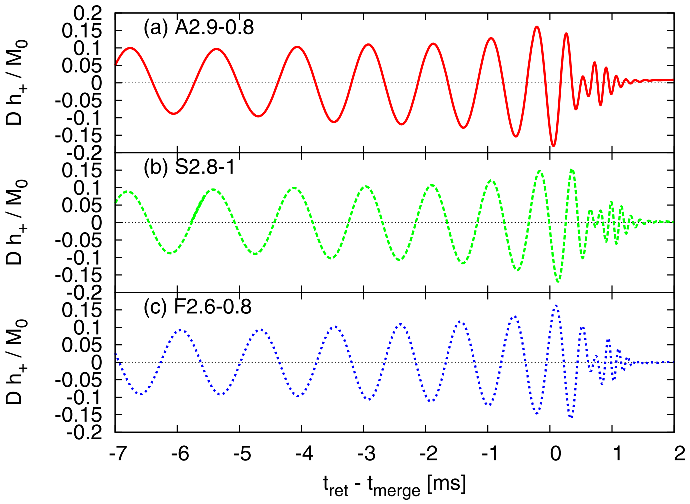

Fig. 3. Waveforms of the + modes of gravitational waves from the merger of three different models of binary neutron star systems. D is the distance from the source to the observer, who is located along the axis perpendicular to the orbital plane. t_{merge} is the time up to merger and M_0 is the total mass of the system.
Reprinted figure with permission from Kiuchi, Sekiguchi, Shibata, Taniguchi, Physical Review Letters 104 (2010) 141101 [28]. Copyright (2010) by the American Physical Society.

energetics of the accretion torus. Setiawan et al. [24] have investigated these points using a three-dimensional hydrodynamic simulation, with pseudo-Newtonian potential [5] to approximate effects of General Relativity. They have computed the time-dependent efficiency for the conversion of gravitational energy into neutrinos, as well as that of the conversion of neutrino energy into a pair-photon fireball. The neutrino energy and total energy release of the torus increase steeply with higher viscosity, larger torus mass and larger black hole spin (in corotation with the disk). Therefore, these models are found to account for most energetic short GRBs if collimation of ultra-relativistic outflow into 1% of the sky is invoked. The question about the collimation mechanism of the polar outflow has been addressed by Aloy et al. [25], with a general-relativistic hydrodynamic code. Their models followed the launch and evolution of relativistic jets and winds driven by the thermal energy deposition in the close vicinity of the black hole–torus system. They show that, even if the torus lifetime is only between 0.01 s and 0.1 s, their models can explain the durations of all observed short GRBs. With two types of models, they can study both the generation phase of a relativistic fireball and its evolution in relatively low-density environments until ~ 0.5 s. This type of study bridges the gap between the simulations of the merger phase and the long-term evolution of the fireball. In particular, this work by Aloy et al. [25] confirms the viability of black hole–torus systems as engines for short GRBs.

3. Gravitational waves from short GRBs

It is clear from Eq. (3) that, if short GRBs are initiated by mergers of binary compact objects, then they should also be associated with the emission of strong gravitational wave signals. These mergers do gather the following three properties: the matter distribution is far from spherical symmetry ($s \sim 1$), they involve relativistic speeds ($v \sim c$) and compact objects, i.e. with radii close to the Schwarzschild ones ($R = R_S$ by definition for a black hole, and $R \sim 3\text{–}5 R_S$ for a neutron star). Gravitational wave emission, although not isotropic, is in principle observable in any direction. An interesting point is that, in principle it is possible to deduce the inclination angle of a binary system (the angle at which one sees its orbital plane from the Earth) from a precise observation of both polarizations of the gravitational waves it produces. This can be seen by taking the lowest-order terms in the post-Newtonian development of the gravitational waveforms from an inspiralling compact binary, as observed from a ground-based detector (see e.g. Section 10.4 of the review by Blanchet [26]):

$$\begin{aligned} H_+^{(0)} &= -\left(1+\cos^2 i\right) H^{(0)} \cos 2\psi \\ H_\times^{(0)} &= -2\cos i\, H^{(0)} \sin 2\psi \end{aligned} \tag{5}$$

