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Electromagnetism in a strongly stratified plasma showing an unexpected effect of the Debye shielding

Électromagnétisme dans les plasmas fortement stratifiés : mise en évidence d'un effet inattendu de l'écrantage de Debye

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ABSTRACT

In the literature, we found 15 references showing, without exception, that the sunspot photospheric magnetic field vertical gradient is on the order of 3–4 G/km, with field strength decreasing with height, whereas the horizontal gradient is nine times weaker, on the order of 0.4–0.5 G/km. This is confirmed by our recent THÉMIS observations. As a consequence, the vanishing of $\mathrm{div}\,\vec{B}$ is not realized, and the present paper is devoted to the investigation of this problem. We point out that the photosphere is a strongly stratified plasma, having different horizontal and vertical characteristic lengths of aspect ratio 1/9 as the different terms contributing to the observed $\mathrm{div}\,\vec{B}$. The velocities are also anisotropic under the stratification effect. As a consequence, the Debye volume is a flattened sphere. We show that in this case $\mathrm{div}\,\vec{B}$ may mathematically depart from zero. Anisotropic shielding constitutes an alternative to the existence of monopoles for being responsible for non-zero $\mathrm{div}\,\vec{B}$. We evaluate that the solar corona is conversely not a strongly stratified plasma, so that the conditions are very different there.

RÉSUMÉ

Nous avons trouvé dans la littérature 15 références montrant sans exception que, dans la photosphère des taches solaires, le gradient vertical du champ magnétique est de 3–4 G/km, l'intensité du champ décroissant avec l'altitude, tandis que le gradient horizontal est neuf fois plus faible, de l'ordre de 0.4–0.5 G/km. Nos récentes observations THÉMIS confirment ces résultats, dont la conséquence est que l'annulation de $\mathrm{div}\,\vec{B}$ n'est pas réalisée. Cet article est consacré à l'étude de ce problème. Nous faisons remarquer que la photosphère est un plasma fortement stratifié. Les longueurs caractéristiques horizontale et verticale sont différentes, dans le rapport d'aspect 1/9, de même que les différents termes qui contribuent à la valeur observée de $\mathrm{div}\,\vec{B}$. Les vitesses aussi sont anisotropes, et font que le volume de Debye est une sphère aplatie. Nous montrons que, dans ce cas, $\mathrm{div}\,\vec{B}$ peut mathématiquement être non nul. L'écrantage anisotrope constitue une alternative à l'existence de monopôles magnétiques pour expliquer la non-nullité de $\mathrm{div}\,\vec{B}$. Nous

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montrons qu'au contraire la couronne solaire n'est pas un plasma fortement stratifié et que les conditions y sont très différentes.

1. Introduction

Analyzing spectropolarimetric observations of a double sunspot obtained with the French–Italian solar telescope THÉMIS, in terms of photospheric magnetic fields, we were faced with the evaluation of $\operatorname{div}\vec{B}$, for which zero was not obtained because we found the vertical gradient $|\partial B_Z/\partial Z|$ on the order of 3 G/km, whereas 0.4 G/km was obtained for the horizontal gradient $|\partial B_X/\partial X + \partial B_Y/\partial Y|$. We then turned to the literature for further investigation. We there found 15 references reporting magnetic field gradient measurements in sunspots, with various lines, instruments and interpretation methods. We restricted to references dated later than 1983. A detailed study of the references shows that they may be classified into two types: those where the vertical gradient is measured, which is found on the order of 3–4 G/km with the field strength decreasing with height, and those where alternatively the horizontal gradient is obtained, which is found on the order of 0.4–0.5 G/km. These orders of magnitude were completely retrieved in our recent THÉMIS observations. This classification *does not suffer any exception*. The two gradients differ by a factor of nine. This problem was pointed out by other authors. In a review paper, Solanki [1] concluded that "no satisfactory solution has been found as yet for the unexpectedly small [horizontal] gradients." As a consequence, the vanishing of $\operatorname{div}\vec{B}$ is not realized, and the present paper is devoted to the investigation of this problem.

After having recalled the physical conditions of the photosphere in Section 2, we discuss different possible causes on the observational side of the question in Section 3. We consider the noise effect, the unresolved structure effect and the line-of-sight and pixel integration effect. As none of these effects has brought the explanation of the phenomenon, we turn to the theoretical side of the question in Section 4. We investigate a plasma effect, the Debye shielding, to which we add the fact that the Debye sphere is flattened under the effect of the medium stratification. Under these conditions, we show that $\operatorname{div}\vec{B}$ may mathematically depart from zero. The stratification of the medium is a necessary condition for the effect being possible. We show that the Debye sphere may be flattened under the effect of the anisotropy of the particle velocities. The photosphere being the surface layer of a star is a strongly stratified plasma where such an anisotropy exists. The effect is discussed in Section 5, and finally a new methodology for the MHD modeling of the photosphere is suggested. We conclude in Section 6.

2. The solar photosphere

The solar photosphere is a superficial layer of small thickness. Taking as reference level ($h = 0$ km) the one where the optical depth in the continuum at $\lambda = 5000$ Å is unity, the photosphere is reckoned to extend up to $h = 320$ km, where this optical depth becomes 5×10^{-3}. This level is denoted as the basis of the chromosphere [2]. The chromosphere is the layer where typically the hydrogen Balmer α line forms, as hydrogen is the most abundant element, and it can be seen in emission when only this layer is visible at the limb under eclipse conditions. This layer then appears as colored, from which its name is issued. The chromosphere is about 2000 km thick. Below, the photosphere is the layer from which the photons of the spectrum continuum are emitted. Most of the solar lines are also formed in the photosphere and appear in absorption in the spectrum. The photosphere is then a few hundred of kilometers thick. It is formed of a plasma of temperature from a bit less than 5000 K to a bit more than 6000 K. This plasma is composed of 90% hydrogen and 10% helium (in particle density), and the other elements as traces. It is weakly ionized. The neutral hydrogen density is between 1×10^{16} and 1×10^{17} cm^{-3}, and the electron (and ion) density ranges from 1×10^{12} to 1×10^{15} cm^{-3}. As mainly composed of hydrogen atoms, the photosphere is paramagnetic, but due to its high temperature the magnetic susceptibility is about 1×10^{-10}, which means that $\mu = \mu_0$ for the permeability, and $\vec{B} = \vec{H}$ for the magnetic induction and field, within this number. The physical conditions of the magnetized stratified fluid dynamics will be clarified along the discussion in Section 5.

