C. R. Physique 12 (2011) 379–386

Contents lists available at ScienceDirect

Comptes Rendus Physique

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Quantum Hall effect / Effet Hall quantique

Measurement of the ratio h/m_{Rb} and determination of the fine structure constant

Mesure du rapport h/m_{Rb} et détermination de la constante de structure fine

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ARTICLE INFO

Article history:
Available online 18 May 2011

Keywords:
Fine structure constant
Quantum Hall effect

Mots-clés :
Constante de structure fine
Effet Hall quantique

ABSTRACT

We present a review of the most precise determinations of the fine structure constant α which are obtained in different domains of physics. We describe the measurement of the ratio h/m_{Rb} between the Planck constant and the mass of Rubidium atom which leads to a precise value of α which is very few dependent of the QED. Finally, we present a review of the different determinations of the von Kitzling constant R_{K}.

RÉSUMÉ

Nous présentons une revue des déterminations les plus précises de la constante de structure fine α qui sont obtenues dans différents domaines de la physique. Nous décrivons la mesure du rapport h/m_{Rb} entre la constante de Planck et la masse de l'atome de rubidium qui conduit à une valeur de α très précise et très peu dépendante de l'électrodynamique quantique. Finalement nous présentons une revue des différentes déterminations de la constante de von Kitzling R_{K}.

1. Introduction

The fine structure constant α characterizes the strength of the electromagnetic interaction. It is defined as:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \tag{1}$$

where ϵ_0 is the permittivity of vacuum, c the speed of light, e the electron charge and $\hbar$ the reduced Planck constant ($\hbar = h/2\pi$). It was introduced by Arnold Sommerfeld in 1916 to explain the fine structure of the levels of hydrogen atom. The fine structure constant is a dimensionless quantity, *i.e.* it is independent of the system of units used. The determinations of α are obtained in different domains of physics, from the quantum Hall effect and Josephson effect in solid state physics, from the combination between the precise measurements of atomic physics and quantum electrodynamics (QED) calculations, and from the measurements of the ratio h/m between the Planck constant and the mass m of a particle. In 2006, the

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1631-0705/$ – see front matter
doi:10.1016/j.crhy.2011.02.002

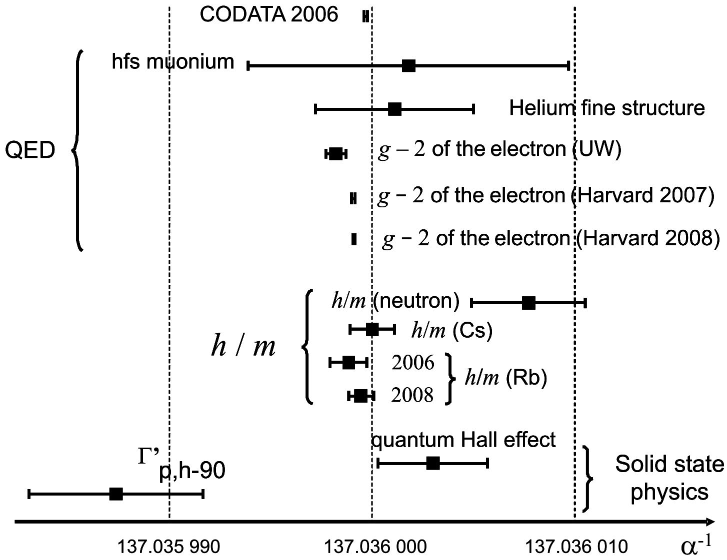

Fig. 1. Determinations of the fine structure constant in different domains of physics. The CODATA value is shifted with respect to the most precise recent results because of an error in the QED calculations of the electron anomaly a_e.

Fig. 1. Déterminations de la constante de structure fine dans différents domaines de la physique. La valeur du CODATA est déplacée par rapport au résultat récent le plus précis à cause d'une erreur dans les calculs QED de l'anomalie de l'électron.

recommended value in the last report of the CODATA (Committee on Data for Science and Technology) [1] was:

$$\alpha^{-1} = 137.035\,999\,679(94) \tag{2}$$

This value was mainly deduced from the combination of the measurement of the electron anomaly a_e made in 2006 by Gabrielse at Harvard University [2] and strenuous QED calculations of a_e by Kinoshita [3]. Since 2006, several progress have been made in the determination of the fine structure constant: an error in the QED calculation of a_e has been detected, Gabrielse has improved his measurement of a_e, and, in our group, we have made a new measurement of the ratio h/m_{Rb} and obtained a new value of α. Moreover, the calculation of the fine structure in helium has been improved by Pachucki and, now, the value of α deduced from this fine structure is in agreement with the other determinations.

The aim of this paper is to relate these different progress. In Section 2 we make a review of the different methods used to determine α. Section 3 presents in detail our measurement of the ratio h/m_{Rb} and, in Section 4 we analyze the α measurements for the determination of the von Kitzling constant R_K.