Here '+' and '×' denote the two polarizations of a gravitational wave, (0) means that these are the lowest-order terms, i is the inclination angle between the direction of the detector as seen from the binary's center-of-mass, and the normal to the orbital plane, and ψ can be related to the binary's time-dependent phase. $H^{(0)}$ is then a term depending on the two masses, relative velocity and distance to the detector (see [26] for details). Note that the term $H^{(0)} \cos 2\psi$ corresponds to the lowest-order part of the h_+ term in Fig. 3. If the angle i could be deduced from the observation of gravitational waves associated to a GRB, it would be possible to constrain the collimation angle of the jet that is supposed to be launched along the binary's rotation axis.

These considerations have led various authors to study the possible link between gravitational radiation and GRB progenitors. For instance, Kobayashi and Mészáros [27] have considered gravitational radiation from most of the proposed scenarios for GRBs. They have evaluated the orders of magnitude of the strain and frequency of the gravitational waves expected from various progenitors at distances based on occurrence rate estimates. Since then, many new results came from the field of numerical relativity, as explained in Section 2 and it is possible to get a much more precise picture, as shown by Kiuchi et al. [28] for the case of a binary neutron star merger.

These authors consider several simulations of binary neutron star mergers in General Relativity and show that the gravitational wave spectrum contains much information, that could constrain the formation mechanism of the central engine of short GRBs. Fig. 3 is taken from this study, and gives gravitational wave signals from double neutron star mergers, for three different rather realistic EOSs. In particular, the authors devise an explicit strategy for the exploration of the merger hypothesis to explain short GRBs, by analyzing gravitational waves. They also show that the massive disk is not formed for equal mass binaries, or when the total mass is greater than $3M_\odot$, which implies that mergers of two neutron stars do not always produce GRBs. For such cases, only gravitational waves will be observed.

A first search for gravitational waves following the observation of a short GRB has been undertaken on the LIGO data [29] S5 run, during which GRB 070201 was observed by Konus–Wind, INTEGRAL and MESSENGER spacecrafts. This event had a sky position coincident with the spiral arms of the Andromeda galaxy (M31). Since no gravitational wave candidate has been found within a 180 s long window around the time of GRB 070201, the authors have excluded most of the possible binary compact system mergers at more than 99% confidence level. They could even exclude any binary neutron star merger closer than 3.5 Mpc. These results imply that, either the GRB was produced by a binary compact star merger at larger distances, or if the GRB source is supposed to be in M31, that the merger scenario cannot apply. In this last option, a GRB coming from a soft γ-ray repeater cannot be excluded. This case shows the difficulty coming from the poor identification of short GRB sources and the wealth of informations that gravitational waves could bring regarding GRB physics.

4. Gravitational collapse of massive stars

From recent observations, long-duration GRBs seem to be tied to the death of massive stars. They thus share some common features with core-collapse supernovae, but with an important difference. Indeed, whereas most of the energy output for an ordinary supernova is in neutrinos, in a long GRB most of the light comes out in about 20 seconds, mainly as gamma-rays focused by relativistic jets into small fraction of the sky. Long-duration GRBs are also less than 1% as frequent as ordinary supernovae. Nevertheless, there still is no complete picture of the processes by which a sizable fraction of the energy involved in a supernova explosion is focused to a relatively narrow channel and produces a GRB. Woosley [30] introduced the *collapsar* model, as the progenitor system for long GRBs. In this model, a massive star between 25 and 35 solar masses collapses when the iron core reaches its maximum mass (Chandrasekhar limit). As for the supernova scenario, a proto-neutron star is formed at the center, but contrary to the supernova case, the accretion rate of higher layers is too high to obtain an explosion and the proto-neutron star possibly collapses to a black hole, surrounded by a torus of dense matter, as in the case of short GRB models (see Section 2). A non-spherical outflow is formed near this system, building up a relativistic jet that can propagate through less dense layers of the star. The formation of such a torus requires very rapid rotation of the core of the progenitor, and that may be what makes the GRBs apart from the rest of supernovae.