3. Observational results

3.1. The ensemble of results

With THÉMIS ('Télescope Héliographique pour l'Étude du Magnétisme et des Instabilités Solaires'), we found that the vertical gradient of the magnetic field in the NOAA 10808 sunspot umbra $|\Delta B_Z/\Delta Z|$ is 3–4 G/km, about nine times larger than the horizontal gradient $|\Delta B_X/\Delta X + \Delta B_Y/\Delta Y|$, which we found to be on the order of 0.4–0.5 G/km. Because the value of $|\operatorname{div}\vec{B}|$ is obtained from the difference between these two numbers, there is a discrepancy between its theoretical value, zero, and its "observed" value, because the difference between these two numbers cannot be zero. In order to investigate this discrepancy, we turned to the literature. We restricted our investigation to results obtained after the 1980s in order to consider only numerical data acquisition systems. In general, only one of the two gradients was derived in a given paper, and there are only a few papers where the difference was detected and discussed. The case of the horizontal gradient is particular, because the authors generally present their result as a vertical gradient indeed derived by applying $\mathrm{div}\,\vec{B}=0$ to the horizontal gradient, which is the true quantity that they measured. It is then necessary to deeply investigate each paper in order to determine the exact nature of the observed gradient. We then divide the papers into two groups, one about the vertical gradient measurements and the other one about the horizontal gradient measurements. Observing the vertical gradient, some authors [3–10] obtain coherent values on the order of 3–4 G/km. Observing the horizontal gradient, other researchers [11–13,9,14] obtain coherent values on the order of 0.4–0.5 G/km. These analyses include visible as well as infrared lines, and a series of different methods and codes for deriving the magnetic field from the measured polarization. Besides all these measurements, the typical order of magnitude can be considered: assuming a penumbral field $B_x=+1500$ G that switches to $B_x=-1500$ G at the opposite side of a typical umbra of diameter 10,000 km, the resulting average horizontal gradient would be 0.3 G/km.

3.1.1. THÉMIS observation results[1]

With the polarization-free telescope THÉMIS, we observed the double sunspot (δ-spot) of the active region NOAA 10808 on 13 September 2005 between 14:25 and 15:25 UT. THÉMIS was operating in MTR ('MulTiRaies') mode where the solar image is scanned over the spectrograph slit. We analyzed the Stokes spectra of the pair of lines Fe I λ 6301.5 and Fe I λ 6302.5 Å. For deriving the magnetic field, we performed the UNNOFIT inversion [15,16] to Fe I λ 6302.5 Å. This is Milne–Eddington inversion, based on Unno's theory [17], further generalized for magneto-optical effects by Rachkovsky [18,19]. For Fe I λ 6301.5 Å, which is not a normal Zeeman triplet line, we applied the more general (but more time consuming) UNNOFIT2 code, also developed by Landi Degl'Innocenti and coworkers.

In order to determine $\mathrm{div}\,\vec{B}$, the formation depth difference between the two lines has to be determined. In the quiet sun, this difference was recently derived from HINODE observations by Faurobert et al. [20], who applied a phase-shift analysis. They obtained as observed value 63.2 ± 0.9 km, corroborated by the value of 69 km derived by the same phase-shift technique applied to theoretical profiles computed with the non-LTE Uitenbroek's code [21]. These values are in full agreement with the 3D magnetoconvection simulation by Khomenko and Collados [22], as it is visible in their Fig. 4. In this figure, it can be seen that, although the difference is well on the order of 60–70 km in the quiet sun, it is rather of 100 km in the active region. As we observed an active region, we applied the value of 100 km to our computations.

In addition, it is visible in Fig. 4 of Ref. [22] that the formation depth difference between these two lines remains constant, even if each formation depth varies. We characterize this feature by saying that these two lines behave in a parallel manner as for their formation depth. Such parallelism probably originates from the fact that these two lines Fe I λ 6302.5 and 6301.5 Å belong to the same multiplet (n. 816) and have different gf values ($gf=0.180$ for 6301.5 and $gf=0.0627$ for 6302.5—data taken from the Kurucz data-basis). Since differential non-LTE effects within multiplets are thought to be very small (as proven by detailed, multi-level, non-LTE computations), this implies that the absorption coefficient of 6301.5 is 3 times larger than the absorption coefficient of 6302.5. There is then no doubt that 6301.5 forms higher than 6302.5.

Given the x and y pixel sizes, which are $\Delta x=1160$ km and $\Delta y=581$ km after a 2×2 pixel binning for noise reduction, we were able to determine $\mathrm{div}\,\vec{B}$. We obtained 3–4 G/km for the vertical gradient $|\Delta B_Z/\Delta Z|$ in the umbra, with field strength decreasing with height, and 0.4–0.5 G/km for the horizontal gradient $|\Delta B_X/\Delta X+\Delta B_Y/\Delta Y|$, after having rotated the results into the solar frame. These two values are respectively in full agreement with the other published results, as described above.

3.1.2. Some modeling results

As for the sunspot magnetic modeling, the modeled vertical gradient is also found about one order of magnitude weaker than the observed vertical gradient. Such behavior is reported by Eibe et al. [23], who derived the observed vertical gradient from longitudinal magnetic field measurements at different wavelengths inside the Na I D_1 line profile observed with THÉMIS/MSDP. These wavelengths along the profile are associated with different heights in the atmosphere via response functions computed with the MULTI code [24]. The modeled vertical gradient is obtained by force-free extrapolation [25]. The authors report a difference of one order of magnitude between the observed and modeled vertical gradients.

In a magnetostatic equilibrium modeling (not force-free), Pizzo [26] reports a modeled vertical gradient of 0.2–0.4 G/km (absolute value—see his Fig. 15 for large tube radii that model sunspots), which is also of the same order of magnitude as the observed horizontal gradient, but one order of magnitude weaker than the observed vertical one.

3.1.3. The particular case of the recent HINODE/SOT/SP observations

Puschmann et al. [27] inverted a HINODE/SOT/SP observation in a portion of penumbra where the ambiguity solution was simple, so that the full magnetic field vector was derived. The inversion code was SIR [28]. In their Fig. 6 (top left), the histogram of the measured $\mathrm{div}\,\vec{B}$ values was plotted and it was obtained that the histogram is clearly not centered on zero, but on 0.2 G/km instead. As this 0.2 value corresponds to the shift of the histogram but not to its width, Puschmann et al. [27] pointed out that the 0.2 does not result from noise. In order to investigate the problem, in Fig. 8 of Ref. [27] were

[1] Based on observations made with the French–Italian telescope THÉMIS operated by the CNRS and CNR on the Island of Tenerife in the Spanish Observatorio del Teide of the Instituto de Astrofísica de Canarias.

plotted individual histograms of the three contributions to $\operatorname{div}\vec{B}$, namely $\partial B_x/\partial x$, $\partial B_y/\partial y$ and $\partial B_z/\partial z$. It was obtained that the shift of 0.2 G/km remains in the $\partial B_z/\partial z$ histogram, whereas the histograms of $\partial B_x/\partial x$ and $\partial B_y/\partial y$ are both centered on zero. This result shows us that, in these observations also, the contribution of the vertical gradient is prevailing in $\operatorname{div}\vec{B}$, as in the ensemble of results cited above. The fact that the 0.2 G/km value of Ref. [27] is smaller than the difference between the generally observed vertical and horizontal gradients comes from the fact that they studied a portion only of penumbra, whereas we reported results for the whole umbra and inner penumbra. But the same inexplicable feature is present in all the results. Puschmann et al. [27] finally ascribed this unexpected value of non-zero $\operatorname{div}\vec{B}$ to unresolved magnetic structures, as suggested by Sanchez Almeida [29], who also detected the problem of inconsistency between different observation results when $\operatorname{div}\vec{B}=0$ is applied in between.

