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Gravitational waves / Ondes gravitationnelles

Gravitational waves and astrophysical sources

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ABSTRACT

In linear approximation to general relativity, gravitational waves can be thought of as perturbation of the background metric that propagate at the speed of light. A time-varying quadrupole of matter distribution causes the emission of gravitational waves. Application of Einstein's quadrupole formula to radio binary pulsars has confirmed the existence of gravitational waves and vindicated general relativity to a phenomenal degree of accuracy. Gravitational radiation is also thought to drive binary supermassive black holes to coalescence – the final chapter in the dynamics of galaxy collisions. Binaries of compact stars (i.e., neutron stars and/or black holes) are expected to be the most luminous sources of gravitational radiation. The goal of this review is to provide a heuristic picture of what gravitational waves are, outline the worldwide effort to detect astronomical sources, describe the basic tools necessary to estimate their amplitudes and discuss potential sources of gravitational waves and their detectability with detectors that are currently being built and planned for the future.

1. Introduction

In 1861 James Clerk Maxwell corrected Ampere's circuital law to account for *the effect due to the elasticity of the medium* [1]. The correction, which was physically motivated, was critical for him to discover in 1865 [2] that electromagnetic phenomena satisfied wave equations and that the velocity of these waves in vacuum was the same as the speed of light. Maxwell marvels at this coincidence between the speed of electromagnetic phenomena and light, and proposes that *light is electromagnetic disturbance propagated through the field according to electromagnetic laws* [2]. Direct and rigorous experimental verification of Maxwell's theory had to wait for 22 years, when Heinrich Hertz conclusively proved the existence of electromagnetic waves [3]. He generated and detected radio waves in the laboratory, successfully showing its transverse nature, measuring its speed, polarisation and intensity, lending support to Maxwell's theory.

It is not an exaggeration to say that Maxwell's theory eventually led to the prediction of gravitational waves. Maxwell's equations of electromagnetism, as noted by Einstein, were instrumental in the discovery of special relativity. Any theory of gravitation consistent with special relativity cannot be an action-at-a-distance theory. Instead, time-varying gravitational fields propagate through space at a finite speed and this retardation of the action of gravity naturally arises in general relativity and in general relativity, this speed is exactly equal to the speed of light. In fact, Einstein discussed generation of gravitational waves as one of the first consequences of this new theory of gravity [4,5].

However, gravitational waves have caused a lot of controversy in the literature and much doubt has been cast of their existence [6]. The difficulty arises as there is no clear way of separating the waves that are part of the spacetime geometry from the background geometry and there is no generally covariant tensor that describes the wave's energy, momentum and their fluxes. Moreover, in Einstein's theory, which is a gauge theory of gravitation, there are degrees of freedom that can be

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transformed away with a suitable choice of gauge making it difficult to understand what are the true physical degrees of freedom.

Ironically, it was Einstein's own work with Rosen that (unsuccessfully) tried to disprove the existence of gravitational waves [7]. The debate about the reality of gravitational waves continued into the late 1950s, culminating in the work of Bondi, Pirani and Robinson [8] that provided exact wave-like solutions of the field equations. The discovery of the Hulse–Taylor binary [9], a system of two neutron stars in orbit around each other, finally laid to rest any remaining doubts about the existence of gravitational radiation. The loss of energy and angular momentum to gravitational waves causes the two stars to slowly spiral in towards each other in this system. General relativity predicts that the binary would emit gravitational radiation and the loss of energy should cause the binary's orbit to shrink. The observed rate of change of the period $\dot{P}_{\rm b}$ agrees with the general relativistic prediction $\dot{P}_{\rm GR} = -2.402531 \pm 0.000014 \times 10^{-12}$ to better than 0.2%: $\dot{P}_{\rm b}/\dot{P}_{\rm GR} = 0.997 \pm 0.002$ [10,11].

Once theoretical impediments in defining the radiation were tackled and observational evidence of the existence of the radiation was firmly established, serious research on astronomical sources of gravitational waves began along side experimental efforts to detect cosmic gravitational waves.

2. Gravitational wave detectors

Joseph Weber was convinced of the reality of gravitational waves years before the Hulse–Taylor discovery. He built resonant mass detectors to detect gravitational waves from Galactic supernovae [12]. It turns out that his resonant bar was not sensitive enough to detect the small amplitudes that we now know are produced by core-collapse supernova.

2.1. Ground-based interferometers

The Hulse–Taylor binary gave the impetus for building more sensitive and wide band detectors, aimed at detecting a greater variety of sources. Today, there is a worldwide network of interferometric detectors of gravitational waves. The network consists of three 4 km arm length detectors at two sites in the US called the Laser Interferometer Gravitational wave Observatory (LIGO) [13], one 3 km arm length detector called Virgo in Pisa, Italy [14], a 600 m arm length interferometer called GEO600 in Hannover, Germany [15] and a 300 m arm length detector called TAMA (now decommissioned), in Tokyo, Japan [16].

These detectors have already taken data at or near their initial sensitivity goal for several years [17], thereby proving that it is possible to construct and reliably operate interferometers with large duty cycles. The data from a number of science runs over the past eight years have led to new grounds in astronomy, including a new best upper limit on the strength of stochastic gravitational waves [18], confirmation that certain extragalactic short-hard gamma-ray sources are probably flares in neutron stars [19] and that less than 2% of the spin-down energy of the Crab pulsar is lost to gravitational waves [20].

After operating for several years, LIGO and Virgo are being upgraded [21,22] to improve their strain sensitivity by an order of magnitude (three orders of magnitude increase in the volume they survey) compared to their initial configurations. It has been proposed that one of the two LIGO interferometers at Hanford be relocated to India [23], in order to increase the global baseline of the detectors, something that will greatly improve the overall science return of the network. The proposal is undergoing the final stages of approval in the US and India and expected to begin construction sometime 2013 and completed in 2022. In Japan funds have been allocated to build a 3 km arm length, underground, cryogenic detector called KAGRA [24]. KAGRA will have sensitivity similar to advanced LIGO and Virgo and become operational around 2017. The kilometre baseline advanced detectors should all be working in about 3 to 10 years. When they begin to operate at design sensitivities, we should be making routine detection of gravitational waves.

Sensitivity of ground-based detectors below a few hertz will be limited by fluctuations in gravity gradients due to variations in the surface density of Earth caused by seismic waves, density of air caused by wind and other environmental factors and, more generally, anthropogenic noise [25]. Some of these noise sources can be tackled by building a detector deep underground where density of air and anthropogenic noise will cease to be problems and the effect of seismic waves is greatly suppressed.

Plans are already underway to go beyond advanced detectors. A recently concluded European study has formulated the design of an underground observatory called the Einstein Telescope (ET) [26,27]. ET will have a triangular topology with 10 km arms and be able to operate three broadband detectors at a single site by using each arm of the triangle twice. With a strain sensitivity that is 10 times better than advanced detectors, ET should be able to take a census of stellar mass binary black holes up to a redshift $z \sim 17$–20, detect intermediate mass black hole binaries at redshifts of $z \sim 5$–7 and binary neutron stars at $z \sim 2$–4 [28].

2.2. Space-based interferometers and pulsar timing arrays

Another solution to low-frequency noise sources is to place a detector in space. The Laser Interferometer Space Antenna (LISA) in Europe and the US [29] and DECIGO in Japan [30], are two projects that aim to have free flying spacecraft in heliocentric orbit, away from the Earth. For example, LISA constitutes a set of three spacecraft, separated from each other by 5 million km, flying in a triangular formation in heliocentric orbit. LISA will be sensitive to sources in the frequency interval of [0.01, 100] mHz. Space antennas like LISA can probe radiation from supermassive black hole binaries from far corners of the Universe as well as Galactic white dwarf binaries [31].

A population of highly stable millisecond pulsars, with timing accuracies of ~ 100 ns over several years, could serve as an array of clocks whose regular ticks would be coherently modulated due to gravitational waves passing by the Earth. There is worldwide effort to observe pulsars and exploit them for detecting gravitational waves, so-called *pulsar timing arrays* (PTAs) [32–36]. Precise timing of an array of pulsars, could detect nano-hertz gravitational radiation that one might expect from *merging* supermassive black hole binaries of masses in the range $[10^9, 10^{10}]\ M_\odot$ but they could also be sensitive to binaries of lower masses at an earlier stage in their evolution [37,38]. More importantly, the array will also be sensitive to stochastic gravitational waves of nano-hertz frequencies [39]. Indeed, the current best constraints on primordial gravitational wave background are obtained by PTAs [40] at frequencies of $\sim 10^{-9}$ Hz to be $\Omega_{\mathrm{GW}} < 2 \times 10^{-8}$, where Ω_{GW} denotes the energy density in gravitational waves compared to the closure density of the Universe.

