Disordered systems / Systèmes désordonnés

Interacting Anderson insulators: The intrinsic electron glass

Isolants d'Anderson en interaction : le verre électronique intrinsèque

Zvi Ovadyahu

Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

A B S T R A C T

The out-of-equilibrium transport properties of Anderson insulators exhibit many glassy features such as slow relaxation, slow approach to a steady state, ageing, and other memory effects. These non-equilibrium features were theoretically anticipated and are believed to result from the interplay between disorder and interactions. Here we review the field-effect experiments that establish the generic nature of electron glasses. It is pointed out that the common feature to all systems that exhibit the characteristic electron-glass properties is their relatively high carrier concentration. Experiments performed as a function of the carrier concentration indicate that below a critical value, the relaxation time becomes very short. It is suggested that this may be the reason why lightly doped semiconductors do not exhibit intrinsic electron-glass effects. The possible connection between the Coulomb gap and the memory dip is discussed.

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R É S U M É

Les propriétés de transport hors équilibre des isolants d'Anderson font apparaître de nombreuses caractéristiques vitreuses: relaxation lente, approche lente vers un état stationnaire, vieillissement, et autres effets de mémoire. Ces caractéristiques hors équilibre ont été anticipées sur le plan théorique et attribuées à une compétition entre désordre et interactions. Nous passons ici en revue les expériences à effet de champ qui permettent d'établir la nature générique des verres électroniques. Nous soulignons que la caractéristique commune à tous les systèmes qui présentent des propriétés caractéristiques du verre électronique est leur concentration en porteurs de charges relativement élevée. Les expériences réalisées en fonction de la concentration en porteurs indiquent qu'en deçà d'une valeur critique, le temps de relaxation devient très court. Il est suggéré que ceci peut être la raison pour laquelle les semi-conducteurs légèrement dopés ne présentent pas d'effet intrinsèque de verre électronique. La possible connexion entre le gap de Coulomb et le *memory dip* est discutée.

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1. Introduction

The interplay between disorder and interactions in Fermi systems has been a source of fascination and controversy for more than five decades. Several seminal papers appeared during this period, starting with the ground-breaking 1958

Anderson paper [1]. The difficulty of dealing with long-range forces was already recognized in this paper. Effects of electron–electron (e–e) interactions on the Anderson insulating phase were the focus of a later study by Fleishman and Anderson [2]. In the interim period, Efros and Shklovskii [3], following Pollak [4], showed that turning on the unscreened Coulomb interaction suppresses the single-particle density of states (DOS), which vanishes at the chemical potential. This led to the concept of the Coulomb gap. New surge of interest in the disorder-induced localization was triggered by the papers of Thouless [5] and Abrahams et al. [6].

At the same time, Altshuler and Aronov [7] and Altshuler et al. [8] found intriguing quantum effects in the diffusive regime arising from e–e correlations. In clean metals, it has been traditionally assumed that screening weakens the effects of the Coulomb interaction and thus e–e effects on transport may be ignored. In the presence of static disorder, however, screening is impaired and effects of interactions can be observed even for relatively clean systems. Such effects contribute to the conductance G, magneto-conductance $\Delta G(H)$, Hall coefficient R_H, as well as to their temperature dependence. Even more dramatic is the appearance of a cusp-like-singularity in the single-particle density of states $N(\varepsilon)$. The latter feature can be observed in tunneling experiments as a characteristic dip centered at the Fermi energy [8,9]. Being a single-particle property, it is not directly related to G, which depends on $\partial n/\partial \mu$, the thermodynamic DOS. The magnitude of the cusp increases with disorder and $N(E_F)$ (E_F is the Fermi energy) eventually vanishes as the Anderson transition is approached.

Davies et al. [10], and Grünewald et al. [11], using the analogy with spin glass, suggested the possibility of an electron-glass phase precipitating in the Anderson insulating regime [12]. In the concluding remarks of their paper, Davies et al. stated: "*This glass state may also appear in the dependence of the electrical conductivity on the sample's history, with difference between samples cooled in an electric field and without.*" This 'field-cooled' vs. 'zero-field-cooled' suggestion further emphasized the conceived analogy with the spin-glass scenario.

Today, in agreement with theoretical expectations, there are quite a few disordered electronic systems that exhibit glassy effects. However, their dynamics differ from that of spin glasses, and the feature that is the hallmark of the electron glass has not been anticipated by theory. This feature, the 'memory dip' (MD), made its appearance when, as sometime happens, the researchers were looking for something else.

The original experiment was designed to distinguish between two mechanisms for magneto-conductance in the hopping regime. The sample was to be placed between two conducting plates connected to a variable voltage source V, and it was expected that the behavior of $G(V)$ will give the desired information. As it turned out, the idea behind this 'critical test' was based on our misunderstanding; it was simply wrong. However, the result was nontrivial and persistent enough not to be denied. It soon became clear that the main feature could be obtained with a much simpler setup, just by applying a voltage V_g between the sample and a conducting plate (gate) separated from it by a band insulator. In either case, there appeared a minimum in $G(V)$ or $G(V_g)$ centered at $V = 0$ or $V_g = 0$. A typical set for $G(V_g)$ for samples near the transition to strong localization is shown in Fig. 1.

The $G(V_g)$ curve of the strongly-localized sample (Fig. 1b) raises the question: '*What is special about zero V_g?*" In hindsight it may seem puzzling that the possibility of non-equilibrium as the explanation for this observation has alluded us for many months. That the anomalous field effect observed in the Anderson-localized phase is due to out-of-equilibrium transport was only halfheartedly acknowledged in the 'notes added in proof' in [13] and these began to be more systematically investigated by Ben-Chorin et al. [14] two years later.

