C. R. Physique 13 (2012) 176–179

Contents lists available at SciVerse ScienceDirect

Comptes Rendus Physique

www.sciencedirect.com

Flavour physics and CP violation / Physique de la saveur et violation de CP

The strong CP problem

Le problème de la conservation de CP par les interactions fortes

Pierre Sikivie

Department of Physics, University of Florida, Gainesville, FL 32611, USA

ARTICLE INFO

Article history:
Available online 30 December 2011

Keywords:
QCD
CP
Axion

Mots-clés :
QCD
CP
Axion

ABSTRACT

QCD depends on an extra parameter $\bar{\theta}$. When $\bar{\theta} \neq 0$, P and CP are violated. Because the strong interactions conserve P and CP, $\bar{\theta}$ must be very small, of order 10^{-10} or less. However, within the Standard Model, $\bar{\theta}$ is expected to be of order one. This puzzle is called the strong CP problem. Possible solutions are briefly described.

RÉSUMÉ

QCD dépend d'un paramètre supplémentaire $\bar{\theta}$. Quand $\bar{\theta} \neq 0$, les symétries P et CP sont brisées. Puisque les interactions fortes conservent P et CP, $\bar{\theta}$ doit être très petit, de l'ordre de 10^{-10} ou moins. Cependant, dans le Modèle Standard, $\bar{\theta}$ est en général de l'ordre de un. Ce puzzle est le problème de la conservation de CP par les interactions fortes. Des solutions possibles sont brièvement décrites.

The strong CP problem raised its head when it was discovered [1] that QCD, the theory of the strong interactions, depends not only on its gauge coupling g_s and quark masses m_j $(j = 1, \ldots, n)$ but also on an angle θ:

$$L_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{j=1}^{n} \left(\bar{q}_j i \gamma^\mu D_\mu q_j - m_j q^\dagger_{Lj} q_{Rj} - m^*_j q^\dagger_{Rj} q_{Lj} \right) + \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \tag{1}$$

The q_j are the quark fields, the $G^a_{\mu\nu}$ are the $\mathrm{SU_c}(3)$ gauge fields $(a = 1, \ldots, 8)$ and $\tilde{G}^{a\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta}$. The last term in Eq. (1) is a 4-divergence and hence only contributes a surface term to the QCD action. Surface terms do not contribute to the classical equations of motion and they do not contribute to the perturbative expansion of the quantum theory. However, they may produce non-perturbative quantum effects and such effects may be important in the strong coupling regime. In the case of QCD there are non-perturbative effects associated with the existence of color instantons [2]. The latter are Yang–Mills configurations describing quantum-mechanical tunneling between inequivalent vacua of QCD [1]. For our purposes, the two following points are the crucial ones:

1. Instanton effects exist and hence the physics of QCD is θ-dependent. We are confident of this because instanton effects and the θ dependence of QCD are necessary to solve the $\mathrm{U_A}(1)$ problem [1]. The $\mathrm{U_A}(1)$ problem [3] is the puzzle why there is no quasi-Nambu–Goldstone boson associated with the spontaneous breaking of the $\mathrm{U_A}(1)$ quasi-symmetry under which $q_j \to \exp^{i\alpha\gamma_5} q_j$ $(j = 1, \ldots, n)$ whereas the π, K and η mesons can be successfully identified as the

E-mail address: sikivie@phys.ufl.edu.

1631-0705/$ – see front matter
doi:10.1016/j.crhy.2011.10.002

quasi-Nambu–Goldstone bosons associated with the spontaneous breaking of $SU_L(3) \times SU_R(3) \to SU_{L+R}(3)$. The non-perturbative effects associated with QCD instantons produce a large explicit breaking of $U_A(1)$ and thus solve the $U_A(1)$ problem. Simultaneously, they make the physics of QCD depend upon the parameter θ.

2. The physics of QCD does not change if one applies the transformations:

$$q_j \to \exp^{i\alpha_j\gamma_5} q_j, \quad m_j \to \exp^{-2i\alpha_j} m_j \quad \text{for } j = 1, \ldots, n$$
$$\theta \to \theta - 2\sum_{j=1}^{n} \alpha_j \tag{2}$$

This result follows from the chiral anomaly [4] in chiral currents

$$\partial_\mu\left(\bar{q}_j\gamma^\mu\gamma_5 q_j\right) = 2im_j\bar{q}_j\gamma_5 q_j - \frac{g_s^2}{16\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \tag{3}$$

where no sum on j is meant. The invariance (2) implies that QCD is periodic in θ with period 2π. It also implies that QCD depends on θ only through the combination

$$\bar{\theta} = \theta - \arg\det m_q \tag{4}$$

and that the dependence of QCD upon θ disappears in the limit where one of the quark masses vanishes.