Stochastic gravitational waves of still lower frequencies (wavelengths as large as the Hubble radius of the Universe) might systematically affect the polarisation patterns of the cosmic microwave background [41]. So far, only upper limits have been set by CMB experiments but the Planck satellite could either detect, or set the most stringent limits, on the strength of primordial gravitational radiation produced in the inflationary era [42].

In summary, there are plenty of opportunities to directly observe gravitational radiation and it is widely expected that this would happen before the end of the current decade. Gravitational wave observations should help answer many puzzles in astronomy and cosmology, but this new window of observation might reveal sources and phenomena no one has imagined before. It is the quest for the unknown that makes the field so exciting.

3. Generation of gravitational waves

Gravitational waves are generated by non-spherical motion of mass. If the motion is spherically symmetric (for example, as it would be in the case of a spherically symmetric pulsations of a star) or axisymmetric (for example, an axisymmetric star rotating about its symmetry axis), there will be no radiation. These conclusions follow from fairly general arguments based on the conservation of mass and momentum, implying that there is no monopole or dipole radiation in general relativity and a time-varying quadrupole distribution of mass-energy is essential for the production of gravitational waves.

3.1. The quadrupole approximation

It is difficult to find exact solutions of Einstein equations for generic non-spherical motions and only approximate analytic solutions are presently known. The most successful general treatment is the post-Newtonian scheme [43], where all the relevant physical quantities (the metric and various potentials) are expanded in the characteristic (non-spherical) velocity in the system. At large distances from a source, gravitational waves can be expressed as small deviations from flat spacetime:

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}, \quad \eta_{\mu\nu} = \mathrm{diag}(-1,\ 1,\ 1,\ 1) \tag{1}$$

where $g_{\mu\nu}$ is the metric of the full spacetime in which gravitational waves are propagating and $h_{\mu\nu}$ describes the departure of the full metric from flat spacetime.[1] $h_{\mu\nu}$ are often referred to as the wave amplitudes.

With the assumption that there exists a coordinate system in which $|h_{\mu\nu}| \ll 1$, it is possible to solve Einstein's equations to linear order in h and obtain an expression for the metric perturbation h, the *wave amplitudes,* in terms of non-spherical motion in the source. There are coordinate and gauge degrees of freedom in general relativity which can be invoked to reduce the number of nonzero components of h. In a suitably chosen gauge, all components of the metric, except the spatial part of h transverse to the direction of propagation of the wave, can be made to vanish. The resulting metric perturbation h_{ij} will also be traceless, having only two independent components in the so-called *transverse-traceless* (TT) gauge. For a wave travelling in the z direction the independent components are $h_+ \equiv h^{\mathrm{TT}}_{xx} = -h^{\mathrm{TT}}_{yy}$ and $h_\times \equiv h^{\mathrm{TT}}_{xy} = h^{\mathrm{TT}}_{yx}$. The wave amplitudes h^{TT}_{ij} are related to the quadrupole tensor Q_{ij} of the source by [46]:

$$h^{\mathrm{TT}}_{ij} = \frac{2G}{c^4 D}\frac{\mathrm{d}^2 Q_{ij}}{\mathrm{d}t^2}, \quad Q_{ij} = \int \rho\left(x_i x_j - \frac{1}{3}\left(x^k x_k\right)\delta_{ij}\right)\mathrm{d}^3x \tag{2}$$

where c is the speed of light, G is Newton's gravitational constant, D is the distance to the source assumed to be much larger than the size of the source and ρ is the source density.

The combination $c^4/G \simeq 1.2 \times 10^{44}$ N has dimensions of force. One can think of the inverse of this quantity as a measure of the coupling of accelerated motion of bodies to the curvature of spacetime. A force of this order of magnitude is required to cause changes of order unity in the geodesics. It just goes to illustrate that the spacetime is extremely stiff and enormous forces are required to change the curvature of spacetime.

[1] A clear exposition of how gravitational waves arise in general relativity can be found in [44–46].

The quantity $\ddot{Q}_{ij}$ has dimensions of energy. It is the energy in the non-spherical motion of the source (non-spherical because Q_{ij} would be identically zero for a spherically symmetric source). For a source in which the entire mass M of the system is in non-spherical motion (a situation that occurs in binary black holes), components of $\ddot{Q}_{ij}$ are of order Mv^2. Furthermore, for self-gravitating sources, the velocity in the system of size R is related to its *compactness* $\sigma = GM/(c^2R)$ by $v^2 \sim \sigma c^2$, giving the maximum one can expect for the amplitude to be

$$h \lesssim \frac{GM}{c^2D}\sigma, \quad \sigma \equiv \frac{GM}{c^2R}$$

Compactness can also be written in terms of the Schwarzschild radius of the source as $\sigma = 0.5R_S/R$. Thus, for a source at a given distance, the amplitude of the waves is the largest for the most compact source. Black holes and neutron stars are the most compact objects in the Universe and interactions involving them are the primary sources of gravitational waves.

3.2. Ripples in the fabric of spacetime

In general relativity, mass-energy curves spacetime and the Riemann tensor encapsulates the curvature of spacetime geometry. In flat spacetime, the Riemann tensor is identically zero and so its components will be zero in all coordinate systems. Close to a massive object, there is no coordinate system in which all the components of the Riemann tensor can be made to vanish – the signature of a curved spacetime. In the same way, the effect of gravitational waves propagating in a flat spacetime background is to render some components of the Riemann tensor nonzero.

In Schwarzschild geometry, in a freely falling frame, the nonzero independent components[2] of the Riemann tensor are proportional to M/r^3 [47]:

$$R^{\hat{r}}{}_{\hat{\tau}\hat{r}\hat{\tau}} = -R^{\hat{\theta}}{}_{\hat{\varphi}\hat{\theta}\hat{\varphi}} = -\frac{2M}{r^3}, \qquad R^{\hat{\theta}}{}_{\hat{\tau}\hat{\theta}\hat{\tau}} = R^{\hat{\varphi}}{}_{\hat{\tau}\hat{\varphi}\hat{\tau}} = -R^{\hat{\theta}}{}_{\hat{r}\hat{\theta}\hat{r}} = -R^{\hat{\varphi}}{}_{\hat{r}\hat{\varphi}\hat{r}} = \frac{M}{r^3}$$

where M is the mass of the black hole and r is the radial distance. This is reminiscent of the tidal field in Newtonian gravity. The effect of this field is to cause tidal deformation of nearby (radial) geodesics, stretching the distance between them in the radial direction and squeezing in the transverse direction due to the opposite signs of $R^{\hat{r}}{}_{\hat{\tau}\hat{r}\hat{\tau}}$ and $R^{\hat{\theta}}{}_{\hat{\tau}\hat{\theta}\hat{\tau}}$. It is important to note that this field drops off with distance as $1/r^3$ and so distant objects have negligible tidal fields. However, this is not so in the case of gravitational radiation.

For a plane wave travelling in the z direction, the nonzero independent components of the curvature tensor in the TT gauge are

$$R^x{}_{0x0} = -\frac{1}{2}\ddot{h}^{\rm TT}_{xx}, \qquad R^y{}_{0x0} = -\frac{1}{2}\ddot{h}^{\rm TT}_{xy}, \qquad R^y{}_{0y0} = -R^x{}_{0x0}$$

from which we can see that the components of the curvature tensor fall off as inverse of the distance from the source since $h \propto D^{-1}$. Even though the stationary part of the field falls off very fast, the radiative part of the field decreases only as inverse of the distance. Moreover, since $R^y{}_{0y0} = -R^x{}_{0x0}$ the effect of gravitational waves is also to cause a tidal deformation of geodesics.

How does an oscillating Riemann curvature tensor affect the motion of free particles? The vector ξ^μ connecting a reference geodesic and its neighbour obeys

$$\frac{d^2\xi^\nu}{d\tau^2} = -R^\mu{}_{\alpha\nu\beta}\, U^\alpha \xi^\nu U^\beta \tag{3}$$

where τ is the proper time and U^α is the four-velocity along the reference geodesic. In flat spacetime, $R^\mu{}_{\alpha\nu\beta} = 0$ and so we can conclude that $d\xi^\nu/d\tau = \text{const}$. If initially $d\xi^\nu/d\tau = 0$ then it will continue to remain zero since there is no acceleration. Thus, the proper distance between two neighbouring geodesics, that are initially parallel, will remain constant. The effect of a passing gravitational wave is to cause the proper distance to oscillate about the mean separation. When the curvature is positive, geodesics get focused [because of the negative sign on the right-hand side of Eq. (3)] and so the proper distance decreases; when the curvature becomes negative, geodesics diverge and the proper distance increases from its mean value.

