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Maxwell: A new vision of the world

Maxwell : Une nouvelle vision du monde

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ABSTRACT

The paper outlines the crucial contributions of James Clerk Maxwell to Physics and more generally to our vision of the world. He achieved 150 years ago a synthesis of the pioneering works in magnetostatics, electrostatics, induction and, by introducing the notion of displacement current, gave birth to Electromagnetics. Then, he deduced the existence of electromagnetic waves and identified light as one of them.

Maxwell equations deeply changed a Newtonian conception of the world based on particle interactions by pointing out the vital role of waves in physics. This new conception had a strong influence on the development of quantum physics. Finally, the invariance of light velocity in Galilean frames led to Lorentz transformations, a key step toward the theory of relativity.

RÉSUMÉ

Nous résumons les contributions cruciales de James Clerk Maxwell à la Physique et plus généralement à notre vision du monde. Tout d'abord, il a effectué, il y a cent cinquante ans, une synthèse des travaux antérieurs en magnétostatique, en électrostatique, en induction et, en introduisant le concept de courant de déplacement, il a donné naissance à l'Électromagnétisme. Il en a déduit l'existence des ondes électromagnétiques et a identifié la lumière comme l'une d'entre elles.

Par ailleurs, les équations de Maxwell ont profondément changé une conception du monde newtonienne basée sur l'interaction entre particules en révélant le rôle essentiel des ondes en physique, ce qui eut une influence déterminante sur le développement de la physique quantique. Enfin, l'invariance de la vitesse de la lumière dans les repères galiléens a entraîné la découverte des transformations de Lorentz, une étape capitale vers la théorie de la relativité.

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1. Introduction

Even though his name cannot be considered as a household name like Einstein or Newton, Maxwell is one of the few scientists who deeply changed our conception of the world. Firstly we will outline his life and his career in the frame of Victorian time. Then, Maxwell contributions to Electromagnetics will be described: synthesis of earlier works on electrostatics, magnetostatics or induction, introduction of the displacement current, discovery of electromagnetic waves, identification of light as one of these waves, prediction of the existence of other kinds of electromagnetic waves.

The developments of some domains of Electromagnetics that had strong consequences in Science, Technology and Industry will be outlined. Finally, we will explain why Maxwell's equations are considered as a crucial step toward the two revolutions of Physics that followed: theory of relativity and quantum physics.

2. Maxwell: a scientist in the Victorian time

2.1. Life

James Clerk Maxwell was born in Edinburgh, Scotland, in 1831. His father, John Clerk, was a lawyer. He shortly adopted the additional surname Maxwell. His mother, who played a crucial role in his early education, died when he was 8 years old and then a tutor was engaged by his father.

After brilliant studies in Edinburgh and Cambridge universities, Maxwell became Professor of Physics and Astronomy at King's College (London) in 1860. Most of his scientific discoveries were achieved during the 5 years time in this university. First, he had the opportunity to discuss with Michael Faraday, then in 1861–1862, he published the eponymous equations [1–4]. From these equations, he discovered in 1864–1865 the existence of electromagnetic waves and calculated their speed with a precision of 5%. In 1965, comparing the speed of these waves to the speed of light measured by the French physicist Hippolyte Fizeau [5], he concluded that light was made of electromagnetic waves. He predicted that electromagnetic waves having other frequencies should exist, a fact confirmed in 1888 by the German physicist Heinrich Hertz who evidenced the existence of radio waves [6]. In 1871, Maxwell became the first Cavendish Professor of Physics. He designed and headed the world famous Cavendish Laboratory. He died in 1879 at the age of 48.

2.2. The Victorian era

The first publication of Maxwell's equations coincides with the unification of Italy and the outbreak of the civil war in the USA. In Britain, the Victorian era included enormous political, scientific, social, and cultural innovation and change, many of them being consequences of the industrial revolution. It was a long period of prosperity, at least before the first crisis of capitalism from 1873 to 1895. This crisis had consequences on scientists, as shown further. In the domain of natural sciences, the theory of evolution was published by Charles Darwin in his book *On the origin of Species* in 1859. Authors like Charles Dickens, Arthur Conan Doyle, the Brontë sisters, Lewis Carroll, Oscar Wilde, George Bernard Show, Robert Louis Stevenson, H.G. Wells created a body of literature that still fascinates readers nowadays.