2. Determination of the fine structure constant

Fig. 1 shows the most precise determinations up to date of the fine structure constant with a relative uncertainty smaller than 10^{-7}. The first determination uses the precise measurement of the ground-state hyperfine splitting of the muonium (an atom formed by a positive muon and an electron). This splitting varies mainly as $\alpha^2 R_\infty$, and it can be calculated very accurately. Indeed the muonium is a purely leptonic system, therefore there is not the difficulty due to the structure of the proton as in the calculation of the hydrogen hyperfine structure. The CODATA report [1] presents the analysis of the measurements carried out in 1982 and 1999 at LAMPF (Los Alamos Meson Physics Facility) [4]. The obtained value is:

$$\alpha^{-1} = 137.036\,0017(80) \tag{3}$$

with a relative uncertainty of 5.8×10^{-8} which is mainly due to the muon-to-electron mass ratio.

The atomic fine structures vary also as $\alpha^2 R_\infty$. Unfortunately, in hydrogen, the measurement of the $2P_{1/2}$–$2P_{3/2}$ splitting (about 11 GHz) is limited by the natural width of the 2P level (100 MHz). Then the uncertainty of the best $2P_{1/2}$–$2P_{3/2}$ measurement [5,6] is only 15 kHz, corresponding to a relative uncertainty of about 1.4×10^{-6}, and providing an α value at the level of 7×10^{-7}. On the other hand, the fine structure of the 2^3P_J states in helium is a more promising case with a 30 GHz splitting and a 1.6 MHz natural width. The corresponding intervals have been accurately measured by radiofrequency spectroscopy by the group of Hessels [7], and by laser spectroscopy of the 2^3S_1–2^3P_J line by the groups of Shiner, Inguscio and Gabrielse [8–10]. The best result has a relative uncertainty of 2.4×10^{-8}. On the theoretical side, the QED calculations of the two electrons system is difficult and, up to recently, there was a disagreement (12 standard deviations) between the theory and the measurements [11]. Recently, Pachucki and Yerokhin have overcome this problem by calculating all the QED terms up to the order $\alpha^5 R_\infty$ [12]. They obtain the value:

$$\alpha^{-1} = 137.036\,0011(39)(16) \tag{4}$$

where the first uncertainty is due to the theory and the second to the experiment. With a relative uncertainty of 3.1×10^{-8}, this value is in perfect agreement with the other α determinations (see Fig. 1).

The most precise determinations of α are deduced from the measurement of the electron anomaly a_e. During twenty years, the best a_e value was the one obtained at the University of Washington with a relative uncertainty of 3.7×10^{-9} [13]. In 2006, this result was superseded by the one of Gabrielse at Harvard University with an uncertainty reduced by a factor of about 6 [2]. Finally, in 2008, Gabrielse has improved again the a_e measurement up to a relative uncertainty of 2.4×10^{-10} [14]. During this period, the QED calculation of the electron anomaly has been continuously improved by Kinoshita who has calculated all the α^4 contributions [3]. In 2006, the combination of these experimental and theoretical results provided the value of the fine structure constant $\alpha^{-1} = 137.035\,999\,710(96)$ with a relative uncertainty of 7×10^{-10}. This result determined the value given by the last report of the CODATA (see Eq. (1) [1]). Nevertheless, in 2007, Kinoshita and collaborators found an error in the calculation of the term in α^4. With this correction, the value of α is shifted to $\alpha^{-1} = 137.035\,999\,070(98)$ [15]. This value is labeled "Harvard 2007" on Fig. 1. Finally, the value of the fine structure constant deduced from the last measurement of a_e is:

$$\alpha^{-1} = 137.035\,999\,084(51) \tag{5}$$

With a relative uncertainty of 3.7×10^{-10}, this result, labeled "Harvard 2008" on Fig. 1, is the present most accurate determination of α.

Another way to obtain the fine structure constant is the measurement of the ratio h/m_X between the Planck constant and the mass of a particle X. Indeed, from the ionization energy of the hydrogen atom ($hcR_\infty = \alpha^2 m_e c^2/2$), one can deduce an expression of α:

$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_e} \tag{6}$$

where m_e is the electron mass. In this equation, c is exactly known and the relative uncertainty of R_∞ is smaller than 10^{-11}. On the contrary there is no precise determination of the ratio h/m_e. To circumvent this limit, one introduces the mass ratio m_X/m_e to write the equation:

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_X}{m_e}\frac{h}{m_X} \tag{7}$$

where the limiting factor is the ratio h/m_X. This method provides values of α which are only slightly dependent on the QED calculations. Actually, it is possible to extract the Rydberg constant from the combination of the radio frequency measurement of the 2S Lamb shift [5] and from the optical frequency measurement of the $2S_{1/2}$–$8D_{5/2}$ made in our group [16,17]. One obtains the frequency interval $2P_{1/2}$–$8D_{5/2}$ (about 771 THz) where the first QED contribution is the Lamb shift of the $2P_{1/2}$ level (about 13 MHz [18]), *i.e.* 1.7×10^{-8} of the total frequency interval. By contrast, the value of α deduced from the measurements of a_e depends wholly on the QED calculations. Consequently, the values of α obtained from the h/m ratio can be used to test the QED calculations of the electron anomaly.