If $\theta \neq 0$ and/or the quark masses are complex, P and CP are in general violated, with C remaining conserved, since $G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$ and $i\bar{q}\gamma_5 q$ are both P odd, CP odd and C even. Using (2) one can shuffle the P and CP violation back and forth between the θ term and the quark masses. However, if $\bar{\theta} \neq 0$ (mod π), one cannot simultaneously set $\theta = 0$ and have all the quark masses real. $\bar{\theta}$ is the parameter that sets the amount of P and CP violation in QCD. From the experimental standpoint, no P or CP violation has been seen in the strong interactions. Hence $\bar{\theta}$ must be small (mod π). The best constraint is provided by the experimental upper limit [5] on the neutron electric dipole moment [6]

$$d_n \sim 5 \times 10^{-16}\bar{\theta}e \text{ cm} < 2.9 \times 10^{-26} e \text{ cm} \tag{5}$$

which implies $\bar{\theta} \lesssim 0.6 \times 10^{-10}$. The strong CP problem is the problem of explaining why $\bar{\theta}$ is so small. Indeed, the quark masses originate in the electroweak sector of the theory which violates P and CP. The Standard Model does not provide a reason why $\arg\det m_q$ should exactly cancel θ in Eq. (4). In fact, with CP violation in the manner of Kobayashi and Maskawa [7], the Yukawa couplings are arbitrary complex numbers and hence $\bar{\theta}$ is expected to be of order one.

Peccei and Quinn [8] proposed a simple and elegant solution to the strong CP problem. The Peccei–Quinn solution and the concomitant axion [9] are described below. Before we get there it is good to mention alternative approaches:

- QCD is $\bar{\theta}$-independent, and hence CP is conserved by the strong interactions, if there is a massless colored fermion. One may consider the possibility that $m_u = 0$. This disagrees, however, with the current algebra relations between the pseudo-scalar meson masses [10] and estimates of the up quark mass from lattice simulations [11]. One may also consider the possibility that the massless fermion is a sextet, octet or larger n-plet of $SU_c(3)$. This alternative proposal is equivalent to the Peccei–Quinn mechanism. It predicts an axion with decay constant of order the binding energy of $\bar{Q}Q$ pairs in the vacuum where Q is the massless colored fermion. Such an axion is ruled out by laboratory searches (see below).
- It is possible that CP is violated spontaneously. In that case $\bar{\theta}$ differs from zero but is calculable in terms of the parameters in the theory, and may be arranged to be small [12]. This approach is generally considered less attractive than the Peccei–Quinn solution, but who knows. Spontaneous CP violation predicts the appearance of domain walls [13] in the early universe. These domain walls must be gotten rid of through cosmological inflation as they would otherwise produce a universe that is too inhomogeneous and that expands too fast.
- Ellis and Gaillard [14] pointed out that in the Standard Model, with CP violation through the Cabibbo–Kobayashi–Maskawa mixing matrix, the infinite renormalization of $\bar{\theta}$ occurs only in very high order of perturbation theory. This means that $\bar{\theta}$ varies only very slowly with energy scale. Therefore, if the (unknown) theory of everything (TOE) sets $\bar{\theta} = 0$ at say the Planck scale, then $\bar{\theta} \ll 10^{-10}$ at the QCD scale. The question how TOE sets $\bar{\theta} = 0$ is left unanswered, but this may be alright for the time being. The point is that the strong CP problem does not necessarily have to be solved in our present low energy effective theory.

The Peccei–Quinn solution to the strong CP problem postulates a global $U_{PQ}(1)$ quasi-symmetry with the following properties:

1. it is a symmetry of the classical theory, i.e. a symmetry of the action density,
2. it is broken explicitly by those non-perturbative QCD effects (instantons and the like) which make the physics of QCD depend upon the parameter θ,

3. it is broken spontaneously.