3. Maxwell's equations: the pioneers and the Maxwell contribution

The papers by Maxwell published in 1861–1862 included 20 equations [1–4], given in an integral form. This number was reduced to 8 in 1865 [7]. These works were gathered into a single paper in 1973 [8]. In fact, the modern form, including four differential equations was given by Heaviside in 1884. Among these equations, three were given previously by pioneers of electromagnetics. On the other hand, Maxwell introduced in Ampère's equation of magnetostatics a new term, the displacement current, which added to the electric current. It will be seen that this new term played a vital role in the birth of Electromagnetics, especially for the discovery of the existence of electromagnetic waves. We briefly outline the origin of Maxwell's equations.

- Maxwell–Gauss magnetic equation:

$$\nabla \cdot \vec{B} = 0 \tag{1}$$

  This equation was discovered (but not published) by the German physicist Carl Friedrich Gauss in 1835. It is sometimes classified as Maxwell–Thomson equation since the physicist of Irish origin William Thomson (Baron Kelvin in 1892) published it in 1840. The field $\vec{B}$ is sometimes called magnetic field, but nowadays this name is generally given to $\vec{H}$ (otherwise termed as magnetic field strength), $\vec{B}$ being classified as the magnetic flux density. The physical meaning of this equation is that magnetic monopoles do not exist. It is the only Maxwell equation to be sometimes contested by some scientists that assert that they have evidenced magnetic monopoles. The interested reader can refer to [9], a paper that shows the existence of magnetic monopoles in spin ice.
- Maxwell–Gauss electric equation:

$$\nabla \cdot \vec{D} = \rho \tag{2}$$

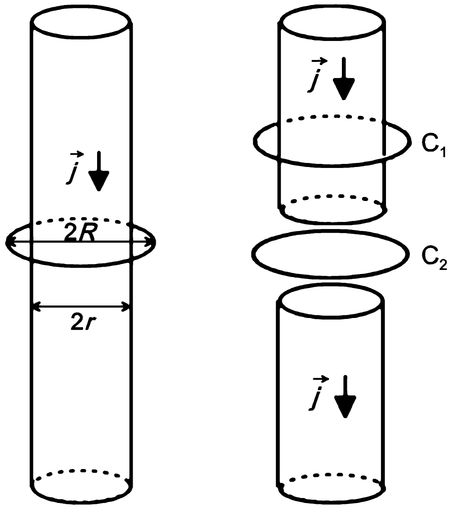

Fig. 1. Ampère law applied to a circular cylinder. Left: continuous cylinder. Right: truncated cylinder.

with $\vec{D}$ being the electric displacement and ρ being the electric charge density. This equation was given by Gauss in 1835. It shows that electric monopoles exist, namely the electric charges.

– Maxwell–Faraday equation:

$$\nabla \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

with $\vec{E}$ being the electric field. It was published by the English physicist Michael Faraday in 1831. It means that a time-variation of the magnetic field creates an electric field.

– Maxwell–Ampère equation:

$$\nabla \cdot \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \tag{4}$$

with $\vec{j}$ being the volume current density. The right-hand-side member of this equation includes two terms. The first one ($\vec{j}$) was included in the law published by the French physicist Jean-Marie Ampère in 1826. On the other hand, the second term, $\frac{\partial \vec{D}}{\partial t}$, the displacement current, was added by Maxwell. Thanks to this new term, Maxwell discovered that a time variation of the electric field creates a magnetic field. Thus, time variations of electric and magnetic fields can support each other. Maxwell deduced from this vital remark the existence of electromagnetic waves. More generally, it must be pointed out that, even though the strong link between the electric and magnetic fields was already shown by the Maxwell–Faraday equation, the symmetric form of Maxwell equations brought by the displacement current has evidenced the fundamental concept of electromagnetic field, in which electric and magnetic fields are gathered together. This breakthrough has unified the domains of electrostatics and magnetostatics.