This scenario has been numerically explored by MacFadyen and Woosley [31], with a two-dimensional hydrodynamics code using the piecewise parabolic MUSCL scheme (PPM), a realistic EOS and the α-prescription for the viscosity in the accretion torus. Neutrino losses where included with thermal losses, and nuclear processes were included using a simplified treatment. Finally, the gravitational interaction was calculated using Newtonian theory, but with a correction following Paczyński and Wiita [5], to account for some general-relativistic effects. The $35M_\odot$ main-sequence star collapses forming a 2–$3M_\odot$ black hole. The authors find that, for low rotation rates material falls into the black hole with no outflow expected. For high rotation rates, the infalling matter is halted by centrifugal force at a distance where neutrino losses are negligible and explosive oxygen burning may power a weak equatorial explosion. For intermediate rotations, a compact disk forms (see Fig. 4) at a radius where the gravitational binding energy can be efficiently radiated as neutrinos. The resulting strong relativistic jets (beamed to about 1.5% of the sky) blow aside the accreting material and can break through the surface, initiating a highly-relativistic flow. The sensitivity of the jet to accretion rate, angular momentum and disk viscosity could explain the variety of long GRB observations. Among the difficulties of such studies, one can cite the various time scales, inducing a very large number of time steps in numerical simulations. It is therefore reasonable to explore only parts of the scenario and the propagation of the relativistic jet through the stellar envelopes is an important one. This point requires relativistic hydrodynamics with a two- or three-dimensional numerical code, on a fixed gravitational background. For example, the recent study of Mizuta and Aloy [32], using an ideal gas EOS of type (2), shows that there can be a strong correlation between the angular energy distribution of the jet, after its eruption from the progenitor surface, and the mass of the progenitors. Lazzati et al. [33] present studies also based on two-dimensional high-resolution hydrodynamic simulations of such jets, where they show that jet emissions at wide angles have properties similar to those of short GRBs with persistent X-ray emission.

When looking at the mechanisms proposed to extract energy out of the central engine, one finds basically two: thermal deposition and hydromagnetic. The thermal mechanism relies, as in the short GRB models, on the conversion of the hot torus accretion energy into a copious flux of neutrinos and antineutrinos, which annihilate in the vicinity of the rotation

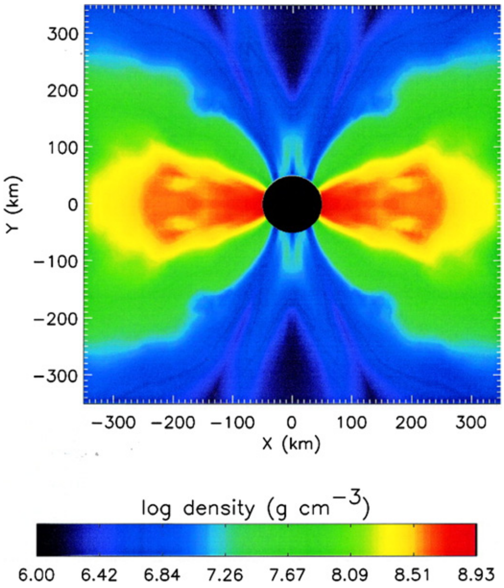

Fig. 4. Density in the central regions of a massive star whose iron core has collapsed to a black hole 7.6 seconds after core collapse. A dense disk (in red) of gas is accreting into the black hole.
Figure from MacFadyen and Woosley (1999) [31], reproduced by permission of the AAS.

axis of the system, just above the poles of the central compact object. From this annihilation a hot e^+e^--plasma is created, yielding the fireball of high energy photons. Some groups have recently incorporated general-relativistic effects in their numerical study of the complex neutrino transfer near the black hole–accretion torus system (see Harikae et al. [34]) and they have found that these effects can increase the local energy deposition rate by an order of magnitude, suggesting that the neutrino pair annihilation is potentially as important as the magneto-hydrodynamic mechanism to form the fireball.

