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CP violation and the matter–antimatter asymmetry of the Universe

Violation de CP et asymétrie matière–antimatière de l'Univers

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ABSTRACT

In our everyday environment one observes only matter. That's quite a fortunate situation! Any sizeable presence of antimatter on Earth, from the enormous energy it would release through annihilation with matter, would prevent us talking about it! For the physicist this fact, at first sight obvious, is nevertheless a kind of surprise: antimatter, which is observed in cosmic rays, in radioactive decays of nuclei, which has been copiously produced and extensively studied in accelerators and which is nowadays currently used in hospitals, turns out to have pretty much the same properties as matter. Moreover, the fact that matter dominates appears to be a general property of our Universe: no evidence of large quantities of antimatter has been observed at any distance from us. Why would matter have taken the advantage on antimatter? In this short review we explain how, through a limited number of basic elements, one can find answers to this question. Matter and antimatter have, in fact, not exactly the same properties: from laboratory experiments CP conservation is known not to be a fundamental law of nature.

RÉSUMÉ

Dans notre vie de tous les jours nous n'appréhendons que de la matière et rien ne nous indique à première vue qu'il puisse exister de l'antimatière. Bienheureux sommes nous ! La présence d'une quelconque quantité macroscopique d'antimatière autour de nous, par l'énergie qu'elle dégagerait en s'annihilant avec la matière, nous empêcherait d'être là pour en parler ! Cet état de chose est cependant une surprise pour le physicien : l'antimatière qui est aujourd'hui bien connue — étant observée et utilisée tous les jours dans les rayons cosmiques, les processus radioactifs, les accélérateurs de particules et les hôpitaux — a des propriétés très similaires à celles de la matière. De plus le fait que la matière domine apparaît être une caractéristique générale de notre univers : aucune trace d'une grande quantité d'antimatière n'a été observée à quelque distance que ce soit. Pourquoi la matière aurait-elle donc pris le pas sur l'antimatière ? Dans ce Fascicule nous expliquons brièvement comment à partir d'un nombre restreint d'éléments de base l'on peut trouver des réponses à cette question, en commençant par un ingrédient crucial : il a été observé en laboratoire que la conservation de la symétrie CP n'est pas une loi de la nature.

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1. Introduction: evidence for the matter–antimatter asymmetry of the Universe

Let us first discuss what is the evidence for the baryon asymmetry of the Universe (BAU).

1.1. Solar system

Obviously there is no trace of any object composed of antimatter in the Solar system. Astronauts apparently did not annihilate incidentally on the moon! All observed meteorites, coming from the Solar system and beyond, do not produce a large flux of γ rays by annihilating on any matter object, etc.

1.2. Antimatter cosmic ray flux: galaxy and cluster of galaxy evidence

The Earth constantly intercepts particles moving freely in the interstellar space. The observed flux of antiparticles, for example of antiprotons $\bar{p}$ and positrons e^+, compared to the particle one, p, e^-,

$$\bar{p}/p \sim 10^{-4} \quad e^+/e^- \sim 10^{-2}\text{–}10^{-1} \tag{1}$$

turns out to be compatible with expectations from production of antiparticles in the Milky Way. Similarly no trace of any γ ray flux, caused by a macroscopic amount of matter–antimatter annihilation, has been observed from our cluster of galaxies, or from galaxy–antigalaxy annihilation further away. As a result there is no trace of an "ambient" population of antimatter particles at any distance from us.

1.3. Annihilation catastrophe

A very strong argument for the matter–antimatter asymmetry is the following. If one goes back in time to an epoch short after the Big Bang, when particles were so compacted that the temperature of the Universe thermal bath was huge, of order the nucleon mass, $T \sim m_N \sim 1$ GeV $\simeq 10^{12}$ K, the density of baryons and antibaryons per unit volume was huge ($n_N \sim n_{\bar{N}} \propto T^3$). Consequently, given the fact that the nucleon–antinucleon annihilation cross section (to particles without baryon number such as pions) is large, $\sigma_{N\bar{N}} \sim \text{GeV}^{-2}$, it is easy to see that this process was in thermal equilibrium at this epoch. This means that N and $\bar{N}$ were constantly annihilating and, through the inverse annihilation process, pair created. For T below m_N the rate of pair creation is nevertheless lower than the rate of annihilation: at these temperatures the number of particles having enough energy to create a nucleon pair is Boltzmann suppressed by a factor $e^{-m_N/T}$. In particular the particles created from a pair annihilation have enough energy to create back a nucleon pair, but soon after they have been produced (i.e. before they have time to encounter another energetic particle to recreate a pair) they loose energy from scattering with the medium (i.e. with less energetic γ, e^-, ν, etc.). As a result, for $T < m_N$, as long as the annihilation process is in thermal equilibrium, the number of baryons (and similarly antibaryons) becomes more and more exponentially suppressed (following the equilibrium Maxwell–Boltzmann number density, $n_N^{Eq} = n_{\bar{N}}^{Eq} \propto e^{-m_N/T}$). Ultimately at a lower $T = T_f$ there are too few N and $\bar{N}$ and the annihilation process stops, i.e. "freezes out", leaving constant N and $\bar{N}$ populations afterwards. These left-over populations are equal to the equilibrium population just before the freeze out. As a result, since T_f turns out to be as low as ~ 22 MeV (due to the fact that $\sigma_{N\bar{N}}$ is large and therefore remains active for a long period), the left number density of baryons, normalized to the photons number density, is tiny: $n_b/n_\gamma = n_{\bar{b}}/n_\gamma \sim e^{-m_N/22\,\text{MeV}} \sim 10^{-18}$. This is many orders of magnitudes smaller than the observed number density of baryons, $\eta \equiv n_B/n_\gamma \equiv (n_b - n_{\bar{b}})/n_\gamma \simeq n_b/n_\gamma \simeq 5 \times 10^{-10}$, see below. To avoid this annihilation catastrophe one therefore concludes that prior to it, baryon and antibaryon had to be present in *unequal numbers*: since the annihilation process conserves the total baryon number, it erases very efficiently the symmetric component but cannot erase any asymmetric component. In other words the fact that we see only baryons around us means that there were more baryons than antibaryons prior to the annihilation catastrophe.

