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The role of lattice QCD in flavor physics

Le rôle de la QCD sur réseau dans la physique de la saveur

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A B S T R A C T

We discuss the current status of the QCD simulations on the lattice and outline the perspectives for precision computation of hadronic quantities relevant to flavor physics phenomenology. High precision results, that have been recently obtained on the lattice for selected quantities, such as the kaon decay constant and the $K_{\ell 3}$ form factor, demonstrated the feasibility of a precision comparison between theory and experiment in the kaon sector. A similar level of accuracy has not yet been reached in the $D_{(s)}$- and $B_{(s)}$-meson decays, but the existing methodology, as well as increased computing resources, allow for substantial improvement in the near future.

R É S U M É

On présente une discussion succincte sur l'état actuel et les perspectives des simulations numériques de la QCD sur réseaux concernant des quantités hadroniques dont la connaissance est essentielle pour la physique de la saveur. Les résultats récemment obtenus pour la constante de désintégration du kaon et le facteur de forme semileptonique $K_{\ell 3}$ à partir des réseaux ont démontrés qu'une comparaison détaillée et précise entre la théorie et l'experience dans le secteur des kaons est possible. Une précision similaire pour des désintégrations des mésons $D_{(s)}$ et $B_{(s)}$ n'a pas encore été atteinte mais la méthodologie déjà existante, en combinaison avec des moyens de calcul numérique toujours plus performants, permettront une réduction significative des erreurs dans ce secteur aussi.

1. Introduction

1.1. Importance of lattice QCD in flavor physics phenomenology

Current experiments at LHC are bound to provide us with an answer to the hierarchy problem in the Standard Model (SM). At the same time, the flavor problem will be experimentally explored at LHCb, as well as at dedicated high intensity experiments, such as NA62 at CERN, and the Super-B factories.

Over the past decade great progress has been made in getting better control over uncertainties involved in determination of the parameters that appear in the SM Lagrangian, namely the quark masses, gauge couplings, and the CKM entries [1].

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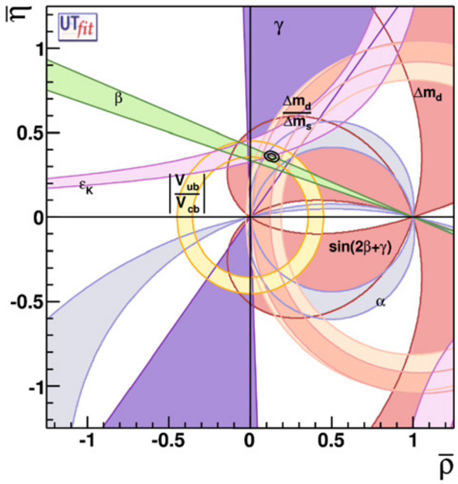

Fig. 1. CKM Unitarity Triangle as obtained from combined analysis in Ref. [3]. Many constraints shown in this plot rely on the theoretical input provided by LQCD.

As a result we now know that the CKM picture of CP-violation is robust, and that no departure from the CKM unitarity has been observed so far (see Fig. 1). To search the signals of physics beyond SM in the flavor sector, it is therefore necessary to increase the theoretical and experimental control over the physical processes that are used to extract various CKM entries. The main theoretical obstacle in that respect is our inability to deal analytically with the non-perturbative aspects of QCD, from the first theoretical principles. To date the only available approach that simultaneously allows one to formulate QCD in the non-perturbative domain, without introducing any extra parameters apart from those appearing in the QCD Lagrangian, and provides us with tools to accurately compute many hadronic quantities, is lattice QCD (LQCD). In the rest of this article we will try to convince our readers that LQCD has reached maturity and that the precision calculation of various hadronic quantities is already possible. A comparable precision could also be reached, in the years to come, in many more hadronic quantities relevant to flavor physics phenomenology.