One can show that in the presence of a $U_{PQ}(1)$ quasi-symmetry the θ parameter becomes a dynamical field. The quantum of this field is a new light scalar particle called the axion [9]. The low energy effective theory has action density

$$L_{\text{stand mod+axion}} = \cdots + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g^2}{32\pi^2}\frac{a(x)}{f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \tag{6}$$

where $a(x)$ is the axion field, and the dots represent the other terms of the Standard Model. f_a is a constant with dimension of energy and of order the magnitude of the vacuum expectation value that breaks $U_{PQ}(1)$. Eq. (6) implies that in a model with $U_{PQ}(1)$ symmetry $\theta = \frac{a(x)}{f_a}$ and hence $\bar{\theta} = \frac{a(x)}{f_a} - \det \arg m_q$ depends on the expectation value of $a(x)$. That expectation value minimizes the effective potential. The strong CP problem is solved because the minimum of the QCD effective potential $V(\bar{\theta})$ occurs at $\bar{\theta} = 0$ [15]. The weak interactions induce a small value for $\bar{\theta}$ [14,16], of order 10^{-17}, but this is consistent with experiment.

The notion of Peccei–Quinn (PQ) symmetry may seem contrived. Why should there be a U(1) symmetry which is broken at the quantum level but which is exact at the classical level? However, the reason for PQ symmetry may be deeper than we know at present. String theory contains many examples of symmetries which are exact classically but which are broken by quantum anomalies, including PQ symmetry [17–19]. Within field theory, there are examples of theories with *automatic* PQ symmetry, i.e. where PQ symmetry is a consequence of just the particle content of the theory without adjustment of parameters to special values.

The first axion models had f_a of order the weak interaction scale and it was briefly thought that this was an unavoidable property of axion models. However, it was soon pointed out [20,21] that the value of f_a is really arbitrary, that it is possible to construct axion models with any value of f_a. A value of f_a far from any previously known scale need not lead to a hierarchy problem because PQ symmetry can be broken by the condensates of a new technicolor-like interaction [22].

The properties of the axion can be derived using the methods of current algebra [23]. The axion mass is given in terms of f_a by

$$m_a \simeq 6 \text{ eV} \frac{10^6 \text{ GeV}}{f_a} \tag{7}$$

All the axion couplings are inversely proportional to f_a. For example, the axion coupling to two photons is:

$$L_{a\gamma\gamma} = -g_\gamma \frac{\alpha}{\pi}\frac{a(x)}{f_a} E \cdot B \tag{8}$$

Here E and B are the electric and magnetic fields, α is the fine structure constant, and g_γ is a model-dependent coefficient of order one. $g_\gamma = 0.36$ in the DFSZ model [21] whereas $g_\gamma = -0.97$ in the KSVZ model [20]. The coupling of the axion to a spin 1/2 fermion f has the form:

$$L_{a\bar{f}f} = i g_f \frac{m_f}{f_a} a \bar{f}\gamma_5 f \tag{9}$$

where g_f is a model-dependent coefficient of order one.

The axion has been searched for in many places but not found. The resulting constraints may be summarized as follows. Axion masses larger than about 50 keV are ruled out by particle physics experiments (beam dumps and rare decays) and nuclear physics experiments [24]. The next overlapping range of axion masses in decreasing order, from about 300 keV to 3×10^{-3} eV, is ruled out by stellar evolution arguments, including the longevity of red giants and the duration of the neutrino pulse from Supernova 1987a [25]. Finally, there is a lower limit, $m_a \gtrsim 10^{-6}$ eV, from cosmology because axions with mass less than this are overproduced in the early universe [26]. This leaves open an “axion window”: $3 \times 10^{-3} > m_a \gtrsim 10^{-6}$ eV.

Acknowledgements

This work was supported in part by the US Department of Energy under contract DE-FG02-97ER41029.

References

[1] G. 't Hooft, Phys. Rev. Lett. 37 (1976) 8;
G. 't Hooft, Phys. Rev. D 14 (1976) 3432;
R. Jackiw, C. Rebbi, Phys. Rev. Lett. 37 (1976) 172;
C.G. Callan, R.F. Dashen, D.J. Gross, Phys. Lett. B 63 (1976) 334.
[2] A.A. Belavin, A.M. Polyakov, A.S. Schwartz, Y.S. Tyuplin, Phys. Lett. B 59 (1975) 85.
[3] S. Weinberg, Phys. Rev. D 11 (1975) 3583.
[4] S. Adler, Phys. Rev. 177 (1969) 2426;
J.S. Bell, R. Jackiw, Nuov. Cim. A 60 (1969) 47.

