C. R. Physique 15 (2014) 509–516

Contents lists available at ScienceDirect

Comptes Rendus Physique

www.sciencedirect.com

Evaluation of Reynolds stress MHD turbulence models using decaying homogeneous turbulence

Évaluation de modèles MHD aux tensions de Reynolds en turbulence homogène en déclin

Hamed Marzougui, Taieb Lili

Département de physique, faculté des sciences de Tunis, campus universitaire El Manar, 1060 Tunis, Tunisia

ARTICLE INFO

Article history:
Available online 17 June 2014

Keywords:
Decaying homogeneous turbulence
MHD turbulence
Low magnetic Reynolds number

Mots-clés :
Écoulement turbulent en décroissance libre
Turbulence MHD
Nombre de Reynolds magnétique faible

ABSTRACT

Direct numerical simulation databases and theoretical analysis for decaying homogeneous turbulent flow in a conducting fluid subjected to an imposed magnetic field are investigated to evaluate the second-order models proposed by Widlund et al. and Kenjeres et al. The case of very small magnetic Reynolds numbers ($Re_m \ll 1$) is considered in the present work. This case corresponds to the quasi-static approximation, which is well suited for most industrial flows involving liquid metals. The results obtained from our calculations show the performance of the model of Widlund et al. in predicting the Lorentz force effects compared with the model of Kenjeres et al.

RÉSUMÉ

Des données issues de simulations numériques directes et d'analyses théoriques relatives à un écoulement turbulent homogène en décroissance libre dans un fluide conducteur sujet à un champ magnétique sont utilisées pour évaluer les modèles de turbulence MHD développés par Widlund et al. et Kenjeres et al. Nous considérons, dans la présente étude, le cas où le nombre de Reynolds magnétique est faible. Ce cas correspond à l'approximation quasi stationnaire. Cette dernière est la plus recommandée dans l'étude des écoulements turbulents industriels. Les résultats de nos calculs montrent la capacité significative du modèle de Widlund et al. à reproduire les effets de la force de Lorentz comparé au modèle de Kenjeres et al.

1. Introduction

The interaction of the turbulence in a conducting fluid with an external applied magnetic field is important in numerous industrial applications in engineering as well as in geophysics and astrophysics. The main difference between these applications is due to the different values of the magnetic Reynolds number Re_m. In astrophysical applications such as interstellar medium, stars, etc., Re_m is very high ($Re_m \gg 1$) and is smaller but still significantly larger than 1 in geophysical applications

http://dx.doi.org/10.1016/j.crhy.2014.05.004
1631-0705/

(Earth dynamo). The case where $Re_\mathrm{m} \ll 1$ occurs in a majority of technological processes where a strong applied magnetic field is imposed on a conducting liquid. Examples of such applications include liquid–metal cooling systems for fusion reactors, growth of semiconductor crystals and the continuous casting of steel and aluminum. For vanishing magnetic Reynolds number ($Re_\mathrm{m} \ll 1$), the quasi-static approximation can be applied to study the turbulence subjected to a magnetic field. In this approximation, the induced magnetic fluctuations are negligible in comparison with the imposed magnetic field, and their characteristic time scale, based on their diffusion, is much weaker than that of turbulence. The magnetic field's fluctuation and the Lorentz force in the quasi-static approximation are expressed as a linear function of the velocity fluctuations and, when we consider that the imposed magnetic field is uniform and vertical: $\vec{\mathrm{B}} = B_z \vec{\mathrm{e}}_z$, these quantities are given, respectively, by [1]:

$$\eta \frac{\partial^2 b_i'}{\partial x_k^2} = -B_z \frac{\partial u_i'}{\partial z} \tag{1}$$

$$f'[v'] = -\frac{\sigma B^2}{\rho} \Delta^{-1} \frac{\partial^2 v'}{\partial z^2} \tag{2}$$

where ρ is the density of fluid, η is the magnetic diffusivity, σ is the electric conductivity, and Δ^{-1} is the reciprocal Laplace operator.

The behavior of the turbulence under the action of the Lorentz force has been studied in analytical [2,3], numerical [4,5], and experimental [6,7] works. These works showed that the principal effects of an imposed magnetic field on turbulence appear clearly in an additional dissipation of kinetic energy via Joule dissipation and becomes anisotropic, its structures being elongated along the magnetic field lines. The Fourier representation of the Joule dissipation rate shows that this rate is anisotropic; it acts at all scales and modifies the standard Kolmogorov phenomenology of the turbulent spectra, which assumes viscous dissipation at small scales [8]. So the anisotropy of the velocity fluctuations for decaying turbulence is due essentially to the Joule dissipation.