Fig. 1 shows how a ring of free particles tidally deforms due to a passing gravitational wave. As the wave amplitude oscillates about a mean value of zero, the components of the curvature tensor too oscillate through positive and negative values. In the presence of true gravitational waves, there is no coordinate system in which all the components of the curvature tensor can be reduced to zero. In this picture, gravitational radiation can be thought of as ripples of spacetime curvature travelling at the speed of light.

3.3. Gravitational wave luminosity

The Isaacson expression gives stress–energy tensor of a gravitational wave field which in the transverse-traceless gauge takes the form [44,48]:

[2] A caret on components indicates that this is a *freely falling* system of coordinates and not the usual Schwarzschild coordinates.

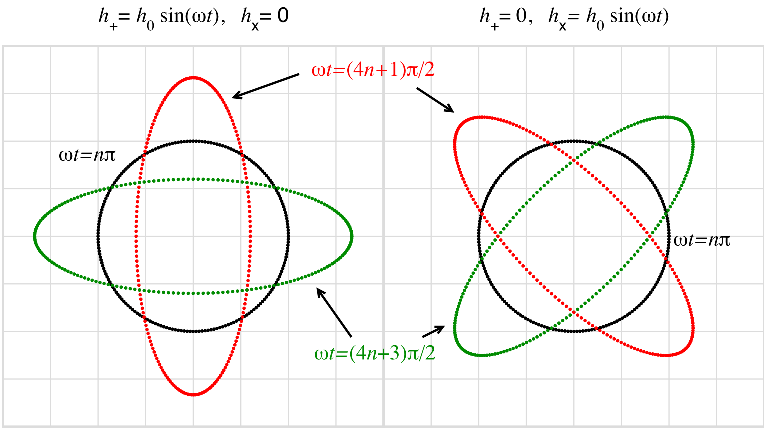

Fig. 1. A ring of free particles gets tidally deformed in response to a gravitational wave passing perpendicular to the plane of the paper. The strain $\delta L/L$, where δL is the change in size L of the ring, is proportional to the amplitude h of the wave. The circular ring deforms through the two oppositely oriented ellipses when the phase ωt of the wave is equal to the indicated value, where n is an integer. The diagram on the left corresponds to a wave of plus polarisation ($h_+ \equiv h_{xx} \neq 0$, $h_\times \equiv h_{xy} = 0$) and the one on the right corresponds to a wave of cross polarisation ($h_+ = 0$, $h_\times \neq 0$).

$$T_{\mu\nu} = \frac{1}{32\pi}\langle h^{\mathrm{TT}}_{mn,\alpha} h^{\mathrm{TT}mn}_{,\alpha}\rangle \tag{4}$$

The flux of gravitational waves in the direction x^k is the $0k$ component of the stress–energy tensor. By integrating the flux over a large sphere, one can obtain luminosity in gravitational waves [44]:

$$L \sim \frac{G}{5c^5}\dddot{Q}_{mn}\dddot{Q}^{mn} \tag{5}$$

This equation can be used to gain a qualitative understanding of the dynamics of a gravitational wave source. Our discussion below is inspired by compact binaries, but the arguments are quite generic and can be applied to other systems with some modification.

Just as in the case of the amplitude, the luminosity can be expressed in terms of the compactness of the source. Let us consider a source of size R and mass M. The time derivatives in the above expression can be written as angular frequency ω of the source, which, for a self-gravitating system, varies as the square-root of its density, $\omega \sim \sqrt{G\rho} \sim \sqrt{GM/R^3}$. Noting that the quadrupole moment is of order $Q \sim \eta M R^2$, where η is a factor that determines the momentum of inertia relevant to the non-spherical motion (which, for binary systems, is the symmetric mass ratio $\eta = m_1 m_2/(m_1+m_2)^2$), and assuming that only two components of Q_{ij} are significant, the source luminosity can be written as

$$L \sim \frac{2G}{5c^5}\epsilon_L M^2 R^4 \omega^6 = \frac{2c^5}{5G}\epsilon_L \sigma^5 \tag{6}$$

where $\epsilon_L \leqslant 1$ is a factor accounting for the efficiency of the source in generating gravitational radiation; for binary systems $\epsilon_L = 16\eta^2$. While the amplitude of the radiation is simply proportional to the source's compactness, luminosity goes as its fifth power. The luminosity drops very rapidly as compactness decreases, which is why non-compact objects are not powerful emitters of gravitational radiation. The constant $c^5/G = 3.6 \times 10^{52}\ \mathrm{J\,s^{-1}} \sim 10^{26}\ L_\odot$, is the factor needed to go from the dimensionless luminosity σ^5, to one that has physical dimensions. This is an enormous luminosity, at least a factor 10^3 larger than the luminosity in visible light of all the stars in the Universe. Since it is multiplied by the fifth power of compactness, most gravitational wave sources never reach this luminosity. However, binary black holes, for which the compactness can be of order unity, the luminosity does reach this stupendous value.

How long can a source of gravitational radiation last? It depends on the total amount of energy that is available to convert to radiation. For example, in a binary system of stars on a circular orbit of size R the available energy is roughly $-G\eta M^2/2R$. A source of luminosity L, with energy $\mathrm{d}E$ will last for a time $\mathrm{d}t = \mathrm{d}E/L$. The time t_{C} it takes for the system's energy to change from E_{i} to E_{f} (size R_{i} to $R_{\mathrm{f}} \ll R_{\mathrm{i}}$) is

$$t_{\mathrm{C}} = \int_{E_{\mathrm{i}}}^{E_{\mathrm{f}}} \frac{\mathrm{d}E}{L} = \int_{R_{\mathrm{i}}}^{R_{\mathrm{f}}} \frac{1}{L}\frac{\mathrm{d}E}{\mathrm{d}R}\,\mathrm{d}R \simeq \frac{5\,G}{256\,c^3}\frac{M}{\eta}\sigma_{\mathrm{i}}^{-4} \tag{7}$$

where $\sigma_{\mathrm{i}} = M/R_{\mathrm{i}}$ is the initial compactness of the source. This equation shows that for non-compact sources ($\sigma_{\mathrm{i}} \ll 1$), the time scale over which gravitational radiation operates is extremely large.

Table 1 lists the luminosity and radiation reaction time scale for several astronomical systems. It is clear that the more compact the object, the greater its luminosity and shorter it lives. The above equations can also be used to deduce that the

Table 1
Compactness of some astronomical sources of gravitational radiation, their GW luminosity and radiation reaction (RR) time scales. For binaries, luminosity and time scale also depend quite critically on the eccentricity of the orbit. Time scales are shorter for eccentric binaries than circular binaries of the same period by a factor $(1-e^2)^{7/2}$, if $e \lesssim 0.6$, and the luminosity is larger by a factor $(1+73e^2/24+37e^4/96)/(1-e^2)^{7/2}$ (see, e.g., Ref. [46]). White dwarf, neutron star and black hole binaries are assumed to be in quasi-circular orbits; the former two are assumed to consist of 1.4 $M_\odot$ stars and the latter 10 $M_\odot$ black holes.

Astronomical system	System's size R	Compactness σ	$L/L_\odot$	RR time scale τ
Sun and Jupiter	7.8×10^{11} m	1.9×10^{-9}	2×10^{-18}	2.4×10^{23} yr
Hulse–Taylor $e\simeq0.62$	2.0×10^{9} m	2.2×10^{-6}	0.041	300 Myr
J0737-3039 $e\simeq0.088$	8.8×10^{8} m	4.4×10^{-6}	0.1	85 Myr
White dwarf binary	10^{7} m	4.2×10^{-4}	9×10^{8}	1 yr
BNS, close to merger	7.26×10^{5} m	0.0057	5×10^{14}	1000 s
BBH, close to merger	2.97×10^{5} m	0.10	8×10^{20}	77 ms