1.4. Inflation

In order to explain that the Universe is so isotropic and homogeneous on large scales, but has so much structure on smaller scales, and the fact that it has a density so close to the critical density (i.e. has no curvature), it is highly probable that, shortly after the Big Bang, the Universe has known a period of exponential growth. If true, this means that any preexisting population of baryons (or quarks at $T \gtrsim 1$ GeV) and antibaryons (or antiquarks), has been exponentially diluted away leaving practically no relevant density at end of inflation, neither symmetric, nor asymmetric. Therefore the observed asymmetry cannot have been an "initial condition" at the Big Bang. Quarks and antiquarks, as well as an asymmetry between them, have been created dynamically after inflation.

1.5. Primordial nucleosynthesis

To determine the baryon number density directly by summing up the amounts of baryons observed in the Universe is not practical because large quantities of them could escape observations. More reliable is to determine it from the observations of the relative abundances of individual light nuclei, ^{3}He, ^{4}He, D, ^{6}Li and ^{7}Li. These observations can be impressively well explained from primordial nucleosynthesis, i.e. from the formation in the Universe thermal bath, of nuclei out of individual nucleons, at a temperature of order the nuclear energy scale, $T \sim$ MeV. These abundances crucially depend on the value of η, for example because photons at these energies can dissociate nuclei. The most reliable determination appears to be from the D/H abundance ratio which gives (for a review see Ref. [1])

$$5.1 \times 10^{-10} < \eta < 6.5 \times 10^{-10} \quad (95\% \, CL) \tag{2}$$

consistently with the ^{4}He abundance (but somewhat inconsistently with the controversial ^{7}Li one).

1.6. Cosmic Microwave anisotropies

Another way to determine η has been made possible recently: from the observation of the anisotropies of the Cosmic Microwave Background light, originating from when the Universe became transparent to photons (when the nuclei and electron combined into electrically neutral atoms at T of order the atomic binding energies $\sim$ eV). The observation of the relative size of the CMB acoustic Doppler peaks gives the rather precise value (for more details see e.g. [2])

$$5.8 \times 10^{-10} < \eta < 6.4 \times 10^{-10} \quad (95\% \, CL) \tag{3}$$

It is remarkable that the BBN and CMB values are in so good agreement. They involve physics at completely different epochs (MeV and eV energy scales respectively) and, moreover, are based on observations at completely different scales. This in turn provides a strong argument for an approximately constant number of baryons since the nucleosynthesis epoch. Note that the baryon number density is often parametrized by $Y_B \equiv n_B/s$ with s the entropy density. This is more convenient because, unless there is an unexpected production of entropy, Y_B does not change after the creation of the baryon asymmetry, unlike η which changes each time n_γ changes, when a particle decouples from the thermal bath. Today $Y_B = \eta/7.03$ so that Eq. (3) corresponds to $8.2 \times 10^{-11} < Y_B < 9.1 \times 10^{-11}$. This translates also into a baryon energy density relative to the critical (i.e. total) Universe density, Ω_B, within the range $4.3\% < \Omega_B < 4.9\%$.

We now can discuss the 3 Sakharov conditions [3] which must be fulfilled to create such an asymmetry: B violation, C and CP violation and creation of the asymmetry out of thermal equilibrium. To this end we will first discuss the GUT and leptogenesis scenarios where the discussion of each condition "factorizes", i.e. they can be considered separately. The discussion of the electroweak baryogenesis scenario, where the interplay of these conditions is more intricate, will come afterwards.

2. B and L violation

To create a B asymmetry out of a symmetric situation, interactions which do not conserve $n_B = n_b - n_{\bar{b}}$ must obviously be operative. This is the first well-known condition for generation of the BAU, formulated by A. Sakharov in 1967. Nowadays this condition has taken a slightly different and more precise formulation: if, as in most models, the BAU is created at temperature above the electroweak scale, $v_{EW} \sim m_W \sim 100$ GeV, the interactions required must not conserve $B - L$. If instead it is created during the electroweak phase transition or after, it is sufficient that they break B no matter how L is violated. This stems from the fact that in the Standard Model (SM) the $B + L$ triangle anomaly spoils the conservation of the left-handed $B + L$ current

$$\partial_\mu \left(J_L^{B\mu} + J_L^{L\mu}\right) = \frac{3g^2}{32\pi^2} \varepsilon_{\alpha\beta\delta\gamma} W_a^{\alpha\beta} W_a^{\delta\gamma} \tag{4}$$

where $W_a^{\alpha\beta}$ $(a = 1, 2, 3)$ are the $SU(2)_L$ gauge fields, g is the weak coupling and $J_L^{B\mu} = \sum_q \bar{\psi}_{qL} \gamma^\mu \psi_{qL}$ ($J_L^{L\mu} = \sum_l \bar{\psi}_{lL} \gamma^\mu \psi_{lL}$) involves a sum over all quark flavors and colors (over all charged lepton and neutrino flavors). Therefore $B + L$ is violated in the SM, but not $B - L$, because the B and L anomalous diagrams cancel out. This results in a non-trivial vacuum structure for the SM gauge field configurations which consists in an infinite series of degenerate vacuum with different $B + L$ number separated by a barrier, E_{sphal}, whose height is set by the electroweak interaction and scale, $E_{sphal} \sim 8\pi v/g$ (with $v =$ 246 GeV the Higgs boson vacuum expectation value and g the weak coupling). As a result at vanishing temperatures (as in colliders and the present Universe) there exist $B + L$ violating processes, but their rates are extremely suppressed by the tunneling effect [4]. However at temperature of order the electroweak scale there is no more suppression as the barrier can be crossed over [5]. The relevant processes are the so-called sphalerons [6] which involve one lepton of each generation and one quark of each generation and each color, i.e. $\Delta B = \Delta L = 3$. In the Universe thermal bath they turn out to be in thermal equilibrium for a temperature above ~ 135 GeV (~ 150 GeV) for a Higgs boson mass equal to 120 GeV (150 GeV) [7]. Consequently any $B + L$ asymmetry created above this temperature is basically washed away: if for example there is a

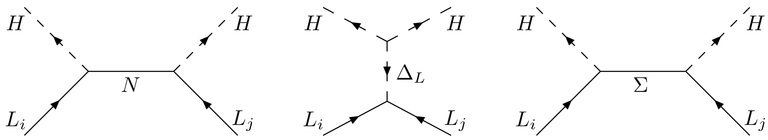

Fig. 1. The 3 basic seesaw diagrams which can induce naturally small neutrino masses.

positive $B+L$ asymmetry there will be statistically more sphaleron processes decreasing $B+L$ than increasing it, until there is no more asymmetry. Therefore the creation of a B asymmetry above the electroweak scale requires $B-L$ violation. From the equalities $(B-L)_{fin}=(B-L)_{in}$ and $(B+L)_{fin}\simeq 0$, the final asymmetry is $B_{fin}\simeq(B-L)_{in}/2$. Note that a pure B or pure L asymmetry results, after sphaleron reprocessing, in a non-vanishing B asymmetry: it gives $B_{fin}\simeq B_{in}/2$ and $B_{fin}\simeq -L_{in}/2$ respectively.