The salient features of this anomalous field effect were slowly accumulated over the last couple of decades. It turned out that these non-equilibrium properties are not consistent with a finite-temperature glass transition. Rather, the glassy phase, much like the Anderson transition, appears to be a quantum phase transition. Moreover, in the few cases where this aspect was studied, the typical relaxation rate of the electron glass did not slow down upon cooling [15]. The electron-glass dynamics then appears to be dominated by quantum tunneling through barriers in phase space. This is in contrast with classical glasses where relaxation is controlled by over-the-barrier Boltzmann statistics. Electron glass is therefore a unique test-bed for exploring the interplay between disorder and interactions in the quantum regime.

In the following, we focus on the results obtained at a fixed temperature, typically 4 K, as this is the temperature where the majority of the experiments were made. For the same reason, the presentation will be limited to field-effect measurements, as this has been the experimental tool employed in all the systems studied to date. Most of the data shown in this paper were taken using thin films of crystalline or amorphous indium oxide and ultra-thin beryllium films. These materials have a relatively low density of states and therefore may be Anderson-localized while still physically continuous structures. Similar non-equilibrium results were obtained by several groups using various granular metals [16–18]. Some aspects that were studied in Anderson-insulator glasses, like the effect of infrared radiation [19] and non-Ohmic fields [20], are yet to be tested in the granular systems; so they are not included here.

2. The main features of the memory dip

2.1. Zero gate-voltage is not special

Our first task is to demonstrate that there is nothing special about $V_g = 0$. The reason that the minimum in the $G(V_g)$ scan in Fig. 1b is observed around $V_g = 0$ is that this was the gate-voltage the sample was allowed to equilibrate at before the scan was taken. Indeed, when the sample is equilibrated under a non-zero value, say V_g^1, the minimum in $G(V_g)$ appears at V_g^1. Furthermore, the system retains a memory of the position where V_g has been held for a certain
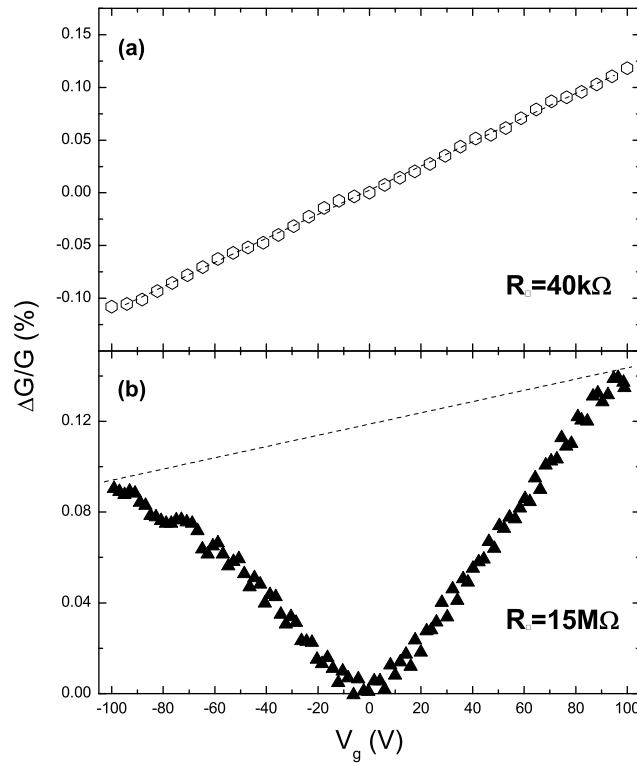
Fig. 1. The relative conductance change versus gate-voltage for amorphous indium oxide films near the transition to strong localization. The samples are labeled by their sheet-resistance at the measurement temperature ($T = 4$ K). Dashed lines depict the (average) slope of the thermodynamic field effect (see text).

duration; Fig. 2 shows $G(V_g)$ sweeps taken at different times. The top curve was taken after the sample was cooled down and allowed to relax for 24 hours while holding the gate-voltage at $V_g = 6.15$ V, showing the minimum at this cool-down voltage. After this initial scan, the gate-voltage is moved to $V_g = -6.15$ V and is kept there between subsequent sweeps. These took several minutes and several hours, respectively, after the new V_g was set, and they reveal now *two* minima; one at $V_g = -6.15$ V with magnitude increasing with time, and another minimum at the old V_g, which slowly fades away. This is the 'two-dip experiment' (TDE), which has been used to get the typical relaxation time of the system, and to demonstrate that the relaxation may be history-dependent [21]. History dependence is manifested by the relaxation law, which is also a function of t_w, the time the system 'waited' in the state previous to its excitation to another state. This type of protocol is sometime referred to as 'ageing' [21–25].

2.2. It is a non-equilibrium feature

The memory dip is peculiar to the strongly-localized regime (which in two dimensions means a sheet-resistance $R_\square > h/e^2$), and it becomes more prominent as the system gets deeper into this regime. For a given $R_\square$, the relative magnitude of the MD $\Delta G/G$ (as defined in Fig. 3) depends on how fast V_g is swept. This dependence is logarithmic, as the inset in Fig. 3 shows. In these experiments, the sample was first allowed to equilibrate for 48 h under $V_g = 0$. Then, G was recorded while the gate-voltage was swept towards $+100$ V at a certain rate, r_i. This trip took a certain time, say t_i. Next, V_g was quickly returned to 0 V and kept there for a time period of $\sim 10 \cdot t_i$ to allow the sample to regain its initial near-equilibrium state. The $G(V_g)$ of the trip to $V_g = -100$ V was recorded at the same r_i to complete the field-effect measurement at this particular rate (again followed by relaxation at $V_g = 0$ for $\sim 10 \cdot t_i$). In the series of experiments depicted in the figure, r_i was varied from 0.02 V/s to 50 V/s with the corresponding sweep times of a few seconds to several hours. With the compulsory 'between-sweeps' relaxation, this experiment is time consuming and requires a rig where a constant bath temperature is maintainable for several weeks. The piecewise manner of recording the MD differs from the one-pass across the equilibrium V_g used in Fig. 2, but the resulting MD has the same shape in either protocol.