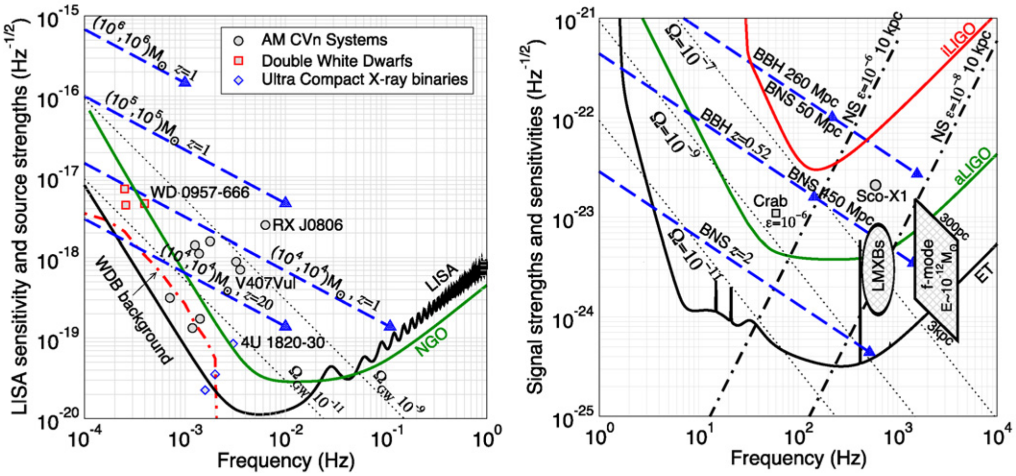

Fig. 2. The panel on the left shows the sensitivity of the Laser Interferometer Space Antenna (LISA) and its variant termed NGO (New Gravitational-Wave Observatory). The panel on the right shows the sensitivity of three generations of ground-based detectors, initial LIGO (iLIGO), advanced LIGO (aLIGO) and Einstein Telescope (ET). Inspiral sources in LISA are for randomly oriented and located binaries while for ground-based detectors they are assumed to be optimally oriented. Both panels show the characteristic amplitude h_c for a number of sources: for burst sources of Fourier amplitude $H(f)$ the characteristic amplitude is $h_c \equiv 2\sqrt{f}H(f)$, for continuous wave sources of strain amplitude h_0 the characteristic amplitude after integrating for a time T is $h_c \equiv \sqrt{T}h_0$ and for stochastic background of spectral density $S_h(f)$ the characteristic amplitude at frequency f after integrating for a time T is $h_c \equiv (Tf)^{1/4}\sqrt{S_h(f)}$. Period of integration is taken to be $T = 1$ yr in all cases. The sensitivity and source strengths are all in units of $\mathrm{Hz}^{-1/2}$.

compactness evolves as function of time as $\sigma(t) = \sigma_i(1-t/t_C)^{-1/4}$. This equation indicates that compactness grows rapidly as $t \to t_C$, but the evolution needs to be corrected for higher order general relativistic effects as $\sigma \to 1$. Compactness was used until now to motivate what sources are the best emitters of gravitational radiation. It is, however, not a useful quantity when describing gauge-independent observational parameters. The frequency of the radiation is what we will use to do so.

4. Astrophysical sources

The most luminous sources of gravitational waves have the greatest compactness. Neutron stars and black holes being the most compact objects in the Universe, they are also the brightest sources of gravitational waves. Gravitational radiation back reaction normally dictates the dynamics of most luminous systems, either driving them to instability that makes them catastrophically bright (e.g., compact binary star coalescences) or driving them to stability by decreasing the non-spherical motion in the system (e.g., rotating neutron stars).

In this section we will discuss different sources of gravitational waves, their typical strengths and how often we might expect to see them. Fig. 2 gives a summary of the sensitivity of a number of detectors, some of which are either already built or currently under construction (iLIGO and aLIGO), and planned for the future (ET, LISA, NGO), together with expected strengths of various sources. For the sake of clarity we have not included the sensitivity curves of initial and advanced Virgo that are similar to iLIGO and aLIGO but with a slightly better low-frequency sensitivity, KAGRA, which will be similar to aLIGO, and GEO600, whose sensitivity above 800 Hz, owing to smaller arms, will be about a factor 3 worse than aLIGO but significantly poorer sensitivity at lower frequencies.

4.1. Compact binaries

Compact binaries consisting of a pair of neutron stars, or black holes or a neutron star and a black hole, are very powerful emitters of gravitational radiation. These systems spend millions of years, like the Hulse–Taylor binary, with very low luminosity but as the two stars get closer, they brighten up to the point that they could be detected at cosmological distances, just minutes to seconds before they merge. They are believed to be very clean systems devoid of any accretion disks or other contaminants and so their dynamics is entirely governed by gravitational radiation back reaction. Using post-Newtonian (PN) approximation to general relativity, it has been possible to calculate the waveforms from these systems very accurately. The plus and cross polarisations of the waves for a binary at a distance D on a quasi-circular orbit take the form:

$$h_+(t) = \frac{4\mathcal{M}^{5/3}\omega^{2/3}}{D}\frac{(1+\cos^2\iota)}{2}\cos[2\phi], \qquad h_\times(t) = \frac{4\mathcal{M}^{5/3}\omega^{2/3}}{D}\cos\iota\sin[2\phi] \tag{8}$$

where ι is the inclination of the binary's orbital plane with the line-of-sight, $\mathcal{M} = \eta^{3/5}M$ is the so-called *chirp mass*, $\omega = \mathrm{d}\phi/\mathrm{d}t = \sqrt{GM/R^3}$ is the angular velocity and ϕ is the orbital phase. We can solve for $\phi(t)$ by using the radiation back reaction equation $\mathrm{d}t = -\mathrm{d}E/L$ and writing $E = -\eta M^2/2R = -\mathcal{M}(\mathcal{M}\omega)^{2/3}/2$. This gives $\dot\omega$ which can be solved to find

$$\dot\omega = \frac{96}{5}\mathcal{M}^{5/3}\omega^{11/3} \quad\Rightarrow\quad \omega(t) = \omega_{\mathrm{i}}\,(1 - t/t_{\mathrm{C}})^{-3/8}, \qquad \phi(t) = -\frac{1}{32}(\mathcal{M}\omega)^{-5/3} \tag{9}$$

where ω_{i} is the angular velocity corresponding to the orbital separation R_{i}. We observe that the amplitude and frequency of the signal monotonically increase as a function of time, producing a characteristic chirp. The chirp rate $\dot\omega$ in Eq. (9) depends on the chirp mass, while the frequency at which the two stars merge depends on the total mass $\omega_{\mathrm{merge}} = 1/(6^{3/2}M)$.

The foregoing equations are the dominant terms in the amplitude, frequency and phase evolution and there are PN corrections, currently known to order $(v/c)^7$ beyond the lowest order, that have additional dependences on the mass ratio of the system. Moreover, if the binary is on an eccentric orbit or the component stars have large spins, the signal's frequency and amplitude will have modulations. Note also that the ratio of the two polarisations contains important information about the orientation of the source (namely, the angle ι). Thus, imprint in the structure of the signal are the various parameters of the source. By a careful analysis it will be possible, in principle, to measure the masses of the components stars, eccentricity of the orbit, spins of the two bodies and the polarisation of the radiation. In reality, however, not all parameters of the source can be measured to a good accuracy as there are strong correlations between some of the parameters (for example, ι and D are strongly correlated).

4.1.1. Stellar mass compact binaries

Hulse–Taylor pulsar and J707-3039 [49] are prototypal binary neutron stars (BNS), whose merger time is far less than the Hubble time. We can, therefore, expect that binary neutron star merger is not an uncommon event in the Universe. How many such events can we expect to occur each year within a given volume of the Universe. Unfortunately, the small number of observed galactic neutron star binaries are only able to provide an answer that is uncertain by a factor of 10^3. Nominally, the rate could be about one event within a volume of 100 Mpc^3 but the rate could be lower by a factor of 100 or larger by a factor of 10 [50].

The other two categories of stellar mass binaries, a pair of black holes (BBH) or a neutron star and a black hole (NSBH), have so far not been observed, although Belczynski et al. argue that high mass X-ray binaries IC10 X-1 and NGC300 X-1 are progenitors of binary black holes [51]. If this is true then statistical arguments similar to the one applied to binary neutron stars, give the event rate of binary black holes to be $R = 3.36^{+8.29}_{-2.92}$ in initial LIGO and roughly 1000 greater in aLIGO [51]. Some authors [52] have explored the effect of metallicity on the formation and evolution of massive stars to deduce that black hole mergers could be far more common in the Universe with rates in excess of several 100s to several 1000s in aLIGO.

In all cases the rates are pretty uncertain. However, a network of advanced gravitational wave detectors could have a distance reach of 200 Mpc for BNS, 800 Mpc for NSBH and 1.5 Gpc for BBH, within which we can not only expect to make the first direct detection of gravitational waves, but also place a firm constraint on astrophysical models of the formation and evolution of compact stars and their merger rates.

4.1.2. Supermassive and intermediate mass black hole binaries

There is now strong observational evidence that galactic centres host supermassive black holes, i.e. black holes of millions to several 10s of billions of solar masses. Decades of observations of stellar orbits close to the Galactic centre has revealed the presence of a black hole of $4\times10^6\ M_\odot$ in the nucleus of our own Milky Way [53,54]. When and how did such black holes form? Did the black holes precede the galaxies or did they form after the galaxies were assembled? What were there initial masses and how did they grow? These are among the most pressing unsolved questions in cosmology. Gravitational wave observations in different spectral windows might be able to answer some of these questions.