The fact that sphalerons break $B+L$ rather than just B has actually a dramatic consequence for the first real baryogenesis scenario which has been proposed.

2.1. GUT baryogenesis

In most of the grand-unified theories (GUT) baryon number is violated in interactions of fermions with heavy GUT scale gauge and Higgs bosons. It is therefore not a surprise that GUT baryogenesis has been proposed already in 1978 [8], not long after these theories have been proposed. Along this scenario, at a temperature above their mass, the heavy Higgs and/or gauge bosons were present in large quantities, eventually in thermal equilibrium with the thermal bath. But once the temperature has dropped below their mass they have progressively disappeared by decaying, in particular to fermions. Since these decays are driven in part by the B-violating interactions, they could in principle produce a B asymmetry. However it has been realized subsequently, with the discovery of the sphalerons, that these models as originally proposed, based in particular on $SU(5)$, cannot produce the BAU. The reason is that not only these scenarios break B but also L, along the $B+L$ direction. Therefore any produced asymmetry which could have been created, quickly will be washout by the sphalerons in thermal equilibrium. As a result we will not discuss anymore this possibility here.[1] Within this decay scenario, the most simple and motivated possibility which avoids this problem is leptogenesis [10]. This scenario can also naturally take place in the framework of GUT theories, in particular in the framework of $SO(10)$ GUT's.

2.2. Leptogenesis

The fact that the neutrinos have tiny but non-vanishing masses, around 10^{-3}–10^{-1} eV, has been discovered in 1998 and on, see the contribution of P. Hernandez in this issue, [9]. The most favorite mechanism to explain these neutrino masses, and the fact that they are tiny, is the seesaw. Along this mechanism the neutrino masses are generated from the tree level exchange of a heavy state between 2 lepton doublets and 2 Higgs doublets, generating an effective interaction $L_{eff}=c_{jk}L_jL_kH^*H^*/\Lambda$ with $L_j=(\nu_{jL},\, l_{jL})^T$, $H=(H^0,H^-)^T$ and c_{jk} numerical coefficients which come from the values of the various coupling constants involved in this exchange diagram. This $SU(2)_L$ gauge invariant interaction contains a component $\nu_{Lj}\nu_{Lk}H^0H^0/\Lambda$ which after electroweak symmetry breaking, replacing H by $v/\sqrt{2}$, gives a Majorana neutrino mass term $\sim c_{jk}\nu_{Lj}\nu_{Lk}v^2/2\Lambda$. The resulting neutrino masses $m_{\nu jk}\sim c_{jk}v^2/2\Lambda$ are naturally tiny if the Λ scale, related to the mass of the particle exchanged, is large. Given the fact that both L and H have hypercharge unity and are doublets of $SU(2)_L$, and since $2\times 2=3+1$ under $SU(2)_L$, there are only 3 types of particles which can be exchanged, as shown in Fig. 1: a right-handed neutrino N (that is to say a hyperchargeless right-handed fermion, singlet of $SU(2)_L$), a scalar triplet with hypercharge 2, Δ_L, or a hyperchargeless triplet of fermion, Σ. These correspond to the so-called type-I, II, III seesaw models. The Majorana neutrino masses induced in this way transform a neutrino ν_k into an antineutrino $\bar{\nu}_j$. These models have therefore an essential property for the generation of a baryon asymmetry: it breaks lepton number by 2 units. In the type-I model (and similarly in type-III) the source of L violation comes from the coexistence in the Lagrangian of the Yukawa interactions, Y_{Nij}, and Majorana masses of the N's, M_{N_i},

$$L=L_{\rm SM}+\bar{N}_i i\partial\!\!\!/ N_i+\left(Y_{Nij}\,H^\dagger\bar{N}_iL_j+\frac{M_{N_i}}{2}N_iN_i+\text{h.c.}\right) \tag{5}$$

On the one hand, to conserve L the Yukawa interactions require that the N_i's have lepton number unity, since the lepton doublet has $L=1$ and the Higgs doublet has vanishing L. On the other hand, the Majorana masses require instead that N_i has a vanishing L number. As a result the $\Delta L=2$ neutrino mass matrix involves both types of terms, $m_{\nu ij}=(Y_N^TM_N^{-1}Y_N)_{ij}v^2/2$. Similarly in the type-II model L is broken by the fact that a scalar triplet can, as in Fig. 1, couple to 2 lepton doublets and 2 Higgs doublets which have $L=2$ and $L=0$, respectively.

[1] This possibility suffers also from the fact that, since it typically occurs at the GUT scale, it requires a very high inflation reheating temperature above this scale, which can lead to various conflicts, for instance overproduction of particles as monopoles or gravitinos in the supersymmetric context.

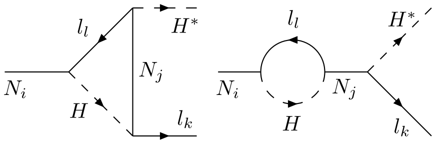

Fig. 2. One-loop diagrams contributing to the CP asymmetry from the N_i decay.