Note that each of the curves in Fig. 3 is a superposition of an anti-symmetric part and a symmetric component (the MD). Up to a constant, the anti-symmetric part of $G(V_g)$ seems to be independent of the sweep rate. Once the scan rate is sufficiently slow, the MD cannot be distinguished from the background $G(V_g)$, which approaches the thermodynamic result. The rate below which the MD is unresolved presumably reflects the characteristic time of relaxation (or the time it takes the MD to form). Therefore, the MD will not show up in systems where the relaxation time is very short, even when
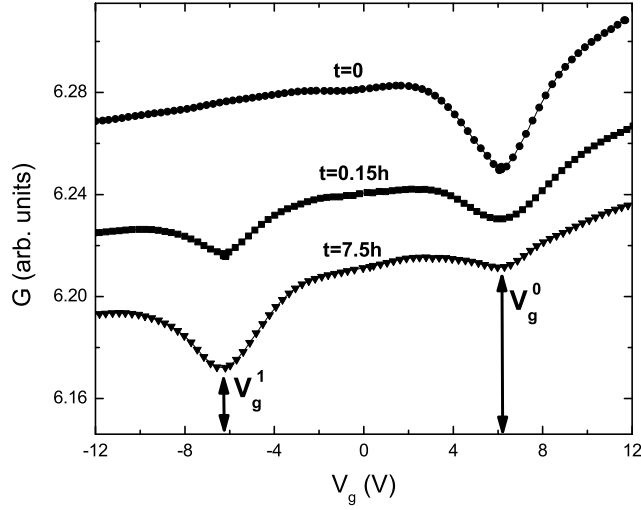
Fig. 2. Conductance versus gate-voltage for $R_\square = 25$ MΩ amorphous indium oxide film. Traces, taken at different times (see text), are displaced for clarity.
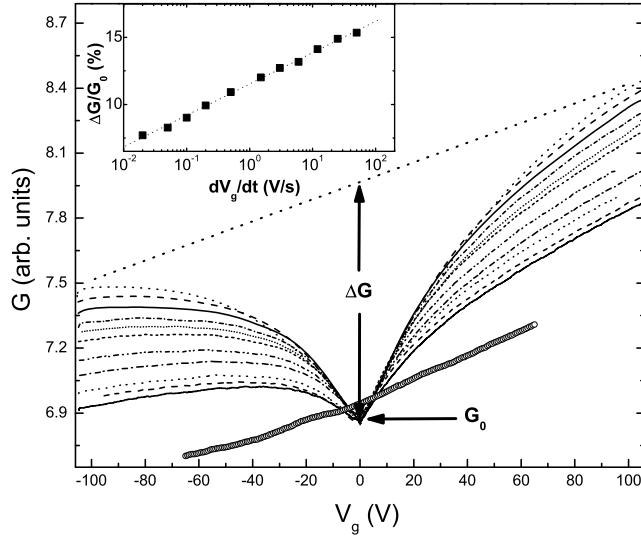

Fig. 3. Conductance as function of the gate-voltage sweep rate, showing $G(V_g)$ traces taken at different rates r_i in the range 0.02 V/s to 50 V/s. The sample is crystalline indium oxide with $R_\square = 26$ MΩ at 4 K. The bottom trace, showing a featureless $G(V_g)$ parallel to the dashed line, is the thermodynamic field effect taken with a very slow scan (about a week from side to side). The inset illustrates the logarithmic dependence of the sweep rate.

strongly localized. This appears to be the case in lightly doped semiconductors [26,27]. This point is elaborated upon in a later section.

A feature shared by all $G(V_g)$ plots shown above is their asymmetric form with respect to the gate-voltage. Each $G(V_g)$ curve can be decomposed into a symmetric component (the MD) and an anti-symmetric part. The latter part of $G(V_g)$ has apparently the same functional form (up to a sweep-rate-dependent constant) as can be seen in Fig. 4. In this figure, two $G(V_g)$ traces with extreme rates are used to illustrate that, after subtracting from each of them the anti-symmetric part, their MD may be scaled by a constant factor to show that their functional form is identical.

This anti-symmetric component of $G(V_g)$ reflects the thermodynamic field effect, which in turn, is controlled by the energy dependence of the conductance. Note that changing the gate-voltage by ΔV_g shifts the Fermi energy (cf. Fig. 5) by $\delta E_F \propto C \Delta V_g / (\frac{\partial n}{\partial \mu})$, where C is the sample-gate capacitance.

The change of conductance may be estimated from the Einstein relation: $\sigma \propto e^2 D \cdot \frac{\partial n}{\partial \mu}$, where D is the diffusivity and $\frac{\partial n}{\partial \mu}$ is the thermodynamic density of states. Both D and $\frac{\partial n}{\partial \mu}$ are functions of energy. In the insulating regime $\frac{\partial D}{\partial \varepsilon}$ has usually the sign of $\frac{\partial}{\partial \varepsilon}(\frac{\partial n}{\partial \mu})$ due to the energy dependence of the localization length. Therefore, the change in the conductance ΔG due to a change in the gate-voltage ΔV_g is essentially controlled by $\frac{\partial n}{\partial \mu}(\varepsilon)$. A common form of $\frac{\partial n}{\partial \mu}(\varepsilon)$ in the electron glasses
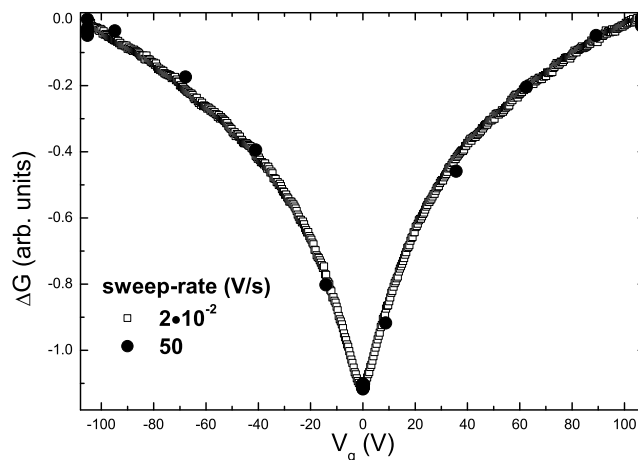
Fig. 4. The collapse of the MD data for the lowest and fastest sweep rates of the $G(V_g)$ curves in Fig. 3 after subtracting the linear part of the field effect and normalizing the amplitude by a constant factor.
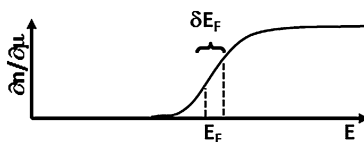