A binary of intrinsic masses $(10^4, 10^4)\ M_\odot$ at $z = 20$ will appear to us as a $(2.1\times10^5, 2.1\times10^5)\ M_\odot$ binary and merge at a frequency of around 10 mHz. The redshift effect on the amplitude is so great that a binary that is visible with a modest signal-to-noise ratio (of, say, 20) at redshift $z = 1$ will continue to be visible until the observed mass is so large that the binary merges outside the sensitivity band of the detector. Even at a redshift of $z = 20$ a randomly oriented $(10^4, 10^4)\ M_\odot$ binary with a random sky position would produce, close to merger, a characteristic amplitude of $h_{\mathrm{c}} \equiv 2\sqrt{f}|H(f)| \sim 1.4\times10^{-19}\ \mathrm{Hz}^{-1/2}$, and will be clearly visible in LISA. These are such high redshifts that the Universe was probably assembling its first black holes at this epoch. LISA can take a census of supermassive black hole binaries in the mass range 10^4–10^7 $M_\odot$ in the entire Universe and provide the necessary input for testing different scenarios of the formation and growth of galaxies [55,56].

The merger rate of supermassive black holes is highly uncertain as there are only a handful of such candidate binaries that would merge within the Hubble time. Detailed modelling of these systems is very difficult due to many unknown astrophysical parameters, including their masses and spins when they formed and how they grew. Predicted merger rates in the Universe in the range of masses that LISA could observe, are of order ~ 30–100 per year, depending on the model that is used for the formation and growth of massive black holes, of which ~ 20–30 should be detectable by LISA [57].

There is as yet no conclusive evidence for the existence of black holes of mass in the range $\sim 10^2$–10^4 $M_\odot$, the so-called *intermediate mass* black holes. However, there are strong indications that certain ultra-luminous X-ray sources, e.g. HLX-1 in ESO 243-49 [58], are host to intermediate mass black holes. If a population of such binaries exists and they grow by merger, then, depending on their masses, ET will be able to explore their dynamics out to $z \sim 6$ and study their mass function, redshift distribution and evolution. Several authors have looked at the possibility that intermediate mass black hole binaries could form and merge in dense stellar clusters. Modelling the growth of seed black holes using different scenarios these authors conclude that ET could observe few to few tens of intermediate mass black hole binaries per year [59–61].

4.1.3. Extreme mass ratio binaries

Binaries we have discussed so far have component stars of comparable masses. When one of the masses is far smaller than its companion, we have the problem of a test body in *near* geodesic motion in black hole geometry. Such binaries are called *extreme mass ratio binaries*, as the mass ratio m_1/m_2, $m_1 \gg m_2$, could get stupendously large. The orbits in this case are near geodesics because in each cycle the test body loses only a small amount of its rotational energy to radiation; recall that the luminosity of a binary source goes as $\eta^2 \simeq (m_2/m_1)^2$.

In general relativity, bound geodesics in Kerr spacetime geometry have a far richer structure than the simple elliptic orbits of Newtonian gravity. Geodesics with eccentricity close to unity and periastron very near the horizon are especially intriguing. A test particle on such an orbit could begin at apastron and when it gets to periastron it might exhibit tens of near-circular orbits, before ending up at an entirely different point in space as apastron. When the particle is close to periastron, it loses far more energy (remember that the luminosity is proportional to fifth power of compactness) than when it is at apastron and hence the emitted gravitational wave signal can have a pretty complex structure.

Moreover, such geodesics could be truly spherical orbits and sample the entire spacetime region near a black hole. Imprint in the signal's phasing is the full multipole structure the black hole spacetime and it should be possible to produce a map of the Kerr geometry of the central object by observing the inspiral of an extreme mass ratio binary [62]. This would be a stringent test of general relativity as it would be possible to check if the spacetime geometry of black holes is truly described by only their mass and spins or if black holes have "hair".

LISA is best suited to observe extreme mass ratio inspirals. Supermassive black holes at galactic nuclei are believed to grow by the infall of stellar mass and intermediate black holes. Such events could be observed by LISA at cosmological distances. For instance, the inspiral of a 10 $M_\odot$ black hole into a 10^6 $M_\odot$ supermassive black hole at $z = 1$ would produce a detectable amplitude in LISA. The rates in this case too are highly uncertain and range from a few to several hundreds per year [59,63].

4.1.4. Gamma-ray bursts

Gamma-ray bursts (GRBs) are extremely bright flashes of energy that last anywhere from milliseconds to several minutes. Discovered in the late 1960s by US spy satellites, they are the most luminous known sources in the Universe. Their rapid variability over short time scales implies that they are very compact sources, likely neutron stars or stellar mass black holes. By measuring the redshift of host galaxies it is now known that most GRBs are cosmological in origin. The flux levels often exceed $10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}}$, implying an isotropic luminosity of $10^{44}\,\mathrm{J\,s^{-1}}$ for bursts at 1 Gpc. This is several orders of magnitude larger than the luminosity of an entire galaxy at all wavelengths. The difficulty in modelling these sources is that it is impossible to produce such stupendously large luminosities from highly compact objects. If the emission is beamed in a narrow cone, however, then the energy requirements can be considerably smaller. Most models assume that the radiation is confined to a cone of opening angle of about 20 degrees.

If the GRBs are compact sources, then it is plausible that gamma-ray emission is accompanied (most likely, preceded) by emission of gravitational waves. GRBs are classified based on the duration of bursts and spectral hardness. Bursts that last for 2 s or more and with soft spectra are called *long GRBs*, and they are associated with core-collapse supernovae that are expected to emit a burst of gravitational waves before the creation of the fireball that leads to GRBs. Bursts lasting for shorter periods of 2 s or less and with hard spectra are termed *short GRBs* and the most popular progenitor model for such bursts is the coalescence of binary neutron stars, which are also the most promising sources for interferometric gravitational wave detectors. It is thought that some of these short GRBs are giant magnetar flares which could also be accompanied by the emission of gravitational radiation.

Observing gravitational waves in coincidence with GRBs will have a tremendous impact on our understanding the progenitors of GRBs and how they are powered. Initial detectors have already placed some impressive constraints on nearby GRBs and set upper limits on the strength of gravitational waves from a population of bursts that occurred during recent science runs [19]. Moreover, such coincident observations will help identify the host galaxy and measure its redshift. If progenitors of short GRBs are binary neutron star mergers then this would help measure both the luminosity distance and redshift to the source, *without* the use of the cosmic distance ladder [64]. Clearly, such observations will have a great potential for precision cosmography [65–68].

GRBs are detected roughly twice per day, of which about a quarter (~ 170) are short GRBs and the rest (~ 500) are long GRBs [69]. The local rate, inferred from redshift measurements of the host galaxies of a subset of the population, for short GRBs is 10 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$, an order of magnitude more than that for long GRBs, 0.5 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$. Most of these events are at redshifts not accessible to advanced detectors. Long GRBs, associated with core-collapse supernovae, will be particularly faint in the gravitational window at distances greater than about a few megaparsecs. Short GRBs, associated with binary neutron stars or neutron star–black hole binaries, could be observed up to distances in the range of 400–800 Mpc depending on the total mass of the binary. Advanced detectors might observe coincidences with short GRBs within about 16 months of observation at modest sensitivities [68]. ET, however, will be sensitive to neutron star coalescences at $z \sim 2$–4 and might observe them in coincidence with GRBs far more frequently [66].

4.1.5. Standard sirens

Compact binary sources are quite unique for astrophysics and cosmology as they are *standard candles* or, perhaps more appropriately, *standard sirens.* Gravitational wave observations can measure both ω and $\dot{\omega}$ if the observation time T is sufficiently long, namely $T > (\dot{\omega}/2\pi)^{-1/2}$. For example, the change in frequency of a $10^6\ M_\odot$ supermassive black hole binary with an orbital frequency of 5 μHz will not be observable, even after a year, while that of a 2.8 $M_\odot$ neutron star binary at 1 Hz would be detectable after roughly 5 minutes. Systems whose frequencies change during the course of observation are called *chirping* binaries. For binaries that chirp, it is possible to measure the chirp mass of the binary, but since we can also measure the two polarisations h_+ and $h_\times$, one can deduce the distance D to the binary from gravitational wave observations alone [64].