3.2. Research of the cause of the observed discrepancy

We can then conclude that, *without any exception*, in the sunspot photosphere the observed vertical gradient is on the order of 3–4 G/km and the horizontal one is on the order of 0.4–0.5 G/km. As a consequence, there is a problem to ensure the vanishing of $|\operatorname{div}\vec{B}|$, whose value is obtained by subtracting these two values. One value is about ten times larger than the other one, so that their difference may not be zero. We analyze below different suggested causes on the observational side, without getting any positive conclusion.

3.2.1. Noise effect

What would be the observation noise effect? If the difference were observation noise, as the difference is about 90% of the largest value $|\Delta B_Z/\Delta Z|$, the noise would be as large as it so that all the values would be within noise. In that case, one would not obtain that the difference is always of the same sign (decreasing field strength with height) and order of magnitude. All the results would be dispersed, which is not the case. One always observes a flux absolute decrease with increasing the height in the atmosphere.

3.2.2. Unresolved structure effect

An unresolved structure effect is frequently suggested as the solution. It is obviously assumed that each unresolved structure obeys $\operatorname{div}\vec{B}=0$. But $\operatorname{div}\vec{B}$ is a linear function

$$\operatorname{div}(\vec{B}_1+\vec{B}_2)=\operatorname{div}\vec{B}_1+\operatorname{div}\vec{B}_2 \tag{1}$$

so that the $\operatorname{div}\vec{B}$ of the average structure is the average of the $\operatorname{div}\vec{B}$ of the individual structures and remains zero. In other words, if the magnetic flux is conserved in elementary structures, one cannot explain why it would not be conserved at a wider scale.

In the case of the sunspot magnetic field, an unresolved structure could be made of small magnetic loops inserted between outgoing vertical field lines, in the umbra. The observed phenomenon is that the magnetic flux at a higher level is weaker than the flux at a lower level, and that this difference is not offset by the flux through the side surfaces. If small loops are considered, which peak lower than the higher altitude considered, they return flux at the lower level, since it is not evacuated at the higher level. But if the two feet of each loop are contained in the lower considered surface, the brought flux is also evacuated at the lower level so that the balance remains zero. Conversely, if the loop does not evacuate the flux through the lower surface, it does that through the side surface having its second foot outside the box, so that this would be detected also in the measurement of the side flux and the balance would also remain zero. Thus, it has not been possible to imagine an unresolved structure capable of generating non-zero $\operatorname{div}\vec{B}$.

A particular kind of unresolved structure is offered by turbulent microstructures. In hydrodynamics where it is $\operatorname{div}\vec{v}=0$ that is expected in incompressible media, Dubrulle and Frisch [30] studied the superposition of a small-scale turbulent perturbation to a large-scale basic structure. They obtain that "in general the [large-scale velocity field] does not [locally] have a vanishing divergence" (due to a contribution of the small-scale perturbation), but they add "in spite of the vanishing of the divergence of [the large-scale velocity field averaged on the small scales]". In other words, the divergence of the large-scale velocity may locally depart from zero due to a contribution of the small-scale turbulent perturbation, but when the large-scale velocity is averaged over the small scales, $\operatorname{div}\vec{v}=0$ as usual. Returning to the solar observations, the average is taken over the unresolved structures so that departure from a vanishing divergence cannot be expected, also due to the unresolved structures.

In order to investigate the unresolved structure effect, we performed several numerical tests suggested to us by J. Heyvaerts. A $256\times256\times256$ mesh cube was constructed, with in each mesh summit a magnetic field vector created by a vertical magnetic dipole placed 2560 mesh-side below the cube. The lack of resolution was then modeled by averaging the pixels by $F=20$ in each x, y, z dimension. Different kind of magnetic perturbations were then added before filtering: first, eight different elementary vertical fields $\vec{e}_z B_k \sin(\vec{k}\cdot\vec{r}+\varphi_k)$, each wavevector $\vec{k}$ being randomly chosen in the horizontal plane with random wavelength between 10 and 100 mesh-size (B_k and φ_k were also randomly chosen); second, we inclined the perturbation by an angle of 10° from the vertical; the third considered perturbation was the one created by eight random thin vertical currents randomly spread in the cube, their magnetic field being derived by applying the Ampère theorem; for the fourth case, we imposed the same sign to all the thin currents. The divergence was computed in a deliberately complex manner, in order to better simulate the observation complexity. The differences were taken between pixels separated by G, smaller or larger than the filter size F. We considered $G = F/2, F, 2F, 4F, 8F$, and we expected this sophisticated divergence to depart from zero when $G > F$. We never obtained such a departure, for any of the four considered perturbations, which are all divergence-free.

To resume, due to the linearity of div $\vec{B}$ and of the Zeeman effect and of the measurement technique, we estimate that unresolved structure effects cannot be retained as the searched for cause.

3.2.3. Line-of-sight and pixel integration

Due to the linearity of the Zeeman effect as previously discussed, the line-of-sight integration can be modeled as

$$\vec{B}(z) = \int_{-\infty}^{+\infty} \vec{B}(z')\varphi(z - z')\,\mathrm{d}z' \tag{2}$$

where φ is the contribution function, and where we have assigned a "depth of formation" z to the final result. This depth of formation is generally close to the maximum of the contribution function, as visible in the examples computed by Bruls et al. [31]. The contribution function acts as a filter in the above equation.

Analogously, the pixel integration in X or Y can be modeled with a convolution by a crenel function.