The seesaw models not only provide a source of L violation well motivated by the observation of the neutrino masses, but if one let operate these interactions at the origin of these neutrino masses in the thermal bath of the Universe, it is expected that they do create a lepton asymmetry, hence a baryon asymmetry. The L to B asymmetry conversion comes from the fact that, unlike original GUT scenarios above, the seesaw interactions violate only L, not B, hence create a pure L asymmetry, hence a $B_{fin} \simeq -L_{in}/2$ asymmetry after the sphaleron thermalization, see above. The way the L asymmetry is produced is from the decay of the heavy seesaw states. For instance at a temperature of order their mass or below, a N_i or a Σ_i will decay both to $L_j H^*$ and to $\bar{L}_j H$, producing an asymmetry if both decay rates are unequal [10]. Similarly, a scalar triplet can decay to LL or HH whereas an antitriplet can decay to $\bar{L}\bar{L}$ or H^*H^*, also producing an asymmetry if both rates to LL and $\bar{L}\bar{L}$ are not same. This is where the crucial CP-violation issue arises.

3. C and CP violation

To have a source of $B-L$ violation, as in the leptogenesis mechanism, or of $B+L$ violation, as in electroweak baryogenesis below, is not enough to effectively produce the BAU. As anticipated above in order that this source can manifest itself, it is also necessary that the rates of production of the particles and antiparticles are unequal. This requires that both C and CP are not conserved. If C is conserved $\Gamma(i \to f) = \Gamma(\bar{i} \to \bar{f})$, and the production rate of particles f is compensated by the corresponding one of its conjugated states $\bar{f}$. Similarly if CP is conserved $\Gamma(i \to f) = \Gamma(\bar{i}_P \to \bar{f}_P)$, the production rate of particles f is compensated by the corresponding one of its mirror conjugated states $\bar{f}_P$ ($\bar{f}_P = P(\bar{f})$). In this case, for example, the rate of production of left-handed antineutrinos ν_L in any process would be compensated by the one of the CP-conjugate antineutrinos $\overline{\nu_L}$ ($CP(\nu_L) \sim \overline{\nu_L}$ which is a right-handed state), even though C could be maximally broken (as in the SM where there are no left-handed antineutrinos, $C(\nu_L) = 0$, remember that in a single spinor field appear the creation or destruction operators of a particle and of its CP-conjugate state, not of its C-conjugate state, see XXXXX contributions of J.-M. Frère [11], A. Stocchi [12], L. Silvestrini and M. Ciuchini [13], and C. Jarlskog [14] in this issue and Ref. [15]).

We now explain how C and CP violation manifest themselves in the leptogenesis scenario considered above. The various possible origins of the neutrino masses in Fig. 1 all involve Yukawa interactions. As in the SM, these Yukawa interactions provide CP-violating phases. For instance in the type-I seesaw framework with 3 right-handed neutrinos there are 6 CP-violating phases in the Lagrangian of Eq. (5), see the contribution of P. Hernandez in this issue. This is more than enough to induce a difference between the $N \to LH^*$ and $N \to \bar{L}H$ partial rates. What quantifies the amount of CP-violation is the CP asymmetry which is nothing but the average ΔL produced each time a right-handed neutrino decays

$$\varepsilon_{N_1} \equiv \frac{\Gamma(N_1 \to LH^*) - \Gamma(N_1 \to \bar{L}H)}{\Gamma(N_1 \to LH^*) + \Gamma(N_1 \to \bar{L}H)} \tag{6}$$

In the following we will consider for simplicity only the asymmetry of the lightest right-handed neutrino N_1. For a given CP asymmetry ε_{N_1} one therefore expects a lepton number asymmetry

$$Y_L \equiv \frac{n_l - n_{\bar{l}}}{s} = \varepsilon_{N_1} \frac{n_N}{s}\bigg|_{T \gg M_{N_1}} \tag{7}$$

where $n_{N_i}|_{T \gg M_{N_1}}$ is the number of right-handed neutrinos there are before they start to decay. At tree level, even if the Yukawa couplings are complex, CP violation cannot manifest itself because all Yukawa couplings appear to be multiplied by their complex conjugated:

$$\Gamma\big(N_1 \to LH^*\big) = \Gamma(N_1 \to \bar{L}H) = \frac{|Y_N Y_N^\dagger|_{jj}}{16\pi} M_{N_1} \tag{8}$$

However at one loop this is not the case anymore and there are 2 diagrams contributing to the CP asymmetry, given in Fig. 2. The interference of the corresponding one-loop amplitude with the tree level amplitude gives the highest order contribution to the CP asymmetry [16]

$$\varepsilon_{N_1} = -\sum_{j=2,3} \frac{3}{2} \frac{M_{N_1}}{M_{N_j}} \frac{\Gamma_{N_j}}{M_{N_j}} I_j \frac{2S_j + V_j}{3} \tag{9}$$

where

$$I_j = \frac{\mathrm{Im}[(Y_N Y_N^\dagger)^2_{1j}]}{|Y_N Y_N^\dagger|_{11}|Y_N Y_N^\dagger|_{jj}}, \qquad \frac{\Gamma^{Tot}_{N_j}}{M_{N_j}} = \frac{|Y_N Y_N^\dagger|_{jj}}{8\pi} \tag{10}$$

$$S_j = \frac{M^2_{N_j}\Delta M^2_{1j}}{(\Delta M^2_{1j})^2 + M^2_{N_1}\Gamma^2_{N_j}}, \qquad V_j = 2\frac{M^2_{N_j}}{M^2_{N_1}}\left[\left(1+\frac{M^2_{N_j}}{M^2_{N_1}}\right)\log\left(1+\frac{M^2_{N_1}}{M^2_{N_j}}\right)-1\right] \tag{11}$$

with $\Delta M^2_{ij} = M^2_{N_j} - M^2_{N_i}$. The I_j factors are the CP-violating coupling combinations entering in the asymmetry for each virtual N_j. They involve a combination of 4 Yukawa couplings, this is the minimum number of couplings we need to get a complex phase invariant quantity,[2] similarly to the Jarlskog invariant in the SM. In order to make the squared amplitude real this imaginary part necessarily must be multiplied by another imaginary part which necessarily must come from the loop integrals. From the optical theorem this "absorptive" term comes from the part of the loop integral where both L and H inside the loop are on-shell. This gives the S_j self-energy and V_j vertex functions. Note that 2 different N are required, the decaying N_1, and a virtual $N_j = N_{2,3}$, otherwise I_j is real. Moreover these 2 N must have a different mass, otherwise the CP asymmetry vanishes.[3]