Until recently, it was thought that while gravitational wave observations of a compact binary inspiral can be used to infer the luminosity distance of its host, it will not be possible to measure the host's redshift. This is because, from the expression for $\dot{\omega}$ in Eq. (9) we see that cosmological redshift in frequency ($\omega \to \omega/(1+z)$, $\dot{\omega} \to \dot{\omega}/(1+z)^2$), causes the source to appear to have a larger chirpmass ($\mathcal{M} \to (1+z)\mathcal{M}$). There is no way to tell if the source is at a lower redshift with an intrinsically larger chirp mass or at a higher redshift with an intrinsically smaller chirp mass. It was, therefore, thought that the only way to break the redshift-mass degeneracy is to identify the host galaxy and measure its redshift. While this is still true for binary black holes, for binaries in which at least one of them is a neutron star it might be possible to infer the intrinsic mass of the system from gravitational wave observations. This is because equation-of-state-dependent tidal effects, which appear at the fifth PN order, depend on the density M/R^3 of the neutron star and not just on the compactness. It turns out that the tidal effect can be used to determine the source's redshift provided the neutron star equation-of-state is known [70]. The effect is completely absent for black hole binaries, weaker in the case of neutron star–black hole binaries and quickly diminishes as the mass ratio η decreases.

4.2. Black hole quasi-normal modes

Coalescence of a compact binary produces a black hole that is initially highly deformed. The energy in the deformation is emitted as a superposition of exponentially damped sinusoidal gravitational waves called quasi-normal modes. The frequency and time constants of these modes depend only on the merged black hole's mass and spin angular momentum, a consequence of the *black hole no-hair* theorem. The modes are indexed by integers similar to the spherical harmonic indices (ℓ, m), $\ell = 2, 3, \ldots,$ $m = -\ell, \ldots, \ell$, as also an overtone index that $n = 0, 1, 2, \ldots,$ where $n = 0$ is the 'fundamental' (i.e., least damped) mode, and so on. In most problems, it is sufficient to consider the $n = 0$ mode and so the two polarisations are given by [71]:

$$h_+ - i\,h_\times = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} Y_{-2}^{\ell m}(\iota, \phi)\, h_{\ell m}, \quad h_{\ell m} = \frac{\alpha_{\ell m} M}{D} e^{-i\,\omega_{\ell m} t - t/\tau_{\ell m}}$$

where (ι, ϕ) refer to the colatitude and the azimuth angle at which the radiation is emitted from the black hole and $Y_{-2}^{\ell m}$ are the -2 spin-weighted spherical harmonics. The mode frequencies and damping times are, as mentioned before, functions of only the final black hole's mass and spin magnitude while the amplitudes $\alpha_{\ell m}$ of the excited modes depend quite critically on the progenitor binary's mass ratio and spins. As a result, by detecting quasi-normal modes, even when the binary itself is not visible (because the inspiral phase lies outside the sensitive band of the detector), it will be possible to measure the parameters of the progenitor binary [72,73].

Quasi-normal modes could also be used to test general relativity by comparing the predictions of numerical solutions to Einstein equations with observations of the spectrum of excited modes and their amplitudes [74,75]. In particular, it should be possible to measure the mass of the system before and after merger and compute the *mass loss* to gravitational waves and see how that compares with predictions of general relativity, to *test the no-hair theorem* by measuring the complex frequencies of different modes and verifying if they depend on extra degrees of freedom other than the object's mass and spin and confirming that the merged object is actually a black hole [76] and not a naked singularity by detecting the presence of quasi-normal modes.

4.3. Neutron stars

Neutron stars in isolation but with a time-varying quadrupole generated by non-axisymmetric rotation or accretion of matter or transfer of energy from a differentially rotating core to crust, can produce gravitational waves that are detectable by ground-based detectors. Any isolated body is bound to have a limited supply of energy available for radiation and so either most of the energy might be emitted in a burst, resulting in a strong source that would be easily discernible, or the energy might leak out slowly over millions of years, giving a long-lived continuous, but weak, source of radiation.

4.3.1. Supernovae: birth of neutron stars

Neutron stars are born in the aftermath of the collapse of a massive star or when the core of a white dwarf becomes more massive than the Chandrasekhar limit of 1.4 $M_\odot$. Both axisymmetric and non-axisymmetric collapse can produce gravitational waves. In fact, first gravitational wave detectors were built to detect Galactic supernova and they are still among the most important sources. Supernovae produce the Universe's dust and heavy elements; their cores are laboratories of complex physical phenomena requiring general relativity, nuclear physics, magneto-hydrodynamics, neutrino viscosity and transport, turbulence, etc. Much of the physics of supernovae is poorly understood: How non-axisymmetric is the collapse? How much energy is converted to gravitational waves and over what time scale? What causes shock revival in supernovae that form a neutron star? Gravitational wave observations could provide some of the clues for solving these questions.

We can make a rough estimate of the amplitude of the radiation if we have a knowledge of the total energy in gravitational waves and the frequency of the emitted waves. The wave amplitude is related to luminosity in gravitational waves [cf. Eqs. (2) and (5)] by $\dot{h}^2 \sim 5GL/(c^3D^2)$. If energy E is converted into gravitational waves and comes out mostly at frequency f in time Δt then using $L \sim \Delta E/\Delta t$ and $\dot{h}^2 \sim 4\pi^2 f^2 h^2$ we get

$$h^2 \sim \frac{5G}{4\pi f^2 c^3 D^2}\frac{\Delta E}{\Delta t}$$

For a Galactic supernova ($D \sim 10$ kpc), assuming $\Delta E = 10^{-8}\ M_\odot$, $\Delta t = 10$ ms and $f = 300$ Hz, the amplitude is $h \sim 5 \times 10^{-22}$. Such an event would produce a characteristic amplitude $h_c \sim h/\sqrt{f} \sim 3 \times 10^{-23}\ \mathrm{Hz}^{-1/2}$. Such large amplitudes should be detectable in advanced detectors. However, supernovae occur only once in about 30 or 100 years in a galaxy of the size of the Milky Way. So the prospect of a Galactic supernova is not so bright. The supernova rate is of order 1 per few years within about 2 Mpc. Advanced detectors will not be sensitive at this distance but ET will be.

4.3.2. Triaxial neutron stars

In many cases, one can think of the neutron star as being a triaxial rotating body. A neutron star rotating at a frequency of f_{rot} emits gravitational waves at $f = 2f_{\mathrm{rot}}$ and the amplitude of the radiation is given by

$$h_+(t) = h_0\frac{1+\cos^2\iota}{2}\cos 2\phi(t), \qquad h_\times(t) = h_0\cos\iota\sin 2\phi(t) \tag{10}$$

where ι is the angle between the star's axis of rotation and the line of sight, h_0 and $\phi(t)$ are the signal's amplitude and phase,

$$h_0 = \frac{4\pi^2 G}{c^4}\frac{\epsilon I_{zz} f^2}{D}, \qquad \phi(t) = \phi_0 + 2\pi f t + \sum_{k=1}^{n}\frac{f_k}{(k+1)!}t^{k+1} \tag{11}$$

Here I_{zz} is the star's moment of inertia with respect to the rotation axis, the ellipticity ϵ is defined in terms of the principal moments of inertia as $\epsilon = (I_{xx} - I_{yy})/I_{zz}$, D is the distance to the star, and ϕ_0 and f_n are, respectively, the phase and the spin-down parameters in the rest frame of the star at the fiducial start time $t = 0$, n being the number of spin-down parameters included in the model (e.g., f_1 is the rate at which the star spins down).

At a spin frequency of $f_{\mathrm{rot}} = 100$ Hz (gravitational wave frequency of $f = 200$ Hz), for a source at 10 kpc and ellipticity $\epsilon = 10^{-6}$, the amplitude of the radiation is $h_0 \simeq 4.2 \times 10^{-27}$. To compute the characteristic strain amplitude h_c produced by such a signal we must assume a time interval over which the signal is integrated; taking this to be one year, we get $h_c = h_0\sqrt{\mathrm{yr}} = 2.3 \times 10^{-23}\ \mathrm{Hz}^{-1/2}$ (see Fig. 2, dash-dotted lines). The amplitude increases as the square of the spin frequency, so for the Crab pulsar (B0531+21, $D = 2$ kpc) with spin frequency of 30 Hz, $h_c \sim 10^{-23}\ \mathrm{Hz}^{-1/2}$, for the same ellipticity. If Crab's ellipticity is ten times higher, then it will be well within the reach of advanced detectors (Fig. 2, right panel). However, it is not clear if neutron stars occur with ellipticities as large as 10^{-5}. Models are mostly able to compute the maximum ellipticity of neutron stars by subjecting the crust to breaking strains. Ellipticities computed in the literature range from values of 10^{-4} (for exotic equations of state) [77] to 10^{-7} for conventional crustal shear [78]. Large toroidal magnetic fields of order 10^{15} G could produce ellipticities of order 10^{-6} [79] and accretion along magnetic fields could produce similar or an order of magnitude larger deformations [80]. The only model that computes the minimum deformation due to a magnetic field predicts a minimum eccentricity of 10^{-12}. The large range in possible eccentricities shows that gravitational wave observations could have a potentially high impact and science return in this area.