The dimensionless parameter introduced to characterize the effects of a uniform magnetic field applied to a decaying turbulence is the magnetic interaction number N. This number represents the ratio of the Lorentz force $\frac{\vec{\mathrm{j}} \wedge \vec{\mathrm{B}}}{\rho}$ to the inertia force $(\vec{\mathrm{u}} \cdot \vec{\nabla})\vec{\mathrm{u}}$: $N = \frac{\sigma B^2 L}{\rho v}$. The results presented in the literature showed that for a vanishing magnetic interaction number ($N \ll 1$), the anisotropy introduced by the magnetic field is negligible. However, when $N \gg 1$, the turbulence becomes anisotropic. The Joule dissipation tends to eliminate the velocity gradients in the direction of the imposed magnetic field and to elongate the turbulent eddies along the lines of the magnetic field. In this case, the turbulence approaches a two-dimensional state completely independent of the coordinate direction aligned with $\vec{\mathrm{B}}$. If N is larger than some critical value N_c, the turbulence is completely in a two-dimensional state and the Joule dissipation vanishes. In this case, the inertia force takes place again and it opposes to the bi-dimensionality and the anisotropy generated by the Lorentz force, which tends to restore isotropy. This picture of the turbulence behavior when $N \gg 1$ was first given by Moffat [2] and later by Sommeria and Moreau [9].

Because MHD turbulence at low magnetic Reynolds numbers is encountered in technological important flows, several models were developed under the quasi-static approximation. In the present work, we restrict our attention to the models of Widlund et al. [10] and Kenjeres et al. [11]. We recall here that the evaluation of the Widlund et al. model showed good agreement with the DNS of Shumann [4] for decaying MHD turbulence. In these simulations, Shumann restricted the attention to short times with magnetic fields corresponding to interaction numbers $N = 0$, 1, 5, and 50.

The purpose of the present paper is to study, for long times, the effects of the Lorentz force on a decaying homogeneous turbulence subjected to a strong magnetic field using the second-order models developed by Widlund et al. and Kenjeres et al.

2. Governing equations and models

2.1. Governing equations

In this section, we focus on the derivation of the evolution equations for the turbulent quantities. To this end, we first recall the Navier–Stokes equations for an incompressible conducting fluid subjected, at a low magnetic Reynolds number, to a magnetic field [12].

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + f_i \tag{3}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{4}$$

In these equations, the u_i are the components of the velocity, p, ρ and ν are respectively the pressure, the density and the kinematic viscosity. f_i is the instantaneous Lorentz force, which is given by:

$$f_i = \frac{1}{\rho}\varepsilon_{ikl} j_k b_l \tag{5}$$

The electric current density j_i is given by Ohm's law:

$$j_i = \sigma(e_i + \varepsilon_{ikn} u_k b_n) \tag{6}$$

where e_i is the electric field and b_i is the magnetic field. e_i can be defined, using the scalar electrostatic potential Φ, by:

$$e_i = -\frac{\partial \Phi}{\partial x_i}. \tag{7}$$

The divergence of Ohm's law together with Eq. (7) and the Kirchhoff continuity condition ($\vec{\nabla} \cdot \vec{j} = 0$) allow us to express the electrostatic potential as follows:

$$\frac{\partial^2 \Phi}{\partial x_i^2} = \varepsilon_{ijk} \frac{\partial (u_j b_k)}{\partial x_i} \tag{8}$$

At this point, it is convenient to decompose any flow variable g into an ensemble of mean and fluctuating parts as follows: $g = \bar{g} + g'$, where, for homogeneous turbulence, the mean $\bar{g}$ can be taken as a spatial average or as a statistically steady turbulence; it can be considered a time average. A direct averaging of Eqs. (3)–(4) yields the mean continuity and momentum equations which are as follows:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{9}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u'_i u'_j}\right) + \overline{f_i} \tag{10}$$