Fig. 5. Schematic illustration of the relation between the shift of the Fermi energy due to a change of the gate-voltage ΔV_g and the density of states $\frac{\partial n}{\partial \mu}$. The systems that show the MD, having relatively high electron-density (and associated relatively large E_F), the maximum $\Delta E_F / E_F$ achievable in the experiments is limited to a few percents.

studied to date is as shown in Fig. 5, where the Fermi energy is within the conduction band, but it is much smaller than the band-width. This usually makes $\Delta G / \Delta V_g > 0$, which is the case in crystalline indium oxide, thallium oxide, and in all versions of the amorphous indium oxide films. The Fermi energy in beryllium happens to lie in a region [28,29] where $\frac{\partial}{\partial \varepsilon}(\frac{\partial n}{\partial \mu}) < 0$. This is also reflected in Hall effect measurements performed on beryllium films, showing the sign of holes [30]. Consistently with these measurements, $\frac{\partial}{\partial \varepsilon}(\frac{\partial n}{\partial \mu}) < 0$ shows up in the field effect of the beryllium samples as $\Delta G / \Delta V_g < 0$ for the thermodynamic component of $G(V_g)$, as illustrated in Fig. 6.

The thermodynamic field effect in the granular systems is usually characterized by $\Delta G / \Delta V_g \approx 0$, and the MD is the dominant part of their $G(V_g)$ [17,18].

2.3. The MD has a characteristic energy width

A protocol commonly used to illustrate glassy dynamics is quench-cooling the system from high temperatures. In polymer glasses, the ensuing dynamics is monitored by measuring the sample volume. In spin glasses, it is the magnetization. The dynamics in the electron glass is reflected in the relaxation of the conductance G. It is a straightforward procedure and may be implemented without a gate being present. Measuring $G(t)$ on a sample without a gate may be accomplished almost from the start of the relaxation process while taking a $G(V_g)$ trace takes about a minute, so it is hard to record the early stages of the process. On the other hand, monitoring the process over a *range* of gate-voltages gives more information about the relaxation from an excited state than just measuring $G(t)$ at a fixed V_g (or when a gate is not present). As may be already clear from the TDE protocol (Fig. 2), the MD takes time to form once new external conditions are imposed on the system. Field-effect measurements, taken at different times, show that the relaxation process involves changes in the conductance limited to a certain range of energy around the Fermi energy (which is determined by the cool-down gate-voltage). Fig. 7a shows how the MD evolves with time after the system is quench-cooled from high temperatures.

There are several things to note here. First, immediately after the quench, the MD magnitude is barely noticeable; only the thermodynamic field effect is observed. Secondly, with time, the depression associated with the MD follows a logarithmic law; $G(V_g^x, t) \propto G_0 - a(x) \cdot \log(t)$. The simple logarithmic relaxation is the form of the history-independent relaxation peculiar to *all* the electron glasses studied to date. Thirdly, the conductance $G(V_g, t)$ depends on time only over a certain range around V_g^1, defined by: $V_g^{min} < x < V_g^{max}$, where $a(x)$ is not negligibly small. This range is one way to define the width of the MD. The width may be given in terms of energy; the translation involves the thermodynamic DOS of the system, and the gate-to-sample capacitance [31].

The MD width Γ is characteristic of the material studied. It is independent of most other parameters. For example, Fig. 7b illustrates that while the magnitude of the MD changes with time after the quench, its shape remains the same. Γ
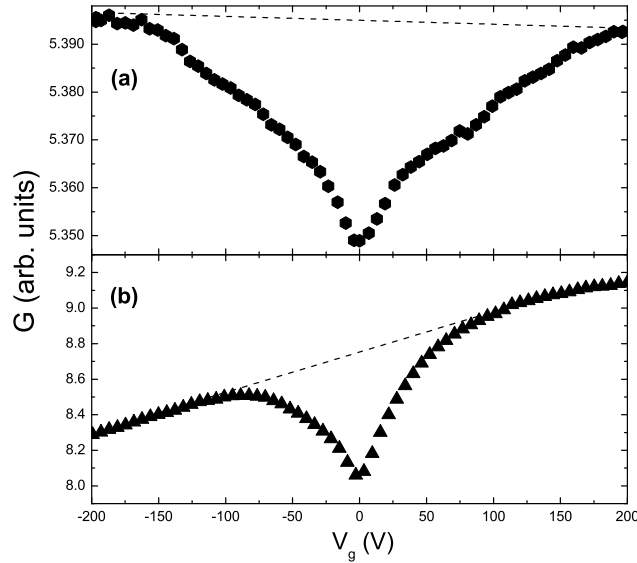
Fig. 6. The field effect for (a) a 18-Å beryllium film with a sheet-resistance of 150 MΩ and (b) a 35-Å film of crystalline indium oxide with a sheet-resistance of 120 MΩ (both at $T = 4$ K). Note the opposite slopes of the thermodynamic component of $G(V_g)$ and the difference in the typical width of the memory dip.
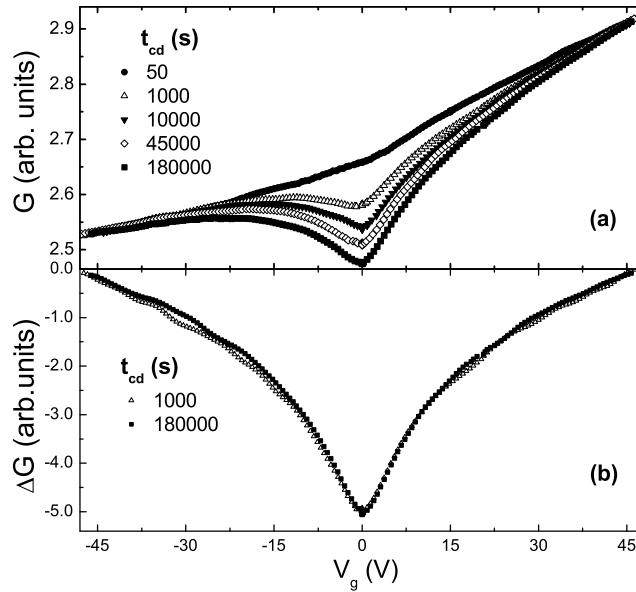