1.2. *LQCD for pedestrians*

LQCD is a way to regularize QCD, i.e. to formulate QCD in a discretized space–time. The main benefit is that on a discretized space–time one can solve the theory numerically, i.e. simulate the non-trivial non-perturbative properties of QCD vacuum at low energies. The result of LQCD simulations is a large set of independent gauge field configurations (snapshots of the QCD vacuum), obtained at fixed lattice spacing a. Since one is interested in the physical results obtained in continuum ($a \to 0$) the simulations are repeated at several small a's and then extrapolated to the continuum limit, $a \to 0$ (*continuum extrapolation*). To make that extrapolation as smooth as possible, one can improve the discretized form of the QCD Lagrangian and eliminate $\mathcal{O}(a, \alpha_s a^2, \ldots)$ effects. Modern LQCD simulations include various forms of such improvement. Methodology to compute various physical quantities – via (Euclidean) Green functions computed on the simulated QCD gauge field configuration s – has been developed in the so-called *quenched approximation*, i.e. by neglecting the effects of the quark–antiquark pairs that polarize the QCD vacuum. These dynamical (or "sea") quarks can be included in the simulations but the computing time to produce a significant number of gauge field configurations becomes huge. Furthermore, the most important effects on physics is expected to come from the light dynamical quarks u, d and s. While the simulations with the strange quark at its physical mass are feasible, those with the very light u, d quarks are prohibitively expensive, especially when we keep the lattice spacing small and the physical lattice volume large. One of the most important goals of modern LQCD is to simulate the QCD vacuum with as low $m_{u,d}/m_s^{\mathrm{phys}}$ as possible and thus reduce the systematic errors that arise from extrapolations of the physical quantities computed at $m_{u,d}/m_s^{\mathrm{phys}} \gtrsim 1/10$ to $(m_{u,d}/m_s)^{\mathrm{phys}} \approx 1/25$ (*chiral extrapolations*).

1.3. *Modern day LQCD*

Before we embark on the discussion of some specific quantities, we would like to point out that in the past decade we witnessed a tremendous progress in LQCD which can be attributed to: (i) judiciously preconditioned algorithms used to invert the Dirac operator on the lattice; (ii) improved LQCD actions; (iii) progress in performance of the computing facilities. These three elements combined together made it possible to simulate QCD with ever lighter dynamical and valence quarks. Improved actions, in particular, were designed to better cope with various theoretical issues such as going faster to the continuum limit, closer to the chiral limit, and simplifying the renormalization of composite operators whose matrix elements are computed on the lattice. As far as the high performance computing facilities are concerned, their steady

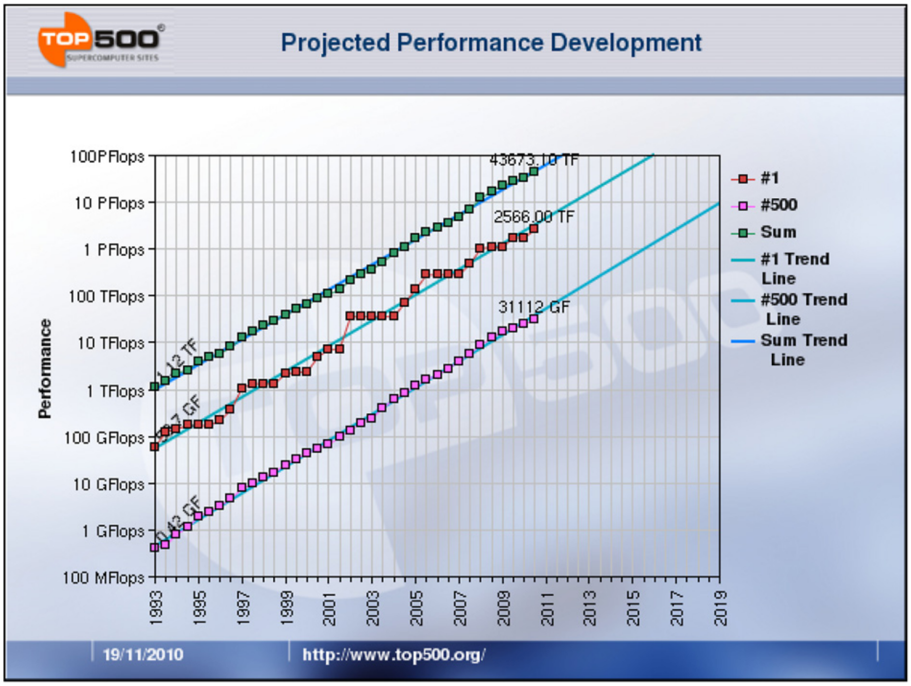

Fig. 2. Performance development of the most powerful computer systems in the world as a function of the year [2]. The top (green) points correspond to the sum of the 500 fastest computer systems, the middle (red) points to the fastest in the world, the bottom (pink) points to the number 500 in the list. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