Since the Yukawa and Majorana terms contribute also to the neutrino masses, there exist neutrino mass constraints on the size of the CP asymmetry. Of course, so far, no CP violation has been observed in the lepton sector, i.e. none of the 3 CP-violating phases in the neutrino mass matrix has been constrained. Moreover one can show that the CP asymmetry turns out to generically depend, not on these 3 "low-energy" phases, but on the 3 "high-energy" phases which decouple from the neutrino mass matrix. Furthermore the CP asymmetry depends on other real parameters which cannot be determined from the knowledge of the neutrino mass matrix, such as the right-handed neutrino masses. But yet there exist neutrino mass constraints on the size of the CP asymmetry in the form of upper bounds. In particular if the spectrum of right-handed neutrinos is highly hierarchical $m_{N_{2,3}} \gg m_{N_1}$ it can be shown that, varying all these unknown parameters in their full possible range, the CP asymmetry is bounded by an expression which depends only on the neutrino masses and M_{N_1} [17]

$$|\varepsilon_{N_1}| \leqslant \frac{3}{16\pi}\frac{M_{N_1}}{v^2}(m_{\nu_3} - m_{\nu_1}) = \frac{3}{16\pi}\frac{M_{N_1}}{v^2}\frac{\Delta m^2_{\mathrm{atm}}}{m_{\nu_3}+m_{\nu_1}} \tag{12}$$

with m_{ν_3} and m_{ν_1} the heaviest and lightest neutrino mass, and experimentally $\Delta m^2_{\mathrm{atm}} \equiv m^2_{\nu_3} - m^2_{\nu_1} \simeq 2.4 \times 10^{-3}$ eV^2. Applying this bound to Eq. (7) and requiring the Y_B asymmetry to be within the observed range, Eq. (3), gives the lower bound

$$M_{N_1} > 5 \times 10^8 \text{ GeV} \tag{13}$$

In other words the bound on the CP-violating asymmetry becomes a bound on the mass of N_1. Its value is quite interesting: it is perfectly in agreement with the mass we typically need to obtain neutrino masses of order the ones observed, i.e. to have a large enough seesaw suppression to explain that the neutrino masses are so tiny. This is the first strong "circumstantial evidence for leptogenesis".

Note that the type-I seesaw model is not the only one in which the BAU can be successfully generated. In the type-III model the way the asymmetry is produced is similar [18]. The diagrams responsible for the CP asymmetry are the same except that the right-handed neutrinos are now replaced by triplets. There are 6 CP-violating phases here too, interplaying in the same way. There is nevertheless an important difference which concerns the third Sakharov condition, see below.

The situation of the type-II seesaw model is somewhat different. With a single scalar triplet (sufficient to fulfill all neutrino mass constraints unlike in the other seesaw models where several heavy states are necessary) all ingredients are there. There are in full generality 3 CP-violating phases in the Yukawa couplings of Fig. 1, matching the 3 CP phases of the neutrino mass matrix (no high-energy decoupling phases here). The average ΔL produced each time a triplet and an antitriplet decay is given by:

$$\varepsilon_\Delta = 2\cdot\frac{\Gamma(\Delta_L^* \to l + l) - \Gamma(\Delta_L \to \bar{l} + \bar{l})}{\Gamma_{\Delta_L^*} + \Gamma_{\Delta_L}} \tag{14}$$

However this CP asymmetry vanishes. There are no vertex diagrams as in Fig. 2 to induce a non-vanishing CP asymmetry because the triplet is not self-conjugated as the N. There is a self-energy diagram, see Fig. 3, but in this diagram all CP-violating phase dependence obviously vanishes because the real and virtual triplet are the same. This shows that to have source of CP violation, even large, is not necessarily sufficient to have leptogenesis operative. However this does not mean that leptogenesis could not be successful from the decay of a triplet. It could be successful easily if there are other heavy states. For instance with another triplet [19], the self-energy diagram of Fig. 3 is operative. Or if there are also right-handed

[2] Phase invariant if we do not rephase the N's, which we do not do because they are Majorana particles and one wants to keep their masses real for the sake of the Feynman amplitude calculation. Otherwise the CP invariant takes a more complicated form.

[3] That would allow one to make an extra 2 by 2 orthogonal transformation on the 2 degenerate N, leaving the N mass matrix invariant, to eliminate the relevant CP-phase in $Y_N Y_N^\dagger$, Eq. (9).

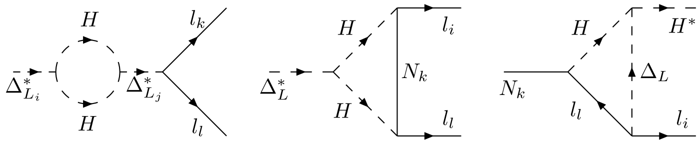

Fig. 3. Diagram contributing to the asymmetry created by the decay of a Δ_L if there are several scalar triplets or an N. The third diagram also contributes to the asymmetry of an N in presence of a heavier scalar triplet.

neutrinos, a vertex diagram with a virtual N in the loop, see Fig. 3, could do the job, see Ref. [20]. To have both a scalar triplet and N's is a particularly natural situation because it is what one has in left–right symmetric model or renormalizable $SO(10)$ GUT.

4. Out-of-equilibrium condition

To produce the BAU the last condition one must fulfill is the following: the CP-violating processes at the origin of the lepton asymmetry as well as all other processes violating $B-L$, must not be too much in thermal equilibrium when the asymmetry is produced, at the temperature of order the mass of the decaying particle. If they are in thermal equilibrium the lepton asymmetry produced, Eq. (7), is suppressed by the extra "efficiency" factor "η" [21]:

$$Y_L \equiv \frac{n_l - n_{\bar{l}}}{s} = \varepsilon_{N_1} \frac{n_N}{s}\bigg|_{T \gg M_{N_1}} \eta \tag{15}$$

To calculate η it is necessary to consider the Boltzmann equations which give the number of N and the lepton asymmetry as a function of the temperature T, or equivalently as a function of $z \equiv M_{N_1}/T$,

$$\frac{s}{z}\frac{\mathrm{d}Y_{N_1}}{\mathrm{d}z} = \left(Y^{Eq}_{N_1} - Y_{N_1}\right) \cdot \frac{\gamma_D}{H(T=M_N) Y^{Eq}_{N_1}} \tag{16}$$

$$\frac{s}{z}\frac{\mathrm{d}Y_L}{\mathrm{d}z} = \varepsilon_{N_1}\left(\frac{Y_{N_1}}{Y^{Eq}_{N_1}} - 1\right) \cdot \frac{\gamma_D}{H(T=M_{N_1})} - \frac{Y_L}{2Y^{Eq}_l} \cdot \frac{\gamma_D}{H(T=M_N)} - 2\frac{Y_L}{Y^{Eq}_l} \cdot \frac{\gamma_{\Delta L=2}}{H(T=M_{N_1})} \tag{17}$$