This method has been implemented for the first time on a neutron beam to determine the ratio h/m_n between the Planck constant and the neutron mass [19]. The principle is to measure together the velocity υ of the neutron beam with a time of flight technique and the de Broglie's wavelength λ_{DB} by reflection on a silicon crystal ($h/m_n = \upsilon\lambda_{DB}$). Consequently this α determination depends also of the measurement of the lattice spacing of silicon. The value obtained on this experiment is [1]:

$$\alpha^{-1} = 137.036\,0077(28) \tag{8}$$

This result, labeled "h/m (neutron)" on Fig. 1, has a relative uncertainty of 2.1×10^{-8}.

The possibility to deduce the ratio h/m_A (m_A is the mass of the atom A) from the measurement of the recoil velocity was pointed out in 1976 by Hall and coworkers [20]. Indeed an atom at rest which absorbs a photon acquires the velocity $\upsilon_r = \hbar k/m_A$, where k is the wave vector of the photon. Nevertheless a precise measurement of the ratio h/m_A requires cold atom techniques and atom interferometry [21]. In a pioneering atom interferometry experiment in the nineties at Stanford University, Chu and colleagues succeeded in doing the first accurate measurement of the recoil velocity of ^{133}Cs atom [22]. As the m_{Cs}/m_e was precisely measured at MIT [23], the deduced value of α is [1]:

$$\alpha^{-1} = 137.036\,0000(11) \tag{9}$$

with a relative uncertainty of 7.7×10^{-9} (labeled "h/m(Cs)" on Fig. 1).

In Paris we have followed the same ideas to measure the ratio h/m_{Rb} between the Planck constant and the mass of ^{87}Rb atom. This experiment is described in the next section. Its originality is the use of Bloch oscillations to transfer to the atom a very large number of photon momenta. In 2006 we obtained a first result without using atom interferometry [24,25]:

$$\alpha^{-1} = 137.035\,998\,84(91) \tag{10}$$

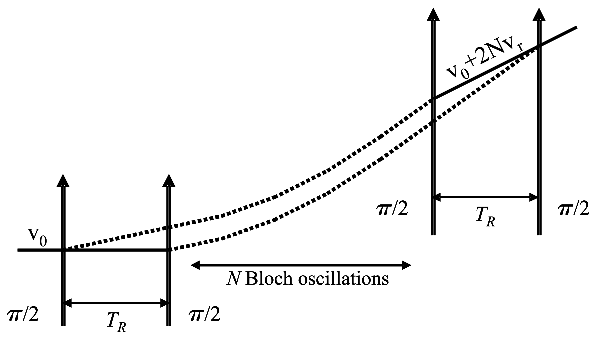

Fig. 2. Scheme of the interferometer used for the measurement of h/m_{Rb}. The first pair of $\pi/2$ pulses produces a fringe pattern in the atomic velocity distribution which is measured by the second pair of $\pi/2$ pulses. Between these two pairs of pulses, the atoms are accelerated upwards or downwards. The solid line corresponds to the atom in the $F = 2$ state, and the dashed line to the $F = 1$ state.

Fig. 2. Schéma de l'interféromètre utilisé pour la mesure de h/m_{Rb}. La première paire d'impulsions $\pi/2$ produit une structure de franges dans la distribution de vitesse des atomes qui est mesurée par la seconde paire d'impulsions $\pi/2$. Entre ces deux paires d'impulsions, les atomes sont accélérés vers le haut où vers le bas. La ligne en trait plein correspond aux atomes dans l'état $F = 2$, et celle en pointillé à ceux dans l'état $F = 1$.

with a relative uncertainty of 6.7×10^{-9}. In 2008, this result was slightly improved by combining Bloch oscillations with atom interferometry and we obtain the value of α [26]:

$$\alpha^{-1} = 137.035\,999\,45(62) \tag{11}$$

With a relative uncertainty of 4.5×10^{-9}, this is the most accurate result after the values deduced from the electron anomaly measurements.

Finally solid state physics allows two precise determinations of α. In the quantum Hall effect, the von Kitzling constant $R_{\mathrm{K}} = h/e^2$ is directly linked to the fine structure constant by the relation $R_{\mathrm{K}} = \mu_0 c/2\alpha$ where μ_0 and c are exactly known in the Système International of units. The measurement of R_{K} has been made in several NMIs (National Metrology Institute) by using the calculable capacitor of Thomson and Lampard. The CODATA report gives a complete review of these results. From the average of these measurements, one obtains the value:

$$\alpha^{-1} = 137.036\,0030(25) \tag{12}$$

with a relative uncertainty of 1.8×10^{-8} [1]. The details of these measurements will be described in Section 4.