Fig. 7. Field-effect sweeps taken at several times t_{cd} (measured after cooling down the sample from $T \simeq 100$ K to $T = 4.1$ K), illustrating the MD slow evolution. The sample is a crystalline indium oxide film with $R_\square = 49$ MΩ. (b) Field-effect sweeps demonstrating that the MD for the $G(V_g)$ sweeps taken at $t_{cd} = 10^3$ s and $t_{cd} = 180,000$ have identical shape. This was achieved by subtracting from the $G(V_g)$ traces the equilibrium field effect and scaling-up the $t_{cd} = 10^3$ s MD data by a constant factor.

is also independent of measurement conditions; as illustrated in Fig. 3, the scan rate affects the magnitude of the MD, but scaling by a constant factor (demonstrated in Fig. 4) shows that the MD shape is invariant.

Most importantly, Γ is independent of the system disorder. It is natural to take the value of the resistance at a given temperature as the measure of disorder when the carrier concentration is fixed. To a good approximation, this is the case in the amorphous and crystalline indium oxide and in beryllium. Fig. 8 illustrates that while the relative magnitude of the MD depends on the sample resistance, its shape remains the same. The insensitivity of Γ to changes in the electron glass' resistance is another property common to all the systems where an independent measurement of n was made. The implication of this empirical finding will be discussed below.
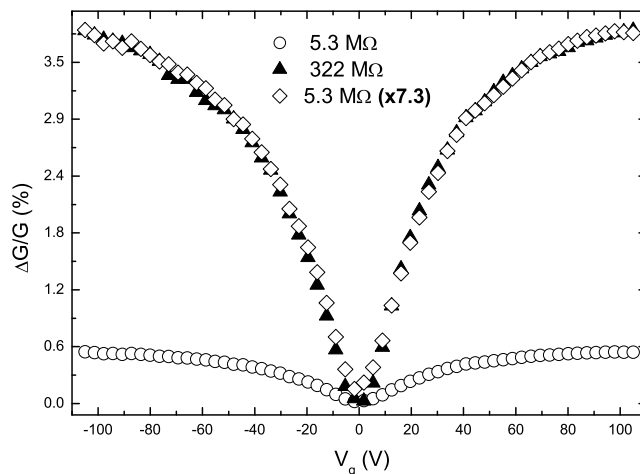
Fig. 8. The memory dips of two amorphous indium oxide samples (actually, the same physical sample and the two different resistances, by which the traces are labeled, were generated by thermal annealing from a specific batch). Note that the relative magnitude of the MD depends on the resistance, but its functional form is the same for both samples.

2.4. What controls the MD width?

The one parameter that systematically affects Γ is the carrier concentration n in the material. Fig. 9 shows $G(V_g)$ traces, taken under similar conditions, for three amorphous indium oxide films having different values of n. The carrier concentrations used here are based on Hall effect measurements at room temperatures. Note that the MD width Γ (defined on the top graph) increases with the carrier concentration. This parameter is controlled in this system by the In/O ratio, which in turn is determined during sample preparation. Unlike the crystalline version, which is subject to crystal chemistry constraints, stable amorphous films with various In/O ratios are feasible. Amorphous indium oxide films can be prepared with carrier concentrations spanning a range of almost four orders of magnitude (10^{18}–10^{22} cm^{-3}). The disorder in a given film may be varied over a considerable range by thermal annealing, a process that entails a minimal change in the carrier concentration, but may change the conductance by orders of magnitude [32].

The correlation between carrier concentration and the width of the MD is not peculiar to amorphous indium oxide – it appears to be a feature common to all intrinsic electron glasses known to date. This is illustrated in Fig. 10 for different materials.

3. Discussion

3.1. No lightly doped semiconductors?

Looking at the systems that are represented in the figure, it is natural to ask: "*Why systems like Si, GaAs, and other semi-conductors that are routinely studied in the hopping regime are absent from the list*?" To date, there is no report of a memory dip in these systems, although MOSFET-like structures in the hopping regime were extensively studied for decades. The reason cannot be that their MD is too narrow to resolve in field-effect experiments. Anderson-localized semiconductors have typically $n \leqslant 10^{18}$ cm^{-3}. Extrapolating the data in Fig. 10 to carrier concentrations typical of semiconductors, one expects the associated MD width Γ to be of the order ≈ 0.1 meV, well within experimental resolution. Therefore, the absence of MD in lightly doped semiconductors has to have another explanation.