If now we compute the divergence of the *observed* magnetic field, we can apply the derivation of a convolution product as recalled below in Section 4. In the case of the line-of-sight integration, this is

$$\vec{\nabla} \cdot \vec{B}(x, y, z) = \int_{-\infty}^{+\infty} \vec{\nabla} \cdot \vec{B}(x, y, z')\varphi(z - z')\,\mathrm{d}z' \tag{3}$$

so that the divergence of the *observed* magnetic field is zero because div $\vec{B}$ is locally zero. Analogous derivation can be made for the filtering with a crenel function for the pixel integration. It results from the derivation property of the convolution product that any filter would lead to the same result, which is that the divergence of the filtered quantity is the filtering applied to the divergence of the local quantity, so that if the last one is zero, the first one is also zero. This result can be more easily derived in the Fourier space, where convolution products are transformed into simple products. If we denote as $\hat{B}_x(\vec{k})$ (resp. y, z) the spatial Fourier transform of the magnetic field component $B_x(\vec{r})$ (resp. y, z), the Fourier transform of the field divergence is:

$$FT[\mathrm{div}\,\vec{B}] = \mathrm{i}k_x\hat{B}_x + \mathrm{i}k_y\hat{B}_y + \mathrm{i}k_z\hat{B}_z \tag{4}$$

We denote as $\varphi(\vec{r})$ the 3D spatial filter to be applied to model the observations, and we accordingly denote as $\hat{\varphi}(\vec{k})$ its Fourier transform. Because

$$\hat{\varphi} \cdot [\mathrm{i}k_x\hat{B}_x + \mathrm{i}k_y\hat{B}_y + \mathrm{i}k_z\hat{B}_z] = \mathrm{i}k_x\hat{\varphi}\hat{B}_x + \mathrm{i}k_y\hat{\varphi}\hat{B}_y + \mathrm{i}k_z\hat{\varphi}\hat{B}_z \tag{5}$$

one has $\varphi[\mathrm{div}\,\vec{B}] = \mathrm{div}\,\varphi[\vec{B}]$, which is the above-mentioned result. This does not assume any isotropy of the filter. Different filter size and types may be assumed in x, y, z, as it is the case of the observations, and the result will be maintained. As a numerical test, we redid the one described at the end of the previous section about the unresolved structures, by applying different filter sizes in x, y on the one hand ($F_x = F_y = 20$) and z on the other hand ($F_z = 4$). The G coefficient was accordingly made anisotropic. $G_x = G_y$ was varied as usual: $G_x = F_x/2, F_x, 2F_x, 4F_x, 8F_x$, whereas G_z was taken fixed: $G_z = F_z$. We did not obtain any departure from zero of the computed sophisticated divergence.

To our opinion, one cannot then ascribe the departure from zero of the divergence of the observed field to the line-of-sight and/or pixel integration effects, as well as to any filtering effect.

As we were unable to solve the problem by investigating the observational side of the question, we turn now to the theoretical side.

4. Theoretical investigation

The magnetic field $\vec{B}$ created at point $\vec{r}$ by the electric current described by the density current $\vec{j}$ at the various points $\vec{r}'$ is given by the Biot & Savart law:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \vec{j}(\vec{r}') \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}\,\mathrm{d}^3\vec{r}' \tag{6}$$

The differential form of the Biot & Savart law is the Ampère law:

$$\vec{\nabla} \times \vec{B} = \mu_0\vec{j} \tag{7}$$

In the vacuum, the Biot & Savart law directly implies div $\vec{B} = 0$ from

$$\frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = -\vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r}'|}\right) \tag{8}$$

but for further investigation we want apply the $\vec{\nabla}$ operator to Eq. (6). In order to determine how this operator has to be applied under the integral, we first remark that Eq. (6) is the sum of elementary convolution products of the form:

$$F(x) = \int_{-\infty}^{+\infty} g(x') f(x - x') \mathrm{d}x' \tag{9}$$

In order to evaluate the derivative, let us evaluate $F(x + \mathrm{d}x)$. One has

$$\begin{aligned} F(x + \mathrm{d}x) &= \int_{-\infty}^{+\infty} g(x') f(x + \mathrm{d}x - x')\, \mathrm{d}x' \\ &= \int_{-\infty}^{+\infty} g(x'' + \mathrm{d}x) f(x - x'')\, \mathrm{d}x'' \end{aligned} \tag{10}$$

by changing the variable $x'' = x' - \mathrm{d}x$. One has then for the derivative $F'(x)$

$$F'(x) = \int_{-\infty}^{+\infty} g'(x') f(x - x')\, \mathrm{d}x' \tag{11}$$

which is that the derivative of the convolution product is the convolution (by the same function) of the derivative.

Applying this result to Eq. (6), where it must be noted that the $\vec{\nabla}$ operator has then to be applied to $\vec{j}(\vec{r}')$ but not to $(\vec{r} - \vec{r}')/|\vec{r} - \vec{r}'|^3$, one obtains:

$$\vec{\nabla} \cdot \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \left[\vec{\nabla} \times \vec{j}(\vec{r}')\right] \cdot \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}\, \mathrm{d}^3\vec{r}' \tag{12}$$

In the following, we investigate the limitation of the Biot & Savart integral by the Debye shielding due to the charges moving in the plasma.

4.1. Effect of the Debye shielding

In a plasma, it is well known that a given charge is shielded by the surrounding ones, so that at a certain distance, called the Debye length, the charge has no effect. Following for instance Meyer-Vernet [32], "[in a plasma] any charge has a dress of size the Debye length L_D which makes it "invisible" from larger distances." It is also well known that the Debye shielding applies to the calculation of the electric field created by the charge, and the Debye shielding in itself is an effect of the electrostatic potential created by each charge. But what about the magnetic field?

Consider the electrostatic potential created by an elementary charge q located at origin

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \tag{13}$$

This charge repels the charges of the same sign and attracts the charges of opposite sign, so that at distances $r > L_\mathrm{D}$, the total potential vanishes. Consider now that this charge has a velocity $\vec{v}$. It then creates the magnetic vector potential

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \frac{q\vec{v}}{r} \tag{14}$$

provided that $v \ll c$, the speed of light. The form of the vector potential is quite the same as the one of the electrostatic potential, in particular as for the r-dependence. The velocity $\vec{v}$ to be considered here is not the thermal velocity, but the fluid or current velocity. The order of magnitude of the Debye length is the micron in the photosphere. Thus it can be assumed that the fluid or current velocity $\vec{v}$ is constant over the Debye volume, so that the velocity can be factorized in the screening formalism. The form of the screening becomes thus the same as for the electrostatic potential.

More precisely, the screened electrostatic potential of the charge q is

$$V_\mathrm{D}(r) = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{r} - \iiint \frac{e[\delta n_\mathrm{e}(r') - \delta n_\mathrm{i}(r')]}{|\vec{r} - \vec{r}'|}\, \mathrm{d}^3\vec{r}'\right] \tag{15}$$

where $\delta n_\mathrm{e}(r')$ and $\delta n_\mathrm{i}(r')$ are respectively the electron and ion density excess or depletion in r' due to the presence of the charge q at origin. Here e is the absolute value of the electron charge. Following Meyer-Vernet [32], this reduces to

$$V_\mathrm{D}(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \exp\left[-\frac{r}{L_\mathrm{D}}\right] \tag{16}$$

As for the vector potential, assuming that all the particles surrounding the charge q undergo the same velocity $\vec{v}$, the total vector potential in r is

$$\vec{A}_{\mathrm{D}}(r) = \frac{\mu_0}{4\pi}\left[\frac{q}{r} - \iiint \frac{e[\delta n_{\mathrm{e}}(r') - \delta n_{\mathrm{i}}(r')]}{|\vec{r}-\vec{r}'|}\,\mathrm{d}^3\vec{r}'\right]\vec{v} \tag{17}$$

which analogously results in

$$\vec{A}_{\mathrm{D}}(r) = \frac{\mu_0}{4\pi}\frac{q\vec{v}}{r}\exp\left[-\frac{r}{L_{\mathrm{D}}}\right] \tag{18}$$

This shows that the magnetic vector potential is also screened, provided that the fluid or current velocity is the same in all the volume under interest. As a consequence, the Debye shielding applies also to the magnetic field in the photosphere, and the Biot & Savart law of Eq. (6) has indeed to be limited to the Debye volume.