In order to achieve closure, we need a model for the Reynolds stress tensor. For a homogeneous turbulent flow, $R_{ij} = \overline{u'_i u'_j}$ is a solution of the transport equations:

$$\frac{\mathrm{d}}{\mathrm{d}t} R_{ij} = -R_{ik}\frac{\partial \bar{u}_j}{\partial x_k} - R_{jk}\frac{\partial \bar{u}_i}{\partial x_k} + \Pi_{ij} - \frac{2}{3}\varepsilon \delta_{ij} + \overline{u'_i f'_j} + \overline{u'_j f'_i} \tag{11}$$

where Π_{ij} and ε are respectively the pressure-gradient velocity and the viscous dissipation rate. We recall here that the interaction between the velocity field and the magnetic field is characterized by the magnetic Reynolds number Re_{m}. This number is defined by: $Re_{\mathrm{m}} = \frac{uL}{\eta}$. Here u is the r.m.s. fluctuating velocity and L is the integral length scale. As mentioned above, in this paper we are interested in a homogeneous MHD turbulent flow at a low magnetic Reynolds number so that the induced magnetic fluctuations b' around the uniformly imposed magnetic field $\vec{\mathrm{B}}$ are small. In this case the electromagnetic fluctuations, under the quasi-static approximation, are:

$$f'_i = \frac{1}{\bar{\rho}}\varepsilon_{ikn} j'_k B_n \tag{12}$$

$$j'_i = \sigma\left(e'_i + \varepsilon_{ikn} u'_k B_n\right) \tag{13}$$

$$e'_i = -\frac{\partial \Phi'}{\partial x_i} \tag{14}$$

$$\frac{\partial^2 \Phi'}{\partial x_i^2} = \varepsilon_{ijk}\frac{\partial (u'_j B_k)}{\partial x_i} \tag{15}$$

Hence a full Reynolds stress closure is achieved in incompressible MHD turbulence if models are provided for:

(i) the viscous dissipation rate ε;
(ii) the pressure–strain correlation Π_{ij};
(iii) the turbulence–Lorentz force interaction terms $\overline{u'_{\mathrm{m}} f'_{\mathrm{l}}}$.

The modeling of the pressure–strain correlation Π_{ij} is one of the central issues in the development of Reynolds stress closure models. Most of the popularly used models are based on the Poisson equation for the fluctuating pressure, which is obtained by taking the divergence of the Navier–Stokes equations [10]:

$$-\frac{1}{\rho}\nabla^2 p' = 2\frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial u'_j}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j}\left(u'_i u'_j - \overline{u'_i u'_j}\right) - \frac{\partial f'_i}{\partial x_i} \tag{16}$$

This equation contains three terms. The first one arises from the mean of strain and its interaction with the turbulence. The second term is generated by a mutual interaction between turbulence components, and the last term represents the contribution of the divergence of fluctuating Lorentz force. The models proposed for pressure strain correlations must reflect the mechanisms implied in the fluctuating pressure field. Thus, the models will be comprised of a rapid part Π_{ij}^{r} representing the linear interaction of turbulence with the mean velocity field, of a slow part Π_{ij}^{s} responsible for the nonlinear interactions between the velocity fluctuations, and of a magnetic part Π_{ij}^{m}, which represents the interaction between turbulence and the Lorentz force:

$$\Pi_{ij} = \Pi_{ij}^{\mathrm{r}} + \Pi_{ij}^{\mathrm{s}} + \Pi_{ij}^{\mathrm{m}} \tag{17}$$

2.2. Literature models for the Lorentz force effects on turbulence

At low magnetic Reynolds numbers, the Lorentz force tends to counteract the turbulence, in the process causing a net dissipation of turbulent kinetic energy called the magnetic dissipation, or Joule dissipation, which reduces the turbulent transport of heat and momentum. The modeling of the Joule dissipation tensor, in physical-space, received little attention. To our knowledge, there is a few authors who worked on this problem. In this section we present the second-order MHD models of Widlund et al., Kenjeres et al. and the theoretical analysis of Davidson [13,14], which are developed under stationary conditions.

2.2.1. The model of Widlund et al. [10]

In the second-order closure model developed by Widlund et al., the components of the Reynolds stress tensor are given by the transport equations:

$$\frac{\mathrm{d}R_{ij}}{\mathrm{d}t} = \left.\frac{\mathrm{d}R_{ij}}{\mathrm{d}t}\right|_{\mathrm{hyd}} - m_{ij} \tag{18}$$

where $\frac{\mathrm{d}R_{ij}}{\mathrm{d}t}|_{\mathrm{hyd}}$ is the hydrodynamic part of the Navier–Stokes equations:

$$\frac{\mathrm{d}}{\mathrm{d}t} R_{ij}|_{\mathrm{hyd}} = -R_{ik}\frac{\partial \bar{u}_j}{\partial x_k} - R_{jk}\frac{\partial \bar{u}_i}{\partial x_k} + \Pi_{ij}^{\mathrm{r}} + \Pi_{ij}^{\mathrm{s}} - \frac{2}{3}\varepsilon\delta_{ij} \tag{19}$$