In 1849, Fizeau was the first to measure the speed of light using a light beam reflected from a mirror 8 km away [5]. The beam propagated through the gaps between the teeth of a rapidly rotating wheel. Increasing the speed of the wheel, Fizeau was able to show that for a critical speed, the returned reflected beam passed through the following gap and could be observed. The speed of light given by Fizeau was 315,000 km/s, which was within about 5% of the exact value. Maxwell was able to calculate the speed of electromagnetic waves from his equations with a similar precision. He noticed that the measured speed of light and the calculated speed of electromagnetic waves coincided to within the experimental and theoretical precisions. He deduced in 1865 that light was made of electromagnetic waves. He also conjectured the existence of other kinds of electromagnetic waves having other frequencies. This guess was confirmed by the German physicist Heinrich Rudolf Hertz in 1988 with the detection of radio waves.

3.1. Why a displacement current?

The displacement current was introduced by Maxwell in order to eliminate the incoherence of the Ampère law of magnetostatics when applied to time-dependent fields. Here we give an example of this incoherence.

The left-hand side of Fig. 1 represents an infinite circular metallic cylinder with radius r. We suppose that a uniform volume current density $\vec{j}$ is running through this cylinder. We apply the Ampère law to the circle of radius R centered on the cylinder axis. This law entails that the path integral of the magnetic field around the circle is equal to the total current crossing the enclosed surface. Since the magnetic field $\vec{H}$ is parallel to the circle, the integral path is equal to $2\pi RH$ while the total current is equal to $\pi r^2 j$ (with H and j modulus of $\vec{H}$ and $\vec{j}$ respectively). It can be deduced that:

$$H = \frac{jr^2}{2R} \tag{5}$$

Let us suppose that at $t = 0$, this cylinder is truncated (right-hand side of Fig. 1), the same volume current density $\vec{j}$ being maintained in the two parts of the metallic cylinder. If Ampère's law is applied to the circle C_1 placed around the upper half-cylinder, the same value of H is obtained. On the other hand, if the path C_2 placed around the gap is considered, the total intensity crossing the enclosed surface vanishes, thus the calculated magnetic field vanishes as well. Since it is difficult to believe that the magnetic field is discontinuous on both sides of the gap, one can try to determinate what physical parameter could replace the total surface current density for path C_2. With this aim, it must be noticed that this problem is not a static one. Indeed, due to the current running through the two half-cylinders, the upper and lower sides of the gap receive positive and negative charges, respectively, in such a way that the gap is nothing else than a capacitor with section $S = \pi r^2$ and with linearly increasing or decreasing charges $\pm Q = \pm jSt$ on both sides of the gap. Assuming that the gap is small, one can consider that the electric field inside this capacitor is uniform, parallel to the cylinder axis and restricted to the cylinder section. The component D along the cylinder axis of the electric displacement is equal to the surface charge density $\sigma = Q/S = jt$:

$$D = \sigma = jt \tag{6}$$

Thus:

$$\frac{\partial D}{\partial t} = j \tag{7}$$

The obvious remark to be made is that for path C_2, the enclosed displacement current $\frac{\partial D}{\partial t}$ has replaced the volume current density j inside C_1. The Maxwell–Ampère equation leads to the continuity of the magnetic field, in contrast with Ampère's equation.

A second remark can be made for understanding the need for the displacement current. Taking the divergence of Maxwell–Ampère equation and bearing in mind that vanishment of the divergence of a curl yields:

$$0 = \nabla \cdot \vec{j} + \frac{\partial \nabla . \vec{D}}{\partial t} \tag{8}$$

Using the Maxwell–Gauss electric equation to express $\nabla . \vec{D}$, we obtain:

$$0 = \nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} \tag{9}$$

which is the classical theorem of electric charge conservation. If the displacement current is canceled in Maxwell–Ampère equation, the term $\frac{\partial \rho}{\partial t}$ disappears and the charge conservation is no more satisfied.