Indeed, the particle velocity $\vec{v}$ is the sum of the thermal velocity $\vec{v}_T$ and of the fluid velocity $\vec{v}_{\mathrm{f}}$

$$\vec{v} = \vec{v}_T + \vec{v}_{\mathrm{f}} \tag{19}$$

The magnetic vector potential is then the sum of both thermal and fluid contributions, for each particle. As for the potential created by the particle under interested and recalled in Eq. (14), the thermal contribution is compensated for by similar contributions of other particles having different and random thermal velocities. The thermal velocity is not responsible for a magnetic field. As for the screening particles whose contribution to the vector potential is given by the triple integral in Eq. (17), the thermal contribution has also to be examined. The thermal velocity being different for each elementary particle of the screen, the thermal velocity does not factorize out of the integral, but on the contrary, the random character of the thermal velocity makes the corresponding triple integral vanishing. Thus, there is no shielding of the vector potential under the effect of thermal velocities. But as for the fluid velocity, it can be considered as constant over the volume integral in its main contributing region, which is on the order of the Debye sphere whose typical radius in the photosphere is 1–2 microns, so that it can be factorized as done in Eq. (17). The fluid velocities are then responsible for shielding the magnetic vector potential.

The typical convolution product entering the Biot & Savart law becomes

$$F(x) = \int_{x-L_{\mathrm{D}}}^{x+L_{\mathrm{D}}} g(x') f(x-x')\,\mathrm{d}x' \tag{20}$$

With the same variable change as before, it can be shown that the derivative of $F(x)$ with respect do x is given by

$$F'(x) = \int_{x-L_{\mathrm{D}}}^{x+L_{\mathrm{D}}} g'(x') f(x-x')\,\mathrm{d}x' \tag{21}$$

Applying integration by parts to Eq. (6) with integral limited to the Debye volume, one obtains after calculation:

$$\vec{\nabla}\cdot\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\iint_{\vec{r}'\in\mathrm{Debye}}\left[\vec{j}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right]\cdot\mathrm{d}\vec{S}' \tag{22}$$

which represents the flux through the Debye volume surface of the elementary field created at the point of interest $\vec{r}$ by the current located on this surface. If the Debye volume is a sphere as usual, $\vec{r}-\vec{r}'$ and $\mathrm{d}\vec{S}'$ are parallel so that $\vec{\nabla}\cdot\vec{B}(\vec{r}) = 0$ whatever the current is. But if the Debye volume is not a sphere, $\vec{\nabla}\cdot\vec{B}(\vec{r})$ may not be zero.

5. Discussion

We have shown above that $\operatorname{div}\vec{B}$ may not be zero when the Debye volume is not a sphere. We think that this is realized in the photosphere, because, as the surface of a star, the photosphere is a stratified medium. Such a medium may be characterized by an *aspect ratio* α [33, see for instance], which is the ratio between the vertical characteristic length and the horizontal one. What are these lengths in the photosphere? The horizontal characteristic length could be the granule typical diameter, 1000 km. The vertical characteristic length could be the pressure scale height, 110 km [2]. The aspect ratio in the photosphere results in being $\alpha = 1/9 = 0.11$. Let us cite other aspect ratios: 0.1–0.01 in the deep ocean, 0.01–0.001 in the terrestrial atmosphere close to the ground and in the thermocline [34], which is the zone of rapid thermic transition between surface and deep water in the oceans. This is only a trivial remark. Returning to the photosphere case, we interestingly remark that this ratio is the same as the one observed between the magnetic field gradients.

The degree of stratification of the medium can be evaluated by computing the horizontal Froude number. Referring to Brethouwer et al. [33] and adopting their notations of U for the typical horizontal velocity, l_{h} and l_{v} for the characteristic horizontal and vertical lengths respectively, we have:

$$\alpha = l_{\mathrm{v}}/l_{\mathrm{h}} \tag{23}$$

we introduce the Brunt–Väisälä frequency N, which is the oscillation frequency of a particle undergoing a vertical motion in the fluid where the gravity acceleration is g and the density ρ is decreasing with height, so that

$$N = \sqrt{\frac{g}{\rho}\left|\frac{\partial \rho}{\partial z}\right|} \tag{24}$$

The horizontal Froude number is then given by

$$F_\mathrm{h} = \frac{U}{l_\mathrm{h} N} \tag{25}$$

and analogously for the vertical Froude number. We find a Brunt–Väisälä frequency of 0.02–0.04 s^{-1} in the photosphere following the VAL C atmosphere model [35] and considering the gas of neutral hydrogen which is the most dense with respect to the free electrons and ions. Considering for U the non-thermal velocity of the particles given by the VAL C model from 0.5 to 1.8 km/s in good agreement with [36], and taking for l_h the typical granule diameter stated above, we derive the horizontal Froude number of the photosphere that we find ranging from 0.01 to 0.09, which is $\ll 1$, so that the medium may be considered as strongly stratified.

On the basis of direct numerical simulations of stratified flows in which the vertical characteristic length l_v is let as a free parameter, Brethouwer et al. [33] show that two regimes may be distinguished following the value of the buoyancy Reynolds number $\mathcal{R}$ number given by

$$\mathcal{R} = \mathrm{Re}\, F_\mathrm{h}^2 \tag{26}$$

where Re is the Reynolds number of the fluid. We have evaluated this number for the gas of neutral hydrogen of the photosphere

$$\mathrm{Re} = \frac{U l_\mathrm{h}}{\nu_\mathrm{HH}} \tag{27}$$

where ν_HH is the collision frequency of the gas particles, which are neutral hydrogen atoms. Following Meyer-Vernet [37], we simply adopt the billiard ball model for evaluating ν_HH for the gas of neutral hydrogen, so that we consider the cross-section $\sigma_\mathrm{HH} = 10^{-19}\ \mathrm{m}^2$. For the typical length entering Re, we have taken the horizontal length scale l_h to be coherent with the horizontal Froude number. We find $\mathrm{Re} \approx 250 \gg 1$ in the photosphere, but $\mathcal{R} \approx 0.14 \ll 1$, a regime that Brethouwer et al. [33] depict as the "viscosity-affected stratified flow regime", more precisely they say that "if $\mathcal{R} \ll 1$ the large-scale dynamics is determined by a balance between inertial and viscous forces due to vertical shearing. Therefore no inertial cascade can develop and the dissipation occurs predominantly at the largest scales [...] Kelvin–Helmholtz type of disturbances cannot be present when $\mathcal{R} \ll 1$." This result was already derived and experimentally confirmed by Godoy-Diana et al. [38]. Such a conclusion can interestingly be brought together with the result obtained by Bommier [39] who has shown that the structure of the quiet sun magnetic field as derived from recent inversions is not turbulent as generally admitted, but organized in connected vertical opening flux tubes.