The reaction density $\gamma_D \propto \Gamma^{Tot}_{N_1} Y^{Eq}_{N_1}$ parametrizes the number of decay/inverse decay occurring per unit time and unit volume. Similarly $\gamma_{\Delta L=2}$ comes from the N mediated $LH^* \leftrightarrow \bar{L}H$ scatterings which can also occur in the thermal bath and are $\Delta L = 2$. $Y^{Eq}_i \equiv n^{Eq}_i/s$ is the thermal equilibrium number density of a particle "i". $H(T)$ is the Universe Hubble expansion rate $H(T) = \sqrt{4\pi^3 g_*/45}\,T^2/M_{\mathrm{Planck}}$ with g_* the number of relativistic degrees of freedom in the thermal bath.

The interpretation of the Boltzmann equations above is the following. The second (first) term in Eq. (16) stems from the fact that each times an N_1 decays (inverse decays) its number decreases (increases) by one unity. The number of decays is proportional to the number of N_1, Y_{N_1}. The number of inverse decays is proportional to the number of L and H which being in thermal equilibrium can be expressed by the thermal equilibrium number density $Y^{EQ}_{N_1}$. Similarly in Eq. (17) each time an N decays (inverse decays) on average a lepton number $\Delta L = \varepsilon_{N_1}$ ($\Delta L = -\varepsilon_{N_1}$) is created. This gives rises to the lepton asymmetry "source" term proportional to ε_{N_1}. On top of the source term in Eq. (17) there are terms which are independent of the CP asymmetry. They reflect the fact that if, for example, there are more L than $\bar{L}$, i.e. $Y_L > 0$, there will be statistically more $LH \to N_1$ inverse decays occurring than $\bar{L}H^* \to N_1$ inverse decays (and similarly for the scattering term). Therefore these terms tend to "washout" the asymmetry by statistically equilibrating it to 0. This inverse decay washout term is at the origin of an efficiency suppression if inverse decays are effective, that is to say if the decay process is in thermal equilibrium. As can be seen in the Boltzmann equation the coefficient of this washout term is proportional to $K_{N_1} \equiv \Gamma_{N_1}/H(T=M_{N_1})$. Therefore the out-of-equilibrium condition to have no suppression of the efficiency from the inverse decays is:

$$\Gamma_{N_1} \lesssim H(T=M_N) \tag{18}$$

This can be understood in the following way: if the Hubble expansion rate is larger than the decay rate, the L and H^* in the thermal bath will not have the opportunity to encounter another H^* and L to form an N, and no inverse decays will occur, hence no washout will be induced. This condition leads to an interesting connection with the neutrino masses. Up to a multiplicative constant, K_{N_1} is given by $\sum_i |Y_{N1i}|^2/M_{N_1}$. This turns out to have a form similar to the neutrino mass matrix induced by N_1, $m^{N_1}_{\nu ij} = \sum_i Y_{N1i} Y_{N_{1j}} v^2/2M_{N_1}$. Consequently, given the general inequality $|Y_N Y^\dagger_N|^2_{jj} v^2/2M_{N_j} \geqslant m_{\nu_1}$, one obtains

$$K_{N_1} \geqslant m_{\nu_1}/10^{-3}\ \mathrm{eV} \tag{19}$$

which is independent of m_{N_1}. The 10^{-3} eV scale has nothing to do with the neutrino masses, it is a function of the electroweak and Planck scales, i.e. $10^{-3}\ \mathrm{eV} \simeq 16\pi^2 v^2 \sqrt{g_*\pi/45}/M_{\mathrm{Planck}}$. It is remarkable that this scale is of the order the neutrino masses. It means that, for a temperature of order their mass, the N_i decay naturally out-of-equilibrium or only slightly in equilibrium, so that a large efficiency is expected. For example if the neutrino masses had been observed not around the 0.1 eV scale but around the keV or MeV scale, K_{N_1} would have been bounded from below by a factor 10^6 or 10^9, in case the washout would have been huge and leptogenesis impossible. This constitutes the second strong "circumstantial evidence" for leptogenesis. Note that if m_{ν_1}, which is still not known experimentally, is smaller than 10^{-3} eV, one can consider situation where there is no washout at all, so that Eq. (13) holds.

In the type-III seesaw case the thermalization issue is similar except for one important difference. Since the triplets, unlike singlets, have $SU(2)_L$ gauge interactions, there is an extra thermalization term in the Boltzmann equation for the number of decaying triplets, from $\Sigma\bar{\Sigma} \to AA, q\bar{q}$ where $A = W^{\pm}, Z^0, \gamma$. These scatterings suppress the efficiency by putting Y_Σ closer to Y_Σ^{Eq}, without inducing any source term in the Y_L Boltzmann equation. This suppression can be large if $T \lesssim 10^{12}$ GeV. As a result the leptogenesis lower bound on a fermion triplet is larger than for a singlet, Eq. (13):

$$M_{\Sigma_1} > 1.5 \times 10^{10}\ \text{GeV} \tag{20}$$

These gauge scattering thermalizing terms also exist in the case of a decaying scalar triplet, so that the lower bound is also higher in this case:

$$M_{\Delta_L} > 2.8 \times 10^{10}\ \text{GeV} \tag{21}$$

5. Electroweak baryogenesis

Instead of invoking a source of B or L violation beyond the Standard Model, as we do above, could the SM sphalerons produce directly an asymmetry? In other words, instead of erasing a previously produced $B + L$ asymmetry as in the GUT baryogenesis mechanism above, or instead of converting an L asymmetry into a B asymmetry, could these $B + L$ anomaly associated processes be the source of the baryon asymmetry? The answer is yes, through the electroweak baryogenesis mechanism (EWBG) [22] which occurs at the electroweak phase transition. This clearly has an advantage over the scenarios above: since it involves electroweak scale physics it is directly testable at colliders. In this section we give a short schematic explanation of its basic ingredients. For this mechanism it is convenient to consider the Sakharov condition in the reverse order.