[5] C.A. Baker, et al., Phys. Rev. Lett. 97 (2006) 131801.
[6] V. Baluni, Phys. Rev. D 19 (1979) 2227;
R.J. Crewther, P. Di Vecchia, G. Veneziano, E. Witten, Phys. Lett. B 88 (1979) 123.
[7] M. Kobayashi, K. Maskawa, Progr. Theor. Phys. 49 (1973) 652.
[8] R.D. Peccei, H. Quinn, Phys. Rev. Lett. 38 (1977) 1440;
R.D. Peccei, H. Quinn, Phys. Rev. D 16 (1977) 1791.
[9] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223;
F. Wilczek, Phys. Rev. Lett. 40 (1978) 279.
[10] J. Gasser, H. Leutwyler, Nucl. Phys. B 250 (1985) 465.
[11] G.T. Fleming, G.W. Kilcup, D.R. Nelson, Nucl. Phys. A 721 (2003) 883c;
C.T.H. Davies, et al., Phys. Rev. Lett. 104 (2010) 132003.
[12] M.A.B. Beg, H.S. Tsao, Phys. Rev. Lett. 41 (1978) 278;
H. Georgi, Hadronic J. 1 (1978) 155;
R.N. Mohapatra, G. Senjanovic, Phys. Lett. B 126 (1978) 283;
G. Segre, A. Weldon, Phys. Rev. Lett. 42 (1979) 1191;
S.M. Barr, P. Langacker, Phys. Rev. Lett. 42 (1979) 1654;
A. Nelson, Phys. Lett. B 136 (1983) 387;
S.M. Barr, Phys. Rev. Lett. 53 (1984) 329;
S.M. Barr, Phys. Rev. D 30 (1984) 1805.
[13] Y.B. Zel'dovich, I.Y. Kobzarev, L.B. Okun, Zh. Eksp. Teor. Fiz. 67 (1974) 3, Sov. Phys. JETP 40 (1975) 1.
[14] J. Ellis, M.K. Gaillard, Phys. Lett. B 74 (1978) 374.
[15] C. Vafa, E. Witten, Phys. Rev. Lett. 53 (1984) 535.
[16] H. Georgi, L. Randall, Nucl. Phys. B 276 (1986) 241.
[17] E. Witten, Phys. Lett. B 149 (1984) 351.
[18] K. Choi, J.E. Kim, Phys. Lett. B 154 (1985) 393;
K. Choi, J.E. Kim, Phys. Lett. B 165 (1985) 689.
[19] P. Svrcek, E. Witten, JHEP 0606 (2006) 051.
[20] J. Kim, Phys. Rev. Lett. 43 (1979) 103;
M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 166 (1980) 493.
[21] M. Dine, W. Fischler, M. Srednicki, Phys. Lett. B 104 (1981) 199;
A.P. Zhitnitskii, Sov. J. Nucl. 31 (1980) 260.
[22] J.E. Kim, Phys. Rev. D 31 (1985) 1733.
[23] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223;
W.A. Bardeen, S.-H.H. Tye, Phys. Lett. B 74 (1978) 229;
J. Ellis, M.K. Gaillard, Nucl. Phys. B 150 (1979) 141;
T.W. Donnelly, et al., Phys. Rev. D 18 (1978) 1607;
D.B. Kaplan, Nucl. Phys. B 260 (1985) 215;
M. Srednicki, Nucl. Phys. B 260 (1985) 689;
P. Sikivie, in: E. Alvarez, et al. (Eds.), Cosmology and Particle Physics, World Scientific, 1987, pp. 143–169.
[24] K. Nakamura, et al., Particle Data Group, J. Phys. G 37 (2010) 075021.
[25] M.S. Turner, Phys. Rep. 197 (1990) 67;
G.G. Raffelt, Phys. Rep. 198 (1990) 1.
[26] L. Abbott, P. Sikivie, Phys. Lett. B 120 (1983) 133;
J. Preskill, M. Wise, F. Wilczek, Phys. Lett. B 120 (1983) 127;
M. Dine, W. Fischler, Phys. Lett. B 120 (1983) 137.