The last point of Fig. 1 is a combination of the low field gyromagnetic ratio measurements and the Josephson and quantum Hall effects. It corresponds to the mean value of the two results deduced from the measurements of the gyromagnetic ratio of the proton and helion (^{3}He nucleus). The principle is to replace in Eq. (6) the ratio h/m_{e} by the product $(h/e) \times (e/m_{\mathrm{e}})$ where the first factor is given by the Josephson constant $K_{\mathrm{J}} = 2e/h$. The detail of this derivation will be described in Section 4. Following the CODATA report [1], the value of the fine structure constant, labeled "$\Gamma'_{\mathrm{p,h\text{-}90}}$" on Fig. 1, is:

$$\alpha^{-1} = 137.035\,9875(43) \tag{13}$$

with a relative uncertainty of 3.1×10^{-8}. This value is in slight disagreement (2.7 standard deviations) with the one deduced from the measurement of the electron anomaly.

3. Measurement of the ratio h/m_{Rb}

We present in this section the Paris experiment in which the ratio h/m_{Rb} is measured. The details are given in Refs. [24–26]. The principle of the experiment is to coherently transfer as many recoils as possible to the atoms at rest (*i.e.* to accelerate them) and to measure the final velocity distribution. In our experiment, the atoms are efficiently accelerated by means of N Bloch oscillations (BO). In our latest measurement, Bloch oscillations are combined with a Ramsey–Bordé interferometer to precisely measure the induced atomic velocity variation (see Fig. 2).

The experiment develops in three steps. (i) Firstly, a pair of $\pi/2$ pulses of a Raman transition transfers the ^{87}Rb atoms from the $F = 2$ hyperfine sublevel to the $F = 1$ one and produces a fringe pattern in the velocity distribution of these atoms. The width of the envelope of this velocity distribution varies inversely with the $\pi/2$ pulse duration τ, while the fringe width varies as $1/T_{\mathrm{R}}$, where T_{R} is the delay between the two $\pi/2$ pulses. (ii) Secondly, we transfer to the selected atoms as many recoils as possible by means of Bloch oscillations. Bloch oscillations have been first observed in atomic physics by the groups of Salomon in Paris and Raizen in Austin [27–29]. Bloch oscillations can be interpreted as Raman transitions in which the atom begins and ends in the same energy level, so that its internal state ($F = 1$) is unchanged while its velocity has increased by $2\upsilon_r$ per Bloch oscillation (see Fig. 3). Bloch oscillations are produced in a one dimension vertical optical lattice which is accelerated by linearly sweeping the relative frequencies of the two counter propagating laser

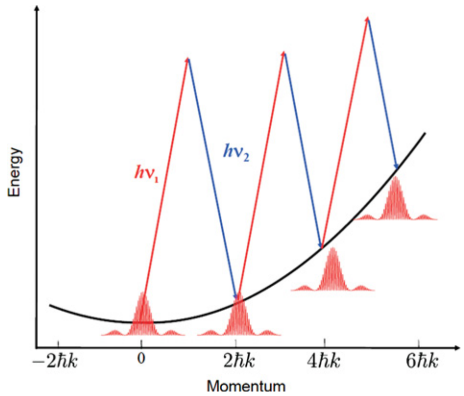

Fig. 3. Acceleration of cold atoms in a frequency chirped standing wave. The variation of the kinetic energy versus the atomic momentum is given by a parabola. This momentum increases by the quantity $2\hbar k$ at each cycle. The Ramsey fringe patterns represent the momentum distribution of the atoms in the $F = 1$ hyperfine level.

Fig. 3. Accélération des atomes dans une onde stationnaire accélérée. La variation de l'énergie cinétique en fonction de l'impulsion de l'atome est représentée par une parabole. Cette impulsion augmente de $2\hbar k$ à chaque cycle. Les franges de Ramsey représentent la distribution des impulsions des atomes dans l'état hyperfin $F = 1$.