progress, as illustrated in Fig. 2, was of course decisive to switch from the simulations of quenched QCD (with no sea quarks) to those in which the dynamical (sea) quarks were included. Nowadays, the main challenge is to lower the u and d quark masses and get closer to the physical (nearly chiral) limit, while keeping the s and c quarks at their physical values. In doing so, one should keep the finite volume effects under control and check for the smoothness of the continuum extrapolation.

2. Quark masses

The determination of the quark masses is one of the benchmark calculations in LQCD, and it is the place where other semi-analytic approaches to QCD can be tested. Since the strange and charm quarks are directly accessible from LQCD simulations, their determination is the least affected by systematic uncertainties. In Fig. 3 we show the current status of $m_s^{\overline{\mathrm{MS}}}$ (2 GeV) as obtained from LQCD simulations in which $N_f = 2$ or $N_f = 2 + 1$ flavors of dynamical quarks were included. The standard method to extract the quark masses from the lattice is to rely on Ward identities and to tune the quark masses until the mass of the corresponding hadron state matches the physical one. One of the most significant ingredients needed for a reliable computation of the quark masses on the lattice is the non-perturbative renormalization of bilinear quark operators and the methods to compute them are known [5]. Being a benchmark computation, many different methods to regularize QCD on the lattice have been employed and the corresponding results in the continuum limit are found to be in very good agreement among themselves. This can be appreciated in Fig. 3. It should be emphasized that the determination of the strange quark mass from LQCD is a genuine prediction.

3. Light quark physics

Besides quark masses, the fundamental parameters appearing on the quark side of the flavor sector of SM are the CKM matrix elements. To extract these couplings one has to compare the experimental measurements of some physical weak interaction processes with their associated theoretical expressions. The latter involve hadronic matrix elements the computation of which requires a control over non-perturbative QCD. In simple physical processes, such as leptonic and semileptonic decays, the initial and final states in the hadronic matrix elements involve at most one hadron and can be accurately computed on the lattice. In particular, for the extraction of $|V_{us}|$ one can either use the leptonic $K \to \ell\nu_\ell$, or semileptonic $K \to \pi\ell\nu_\ell$ decay mode. Their hadronic matrix elements are expressed in terms of the decay constant f_K and of the form factor $f_+(0)$, respectively. In recent years, great progress has been achieved in evaluating these quantities on the lattice and this computation has reached the level of high accuracy that is comparable with current experiments. In Fig. 4 are shown the results for $f_+(0)$ obtained by using several regularizations of LQCD. The results display an impressive

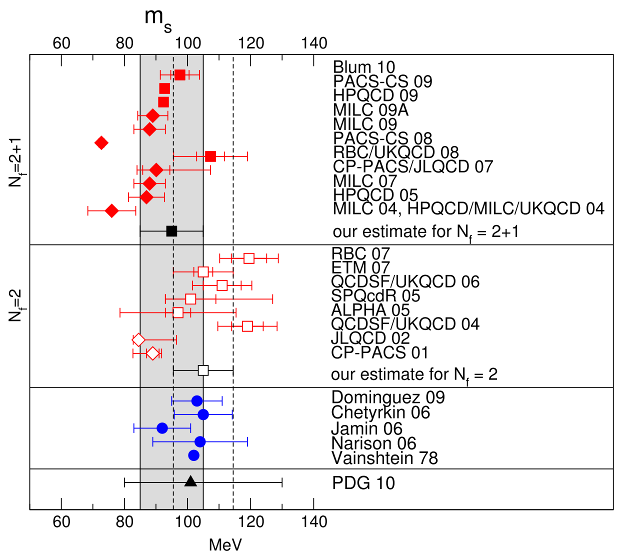

Fig. 3. Results for the strange quark mass, as obtained from LQCD with $N_f = 2$ sea quarks (empty symbols) and $N_f = 2 + 1$ (full symbols), are compared with those obtained by using the QCD sum rules (blue symbols). Reported results refer to the $\overline{\text{MS}}$ renormalization scheme and the scale $\mu = 2$ GeV. See Ref. [4] for more details.