Moreover, Brethouwer [33] show that in the case of the viscosity-affected stratified flow regime, the vertical characteristic length l_v that was let free in the simulation results is

$$l_\mathrm{v} = \frac{l_\mathrm{h}}{\sqrt{\mathrm{Re}}} \tag{28}$$

so that the theoretical value of the aspect ratio α_th is

$$\alpha_\mathrm{th} = \frac{1}{\sqrt{\mathrm{Re}}} \tag{29}$$

We have computed α_th for the photosphere and we obtain a theoretical value of between 1/10 and 1/20, in very good agreement with the empirical one of 1/9 that we determined from the granule typical diameter and the pressure scale height, and that we observed in $\mathrm{div}\,\vec{B}$.

Concerning the anisotropy of the velocities, Brethouwer et al. [33] show that when the horizontal velocity scales as U, the vertical one scales as $U F_\mathrm{h}^2/\alpha$, so that the theoretical aspect ratio for the velocities is

$$(\alpha_\mathrm{th})_\mathrm{vel} = \frac{F_\mathrm{h}^2}{\alpha} \tag{30}$$

We obtain values ranging from 14 to 192, so that the ratio horizontal/vertical is even higher for the velocities than for the typical lengths. It can then be concluded that the velocities are highly anisotropic in the photosphere.

Meyer-Vernet [32] has recomputed the Debye length, discarding the usual hypothesis of the thermodynamical equilibrium, and basing the calculation on the charge velocity. The model assumes isotropic velocities, but it can be generalized to anisotropic ones, by performing an affinity along one of the reference axes, which leads to different Debye lengths L_h and L_v for the different vertical and horizontal typical velocities v_h and v_v:

$$\begin{cases} L_\mathrm{h}^2 = \dfrac{\varepsilon_0 m_\mathrm{e} v_\mathrm{h}^2}{N_\mathrm{e} q^2} \\ L_\mathrm{v}^2 = \dfrac{\varepsilon_0 m_\mathrm{e} v_\mathrm{v}^2}{N_\mathrm{e} q^2} \end{cases} \tag{31}$$

provided that the hydrogen atom velocity ratio be transferred to the charges by their collisions between themselves. Indeed, the particle velocity is the sum of the thermal velocity and of the fluid velocity, as in Eq. (19). The anisotropy described above is the one of the fluid velocity. The thermal velocity is isotropically distributed, but the total velocity remains anisotropic, leading to the different horizontal and vertical Debye lengths derived above. As a result, the Debye volume is not a sphere and $\operatorname{div}\vec{B}$ may mathematically depart from zero. Thus $\operatorname{div}\vec{B}$ is not zero as a consequence of the stratification, as observed. The aspect ratio is transferred to $\operatorname{div}\vec{B}$.

The magnetic Reynolds number, computed by taking as typical length the horizontal length l_h in coherence with the Froude and Reynolds numbers,

$$\mathrm{Re}_\mathrm{m} = \mu_0 \sigma U l_\mathrm{h} \tag{32}$$

is obtained and is on the order of 5×10^5. The magnetic Reynolds number makes intervene the Coulombian electron–ion collisions via the electric conductivity σ, whereas the Reynolds number Re is concerned by the collisions between the neutral hydrogen atoms (see Eq. (27)). The photosphere plasma is a complex medium, having the neutral particle density four orders of magnitude higher than the charged particle density. All these particle collide, the neutral particles with the neutral particles, the charged particles with the charged particles, but also the charged particles with the neutral ones. Due to the ensemble of collisions, the fluid properties should be common for the neutral and charged particles. The relative effect of the magnetic forces is scaled by the magnetic Prandtl number, which is simply the ratio between the magnetic Reynolds number and the Reynolds number $P_\mathrm{m} = \mathrm{Re}_\mathrm{m} / \mathrm{Re}$, which we then found on the order of 2×10^3. This number represents also the ratio between the kinematic viscosity and the magnetic diffusivity. We get that this number is large with respect to unity in the photosphere. This means that the dynamical viscosity dominates the magnetic diffusivity, so that the photosphere can effectively be considered as a highly stratified medium, even in the presence of magnetic field. As for the plasma β coefficient, which is the ratio between the gas pressure and the magnetic pressure

$$\beta = \frac{N_\mathrm{H} k_\mathrm{B} T}{B^2/2\mu_0} \tag{33}$$

we get values about unity in the photosphere, by assuming a magnetic field $B = 1000$ G as a typical value for sunspot umbra and penumbra. The fact that β is of the order of unity also shows that the medium's structure is not dominated by the magnetic field and that the stratification acts besides.

5.1. A proposal for MHD modeling in the photosphere

It could be considered that in reduced spatial scales, where the horizontal scales are multiplied by α, the Debye volume becomes spherical so that $\operatorname{div}\vec{B} \approx 0$. This is true for the observations reported at the beginning of this paper, having the vertical gradient as 3–4 G/km and the horizontal one as 0.4–0.5 G/km. If the vertical gradient is multiplied by α, it results in a value of 0.33–0.44 G/km, of the same order of magnitude as the horizontal gradient, so that $\operatorname{div}\vec{B} \approx 0$.

Thus we are led to suggest that the usual methods of the MHD could nevertheless be applied for calculations in a stratified medium as the photosphere, by considering that:

- the div operators have to be computed in the reduced length scale;
- the curl operators have to be computed in the normal length scale. The magnetic field remains in any case given by the Biot & Savart law (limited to the Debye volume) that reduces locally to the Maxwell–Ampère law of Eq. (7), where the curl is in the normal scale.

As an example, one can consider the magnetostatic sunspot model by Pizzo [26]. As it is visible in his Fig. 7, the prototype spot model has a diameter of about 2000 km, whereas the average diameter is 10,000 km in the observations [2]. If the reduced length scale had been used in the computation, the spot model diameter would be $1/\alpha$ times larger, so that it would become of the same order of magnitude as the average observed spot diameter.