The emitted radiation is mostly a monotonic signal, but the radiation back reaction and energy lost to electromagnetic radiation and particles could cause the frequency to slowly drift in time. Comparing Eqs. (2) and (5) we observe that, as an order of magnitude, $L = (D\dot{h})^2/20$. Writing $\dot{h} = 2\pi f h$, where f is the frequency of the source, the luminosity can be computed from the amplitude using $L = (\pi/5)(h_0 f D)^2$ [31]. The luminosity for the example we considered above is roughly 40 $L_\odot$. If we assume that the rotational energy $E = I\omega^2/2$ of the star powers the radiation, then the minimum time scale over which the energy is exhausted is roughly $\tau \sim E/L \sim 5 \times 10^8$ yr. Newly born neutron stars emitting in the gravitational window will take 100s of millions of years to exhaust their source of energy and are essentially continuous wave (CW) sources.

The motion of the detector with respect to the source (due to Earth's rotation and orbital motion) causes amplitude and frequency modulation of the signal. Imprint in this modulation is the signal's position on the sky and so it will be possible to resolve the source's location subject to Rayleigh criterion, $\delta\theta = 2\pi\lambda/L$, where $\delta\theta$ is the angular resolution, λ is the wavelength of the radiation and $L = 2$ AU is the diameter of Earth's orbit. At a frequency of 100 Hz, $\delta\theta \sim 2''$. Moreover, the amplitude of the radiation will help constrain the product of the star's ellipticity and moment of inertia, which is one of the main ingredients that goes into determining its equation-of-state.

Observing a representative sample of the Galactic population of neutron stars could transform astrophysical studies of compact objects. A catalogue of CW sources would help understand the galactic supernova rate, their demographics will lead to insights on evolutionary scenarios of compact objects, their amplitudes and distances[3] can be used to constrain the equation-of-state. Constraints on the range of ellipticity could help understand crustal strengths and test models of the structure and composition of neutron stars.

4.3.3. Pulsar glitches and magnetar flares

Radio pulsars have very stable spins and their periods (P) change very slowly over time. Their small spin-down rate ($\dot{P} \lesssim 10^{-12}$ s s^{-1}), is occasionally marked by a sudden increase in angular frequency Ω, an event that is called a *glitch* [81]. To date more than 300 glitches have been observed in about 100 pulsars [82].[4] Vela (B0833-45) is a nearby ($D \sim 300$ pc) pulsar in which 16 glitches have been observed since its discovery in 1969. The magnitude of a glitch is measured in terms of the fractional change in the angular velocity which is in the range $\Delta\Omega/\Omega \sim 10^{-5}$–$10^{-11}$. Some time after a glitch, the pulsar returns to its regular spin-down evolution. The origin of pulsar glitches is not a settled matter, although the most favoured explanation is that it is due to transfer of angular momentum from a differentially rotating core to crust [81].

Glitches are not the only transient phenomena observed in neutron stars. Sources of giant X- and gamma-ray flashes are believed to be highly magnetised ($B \sim 10^{15}$–10^{16} G) neutron stars called *magnetars*. The source of high energy radiation is believed to be decay of the magnetic field associated with a stellar quake.

Star quakes could excite normal mode oscillations of the ultra dense core. The energy in the modes is emitted as gravitational waves with a characteristic frequency and decay time, similar to quasi-normal modes of black holes. Unlike black holes, however, the complex mode frequencies depend on the mass and radius of the neutron star.[5] One can get an estimate of the amplitude of gravitational waves from a glitch by noting that for a star with rotational velocity Ω the glitch energy is $\Delta E \simeq I\Omega\,\Delta\Omega$, where I is the star's moment of inertia and $\Delta\Omega$ is the change in angular velocity. For Vela, whose spin frequency is $f_{\rm rot} \simeq 11$ Hz, the largest glitch has $\Delta\Omega/\Omega \sim 3 \times 10^{-6}$ [82]. The corresponding glitch energy is $\Delta E \sim 8 \times 10^{-12}\ M_\odot$. If a tenth of this energy is available to normal modes, which can then emit gravitational waves at the fundamental mode frequency of $f \sim 2$ kHz, then we can expect a strain amplitude of

$$h_0 \sim \frac{1}{\pi D}\sqrt{\frac{E_{\rm GW}}{f}} \simeq 7 \times 10^{-22} \left(\frac{300\ \text{pc}}{D}\right)\left(\frac{E_{\rm GW}}{8 \times 10^{-13}\ M_\odot}\right)^{1/2}\left(\frac{2\ \text{kHz}}{f}\right)^{1/2}$$

and a characteristic amplitude of $h_c = h_0/\sqrt{f} \simeq 10^{-23}$ Hz$^{-1/2}$ [31]. Third generation detectors like ET should be able to detect such amplitudes in coincidence with radio observations. Fig. 2 shows plausible characteristic amplitudes produced by normal modes of energy $10^{-12}\ M_\odot$, for mode frequencies in the range of 1.5–4 kHz and neutron star distances in the range of 0.3–3 kpc.

The frequency and decay time of normal modes have different dependences on the mass and radius of the star. Thus, by measuring the complex mode frequency of, say, the fundamental mode one can infer both the mass and radius of the star. The size of a neutron star of a given mass depends critically on the supranuclear equation-of-state of matter, which is currently highly uncertain. Gravitational wave observations of glitches will help directly measure the equation-of-state of matter under extreme conditions of density, pressure and magnetic fields, one of the most important unsolved problems in astronomy and nuclear physics.

4.3.4. Low-mass X-ray binaries

Gravity in the vicinity of a compact star is so large that particles falling into it can get accelerated close to the speed of light. Charged particles accelerated in this way are responsible for intense flashes of X-rays in binary systems in which one of the stars is a compact object that accretes matter from a low-mass star, or a main sequence star, or a giant. Low-mass

[3] Gravitational wave observations alone cannot determine the distance to CW sources. However, it should be possible to measure the distance to a subset of them from radio observations.

[4] Jodrell Bank Observatory maintains a glitch catalogue at http://www.jb.man.ac.uk/pulsar/glitches.html.

[5] Although the modes would depend on the spin of the star, at a first approximation it can be neglected.

X-ray binaries (LMXBs) are systems where the donor star is less massive than the compact object. LMXBs are thought to have spun up millisecond pulsars.

LMXBs emit bursts of X-ray flashes (which last about 10 s and repeat once in a few hours to days) in which millisecond oscillations in burst intensity are observed. X-ray bursts are believed to be caused by thermonuclear burning of infalling matter while oscillations are suspected to be the spin frequency of the neutron star. About 100 galactic LMXBs are known to-date as also many extragalactic ones. Inferred spin frequencies of neutron stars in LMXBs in all these cases seem to have an upper limit of about 700 Hz. The centrifugal breakup of neutron star spins for most equations of state is far higher, about 1500 Hz. It has therefore been a puzzle as to why neutron star spin frequencies in LMXBs are stalled. One reason for this could be that some mechanism operating in the neutron star is emitting gravitational waves and the resulting loss in angular momentum explains why neutron stars cannot be spun up beyond a certain frequency.

Stalled spin frequencies in LMXBs can be explained if the effective ellipticity in a neutron star is $\sim 10^{-8}$. This "ellipticity" could be produced by a time-varying, accretion-induced quadrupole moment, or by relativistic instabilities (e.g., r-modes), or by large toroidal magnetic fields. Assuming that the gravitational wave luminosity is related to the X-ray luminosity the expected characteristic amplitude of gravitational radiation is shown in Fig. 2 for the well-known LMXB Sco-X1 as well as for the known Galactic population of LMXBs. Advanced detectors should detect Sco-X1 if the system is losing all of its accreted angular momentum to gravitational waves. Such a detection could help understand what is the mechanism behind limiting spin frequencies and in turn provide deeper insights into models of LMXBs.

4.4. Stochastic backgrounds

Another class of continuous gravitational waves is an ever present stochastic gravitational wave background. Such a background might have been produced by physical processes in the early Universe (just as cosmic microwave background was produced at the Big Bang) [83] or by random superposition of a population of point sources throughout the Universe [84]. Detecting such a background and measuring its spectral features could provide insight into the physical processes in the very early Universe accessible in no other way or provide a census of sources at cosmological distances.