Apparently, there is something that the systems listed in Fig. 10 share, which lightly doped semiconductors do not. The question therefore is what is the difference between the two groups.

A possible difference between traditional semiconductors and the systems that exhibit MD might be the type of disorder that causes the system to be localized. There were some attempts to account for the MD by an 'extrinsic' mechanism [33] associated with structural aspects of the sample. This approach, however, is not consistent with the experimental facts. Note first that the defects that are assumed to be the source of glassiness are also responsible for the $1/f$ noise [34]. Yet, no correlation was found between $1/f$ noise and the electron-glassy effects. In particular, $1/f$ noise is quite prominent in strongly-localized semiconductors like doped Si and doped Ge [35], while none of the electron-glass phenomena is observed in them. Secondly, the universal electron-glass properties described above can hardly be reconciled with structural details. In terms of microstructure, there are in the samples depicted in Fig. 10 representatives of all types of disordered structures; polycrystalline (In$_2$O$_{3-x}$, Tl$_2$O$_{3-x}$, and beryllium), granular or discontinuous (Al, Pb, Au), and amorphous (In$_x$O). Some of these systems contain oxide, whether as an intrinsic part of the material (In$_2$O$_{3-x}$, In$_x$O), or to stabilize a granular structure (e.g., Al). However, the discontinuous Au films, being prepared and measured under high-vacuum conditions, are oxygen free
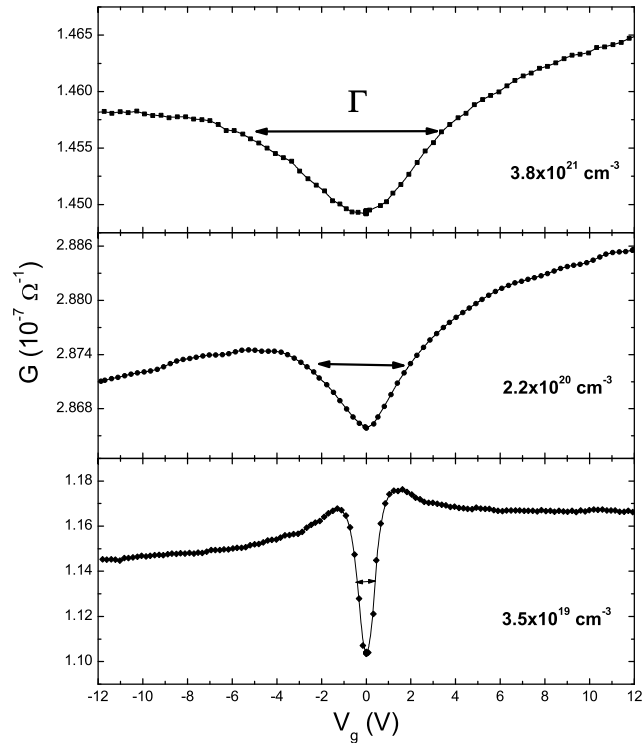
Fig. 9. The field effect for three films of amorphous indium oxide having different carrier concentrations. Note the dependence of the MD width Γ (defined here as the width at half-height of the MD amplitude and marked by the arrows) on the carrier concentration.
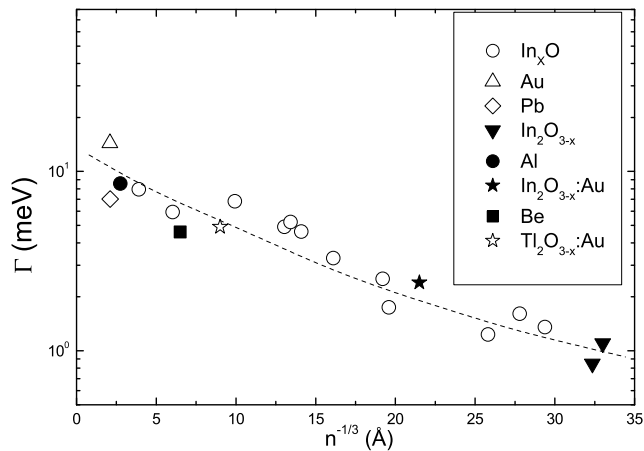

Fig. 10. The typical width of the MD for eight different materials. The carrier concentrations are based on Hall effect measurements for the amorphous and crystalline indium oxide, thallium oxide, and beryllium films. For the granular systems, n is based on the carrier density in the metal. This does not take into account the effect of the insulating part, which may account for the scatter in these data.

and exhibit the memory dip as in the other intrinsic electron glasses [18]. Finally, it is hard to reconcile the dependence of the MD width Γ on the carrier concentration in the material (while being *independent of disorder)* with structural aspects.

3.2. More is better

Other properties of the systems that exhibit electron-glass characteristics are also quite diverse; all these systems obey some form of activated conductivity, $G(T) \propto \exp[-(\frac{T_0}{T})^{\alpha}]$ with $0.3 \leqslant \alpha \leqslant 1$; however, no single value of α is singled out in the group. In other words, the conductivity versus temperature law $G(T)$ exhibited by these systems does not set this group apart from other hopping systems that do not exhibit an MD; many disordered semiconductors obey a $G(T)$ law that cannot be distinguished from one of the hopping laws above.
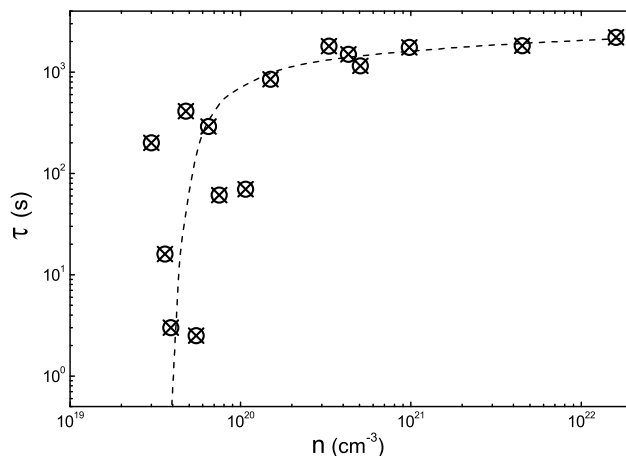
Fig. 11. The typical relaxation time, based on the two-dip experiment (see text), for a series of amorphous indium oxide. Note the sharp drop in τ for $n \leqslant 10^{20}$ cm^{-3}.