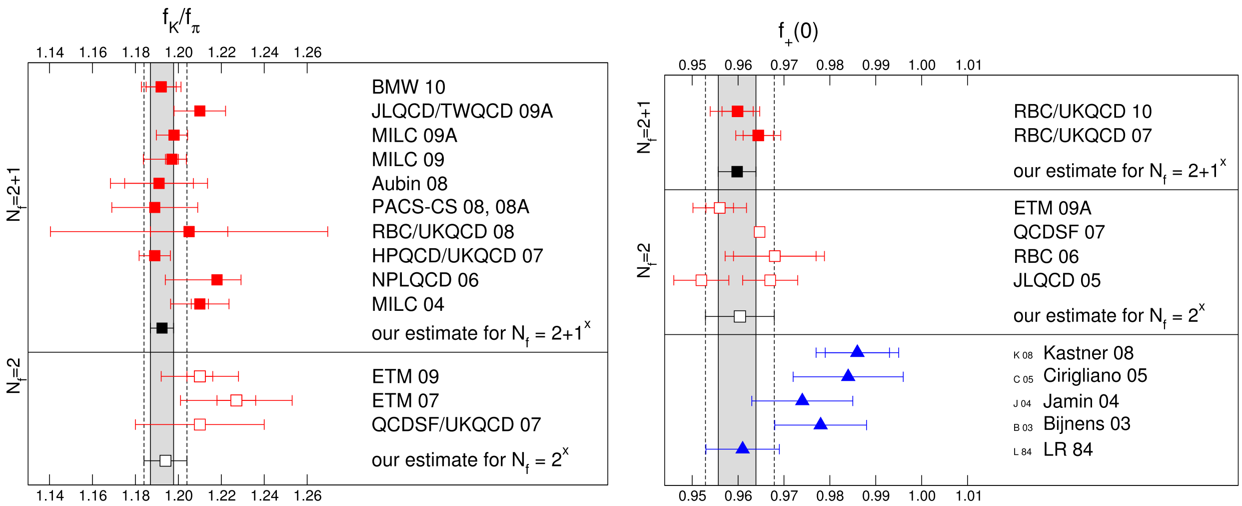

Fig. 4. High precision unquenched lattice results for the decay constant ratio f_K/f_π and for the $K_{\ell 3}$ decay form factor. In the right plot the lattice results are compared with those obtained with the semi-analytic approaches. For more details and a full list of references see Ref. [4].

agreement with one another. Notice that the lattice results are slightly different from most of those obtained by using analytic approaches. The source of that discrepancy most likely lies in the model dependence involved in the analytic evaluation of $f_+(0)$.

Many more LQCD computations have been performed in estimating f_K and f_K/f_π. A compilation of the latter results is also presented in Fig. 4. Importantly, the values of $|V_{us}|$ extracted from leptonic and from semileptonic kaon decays are completely consistent with each other. Furthermore the resulting $|V_{us}|$, when combined with $|V_{ud}|$, verifies the CKM unitarity to a level of a few per-mil, which is a very stringent test of SM.

Another quantity that is particularly relevant to the new physics searches is ε_K, which parameterizes the amount of indirect CP-violation in the $K^0 - \bar{K}^0$ system. Even though the lattice computation of the matrix element of the relevant four-quark ($\Delta S = 2$) operator, conveniently parameterized by the so called bag parameter $\hat{B}_K$, is more involved, the current results have reached the 5% precision level and a further improvement is possible [6].

4. Heavy quark physics

Concerning the heavy quarks on the lattice, current simulations can accommodate the charm quark directly, while the b-quark mass is far too heavy to be treated on equal footing with lighter quarks.