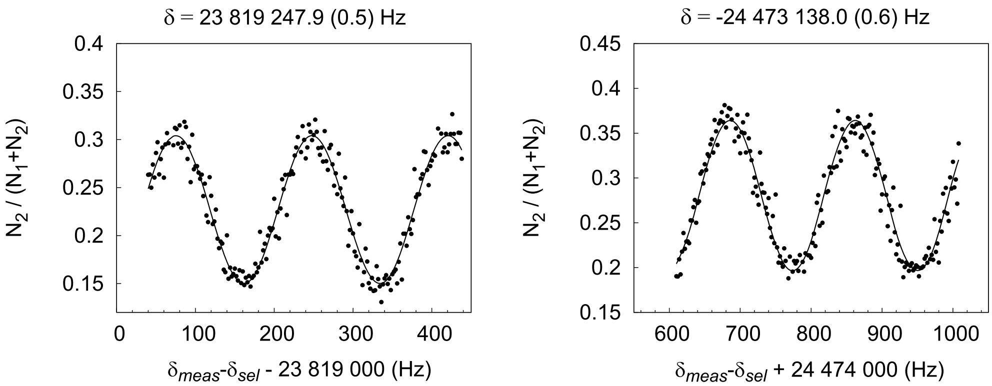

Fig. 4. Velocity spectra obtained when the atoms are accelerated downwards and upwards. The spectrum on the left corresponds to the downwards acceleration (800 Bloch oscillations) and on the right to the upwards acceleration (800 Bloch oscillations). The frequency difference between these spectra corresponds to 3200 recoil velocities.

Fig. 4. Franges d'interférence obtenues quand les atomes sont accélérés vers le bas et vers le haut. Le spectre de gauche correspond à une accélération vers le bas (800 oscillations de Bloch) et celui de gauche à une accélération vers le haut (800 oscillations de Bloch). La différence de fréquence entre ces deux spectres correspond à 3200 vitesses de recul.

beams (frequencies ν_1 and ν_2). This leads to a succession of rapid adiabatic passages between momentum states differing by $2\hbar k$. (iii) Finally, we measure the final velocity of the atoms by a second pair of $\pi/2$ pulses which transfers the atoms from the $F = 1$ to the $F = 2$ hyperfine level. The frequency difference between the two pairs of $\pi/2$ pulses is scanned to obtain a fringe pattern from which we can deduce the velocity variation between the two pairs of $\pi/2$ pulses.

In the vertical direction, an accurate determination of the recoil velocity would require an accurate measurement of the gravitational acceleration g. In order to circumvent this difficulty, we make a differential measurement by accelerating the atoms in opposite directions (upward and downward trajectories) keeping the same delay between the two pairs of $\pi/2$-pulses. Thus the photon-recoil measurement is a determination of the frequency difference between the central fringes of two opposite interferometers (upward and downward). Fig. 4 shows two records obtained with 800 Bloch oscillations between the pairs of $\pi/2$ pulses. The frequency difference between the two spectra corresponds to 3200 recoil velocities. Moreover, the contribution of some systematic effects (energy level shifts) changes sign when the direction of the Raman beams is exchanged: for each up or down trajectory, the Raman beams directions are reversed and we record two velocity spectra. Finally, each determination of h/m_{Rb} and α is obtained from 4 velocity spectra.

Our determinations of h/m_{Rb} and α have been derived from 221 experimental data points. Each point corresponds to a 20 minute measurement. The total number of Bloch oscillations $N^{up} + N^{down}$ has been varied from 200 to 1600. The dispersion of these $n = 221$ measurements is $\chi^2/(n-1) = 1.85$ and the resulting relative statistical uncertainty on α is 3×10^{-9}. The systematic effects have been analyzed in detail in Ref. [25]. They are listed in Table 1 with the corresponding contributions and uncertainties. The two main effects are due to the geometry of the laser beams and to the second order Zeeman effect. To evaluate these effects, we have measured the wave front curvatures with a Shack–Hartmann wave front

Table 1
Error budget on the determination of $1/\alpha$.

Source	Correction	Relative uncertainty (parts in 10^9)
Laser frequencies		0.4
Beams alignment	−2	2
Wavefront curvature and Gouy phase	−11.9	2.5
2nd order Zeeman effect	4.9	1
Quadratic magnetic force	−0.59	0.2
Gravity gradient	−0.07	0.02
Light shift (one photon transition)		0.1
Light shift (two photon transition)		0.01
Light shift (Bloch oscillation)	0.48	0.2
Index of refraction atomic cloud		0.3
Index of refraction background vapor	−0.36	0.3
Rydberg constant and mass ratio [1]		0.23
Global systematic effects	−9.54	3.4

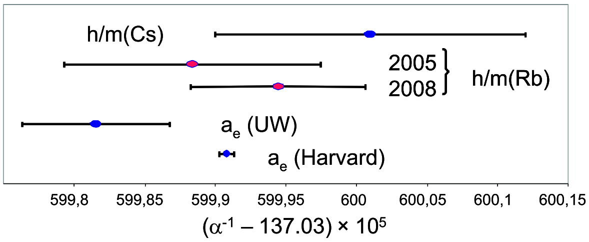

Fig. 5. The five most accurate determinations of the fine structure constant.

Fig. 5. Les cinq déterminations les plus précises de la constante de structure fine.

analyzer and mapped the frequency shift due to the magnetic field following the procedure described in Ref. [30]. Finally the relative uncertainty due to the systematic effects is 3.4×10^{-9} and we obtain for α the value given by Eq. (11).