As mentioned above, our principal object is to study the decaying homogeneous MHD turbulent flow. In this case, the turbulent production and the rapid part of the pressure–strain correlation vanish. For the slow part Π_{ij}^{s}, we admit the Rotta model [15]: $\Pi_{ij}^{\mathrm{s}} = -3\varepsilon b_{ij}$. In the Widlund et al. model, the Lorentz force effects are incorporated into a Joule dissipation tensor m_{ij} defined as:

$$m_{ij} = -u_i' f_j' - u_j' f_i' - \Pi_{ij}^{\mathrm{m}} \tag{20}$$

When we consider l_i, the unit vector parallel with the imposed magnetic field, the model of Widlund et al. is given by:

$$m_{ij} = \frac{2\sigma B^2}{\rho}\left[G(\alpha, I_{\mathrm{la}})R_{ij} + \frac{H(\alpha)}{2}(l_i l_k R_{kj} + l_j l_k R_{ki})\right] \tag{21}$$

where

$$H(\alpha) = -\frac{27}{10}\alpha^2(1-\alpha) \tag{22}$$

$$G(\alpha, I_{\mathrm{la}}) = \alpha - 2H(\alpha)\left(I_{\mathrm{la}} + \frac{2}{3}\right) \tag{23}$$

$$I_{\mathrm{la}} = \frac{l_i l_k R_{ki}}{K} - \frac{2}{3} \tag{24}$$

K is the turbulent kinetic energy and $\alpha = l_i l_j Y_{ij}/2K$ is the normalized dimensionality anisotropy variable of Y_{ij}. The last one is defined by: $Y_{ij} = \int \frac{k_i k_j}{k^2}\Phi_{nn}(\vec{k})\mathrm{d}^3\vec{k}$. Further details about this tensor could be found in Ref. [10]. In the limit of two-dimensional turbulence, α is equal to zero and takes the value 1/3 for isotropic turbulence. Widlund et al. proposed for α a phenomenological transport equation. This equation, in homogeneous turbulence, can be written as:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = P_\alpha + \Pi_\alpha + \pi_\alpha - m_\alpha. \tag{25}$$

The source terms in the α model equation are:

$$P_\alpha = -\frac{1}{K} S_{im} R_{ki} \big[(G+\alpha)\delta_{mk} + H l_m l_k \big] \tag{26}$$

$$\Pi_\alpha = \alpha(1-\alpha)\left[-\frac{9}{5}(1-3\alpha) - \frac{27}{5}\alpha \right] S_{ik} l_k l_i \tag{27}$$

$$\pi_\alpha = C_{\alpha_2} \frac{\varepsilon}{K} \left(\frac{1}{3} - \alpha \right) \tag{28}$$

$$m_\alpha = C_{\alpha_1} \frac{2\sigma B^2}{\rho} \alpha^2 \tag{29}$$

where $S_{ij} = 0.5(\bar{u}_{i,j} + \bar{u}_{j,i})$ is the mean strain rate and $C_{\alpha_1} = 1.2$ and $C_{\alpha_2} = 0.1$.

Finally, to close these equations, the model equation given by Widlund et al. for the viscous dissipation rate takes the form:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -C_{\varepsilon_1} \frac{\varepsilon}{K} \overline{u_i' u_j'} \bar{u}_{i,j} - C_{\varepsilon_2} \frac{\varepsilon^2}{K} - C_{\varepsilon_\alpha} \frac{2\sigma B^2}{\rho} \varepsilon\alpha \tag{30}$$

in the case of a homogeneous turbulence, where $C_{\varepsilon_1} = 1.44$, $C_{\varepsilon_2} = 1.83$, and $C_{\varepsilon_\alpha} = 0.5$.