Finally, some of the MD systems are made of materials that may show superconductivity (Pb, the electron-rich version of amorphous indium oxide, Be, Al), but no superconductivity is observable in crystalline indium oxide, thallium oxide, and the low-n version of amorphous indium oxide. Superconductivity is not likely to matter anyhow as the MD is observable at temperatures far above the transition temperature of any of the materials used.

The one feature that *all* the materials that show MD share is a relatively high carrier concentration, and this is the feature that distinguishes these systems from lightly doped semiconductors.

It has been suggested [36] that the reason that the MD is not observable in lightly doped semiconductors is that their relaxation time τ is short. As discussed in connection with the data in Fig. 3, once the sweep rate becomes shorter than the typical relaxation time, the MD will not be resolved in the $G(V_g)$ trace. Observability of the MD favors long relaxation time. The large resistances and sample-gate capacitances encountered in these experiments make it practically impossible to observe the MD in a system once its relaxation time is shorter than a second. That low carrier concentration may result in very short relaxation times may be inferred from the experiments discussed next.

The relaxation time τ of amorphous indium oxide was measured in a series of samples as a function of their carrier concentrations. This was based on the two-dip protocol (an example is illustrated in Fig. 2), where τ was defined as the time the 'old dip' magnitude equals that of the newly formed dip. This procedure is meaningful only when the dynamics of the old dip differs from the dynamics of the new dip, as is the case in In$_2$O$_{3-x}$ [21]. In granular aluminum, however, the dynamics of the old and new dips seems to be identical, and it was demonstrated that the τ derived from the TDE depends logarithmically on the equilibration time, which appears as a saturation for long relaxation times [37].

The In$_x$O samples used in the experiments presented in Fig. 11 were heat treated to keep their resistances at the measurements temperature approximately constant while having different carrier concentrations. The results of these experiments are summarized in Fig. 11. Note that for amorphous indium oxide films, once the carrier concentration falls below $\approx 10^{20}$ cm^{-3}, the relaxation time decreases *extremely* fast. This is where the magnitude of the old dip diminishes considerably faster than the build-up of the new dip.

The carrier concentration is probably not the only relevant parameter for τ, but the sharp dependence on n suggests that it is a dominant one. Extrapolating to densities typical of semiconductors in their strongly-localized regime (as done above for Γ), it is not unlikely that the associated τ is shorter than a second, perhaps much shorter. This could account for the absence of a memory dip in field-effect experiments of lightly doped semiconductors. The conjecture that in these systems τ is very short is supported by a recent experiment on Anderson-localized phosphorous-doped silicon, where ultra-fast relaxation was found [27] consistent with the trend observed in Fig. 11. On the other hand, a corollary of this conjecture is that a substance of metallic density, once strongly localized, should show the MD and all the associated electron-glass properties (assuming of course that disorder and interactions are comparable in magnitude and the temperature is low enough). Every system composed of a material with $n \geqslant 10^{20}$ cm^{-3} that was tested so far confirmed this expectation.

3.3. The relation between the memory dip and the Coulomb gap

A natural candidate to consider as the origin of the memory dip is the Coulomb gap. The Coulomb gap (CG), first discussed by Pollak [4] and Srinivasan [38], is a consequence of the loss of screening in the disordered medium, thus exposing the long-range nature of the Coulomb interaction. Two main features of the CG make this approach promising. First, the CG is a depression in the single-particle density of states $N(\varepsilon)$, with its minimum pinned to the Fermi energy E_F. This immediately accounts for the observation that the MD is centered at the gate-voltage at which the system equilibrated.

Secondly, the correlation between the MD width and carrier concentration (Fig. 7) is in full accord with the CG scenario; it is expected that the energy scale due to interactions will be larger in systems with higher carrier concentrations.

The possible relevance of the Coulomb gap to the non-equilibrium features observed in the intrinsic electron glasses was first discussed by Clare Yu [39,40] and by Lebanon and Müller [41,42]. Yu has adopted the Efros–Shklovskii approach according to which the Coulomb gap determines the system conductance. This allowed her to reproduce the evolution with time of the MD following a quench from high temperatures [39]. Lebanon and Müller, based on similar ideas, developed a model that accounts for many of the observed electron-glass features. In particular, they gave a plausible interpretation to the temperature dependence of the MD shape [41]. This is one of the features that distinguishes the MD of the electron-glass from the cusp that appears in the structural two-level system [43,44]; the shape of the cusp in the structural two-level system does not change with temperature, whereas the MD's shape is very sensitive to temperature [22].

The consistency of these theoretical approaches with the main features of the experiments is encouraging. It does look as a step in the right direction. At the same time, there are questions of fundamental nature that should be addressed. The first question is the subtle issue of the connection between a single-particle DOS and any equilibrium (or near-equilibrium) quantity like conductivity. Patrick Lee has commented on this issue with regard to the Thomas–Fermi screening length and the use of the Einstein relation [45]. He correctly pointed out that in both cases it is the thermodynamic DOS $\frac{\partial n}{\partial \mu}$ that is the relevant quantity. The single-particle DOS is relevant for processes associated with insertion (removal) of a particle into (from) the system, while no relaxation of other particles takes place (such as tunneling or photoemission). The field effect is usually conceived in the same category as conductance in this respect.