The most precise determinations of the fine structure constant are reported on Fig. 5. There is a very good agreement between our measurements (labeled "h/m(Rb)") and the value deduced from the electron anomaly measurement. Even if the uncertainty on this value is 10 times smaller than our result, the comparison of these two results provides the most stringent test of the QED calculations or, assuming these calculations exact, it gives a limit to test a possible internal structure of the electron, or the existence of low-mass dark-matter particles [31].

4. Determination of von Kitzling constant

The two determinations of the fine structure constant derived by solid state physics assume the exactness of the relations $R_K = h/e^2$ and $K_J = 2e/h$. To test this validity we introduce two small deviations characterized by ϵ_K and ϵ_J [32]:

$$R_K = \frac{h}{e^2}(1+\epsilon_K), \qquad K_J = \frac{2e}{h}(1+\epsilon_J) \tag{14}$$

In this section we present the different determinations of R_K which correspond to the values of the fine structure constant presented in Section 2. This analysis is illustrated in Fig. 6. The first group of R_K values corresponds to the direct measurements of the von Kitzling constant with the Lampard capacitor. The values obtained in the different NMIs are detailed on this figure [33–37]. These determinations of R_K are in good agreement. The weighted mean value of these results is:

$$R_K = 25\,812.808\,18(47)\ \Omega \tag{15}$$

The value of α given in Eq. (12) is directly obtained from this value of R_K by the relation $\alpha = \mu_0 c/2R_K$.

A second method to obtain the von Kitzling constant R_K is to use the relation:

$$R_K = \frac{(1+\epsilon_K)\mu_0 c}{2\alpha} \tag{16}$$

with the values of α deduced from the h/m_X ratio or from the electron anomaly. Assuming that $\epsilon_K = 0$, we have reported in Fig. 6 the values of R_K obtained from our measurements of h/m_{Rb} and the last measurement of the electron anomaly. These results are in good agreement with the direct measurements of R_K.

Finally, it is also possible to deduce R_K from the gyromagnetic ratio measurements. From the proton gyromagnetic ratio, we deduce the fine structure constant:

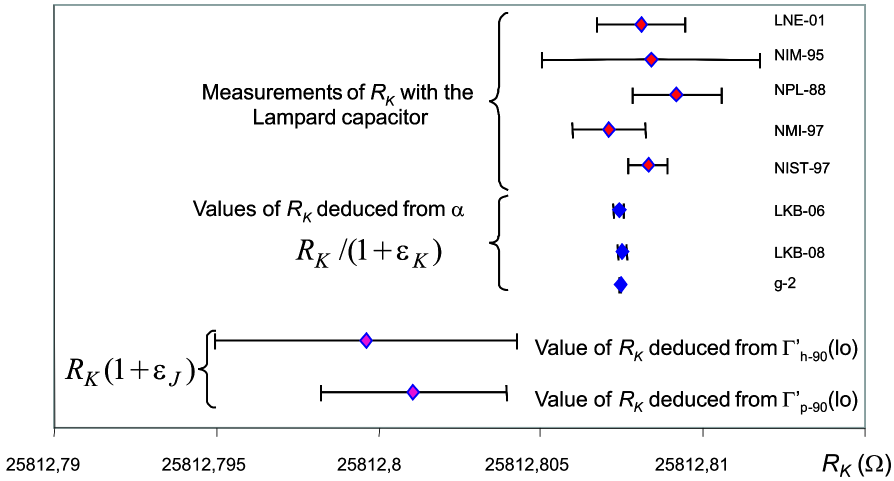

Fig. 6. Determination of the von Kitzling constant R_K.

Fig. 6. Détermination de la constante de von Kitzling R_K. LNE : Laboratoire National d'Essais, France ; NIM : National Institute of Metrologie, People's Republic of China ; NPL : National Physical Laboratory, United-Kingdom ; NMI : National Metrology Institute, Australia ; NIST : National Institute of Standards and Technology, USA.

$$\alpha^{-2} = \frac{c}{4R_\infty} \times \frac{\mu'_p}{\mu_B} \times \frac{2e/h}{\gamma'_p} \tag{17}$$

where μ'_p and γ'_p are the magnetic moment and the gyromagnetic ratio of the shielded proton. Taking into account Eq. (14), one deduces:

$$\alpha^{-2} = \frac{c}{4R_\infty} \times \frac{\mu'_p}{\mu_e} \times \frac{g_e}{2} \times \frac{K_J/(1+\epsilon_J)}{\gamma'_p} \tag{18}$$

where the electron g-factor and the ratio μ'_p/μ_e are precisely well known [14,38]. The gyromagnetic ratio of the shielded proton has been measured in low field in terms of the conventional electrical units V_{90} and Ω_{90} with a relative uncertainty of 1.1×10^{-7} [39]. The values of the gyromagnetic ratio of the shielded proton in terms of SI units (γ'_p) and in terms of the conventional units ($\Gamma'_{p\text{-}90}(\mathrm{lo})$) are linked by the relation [1]:

$$\gamma'_p = \Gamma'_{p\text{-}90}(\mathrm{lo}) \times \frac{K_J R_K}{K_{J\text{-}90} R_{K\text{-}90}} \tag{19}$$

where $K_{J\text{-}90}$ and $R_{K\text{-}90}$ are the conventional values of the Josephson and von Kitzling constants expressed in Hz/V_{90} and Ω_{90}. Then one obtains:

$$\alpha^{-2} = \frac{c}{4R_\infty} \times \frac{\mu'_p}{\mu_e} \times \frac{g_e}{2} \times \frac{K_{J\text{-}90} R_{K\text{-}90}}{R_K \Gamma'_{p\text{-}90}(\mathrm{lo})} \times \frac{1}{1+\epsilon_J} \tag{20}$$

and deduces R_K:

$$R_K = \frac{c}{4R_\infty} \times \frac{\mu'_p}{\mu_e} \times \frac{g_e}{2} \times \frac{K_{J\text{-}90} R_{K\text{-}90}}{\Gamma'_{p\text{-}90}(\mathrm{lo})} \times \alpha^2 \times \frac{1}{1+\epsilon_J} \tag{21}$$

where the limiting factor is the measurement of $\Gamma'_{p\text{-}90}(\mathrm{lo})$. We can obtain a similar expression from the measurement of the gyromagnetic ratio of the shielded helion $\Gamma'_{h\text{-}90}(\mathrm{lo})$:

$$R_K = \frac{c}{4R_\infty} \times \frac{\mu'_h}{\mu'_p} \times \frac{\mu'_p}{\mu_e} \times \frac{g_e}{2} \times \frac{K_{J\text{-}90} R_{K\text{-}90}}{\Gamma'_{h\text{-}90}(\mathrm{lo})} \times \alpha^2 \times \frac{1}{1+\epsilon_J} \tag{22}$$

where μ'_h is the magnetic moment of the shielded helion.

The two values of $R_K(1+\epsilon_J)$ deduced from Eqs. (21) and (22) are reported in Fig. 6. If we suppose $\epsilon_J = 0$, these values are in slight disagreement with the other determinations of R_K. This result suggests a non null value of ϵ_J as already mentioned in Ref. [32].

5. Conclusion

We have presented a review of the most accurate determinations of the fine structure constant and described in detail the measurements of the h/m_{Rb} ratio. This last experiment leads to a value of α which is the most precise after the determinations of α deduced from the electron anomaly. The agreement between these two results is the more stringent test of the QED. We have built an improved experimental setup and we expect to reduce the uncertainty of α to 1 ppb. A review of several determinations of the von Kitzling constant R_{K} is also presented. There is a slight disagreement for the values deduced from the gyromagnetic ratios of the proton and the helion (relative difference of 2.5×10^{-7} and 3×10^{-7} respectively). This suggests a non exactness of the relation between the Josephson constant K_{J} and $2e/h$. It would be advisable to understand the origin of this discrepancy before the change of the definition of the units of the Système International.

Acknowledgements

This experiment is supported in part by IFRAF (Institut Francilien de Recherches sur les Atomes Froids), and by the Agence Nationale pour la Recherche, FISCOM Project-(ANR-06-BLAN-0192).