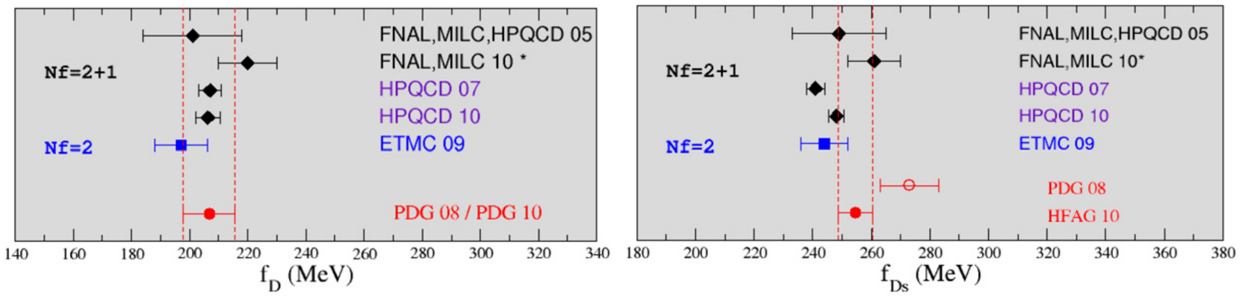

Fig. 5. Recent lattice results for the D and D_s meson decay constants.

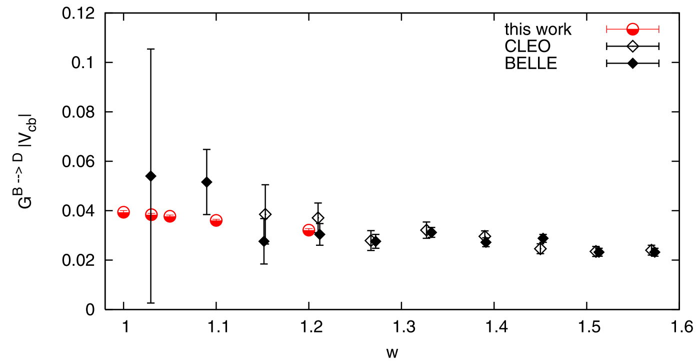

Fig. 6. $B \to D\ell\nu_\ell$ decay form factor obtained from high precision quenched lattice QCD at various values of momentum transfer (red symbols) are compared with those extracted from experiments (black symbols). For more details see Ref. [9].

In the charm sector, LQCD was particularly focused onto evaluating the hadronic matrix elements describing the leptonic $[D_{(s)} \to \ell\nu_\ell]$ and semileptonic $[D_{(s)} \to K(\pi)\ell\nu_\ell]$ decays [7]. The current situation, with the $D_{(s)}$ meson decay constants, computed in unquenched QCD and extrapolated to the continuum limit, is shown in Fig. 5. These results were genuine predictions of LQCD and have recently been confirmed by the experiments [8].

As far as the semileptonic decays are concerned, an additional difficulty with respect to the kaon decays arises from the fact that the kinematical region of the accessible momenta transferred to leptons is larger and therefore the form factors should be computed at many more different values of q^2's. Besides, the analysis of kaon decays benefits from information provided by chiral perturbation theory, while this is not the case with the charm decays. Although the level of accuracy in the charm sector on the lattice is not comparable to what has been achieved with kaons, there is room for substantial improvement and the overall errors can be reduced to a 5% level. In addition, penguin induced charm decays can also be studied on the lattice. Although strongly suppressed by the GIM mechanism their potential observation could help in discerning among various scenarios of physics beyond Standard Model. For that task a few more form factors should be computed on the lattice. Several such projects are underway.

For the weak interaction phenomenology, however, B-physics offers much more information but it is also more challenging for LQCD. One can either simulate a fictitious quark heavier than charm and lighter than b and then extrapolate the corresponding results to the physical b quark mass, or to use the effective field theory approaches on the lattice and match the obtained results with continuum QCD. Both ways are used in practice and lead to physics information that is obviously less accurate than those in the kaon or charm sectors. Reducing the current uncertainties within these approaches, nevertheless, would give valuable information relevant to new physics searches at LHCb and Super-B.

The prospects for high precision computation in the b quark sector were possible after the method of Ref. [10] was proposed and implemented in quenched approximation ($N_f = 0$). It starts by simulating directly the B-meson on the lattice with small volume, and then the finite volume effects are eliminated by successive simulations in larger volumes but for quarks lighter than the physical b-mass. The successive results scale much better with the heavy quark mass than the physical quantities themselves and after only a few enlarged volumes the finite volume effects become negligible. A drawback of the method is that it is numerically very costly. The implementation of this method in computing the $B \to D^{(*)}\ell\nu_\ell$ decay form factors was particularly successful because it showed that it was possible to find a range of momentum transfers that are directly accessible from both LQCD and the experiments, cf. Fig. 6. Therefore one can envisage extracting $|V_{cb}|$ from exclusive decays without relying on the heavy quark expansion (particularly dangerous for the charm quark). The method of Ref. [10] can in principle be used for studying other B-physics observables, such as the $B \to \pi\ell\nu_\ell$ form factors whose values are essential for the extraction of $|V_{ub}|$.