3.2. *Constitutive relations and Lorentz force equation*

The solution of problems of electromagnetics needs the use of constitutive relations, which establish the links between the electromagnetic fields. These constitutive relations depend on the material in which the electromagnetic field propagates and can take approximate and complicated forms, for example in non-linear or chiral media. However, in vacuum, constitutive relations are rigorous and quite simple:

$$\vec{D} = \varepsilon_0 \vec{E} \tag{10}$$

$$\vec{B} = \mu_0 \vec{H} \tag{11}$$

Using these relations, a problem of Electromagnetics reduces to the determination of two fields only: $\vec{E}$ and $\vec{H}$. A last equation must be included in the set of elementary laws of electromagnetics: the Lorentz force equation which express the force $\vec{F}$ on a particle of charge q traveling an electromagnetic field with speed $\vec{v}$:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{12}$$

4. The first developments of Electromagnetism

4.1. *Radioelectricity*

After studying Maxwell's equations, Hertz tried between 1886 and 1870 to conduct experiments in order to validate the existence of electromagnetic waves. He was able to detect spark-gap radio waves using another spark gap located at short distance acting as an unpowered antenna. In order to show that radio waves behave like light, he focused radio waves using a parabolic reflector. Amazingly, he was not convinced that the discovery of radio waves was important and, asked about the practical consequences of these waves, he answered "I do not think that the wireless waves will have any practical application".

In 1890, the French physicist Édouard Branly made the coherer, a first form of sensitive radio-signal detector. It is made of a tube containing two electrodes placed close to each other, with metal filings between them. When a radio-frequency signal illuminates the device, the high resistance of the filings reduces, allowing an electric current to flow between the electrodes. The reason for the strong increase of conductivity of the filings is that electromotive forces are generated by the radio waves, that bring the filings more closely together. The coherer was a widespread tool to detect radio signals in wireless telegraphy and radio reception until 1910. Then, it was replaced by more sensitive electrolytic or crystal detectors.

In 1893, the Serbo-American physicist Nikola Tesla gave at St Louis, Missouri, the first public demonstration of radio waves transmission.

The Russian physicist Alexander Popov built in 1894–1995 the first stable radio receiver by improving the coherer along the lines suggested by the English physicist Oliver Lodge. On May 7, 1895—the day celebrated in Russia as "Radio Day"—Popov gave a public demonstration of transmission and reception of radio waves.

The Italian physicist Guglielmo Marconi developed electric wave telegraphy in Britain, in such a way that at the end of 1898, the utility of radio waves devices was fully acknowledged, especially for communication between ships or between ship and shore. In 1899, he transmitted messages between France and England. Marconi and the German physicist Karl Ferdinand Braun (who introduced a closed tuned circuit in the generating part of the transmitter) received the 1909 Nobel Prize in Physics for their contributions to the radio sciences.

4.2. Electromagnetic optics

In contrast with radio electricity, the first developments of electromagnetic optics have been achieved almost in totality by a single scientist. The English physicist John William Strutt was born in 1842. He was the son of John James Strutt, second Baron Rayleigh, in a family with little interest in science since they were mostly landowners with interests in the countryside. At the end of his studies in 1865, Strutt decided to apply Mathematics to Optics and more generally to the domain of waves. With this aim, he established his laboratory at the family home.

In 1871 he published the well-known theory of light scattering by small particles, now classified as Rayleigh scattering [10]. Using this theory, he explained the blue color of the sky. Rayleigh scattering is caused by the electric polarizability of the particles. The sinusoidal electric field of light acts on the charges within a molecule. Consequently, they move at the same frequency and therefore the molecule becomes a small radiating dipole that scatters light. Rayleigh scattering of sunlight in the atmosphere generates diffuse light. Shorter (blue) wavelengths are scattered more strongly than medium and longer wavelengths. Thus, the indirect light coming from all regions of the sky is blue while the direct light (the sun itself) has the complementary color. The blue color of eyes can be explained by similar considerations.