The strength of a stochastic background is measured not by the amplitude of the radiation (as we cannot follow the amplitudes of individual waves) but by the power spectrum that it produces. An equivalent, but more popular, way of characterising the strength of a background is to specify the energy density in the radiation, as a function of frequency, relative to the closure density of the Universe:

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{\rm C}} \frac{{\rm d}\rho_{\rm GW}(f)}{{\rm d}\log f}$$

where $\rho_{\rm C} = 3H_0^2c^2/(8\pi G) \simeq 8.8 \times 10^{-10}\ {\rm J\,m^{-3}}$ is the closure density, $H_0 = 72\ {\rm km\,s^{-1}\,Mpc^{-1}}$ is the present value of the Hubble parameter. The dimensionless quantity $\Omega_{\rm GW}$ is related to the strain power spectrum $S_{\rm GW}(f)$ (which has dimensions of ${\rm Hz}^{-1}$) by [85]:

$$S_{\rm GW}(f) = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{\rm GW}(f)}{f^3}$$

Since detector sensitivities and source strengths are often compared on a plot of strain amplitudes, it is useful to define $h_{\rm GW}(f) \equiv \sqrt{S_{\rm GW}(f)}$. $h_{\rm GW}$ has units of ${\rm Hz}^{-1/2}$ but it is still not the relevant quantity if we wish to understand detectability of a given stochastic background.

Stochastic signals are detected by cross-correlating the data from a network of two or more detectors over a bandwidth Δf for a time duration T. For a pair of detectors the signal-to-noise ratio of the background power spectrum grows as $\sqrt{T\Delta f}$ and so the amplitude grows as $(T\Delta f)^{1/4}$. Thus, the characteristic amplitude of a background at frequency f is[6] $h_{\rm c}(f) = h_{\rm GW}\ (T\Delta f)^{1/4}$. At 100 Hz, where ground-based detectors have their best sensitivity, the characteristic amplitude for an integration period of 1 year and bandwidth $\Delta f = f$ is

$$h_{\rm c}(f) \simeq 3.0 \times 10^{-24} \left(\frac{T\Delta f}{3\times 10^9}\right)^{1/4} \left(\frac{f}{100\ {\rm Hz}}\right)^{-3/2} \left(\frac{\Omega_{\rm GW}}{10^{-9}}\right)^{1/2}\ {\rm Hz}^{-1/2}$$

The left-hand panel of Fig. 2 plots (in dotted lines) $h_{\rm c}(f)$ for several values of $\Omega_{\rm GW}$ assumed to be independent of f. Advanced ground-based detectors should detect $\Omega_{\rm GW} \geqslant 10^{-9}$.

In the case of PTA the detection technique is essentially similar. Instead of just a pair of detectors one looks at the timing residuals of many stable millisecond pulsars to improve the sensitivity. For an integration time of 5 years and 20 millisecond pulsars, PTA could reach a sensitivity level of

[6] Some authors (see, e.g., Sesana, Vecchio and Colacino [39]) choose to divide their characteristic amplitude by the factor $(T\Delta f)^{1/4}$. This is because their characteristic amplitude is a measure of the PTA sensitivity while our characteristic amplitude refers to the signal strength. Note also that these and many other PTA authors use a dimensionless characteristic amplitude: $h_{\rm c}^{\rm PTA} = \sqrt{f}\, h_{\rm c}^{\rm Here}$.

$$h_c(f) \simeq 2.4 \times 10^{-11} \left(\frac{T\Delta f}{1.6}\right)^{1/4} \left(\frac{f}{6\ \mathrm{nHz}}\right)^{-3/2} \left(\frac{\Omega_{GW}}{2.5 \times 10^{-10}}\right)^{1/2} \mathrm{Hz}^{-1/2}$$

This corresponds to the dimensionless amplitude of $h_c^{PTA} \simeq 2 \times 10^{-15}$ at a frequency of $(5\ \mathrm{yr})^{-1} \simeq 6 \times 10^{-9}$ Hz.

4.4.1. Populations of point sources

The most certain source of stochastic gravitational wave background is the one produced by the Galactic white dwarf binary population. White dwarf binaries with orbital periods in the range of few hours to few minutes are abundant in the galaxy. The combined effect of hundreds of millions such systems is a stochastic background radiation. This white dwarf binary (WDB) background should be visible in the frequency range of 0.1 to 2 mHz in LISA (see Fig. 2) [86]. This would correspond to energy density in gravitational waves of $\Omega_{GW} = 10^{-12}$ at 1 mHz [87]. Some close white dwarf binaries, AM CVn systems and ultra compact X-ray binaries should be detectable above the confusion background of WDB as shown in [86].

Inspiralling compact binaries at cosmological distances will also cause a confusion background. Astrophysical observations guarantee the presence of two such populations: binary neutron stars, which should be observable in ground-based detectors, and binary supermassive black holes, which should be observable by PTAs. In both cases the lack of precise knowledge about the underlying population of sources and their coalescence rate as a function of redshift makes it hard to predict the precise strength of the background. Regimbau and Mandic estimate that the binary neutron star (BNS) population could produce a background strength of [88,89]:

$$\Omega_{GW}^{BNS}(f) \simeq 2.5 \times 10^{-10} \left(\frac{f}{100\ \mathrm{Hz}}\right)^{2/3}$$

This is out of reach of advanced detectors but ET should be able to quite easily detect such a background. It should be noted, however, that unlike WDB population in LISA, binary neutron star population will not cause a confusion background above the detector noise spectral density [90]. Sesana [91] considers a number of different mechanisms for the formation and evolution of supermassive black hole binaries (SMBBH) and computes a median energy density of

$$\Omega_{GW}^{SMBBH}(f) \simeq 2 \times 10^{-10} \left(\frac{f}{10\ \mathrm{nHz}}\right)^{2/3}$$

This is only a factor of about 10 in Ω (which translates to a factor of 3 in amplitude) below the current best upper limits [92] and could be reachable within the next 5 to 10 years.

Stochastic background could be produced by any population of point sources including stellar mass binary black holes, supernovae and magnetars. In most cases the uncertainty in event rates is so high that it is difficult to predict reliable estimates (for reviews see [84,88]).

4.4.2. Early Universe

Physical processes in the very early Universe can lead to a stochastic gravitational wave background. On generic grounds, we should expect that just as the electromagnetic stochastic radiation (the cosmic microwave background) was produced at the birth of the Universe, a background of gravitational radiation was also generated. While the electromagnetic radiation was in thermal equilibrium with relativistic particles for about 300,000 years after the Big Bang, gravitational waves, due to their weak interaction with matter, decouple from all particles and radiation a tiny fraction of a second after the birth of the Universe. They should, therefore, carry the signature of physical processes when quantum gravity effects were important. Observing relic gravitons from the early Universe is undoubtedly the most important goal for gravitational astronomy.

The primary mechanism for the generation of primordial gravitational waves is the parametric amplification (by the background gravitational field) of gravitational waves generated by quantum fluctuations in the inflationary era [83,93,94]. This is the same mechanism that is believed to have produced the scalar density perturbations that led to the formation of large scale structure in the Universe. The standard de Sitter inflationary model predicts that the energy density Ω_{GW} is independent of frequency for $f < f_0$, where $f_0 \simeq 10^{-16}$ Hz, raising as a power-law for $f < f_0$ [84]:

$$\Omega_{GW}^{Inflation} = \begin{cases} \Omega_0 (\frac{f}{f_0})^{-2}, & f \leqslant f_0 \\ \Omega_0, & f > f_0 \end{cases}$$

where the value of Ω_0 is uncertain and could be in the range $10^{-13} \leqslant \Omega_0 \leqslant 10^{-14}$. The transition frequency f_0 corresponds to the horizon scale at the time of matter and radiation equality redshifted to the current epoch. Gravitational waves with $f > f_0$ today were much smaller than the horizon scale at matter radiation equality and hence were not amplified. COBE observations place a bound of $\Omega_{GW} \simeq 2 \times 10^{-12}$ (for $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$). This is at a level that might be detectable by ET and LISA. The more popular slow roll inflation predicts background density two orders of magnitude smaller and hence unreachable by ground- and space-based detectors or PTA.

Many other interesting sources of stochastic background have been studied including cosmic strings, phase transitions in the early Universe and processes during the re-heating phase (see Maggiore [84] for a detailed account). Of these, cosmic strings are possibly the most interesting that could produce energy densities of order $\Omega_{GW} \sim 10^{-8}$–10^{-7} that is flat in the frequency range of 10^{-8} to 10^{10} Hz.

5. Conclusions

Heinrich Hertz was of the opinion that his experiment was of no utility whatsoever other than proving the correctness of Maxwell's theory. Today we might almost say the same about gravitational waves. We do not expect gravitational radiation to be technologically useful due to the difficulty of producing them in the laboratory. Nevertheless, observing the radiation from astronomical sources will undoubtedly help us learn a great deal about fundamental theories of nature, verify if astrophysical black holes have the properties predicted by general relativity, measure the large scale geometrical and topological properties of the Universe, and infer the very early history of the Universe by detecting or constraining the stochastic background that would have been produced in the early Universe.

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