6. Conclusion

We have shown that, *without any exception*, the observed value of the so-called vertical gradient of the sunspot photospheric magnetic field $|\Delta B_Z/\Delta Z|$ is 3–4 G/km, with field strength always decreasing with height, whereas the horizontal gradient $|\Delta B_X/\Delta X + \Delta B_Y/\Delta Y|$ is 0.4–0.5 G/km. These observations involve various lines, visible or IR, various ground-based or spaceborne instruments, various interpretation methods. In some cases, the same inversion code performs the magnetic field and depth difference determination, in other cases they result from two different codes. The fact that no exception was found in spite of the variety of lines, instruments and methods, argues in favor of a true difference and not of a result of noise, together with the fact that it is always a flux loss and never a flux gain that is observed with increasing height. Moreover, their ratio is on a factor of 9, which is rather high. As, also, the difference is large and nearly as large as the largest gradient, ascribing the difference to noise would imply that all the measurements would be not larger than noise. This is not compatible with the fact that the difference is always of the same sign.

This difference implies that $\operatorname{div}\vec{B}$ derived from these two gradients would not be zero. This is usually ascribed to unresolved magnetic field structures. But we are not convinced, because if each elementary unresolved structure has $\operatorname{div}\vec{B} = 0$, we do not see how the field divergence of the whole region could be non-zero, because the divergence operation is linear. In other words, how the field could be flux non-conservative at large scale together with resulting of the combination of smaller flux conservative structures?

On the theoretical side of the problem, $\operatorname{div}\vec{B} = 0$ is a direct consequence of the Biot & Savart law. In this paper, we propose to solve the discrepancy by considering a plasma effect, the Debye shielding of the charges. Without challenging the Biot & Savart law, we remark that in the plasma the charge effect is shielded so that the integral of the Biot & Savart law has to be limited to the Debye volume. But when the volume is a sphere, we obtain that nevertheless $\operatorname{div}\vec{B} = 0$. This is only when the volume is not a sphere that $\operatorname{div}\vec{B}$ may depart from zero. We show that this could be the case of the solar photosphere, because this is the surface layer of a star, and thus it is a stratified medium. We verify this point by computing the horizontal Froude number. We evaluate its *aspect ratio* as 1/9 by considering the granule typical radius as the horizontal characteristic length, and the pressure scale height as the vertical characteristic length. Their ratio is 1/9, interestingly the same as the one of the observed horizontal and vertical magnetic field gradients. This observed value is interestingly in very good agreement with the theoretical one derived following Brethouwer et al. [33]. Following these authors, we obtain also that the photosphere ranges in the “viscosity-affected stratified flow regime”, where the inertial cascade cannot develop. As a result of the stratification, the horizontal velocity is highly different from the vertical one. It is possible to generalize the result obtained by Meyer-Vernet [32] that the Debye length is proportional to the charge velocity to such anisotropic velocities by performing an affinity of one of the reference axes. Thus, the Debye volume is a flattened sphere and $\operatorname{div}\vec{B}$ may depart from zero in the photosphere, as observed.

A magnetic monopole can be the source of non-zero $\operatorname{div}\vec{B}$. But magnetic monopoles have not been observed. The logical assertion is that the presence of a monopole implies non-zero $\operatorname{div}\vec{B}$. The negative assertion is that $\operatorname{div}\vec{B} = 0$ implies the absence of any magnetic monopole. But the negative assertion is not that the absence of detection of any monopole would imply $\operatorname{div}\vec{B} = 0$, as generally believed. In other words, the presence of a magnetic monopole is a sufficient condition, but not a necessary condition, for $\operatorname{div}\vec{B} \neq 0$. Other sufficient conditions may exist. Anisotropic shielding constitutes an alternative to the existence of monopoles for being responsible for non-zero $\operatorname{div}\vec{B}$ in a medium.

It can be concluded that such an effect of $\operatorname{div}\vec{B}$ departing from zero is very specific of the photosphere, because it is a combined effect of plasma and stratification.

Finally, this leads us to introduce a spatial scaling by the aspect ratio, where $\operatorname{div}\vec{B} \approx 0$. We then propose nevertheless this scaling method to perform MHD modeling in the photosphere, where the div operators have to be computed in the reduced length scale, and the curl operators have to be computed in the normal length scale.

We have also evaluated the Froude number at the basis of the solar corona and found it to be 3.6, increasing with height, which shows that the corona is conversely not at all a stratified plasma, so that the conditions are very different there.

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References

[1] S.K. Solanki, Sunspots: an overview, Astron. Astrophys. Rev. 11 (2003) 153–286, http://dx.doi.org/10.1007/s00159-003-0018-4.
[2] C.W. Allen, Astrophysical Quantities, third edition, The Athlone Press, University of London, 1973.
[3] C. Westendorp Plaza, J.C. del Toro Iniesta, B. Ruiz Cobo, V. Martínez Pillet, B.W. Lites, A. Skumanich, Optical tomography of a sunspot. II. Vector magnetic field and temperature stratification, Astrophys. J. 547 (2001) 1130–1147, http://dx.doi.org/10.1086/318376.
[4] H. Balthasar, W. Schmidt, Polarimetry and spectroscopy of a simple sunspot. 2: On the height and temperature dependence of the magnetic field, Astron. Astrophys. 279 (1993) 243–250.
[5] W. Schmidt, H. Balthasar, Polarimetry and spectroscopy of a simple sunspot. 3: Velocity and magnetic field of sunspot umbral dots, Astron. Astrophys. 283 (1994) 241–246.
[6] K.-D. Pahlke, E. Wiehr, Magnetic field, relative Doppler shift and temperature for an inhomogeneous model of sunspot umbrae, Astron. Astrophys. 228 (1990) 246–252.
[7] M. Collados, V. Martinez Pillet, B. Ruiz Cobo, J.C. del Toro Iniesta, M. Vazquez, Observed differences between large and small sunspots, Astron. Astrophys. 291 (1994) 622–634.
[8] J.H.M.J. Bruls, S.K. Solanki, R.J. Rutten, M. Carlsson, Infrared lines as probes of solar magnetic features. VIII. MgI 12 μm diagnostics of sunspots, Astron. Astrophys. 293 (1995) 225–239.