References

[1] P.J. Mohr, B.N. Taylor, D.B. Newell, Rev. Mod. Phys. 80 (2008) 633–730.
[2] B. Odom, D. Hanneke, B. D'Urso, G. Gabrielse, Phys. Rev. Lett. 97 (2006) 030801.
[3] G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, B. Odom, Phys. Rev. Lett. 97 (2006) 030802.
[4] W. Liu, M.G. Boshier, S. Dhawan, O. van Dyck, P. Egan, X. Fei, M. Grosse Perdekamp, V.W. Hughes, M. Janousch, K. Jungmann, D. Kawall, F.G. Mariam, C. Pillai, R. Prigl, G. zu Putlitz, I. Reinhard, W. Schwarz, P.A. Thompson, K.A. Woodle, Phys. Rev. Lett. 82 (1999) 711.
[5] S.R. Lundeen, F.M. Pipkin, Phys. Rev. Lett. 46 (1981) 232.
[6] E.W. Hagley, F.M. Pipkin, Phys. Rev. Lett. 72 (1994) 1172.
[7] J.S. Borbely, M.C. George, L.D. Lombardi, M. Weel, D.W. Fitzakerley, E.A. Hessels, Phys. Rev. A 79 (2009) 060503(R).
[8] J. Castillega, D. Livingston, A. Sanders, D. Shiner, Phys. Rev. Lett. 84 (2000) 4321.
[9] G. Giusfredi, P.C. Pastor, P.D. Natale, D. Mazzotti, C. de Mauro, L. Fallani, G. Hagel, V. Krachmalnicoff, M. Inguscio, Can. J. Phys. 83 (2005) 301.
[10] T. Zelevinsky, D. Farkas, G. Gabrielse, Phys. Rev. Lett. 95 (2005) 203001.
[11] K. Pachucki, Phys. Rev. Lett. 97 (2006) 013002.
[12] K. Pachucki, V.A. Yerokhin, Phys. Rev. Lett. 104 (2010) 070403.
[13] R.S. Van Dick, P.B. Schwinberg, H.G. Dehmelt, Phys. Rev. Lett. 59 (1987) 26.
[14] D. Hanneke, S. Fogwell, G. Gabrielse, Phys. Rev. Lett. 100 (2008) 120801.
[15] G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, B. Odom, Phys. Rev. Lett. 99 (2007) 039902.
[16] B. de Beauvoir, F. Nez, L. Julien, B. Cagnac, F. Biraben, D. Touahri, L. Hilico, O. Acef, A. Clairon, J.J. Zondy, Phys. Rev. Lett. 78 (1997) 440.
[17] B. de Beauvoir, C. Schwob, O. Acef, J.-J. Zondy, L. Jozefowski, L. Hilico, F. Nez, L. Julien, A. Clairon, F. Biraben, Eur. Phys. J. D 12 (2000) 61.
[18] U. Jentschura, K. Pachucki, Phys. Rev. A 54 (1996) 1853.
[19] E. Krüger, W. Nistler, W. Weirauch, Metrologia 36 (1999) 147.
[20] J.L. Hall, Ch.J. Bordé, K. Uehara, Phys. Rev. Lett. 37 (1976) 1339.
[21] C.J. Bordé, Phys. Lett. A 140 (1989) 10.
[22] A. Wicht, J.M. Hensley, E. Sarajilic, S. Chu, Physica Scripta T 102 (2002) 82.
[23] M.P. Bradley, J.V. Porto, S. Rainville, J.K. Thompson, D.E. Pritchard, Phys. Rev. Lett. 83 (1999) 4510.
[24] P. Cladé, E. de Mirandes, M. Cadoret, S. Guellati-Khélifa, C. Schwob, F. Nez, L. Julien, F. Biraben, Phys. Rev. Lett. 96 (2006) 033001.
[25] P. Cladé, E. de Mirandes, M. Cadoret, S. Guellati-Khélifa, C. Schwob, F. Nez, L. Julien, F. Biraben, Phys. Rev. A 102 (2006) 052109.
[26] M. Cadoret, E. de Mirandes, P. Cladé, S. Guellati-Khélifa, C. Schwob, F. Nez, L. Julien, F. Biraben, Phys. Rev. Lett. 101 (2008) 230801.
[27] M. Ben Dahan, E. Peik, J. Reichel, Y. Castin, C. Salomon, Phys. Rev. Lett. 76 (1996) 4508.
[28] E. Peik, M. Ben Dahan, I. Bouchoule, Y. Castin, C. Salomon, Phys. Rev. A 55 (1997) 2989.
[29] S.R. Wilkinson, C.F. Bharucha, K.W. Madison, Qian Niu, M.G. Raizen, Phys. Rev. Lett. 76 (1996) 4512.
[30] M. Cadoret, E. de Mirandes, P. Cladé, S. Guellati-Khélifa, C. Schwob, F. Nez, L. Julien, F. Biraben, Eur. Phys. J. Special Topics 163 (2008) 101.
[31] C. Boehm, J. Silk, Phys. Lett. B 661 (2008) 287.
[32] P. Mohr, B.N. Taylor, Rev. Mod. Phys. 77 (2005) 1.
[33] G. Trapon, O. Thévenot, J.-C. Lacueille, W. Poirier, H. Fhima, G. Genevès, IEEE Trans. Instrum. Meas. 50 (2001) 572.
[34] Z. Zhang, X. Wang, D. Wang, X. Li, Q. He, Y. Ruan, Acta Metrologia Sinica 16 (1995) 1.
[35] A. Hartland, Metrologia 29 (1992) 175.
[36] G.W. Small, B.W. Ricketts, P.C. Coogan, B.J. Pritchard, M.M.R. Sovierzoski, Metrologia 34 (1997) 241.
[37] M.E. Cage, R.F. Dziuba, R.E. Elmquist, B.F. Field, G.R. Jones Jr., P.T. Olsen, W.D. Phillips, J.Q. Shields, R.L. Steiner, B.N. Taylor, E.R. Williams, IEEE Trans. Instrum. Meas. 38 (1989) 284.
[38] W.D. Phillips, W.E. Cooke, D. Kleppner, Metrologia 13 (1977) 179.
[39] P.J. Mohr, B.N. Taylor, Rev. Mod. Phys. 72 (2000) 351–495.