In 1872, Strutt had an attack of rheumatic fever and was advised to travel to Egypt. The trip on the Nile river was a very profitable trip for his health but also from a scientific point of view. He wrote a quite remarkable text, *The Theory of Sound*, while on the trip [11]. Upon the death of his father in 1873, Strutt inherited the family title and became the third Baron Rayleigh. In 1979, due to the decrease of farm incomes caused by the economic crisis, he asked for a position and was appointed to follow Maxwell as Professor of Experimental Physics and Head of the Cavendish Laboratory at Cambridge. After 1885, he spent the rest of his life working at family home. His discovery of argon in 1895 earned him a Nobel Prize in 1904.

In 1907, Rayleigh published the electromagnetic theory of diffraction gratings [12]. This work is based on the so-called Rayleigh hypothesis, that remained unquestioned for almost 50 years but provoked considerable controversy thereafter. Indeed, the implementation of the Rayleigh theory on the first powerful computers led to unsatisfactory numerical results, except for shallow gratings. Nevertheless, this theory is considered as the first achievement of modern electromagnetic optics.

5. Contribution to modern physics

5.1. Contribution to special relativity

In order to establish the existence of electromagnetic waves, one can use Maxwell's equations and constitutive relations. In vacuum, a straightforward calculation leads to the wave equation:

$$\nabla \cdot \nabla \cdot \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{13}$$

From this wave equation, one can deduce that the light speed c is given by:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{14}$$

If we assume that Maxwell's equations and constitutive relations do not depend on the Galilean frame of the observer, we are led to the conclusion that the speed of light is a constant. This crucial property of light was evidenced by the American physicists Michelson and Morley in 1887, using a sophisticated interference device [13].

However, this conclusion was in contradiction with the classical rules of change of the Galilean reference frame: if the speed of light in a given frame is equal to c, it is equal to $c - v$ for an observer traveling at speed v (in the same direction as the light speed) in this frame. The Dutch mathematician and physicist Hendrik Antoon Lorentz published his coordinate transformations in 1899 [14] in order to guarantee the constancy of the speed of light in a change of Galilean frame. According to Lorentz himself, the mathematical derivation of these transformations was not completely satisfactory. The French mathematician and physicist Henry Poincaré modified the algebraic formulations, introduced the famous "Poincaré group" and attributed the transformations to Lorentz with the name "Lorentz transformations" in 1905 [15].

Poincaré's formulation of the transformation was almost identical to that published by the German born American physicist Albert Einstein later in the same year in his theory of special relativity [16]. The role of Lorentz, Poincaré (who was the first to use the word "relativity") and Einstein in the development of special relativity has been (and remains) the subject of strong controversies. Anyway, the Lorentz transformations have been a crucial step towards special relativity and this step was a direct consequence of Maxwell's equations.

5.2. Contribution to quantum physics

"We can say: Before Maxwell people thought of physical reality—in so far as it represented events in nature—as material points, whose changes consist only in motions which are subject to total differential equations. After Maxwell they thought of physical reality as represented by continuous fields, not mechanically explicable, which are subject to partial differential equations. This change in the conception of reality is the most profound and the most fruitful that physics has experienced since Newton" claimed Einstein in 1931, celebrating the centenary of Maxwell's birth.

Einstein's appreciation points out the crucial role played by Maxwell in the new vision of reality brought by quantum physics. According to Schrödinger himself, his eponymous equation was derived by taking the Helmholtz equation ($\nabla^2 F + k^2 F = 0$, combination of Maxwell's equations) as a model. The mathematical forms of Helmholtz and Schrödinger equations are very close to each other, which explains why theorems or phenomena like Heisenberg uncertainty principle or tunneling effect, for example, can take similar forms in Electromagnetism. However, it must be recalled that the use of quantum optics may be needed in problems of Electromagnetism in which the size of the objects has the same order of magnitude as the wavelength of the light, especially in nanophotonics.

6. Conclusion

Maxwell's name is not world-famous in the non-scientific world. Nevertheless, his work remains among the most fruitful scientific achievements of all time. Nowadays, the communication society stems widely from the developments of Electromagnetics and Optics enabled by Maxwell's work like television, radio, optical and satellite communications, high-speed internet connections, X-rays.... His contributions to the human progress should be considered to be of the same importance as those of his celebrated peers Isaac Newton and Albert Einstein.

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