On the other hand, massive stellar cores endowed with large angular momentum at the time of collapse should experience the magneto-rotational instability [35], as these cores have the potential to exponentially amplify weak initial fields on a rotational timescale. Within this context, Dessart et al. [36] have used a two-dimensional multigroup, flux-limited-diffusion magneto-hydrodynamics code to study the collapse, bounce and immediate post-bounce phases of a $35M_\odot$ collapsar model. They find that, provided the magneto-rotational instability operates in the surface layers of the proto-neutron star, a magnetically-driven jet is launched whereas the black hole does not form. This is coherent with the now forming picture that, in the iron core-collapse context, the black hole formation is never prompt, since it takes a finite time (of the order of seconds) for the proto-neutron star to accumulate the critical mass at which it collapses.

The gravitational waves from collapsars may be not as powerful as the ones from binary compact star mergers. Indeed, in the stellar collapse case the asymmetry parameter s in Eq. (3) is much smaller than 1, due to the spherical shape of the collapse. Collapse, bounce and post-bounce phases must show some strong non-spherical features in order for the gravitational waves to be efficiently emitted. If the collapsed remnant is a black hole, gravitational waves will be radiated when the accretion of stellar matter distorts the black hole geometry. This is called *black hole ringdown*, and will end when the black hole radiates away the distortion to end up in a stationary state (Kerr geometry). Sekiguchi and Shibata [37] have studied the collapse of a massive star to a black hole, and have computed the resulting gravitational waveform. Although they have used a fully-relativistic code to perform the simulations, they have extracted the gravitational radiation using the standard quadrupole formula, which has limited the waveforms to the time interval before the appearance of the black hole. The accretion torus around the black hole may also result in a strong emission of gravitational waves, in particular if bar-mode or spiral arm instabilities develop. The emission of gravitational waves from such perturbed tori (and possibly magnetized) has been extensively studied by Montero et al. (see [38] and references therein). They find however, that the signal from the accretion torus is rather weak to be detectable from an extra-galactic source by current detectors. Other mechanisms for the emission of gravitational waves from the stellar core collapse include anisotropic neutrino emission and magneto-rotational processes. In this last case, if the pre-collapse magnetic field is stronger than $\gtrsim 10^{12}$ G the overall dynamics of the collapse can be influenced. Gravitational wave amplitudes are then affected by the time-dependent magnetic field, which considerably contributes to the overall energy density, and by the bipolar jet outflows which give rise to a gravitational wave signal with memory [39]. Physically, such a memory effect in the gravitational wave signal arises from the temporal history of asymmetric matter outflow, leaving behind a constant offset in the amplitude. However, since the memory effect in the amplitude appears on the rather long timescale of several tens of milliseconds, it is out of the high-frequency Earth-based gravitational detectors (LIGO/Virgo, ...) and may only be detectable by the planned space-based DECIGO instrument [40] (it would also be out of the range of the other planned space-based detector LISA).

5. Conclusions

The numerical models of GRB central engines have benefited in the last years from the progresses made by the field of numerical relativity in developing stable and reliable codes. However, in order to be more relevant in this type of simulations, the general-relativistic codes need to incorporate more detailed microphysics that have been used until now in Newtonian-like simulations. These include the EOS, neutrino physics and magnetic field. The computational requirements are huge for these "complete" models, but many groups are on the way. In particular, it has been realized that the inclusion of General Relativity in GRB central engine simulations is quite important, and even compulsory for the correct modeling of black hole properties. Future studies should thus combine historically different approaches of two different communities: numerical relativity and computational astrophysics. The interplay with observations will certainly also play a major role in the building of numerical models and the understanding of both long and short GRB central engines. Gravitational waves are very important in that respect, since they can bring us unaltered information from these regions otherwise opaque to electromagnetic signals.