In the current situation, we are in an uncharted territory; the process of sweeping the gate-voltage involves inserting (removing) particles, while "*some* fast things have happened but *some* slow things have not", which means: *things are happening even as we sweep*. Under these non-equilibrium conditions, it is not clear whether the use of the Einstein relation is justified. One may argue that if the sweep duration is short relative to the relaxation time (yet keeping $\Delta V_g / \Delta t$ small enough to minimize the risk of structural changes that may be caused by the sweep such as displacing charged defects in the insulating layer separating the sample from the gate), the situation is no different than the near-equilibrium conditions encountered in real-life situations. At least with respect to the anti-symmetric part of the field effect, this argument is in agreement with experiments. Nonetheless, this conjecture ought to be justified by a proper theory. It is a challenge to theory to justify the use of equilibrium ideas to account for the MD appearance in the field-effect experiments in these situations.

Even if it is the single-particle DOS that modulates $G(V_g)$, it is not clear what functional form should be taken for it. The problem is not only the multitude of forms derived for the Coulomb gap by different authors [46–52], it is rather the basic model used; essentially all Coulomb-gap models assume *single* occupation of sites, while the one common features of all the systems that exhibit MD is that their localized states contain *many* particles. There were several attempts to include quantum effects to assess their modifications to the Coulomb gap [53–57]. There were also some attempts to consider double-occupation of the localized states that already introduced new effects that are manifested in magneto-conductance measurements [58]. However, to our knowledge, no treatment of the Coulomb gap for the multi-occupation scenario has been attempted.

3.4. Quantum effects

There are by now eight different systems that show a memory dip with a characteristic width determined by the carrier concentration in the sample, and logarithmic conductance relaxation from an excited state. The common, out-of-equilibrium features that all these electron glasses exhibit are suggestive of a generic mechanism of intrinsic nature. Another attribute of this glass that makes its study even more appealing is the quantum nature of its dynamics.

Glasses in general are characterized by a slow approach to equilibrium, a process often described as a journey through a phase space composed of many local energy minima that are separated by barriers [59]. Motion through this phase space is hindered by these barriers; the system is trapped for a relatively long time in one of the local minima before it manages to cross a barrier and move to another metastable state. At finite temperatures, crossing a barrier is usually assumed to be controlled by thermal activation, as seems to be the case in most glasses. Alternatively, the barriers may be crossed by quantum-mechanical tunneling (which occurs simultaneously with a change in the phonon occupation, to get a real transition) and this is the dominant mechanism at sufficiently low temperatures. The quantum-mechanical regime may be distinguished from the thermally activated regime by the dependence of the glassy dynamics on temperature. Dynamics in the classical glass speeds up as the temperature increases, and this feature is observed in many systems. The situation in the quantum scenario is less clear. It is commonly believed that, as long as the barrier does not change, tunneling is temperature independent. However, in a dissipative environment, which seems a relevant consideration for the situation at hand [60], the tunneling rate, γ, may depend on temperature; γ may increase or decrease with T, depending on the interplay between the barrier parameters, the coupling of the tunneling object to the environment, and the temperature range [60]. Experiments on indium oxide films show nontrivial dynamics; over some range of temperatures and sample parameters, the dynamics actually slowed down with temperature [15]. Such a behavior reflects the dynamics expected of a quantum glass where diffusion in phase space is controlled by tunneling in the presence of dissipation.

Coupling to the environment is known to have nontrivial effects on the dynamics of quantum systems. This has been discussed in the context of a two-state system coupled with an oscillator bath [61]. Much less is understood in real systems,

most notably in systems that contain many, and possibly strongly interacting two-state systems [60]. The dynamics of crystalline indium oxide films were found to exhibit some surprising similarities with the behavior expected of a single two-state system coupled with an electronic bath [15]. These include an unusual temperature dependence of dynamics, and a strong suppression of relaxation rates caused by coupling with a dissipative bath. These results may indicate the relevance of the Anderson orthogonality catastrophe [62–64] to the slow relaxation in electron glasses. In these films, the carrier concentration of the system plays a unique role in determining the temperature dependence of its dynamics as well as its "viscosity".

3.5. Summary

We reviewed the results of field-effect measurements on out-of-equilibrium Anderson insulators. The central feature in these experiments is the appearance of a memory dip that is the identifying signature of the electron glass. Another distinct attribute of this group is the logarithmic relaxation law. This unambiguous law, obeyed by all electron glasses, including the granular systems, makes this glass unique among all other groups of glasses.

It is remarkable that the same glassy effects are observed over a large range of parameters and many orders of magnitude in resistance. This suggests that interaction and disorder in the insulating regime are interdependent; the glassy phase results from a competition that is most effective when the disorder strength is comparable to that of the interaction. It would then be interesting to see what happens when the diffusive regime is approached from the insulating side where the interaction may be greatly weakened due to the divergence of the localization length, while the resistance may be made large enough by going to low temperatures. Similarly, the interaction may be much weaker than disorder in systems with unusually large dielectric constants.

A common feature shared by the systems that exhibit intrinsic electron-glass behavior is high carrier concentration. This is what distinguishes them from the lightly doped semiconductors. Empirically, Anderson insulators with higher carrier concentration tend to exhibit long relaxation times. This observation led to the conjecture that the absence of lightly doped semiconductors from the group of electron glasses is their short relaxation time.

To further test the empirical connection between carrier concentration and relaxation time, it is desirable to include more materials in the experimental study. Both low and high carrier concentration should be tested. Systems that look promising to show electron-glass features include high-Tc compounds, and transition metal semiconductor alloys, provided that they are sufficiently deep into the hopping regime. On the theoretical side, there is the challenge of dealing with the consequences to hopping transport due to multi-occupation of electronic states, and the connection between the single-particle DOS and conductivity in non-equilibrium situations.

Acknowledgements

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