[9] S.K. Mathew, A. Lagg, S.K. Solanki, M. Collados, J.M. Borrero, S. Berdyugina, N. Krupp, J. Woch, C. Frutiger, Three dimensional structure of a regular sunspot from the inversion of IR Stokes profiles, Astron. Astrophys. 410 (2003) 695–710, http://dx.doi.org/10.1051/0004-6361:20031282.
[10] A. Berlicki, P. Mein, B. Schmieder, THEMIS/MSDP magnetic field measurements, Astron. Astrophys. 445 (2006) 1127–1132, http://dx.doi.org/10.1051/0004-6361:20053295.
[11] H. Balthasar, Vertical current densities and magnetic gradients in sunspots, Astron. Astrophys. 449 (2006) 1169–1176, http://dx.doi.org/10.1051/0004-6361:20054358.
[12] M.J. Hagyard, D. Teuber, E.A. West, E. Tandberg-Hanssen, W. Henze Jr., J.M. Beckers, M. Bruner, C.L. Hyder, B.E. Woodgate, Vertical gradients of sunspot magnetic fields, Sol. Phys. 84 (1983) 13–31, http://dx.doi.org/10.1007/BF00157439.
[13] Y. Liu, J. Wang, Y. Yan, G. Ai, Gradients of the line-of-sight magnetic fields in active region NOAA 6659, Sol. Phys. 169 (1996) 79–89, http://dx.doi.org/10.1007/BF00153835.
[14] H. Balthasar, M. Collados, Some properties of an isolated sunspot, Astron. Astrophys. 429 (2005) 705–711, http://dx.doi.org/10.1051/0004-6361:20041451.
[15] M. Landolfi, E. Landi Degl'Innocenti, P. Arena, On the diagnostic of magnetic fields in sunspots through the interpretation of Stokes parameters profiles, Sol. Phys. 93 (1984) 269–287, http://dx.doi.org/10.1007/BF02270839.
[16] V. Bommier, E. Landi Degl'Innocenti, M. Landolfi, G. Molodij, UNNOFIT inversion of spectro-polarimetric maps observed with THEMIS, Astron. Astrophys. 464 (2007) 323–339, http://dx.doi.org/10.1051/0004-6361:20054576.
[17] W. Unno, Line formation of a normal Zeeman triplet, Publ. Astron. Soc. Jpn. 8 (1956) 108.
[18] D.N. Rachkovsky, Izv. Ordena Trudovogo Krasnogo Znameni Krym. Astrofiz. Obs. 27 (1962) 148.
[19] D.N. Rachkovsky, Izv. Ordena Trudovogo Krasnogo Znameni Krym. Astrofiz. Obs. 37 (1967) 56.
[20] M. Faurobert, C. Aime, C. Périni, H. Uitenbroek, C. Grec, J. Arnaud, G. Ricort, Direct measurement of the formation height difference of the 630 nm Fe I solar lines, Astron. Astrophys. 507 (2009) L29–L32, http://dx.doi.org/10.1051/0004-6361/200913330.
[21] C. Grec, H. Uitenbroek, M. Faurobert, C. Aime, Measuring line formation depths by cross-spectral analysis. Numerical simulations for the 630 nm Fe I line pair, Astron. Astrophys. 514 (2010) A91, http://dx.doi.org/10.1051/0004-6361/200811455.
[22] E. Khomenko, M. Collados, On the Stokes V amplitude ratio as an indicator of the field strength in the solar internetwork, Astrophys. J. 659 (2007) 1726–1735, http://dx.doi.org/10.1086/512098.
[23] M.T. Eibe, G. Aulanier, M. Faurobert, P. Mein, J.M. Malherbe, Vertical structure of sunspots from THEMIS observations, Astron. Astrophys. 381 (2002) 290–299, http://dx.doi.org/10.1051/0004-6361:20011495.
[24] M. Carlsson, A computer program for solving multi-level non-LTE radiative transfer problems in moving or static atmospheres, Tech. Rep. Report No 33, Uppsala Astronomical Observatory, 1986.
[25] P. Demoulin, L.G. Bagala, C.H. Mandrini, J.C. Henoux, M.G. Rovira, Quasi-separatrix layers in solar flares. II. Observed magnetic configurations, Astron. Astrophys. 325 (1997) 305–317.
[26] V.J. Pizzo, Numerical solution of the magnetostatic equations for thick flux tubes, with application to sunspots, pores, and related structures, Astrophys. J. 302 (1986) 785–808, http://dx.doi.org/10.1086/164041.
[27] K.G. Puschmann, B. Ruiz Cobo, V. Martínez Pillet, A geometrical height scale for sunspot penumbrae, Astrophys. J. 720 (2010) 1417–1431, http://dx.doi.org/10.1088/0004-637X/720/2/1417, arXiv:1007.2779.
[28] B. Ruiz Cobo, J.C. del Toro Iniesta, Inversion of Stokes profiles, Astrophys. J. 398 (1992) 375–385, http://dx.doi.org/10.1086/171862.
[29] J. Sanchez Almeida, Optically thin irregularities in the penumbrae of sunspots, Astrophys. J. 497 (1998) 967, http://dx.doi.org/10.1086/305511.
[30] B. Dubrulle, U. Frisch, Eddy viscosity of parity-invariant flow, Phys. Rev. A 43 (1991) 5355–5364, http://dx.doi.org/10.1103/PhysRevA.43.5355.
[31] J.H.M.J. Bruls, B.W. Lites, G.A. Murphy, Non-LTE formation heights of Stokes profiles of Fe I lines, in: L.J. November (Ed.), Proceedings of the Eleventh NSO/Sac. Peak Summer Workshop, Solar Polarimetry, 27–31 August 1990, 1991, pp. 444–456.
[32] N. Meyer-Vernet, Aspects of Debye shielding, Am. J. Phys. 61 (1993) 249–257, http://dx.doi.org/10.1119/1.17300.
[33] G. Brethouwer, P. Billant, E. Lindborg, J.-M. Chomaz, Scaling analysis and simulation of strongly stratified turbulent flows, J. Fluid Mech. 585 (2007) 343, http://dx.doi.org/10.1017/S0022112007006854.
[34] P. Augier, Turbulence en milieu stratifié, Étude expérimentale et rôle de l'instabilité zigzag, Master's Thesis, École Normale Supérieure de Lyon, 2008, www.ens-lyon.fr/DSM/SDMsite/M2/stages_M2/Augier.pdf.
[35] J.E. Vernazza, E.H. Avrett, R. Loeser, Structure of the solar chromosphere. III – Models of the EUV brightness components of the quiet-sun, Astrophys. J. Suppl. Ser. 45 (1981) 635–725, http://dx.doi.org/10.1086/190731.
[36] V. Bommier, M. Derouich, E. Landi Degl'Innocenti, G. Molodij, S. Sahal-Bréchot, Interpretation of second solar spectrum observations of the Sr I 4607 Å line in a quiet region: turbulent magnetic field strength determination, Astron. Astrophys. 432 (2005) 295–305, http://dx.doi.org/10.1051/0004-6361:20035773.
[37] N. Meyer-Vernet, Basics of the Solar Wind, Cambridge Atmospheric and Space Science Series, Cambridge University Press, 2007.
[38] R. Godoy-Diana, J.-M. Chomaz, P. Billant, Vertical length scale selection for pancake vortices in strongly stratified viscous fluids, J. Fluid Mech. 504 (2004) 229–238, http://dx.doi.org/10.1017/S0022112004008067.
[39] V. Bommier, The quiet Sun magnetic field: statistical description from THEMIS observations, Astron. Astrophys. 530 (2011) A51, http://dx.doi.org/10.1051/0004-6361/201015761.