5.1. Out-of-equilibrium condition

As long as the temperature is well above the electroweak scale, sphalerons are in thermal equilibrium and cannot create an asymmetry, see the discussion above. Similarly well after the phase transition the sphalerons rate is hugely exponentially suppressed and cannot do any relevant job. But during the phase transition an asymmetry could be created. This crucially depends on the order of the electroweak phase transition. If the transition is of second order (i.e. if the Higgs vev smoothly develop its value from 0 to v around the electroweak scale) no asymmetry can be created. In this case, as the temperature drops around the electroweak scale the sphalerons become less and less effective in erasing any $B + L$ asymmetry until they get out-of-equilibrium and have no more effects. However, if the phase transition is of first order, the situation is different. In this case it proceeds through tunneling between 2 vacua, i.e. via nucleation of bubbles inside (outside) which the electroweak symmetry is broken (unbroken). Bubble nucleation is obviously an out-of-equilibrium process, as there is no equivalent inverse process, allowing to satisfy the out-of-equilibrium condition when the sphalerons are still active in the vicinity of the bubble wall, especially towards the front of the bubble wall where electroweak symmetry is still unbroken.

5.2. Creation of a CP asymmetry

Let us consider a straightforward case [23] for EWBG: the 2 Higgs doublet model. In this model there is a source of CP violation directly in the scalar potential, in the term

$$L \ni -\lambda_5\big[\big(H_1^\dagger H_2\big)^2 + \text{h.c.}\big] \tag{22}$$

whose phase in the quartic coupling, $\lambda_5 = |\lambda_5| e^{i\phi}$, cannot be removed by a redefinition of the Higgs doublet fields without reintroducing it elsewhere. Determining the Higgs field configuration, as a function of the distance "z" to the wall, by minimizing the energy, leads to a "vacuum expectation value" of the form $H_{1,2}(z) = \frac{1}{\sqrt{2}} h_{1,2}(z) e^{i\theta_{1,2}}(z)$, which is complex. The modulus goes smoothly from 0 to v through the wall, while the phase $\theta_{1,2}$ goes from the value $-\phi$ at $z = \infty$ to another value inside the bubble. This space dependent vev leads to Yukawa induced space dependent complex quark masses, $m_q \propto Y_{q1} v_1 + Y_{q2} v_2$ (and similarly for leptons), whose phase cannot be removed by a space dependent rephasing of the left-handed and right-handed quark components (since it would not leave invariant the derivative terms in the Lagrangian, i.e. the quark kinetic terms). This CP-violating phase does not contribute in the same way to the Dirac equation of left-handed and right-handed fields. The Dirac equation reads

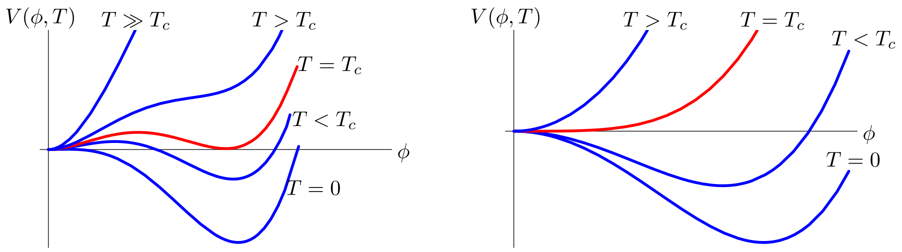

Fig. 4. Temperature dependence of a potential leading to first or second order electroweak phase transition respectively. Schematic illustration of the Higgs potential evolution with temperature for first (left) and second (right) order phase transition. The red curve is the potential at the critical temperature T_c.

$$(E - is\partial_z)q_L^s = mq_R^s$$
$$(E + is\partial_z)q_R^s = m^* q_L^s \tag{23}$$

where $s = \pm 1$ refers to the spin direction along the z axis. As a result around the wall the "wave function" solutions of these equations are not the same for left-handed and right-handed fields. In other words they propagate differently through the bubble wall: for an appropriate choice of the phase the $\overline{q_L}$ and q_R can reflect more on the wall than $\overline{q_R}$ and q_L, so that the former species are more abundant in the front of the wall than the later ones. Nevertheless, at this level there is still no creation of a B asymmetry since none of the interactions invoked so far (Higgs potential and quarks Yukawa couplings) break B.

5.3. Conversion of the CP asymmetry into a B asymmetry

Here comes the subtlety: sphalerons which are still deeply in thermal equilibrium outside the wall (where $v_{1,2} = 0$) are interactions which involve only the left-handed quarks but not the right-handed ones since the later do not couple to the W and Z. As a result they will tend to decrease the $\overline{q_L}$–q_L asymmetry, leaving the $\overline{q_R}$–q_R one unchanged. This creates the baryon asymmetry. If such an asymmetry remains outside the wall this asymmetry will nevertheless be erased because there are various interactions which can change the chirality of the fermions (Yukawa interactions and non-perturbative strong $SU(3)$ sphalerons). Fortunately as the bubble expand quickly part of this asymmetry will cross the wall before it can undergo any interaction of this kind. The last issue is then: can the B asymmetry be erased inside the bubble by the sphalerons? The answer is no if the phase transition is sufficiently of first order, that is to say if the transition occurs at a critical temperature T_c (Fig. 4) (when tunneling can begin to operate, both Higgs potential minima having the same energy) which is already below the sphaleron potential barrier, E_{sphal}. In such a case, as for any tunneling process, the sphalerons rate is suppressed by an $e^{-E_{sphal}/T}$ exponential factor. In practice, since $E_{sphal}/T \simeq \frac{8\pi}{g}\frac{v(T)}{T}$, see above, the condition is [24]

$$v_c/T_c > 1 \tag{24}$$

with $v_c \equiv \sqrt{v_{1c}^2 + v_{2c}^2}$, the value of the scalar vev at this temperature. The value unity in this equation stems from a comparison of the sphaleron rate with the Hubble rate. Note that this condition is meaningful only inside the bubble. Outside the bubble, the sphalerons have no barrier to cross since the scalar fields have no vev, so that they can act on the q_L–$\overline{q_L}$ asymmetry as described above.