It has been recalled here that short GRBs should be associated with stronger gravitational wave sources and that first searches of coincidences γ-ray/gravitational waves using current Earth-based gravitational wave detectors, such as LIGO, have already begun. This is very important for gravitational wave detectors, which need some additional information to help them in the data analysis. As shown in Section 3, gravitational radiation polarization from the merger of a binary system of compact stars can in principle provide us with information on the angle between the binary orbital plane and the direction of observation. Similar results can also be obtained for gravitational waves from collapsar. One can therefore expect that coincident observations in gamma-rays and gravitational waves would constrain the GRB models and in particular the collimation angle. Furthermore, with future ground-based gravitational wave detectors, one can hope to observe enough double neutron star and neutron star–black hole signals, among which some may be associated with GRBs not seen from the Earth. This is possible because the beaming which is invoked in current models for GRBs, may cause some of the progenitor events not to be observable in gamma-rays from the misalignment between the observer and jet axis, whereas gravitational radiation from these events should be "isotropic" within a factor 2 in amplitude.

If one compares simulations of gravitational waves from binary compact star mergers (commonly associated to short GRBs) and from massive stellar collapses (associated to long GRBs), one can see that: (i) because of a stronger deviation from spherical symmetry, short GRBs events should emit stronger gravitational waves and therefore be easier to observe; (ii) the short GRB progenitors are better understood and numerically modeled. This comes in particular from the initial data: for the binary systems, they are quite well understood and represent a "clean" system of two compact objects in a quasi-stationary state. In the massive stellar collapse, the progenitor models come from stellar evolution models, where in particular the rotation profiles and magnetic fields are difficult to model, moreover the system is rather far from stationarity and subject to turbulence and/or convection.

Future directions in the numerical modeling of GRB central engines therefore include improvements of initial models. For binary systems, the main problem comes from the fact that, for the moment gravitational waves are completely discarded from the initial data. The inclusion of realistic gravitational wave content is not an easy task because it requires the knowledge of the gravitational and dynamical history of the binary. However, some information can perhaps be deduced from post-Newtonian computations. For massive stars, one is dealing with the main-sequence evolution of such stars, which is a very demanding task. In particular, the poor knowledge of initial rotation profiles and the lack for two-dimensional models represents the major uncertainty on the supernova/hypernova simulations and on the prediction of the resulting gravitational waveforms.

The effect of magnetic field, and particularly the magneto-rotational instability must be taken into account by future numerical models. While some recent simulations already include ideal magneto-hydrodynamics, it seems necessary to go beyond this approximation. This opens new questions about the knowledge of realistic resistivity and heat conduction in very dense matter. Just as simulations of the instability itself, it requires a very high spatial resolution, which is impossible to reach at this moment. Indeed, numerical viscosity and resistivity are always present in the simulations at much higher level than the expected values for the considered systems. Another improvement requiring a real increase of available computer power and/or qualitative enhancement of numerical algorithms, is the neutrino treatment. This is a vital point because in most current models, neutrinos are emitted by the central engine and deposit energy at the origin of the relativistic jet. On the other hand, in order to get the full neutrino dynamics, one has to solve the Boltzmann equation, which covers a six-dimensional phase space (in addition to the time coordinate!). Additionally, neutrino interactions, realistic resistivities and heat capacities are to be computed in a coherent manner together with the EOS. In order to be able to follow the collapse to a black hole, the EOS must also be extended to higher densities and temperatures, where additional particles (pions, hyperons, ...) appear, with eventual phase transitions that may have an imprint on the overall process. The computation of such EOSs is very active now and in a foreseeable future, some numerical models should be able to use new and more detailed EOSs from microphysical computations. Finally, the models considered here will gain in coherence and capacity to predict accurate observables, if they take into account the dynamical interplay between the black hole and the accretion torus, which are common to many central engine models. As this interaction is essentially (general-)relativistic, the approach should use either a puncture or excision and would therefore require a good comprehension of black hole geometrical and physical properties. This is not a surprise because black holes are today considered as the most plausible central engines for a vast majority of GRBs.

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