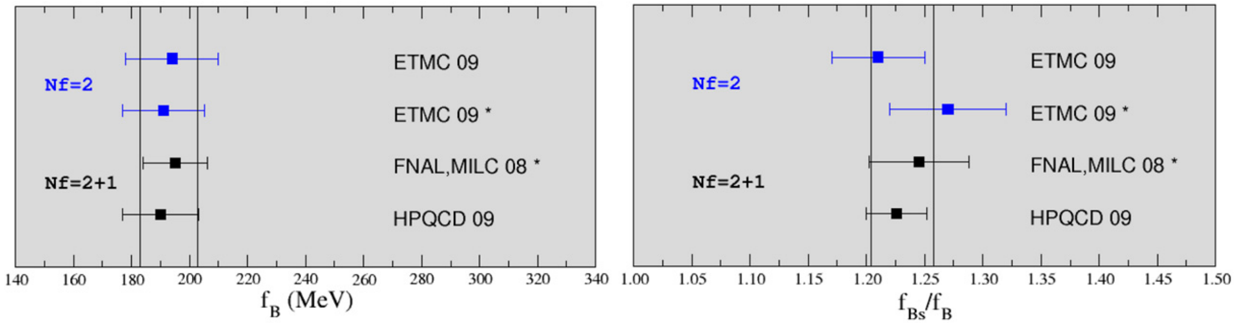

Fig. 7. Recent lattice results for the B meson decay constants and for the ratio f_{B_s}/f_B.

Flavor changing neutral currents (FCNCs) are particularly powerful way for studying the effects of physics beyond SM in the flavor sector. Within the SM, FCNCs are used to extract $|V_{td}|$ and $|V_{ts}|$ from the experimentally measured $B_{(s)}$ decays and mixing. In that respect, particularly important are the LQCD calculations of the $B^0_{(s)} - \bar{B}^0_{(s)}$ mixing amplitudes, both in and beyond the SM. In the most general case they involve a set of 5 bag parameters, in addition to the decay constants $f_{B_{(s)}}$. Those decay constants are also relevant to the leptonic decays $B \to \ell\nu$ and $B_{(s)} \to \ell^+\ell^-$, that are very sensitive to new physics contributions and are currently being explored at LHCb. They have been computed on the lattice for many years but only a few such computations are performed with dynamical light quarks and at several lattice spacings (see Fig. 7).

5. Summary

Modern lattice QCD simulations include the effects of light dynamical quarks and are performed at several lattice spacings to verify the smoothness of extrapolation to the continuum limit. A particular attention is devoted to exploring the light quark physics and probing the region ever closer to the physical u and d quarks (i.e. physical pions). Many studies of phenomenologically relevant quantities related to kaon physics made so far, showed an impressive agreement among various discretization schemes of QCD. These results already made a major impact on the flavor physics phenomenology and allowed for testing the CKM unitarity at the per-mil level.

Further exploration of flavor physics relies on a similar success with heavier quarks. The methodology to meet the precision comparable to that reached with kaons is available for the charm sector. Not as many such computations have been made so far, but quite a number of them are underway. Even though most of the methods used to simulate b-quarks on the lattice can provide results with more limited precision, the target accuracy of 5% for several hadronic matrix elements is very welcome for the new physics searches at LHCb. It should be emphasized that a method to perform high precision lattice measurement of B-physics quantities does exists but, as of now, it is computationally very costly for extensive unquenched studies.

The quantities we mentioned in this contribution are only emblematic for both flavor physics and lattice studies. Many more but similar quantities can be and are studied on the lattice. Improving their precision will certainly be helpful in confronting the experimentally measured rare decays at LHCb and especially at Super-B factories. Such studies are expected to provide indirect clues about physics beyond Standard Model, which are complementary to the direct searches at LHC.

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