5.4. EWBG in the Standard Model

In the SM CP violation comes from the CKM phase. At zero temperature this phase can be transferred from the quark masses to the charged current interaction. This however cannot be done in the vicinity of the wall where the Higgs vev and therefore the quark masses have a space dependent phase. They can therefore contribute to the electroweak baryogenesis mechanism as explained above. In order to have a first order phase transition one needs a bump in the Higgs scalar potential between 2 vacua. This requires a Higgs field trilinear term which can come only at the loop level in the effective potential. At this level the potential has the form

$$V_{eff} \sim m_H^2(T)H^2 - \rho(T)H^3 + \lambda H^4 \tag{25}$$

The cubic coefficient is $\rho(T) \simeq 3Tg^3/(8\sqrt{2}\pi)$ (neglecting the contributions of the hypercharge g' and Higgs quartic coupling λ). $m_H^2(T) = -\lambda v^2 + aT^2$ is the thermal Higgs boson mass with $a \simeq 9g^2/16 + y_t^2/4$ with y_t the top Yukawa coupling. Imposing to this potential to have 2 degenerate minima at a given temperature, defined as the critical temperature, one finds $v_c/T_c \simeq 3g^3/(16\pi\lambda)$. Eq. (24) gives then $\lambda < 3g^3/16\pi$, which translates in an upper bound on the Higgs boson mass

$$m_H < 40\ \text{GeV} \tag{26}$$

Given the LEP experimental lower bound $m_H^2 \gtrsim 114$ GeV, one concludes that in the Standard Model the electroweak phase transition does not proceed through bubble nucleation but occurs through a simple "crossover". Consequently, electroweak baryogenesis, as attractive as it is, does not work in the SM [22]. Moreover a proper calculation of the produced CP asymmetry shows that it is heavily suppressed by the GIM mechanism (i.e. essentially by the small SM Jarlskog CP invariant). Therefore even if the transition was sufficiently first order the baryon asymmetry produced would be many orders of magnitude too small. A first order phase transition can be achieved easily, for example by simply adding a scalar singlet to the SM. To enhance sufficiently the amount of CP violation requires more.

5.5. EWBG in the MSSM

In the minimal supersymmetric standard model the situation is different, EWBG can work [25] but is close to be excluded. First of all in the MSSM the phase transition can be of first order because there are extra contributions to the Higgs scalar potential which can enlarge the cubic term, so that Eq. (24) can be satisfied. The important contribution is from the "right-handed" stop which gives a contribution $(m^2_{\tilde{t}_R})^{3/2}$ to the Higgs effective potential. In the MSSM the stop mass receives essentially 3 contributions (neglecting $\tilde{t}_L$–$\tilde{t}_L$ mass mixing), $m^2_{\tilde{t}_R} = \tilde{m}_U^2 + y_t^2|H_2|^2 + cT^2$, the $\tilde{m}_U$ soft mass one, the usual Yukawa one and the thermal one with $c = g_s^2 + y_t^2(1+\sin^2\beta)$ a coefficient driven by the QCD and top Yukawa couplings. Therefore to get a large enough cubic term one needs a cancellation between the thermal mass term and the soft mass (which consequently must be negative). This negative soft mass leads automatically to a light right-handed stop: $m_{\tilde{t}_R} < 120$ GeV [26]. As a result the SM bound of Eq. (26) is relaxed to a larger value, $m_H < 127$ GeV [26], but still this value is not far above the LEP lower bound $m_H \gtrsim 114$ GeV. Furthermore since in the MSSM the Higgs mass is given by

$$m_H^2 \simeq m_Z^2 + c\frac{m_t^4}{v^2}\ln\left(m_{\tilde{t}_R} m_{\tilde{t}_L}/m_t^2\right)$$

this LEP bound requires a multi TeV left-handed stop mass, $m_{\tilde{t}_L} > 7$ TeV, hence a corresponding supersymmetric breaking scale. As for CP violation, in the MSSM there are new CP-violating phases which can be invoked. The main ones are the phase of the $\mu H_1 H_2$ term and of the gaugino soft mass $m_{\tilde{W}}$. These terms imply a different reflection on the bubble wall of gauginos and Higgsinos. However, since these particles have no L or B quantum numbers and, moreover, do not undergo sphaleron interactions, their effect is only indirect. The higgsino–gaugino CP asymmetry has first to be transfered into a quark CP asymmetry through higgsino–gaugino–(s)quarks interactions. Enough baryon asymmetry can be created only if $m_{\tilde{W}}$ is close to μ, and if the phase of μ is close to maximal. Such a large phase is expected to induce at one loop a too large electric dipole moment of the light quarks, unless the mass of the light generation squarks intervening in the loop are beyond the TeV scale (contrasting, but not in contradiction, with what we need for the third generation $\tilde{t}_R$ squark above). Similarly a too large electron EDM is generally induced (at 2 loops) unless the charged Higgs is also quite heavy. To sum up EWBG in the MSSM is not strictly speaking excluded but it would be miraculous that all these constraints are satisfied.

5.6. EWBG in the NMSSM

If one adds a scalar singlet S to the MSSM the situation is considerably eased [27], simply because in this model a Higgs cubic term is already induced directly at tree level from an interaction of the form SH_1H_2 in the superpotential. Second because, together with both Higgs doublets, it allows for extra CP-violating phases directly in the Higgs potential. EWBG is perfectly allowed in this framework, as in the 2 Higgs doublet model.

6. Conclusion

Thanks to CP violation our Universe can have developed a baryon asymmetry. The CP violation which could be responsible for this "vital" phenomenon can take a wide variety of forms beyond the Standard Model, depending on the mechanism invoked. The two most motivated ones appear to be leptogenesis and electroweak baryogenesis. The first one because it invokes CP violation in the lepton sector which is anyway expected to exist, in the Yukawa couplings responsible for the observed neutrino masses. The second one because, for what concerns baryon number violation, it relies on nothing else than the Standard Model. Leptogenesis can work very easily, comforted by a series of circumstantial evidences. The principal criticism we could do to it nevertheless is not theoretical but phenomenological: it is expected to work at a very high scale not directly probable experimentally. The electroweak baryogenesis mechanism can also work very well but not in the Standard Model and hardly in the MSSM. It typically requires the adjunction to the SM or MSSM of extra singlet(s) or doublet(s) scalar fields.

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