2.2.2. *The model of Kenjeres et al. [11]*

The second-order MHD model of Kenjeres et al. is based on the analysis of the direct numerical simulations of Noguchi et al. [16] for a turbulent flow in an infinite plane channel subjected to a uniform magnetic field. This model produces results that are in good agreement with both the DNS and the experimental data for channel turbulent flows. For a free homogeneous turbulent flow, this model can be written as:

$$\frac{\mathrm{d}R_{ij}}{\mathrm{d}t} = \left.\frac{\mathrm{d}R_{ij}}{\mathrm{d}t}\right|_{\mathrm{hyd}} + \Pi_{ij}^{\mathrm{m}} + Q_{ij}^{\mathrm{m}} \tag{31}$$

where Π_{ij}^{m} and Q_{ij}^{m} are respectively the magnetic part of the pressure–strain correlation and the production of the stress tensor due to magnetohydrodynamic interactions. These terms are given by:

$$\Pi_{ij}^{\mathrm{m}} = -C_4 \left(Q_{ij}^{\mathrm{m}} - \frac{1}{3} Q_{ii}^{\mathrm{m}} \delta_{ij} \right) \tag{32}$$

$$Q_{ij}^{\mathrm{m}} = -\frac{\sigma}{\rho} \left(\varepsilon_{ikl} B_l \overline{u_i' \frac{\partial \Phi}{\partial x_k}} + 2K B_k^2 - B_i B_k R_{ij} \right) + \frac{\sigma}{\rho} \big(B_i B_k R_{jk} + B_j B_k R_{ik} - 2 B_k^2 R_{ij} \big) \tag{33}$$

$$\overline{u_i' \frac{\partial \Phi}{\partial x_j}} = C_\lambda \varepsilon_{jkl} B_l R_{ik}, \tag{34}$$

In the Kenjeres et al. model, the equations for the hydrodynamic part of the Reynolds stress tensor $\frac{\mathrm{d}R_{ij}}{\mathrm{d}t}|_{\mathrm{hyd}}$ are closed by the following equation model of the viscous dissipation rate:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -C_{\varepsilon_1} \frac{\varepsilon}{K} \overline{u_i' u_j'} \bar{u}_{i,j} - C_{\varepsilon_2} \frac{\varepsilon^2}{K} + \frac{C_{\varepsilon 4}}{2} \frac{\varepsilon}{K} Q_{ii}^{\mathrm{m}} \tag{35}$$

this model involves eight constants and their values are given by: $C_{\varepsilon_1} = 1.44$, $C_{\varepsilon_2} = 1.92$, $C_\lambda = 0.6$, $C_4 = 0.6A^{1/2}$, $C_{\varepsilon_4} = 6.5 \min(A_2, 0.25)$, $A = 1 - 9/8(A_2 - A_3)$, $A_2 = 4b_{ij}b_{ji}$, $A_3 = 8b_{ij}b_{jk}b_{ki}$.

2.2.3. *The theoretical analysis of Davidson [13,14]*

In the present study, the turbulent kinetic energy decay, for high magnetic interaction numbers and long times, obtained with the second-order models of Widlund et al. and Kenjeres et al., is compared with the theoretical analysis developed by Davidson. In this analysis, the energy flux is modeled according to the integral length scales parallel (L_{p}) and normal (L_{n}) to the imposed magnetic field. For Davidson, the energy decay is given by:

$$\frac{\mathrm{d}K}{\mathrm{d}t} = -\beta_1 \frac{K^{3/2}}{L_{\mathrm{n}}} - \beta_2 \left(\frac{L_{\mathrm{p}}}{L_{\mathrm{n}}} \right)^2 \frac{K}{\tau} \tag{36}$$

where $\tau = (\sigma B^2/\rho)^{-1}$ is the Joule dissipation time, and β_1, β_2 are dimensionless coefficients of order unity.

$L_{\mathrm{p}} = L_{\mathrm{n}}$ for turbulent flow in isotropic state. When the magnetic field is applied, the turbulence becomes anisotropic, with $L_{\mathrm{p}} > L_{\mathrm{n}}$. In the analyses of Davidson, the ratio $L_{\mathrm{p}}/L_{\mathrm{n}}$ increases with the increase of the initial magnetic interaction number. The model equation has been tested against direct numerical simulations for Saffman turbulence, in which $E(k \to 0) \sim k^2$, and for Batchelor turbulence, in which $E(k \to 0) \sim k^4$, by Davidson et al. [17]. This test shows that Eq. (36) is a good approximation for fully developed turbulence. The same equation has been integrated by Davidson, and gives for the previous cases the following expressions, respectively:

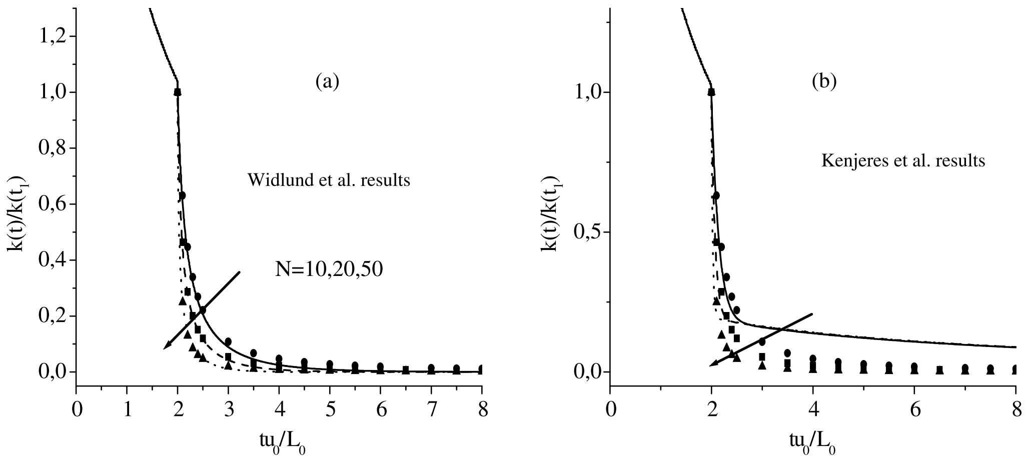

Fig. 1. Time evolution profile of the turbulent kinetic energy.

- Saffman turbulence [13]

$$K/K_0 = t^{*-1/2}\left[1 + \frac{5\beta_1}{9\beta_2}\left(t^{*3/4} - 1\right)N_0\right]^{-6/5} \tag{37}$$

$$L/L_\mathrm{p} = t^{*-1/2}\left[1 + 5\beta_1/9\beta_2 N_0\left(t^{*3/4} - 1\right)\right]^{-2/5} \tag{38}$$

- Batchelor turbulence [14]

$$K/K_0 = t^{*-1/2}\left[1 + \frac{7\beta_1}{15\beta_2}\left(t^{*3/4} - 1\right)N_0\right]^{-10/7} \tag{39}$$

$$L/L_\mathrm{p} = t^{*-1/2}\left[1 + 7\beta_1/15\beta_2 N_0\left(t^{*3/4} - 1\right)\right]^{-2/7} \tag{40}$$

where N_0 is the initial magnetic interaction number, $t^* = 1 + 2\beta_2 t/\tau$ is a dimensionless time and L is the integral length scale for a turbulent flow in its isotropic state.

3. Results and discussion

The transport equations incorporating the Widlund et al. and Kenjeres et al. models discussed above are solved numerically for homogeneous MHD turbulence using a fourth-order Runge–Kutta numerical integration scheme. In the present work, we consider a decaying homogeneous turbulence in a conducting fluid at a low magnetic Reynolds number, which at time $t_1 = 2$ is exposed to a uniform magnetic field $B = B_i\delta_{i3}$. In this case, the x_1 and the x_2 directions are indistinguishable. The axisymmetry around the direction of the magnetic field allows us to distinguish only between components in directions parallel to the magnetic field, which are designed by letter p, and to any axes, indicated by letter n, normal to the magnetic field. Five simulations are performed in this study with a magnetic fields corresponding to magnetic interaction numbers $N = 10$, 20, 50, 100, 150. In Figs. 1a and 1b, the time evolution of the turbulent kinetic energy predicted by the Widlund et al. and Kenjeres et al. models are compared with the theoretical analysis of Davidson [13]. Before $t_1 = 2$, the turbulent kinetic energy decreases with time; this is obvious since the turbulence evolves without turbulent production terms. When the turbulence is subjected to a magnetic field, we remark that both models predict an increase in the reduction rate of the turbulent kinetic energy when increasing the initial magnetic interaction number. These results show the dissipative character of the Lorentz force at low magnetic Reynolds numbers; the Lorentz force dissipates the velocity fluctuations via Joule dissipation. This result is observed in the numerical experiences of Schumann [4] and Vorobev et al. [1] for short times. It is also clear from these figures that the Widlund et al. model is in excellent agreement with the theoretical analysis of Davidson. While the Kenjeres et al. model is unable to predict correctly the effect of a strong magnetic interaction number on the decay of the turbulent kinetic energy, the increase in N has little effect, localized near the moment when the magnetic field is applied.

The time evolution of the ratio between the Joule dissipation and the viscous dissipation rates obtained with the Widlund et al. and Kenjeres et al. models are shown respectively in Figs. 2a and 2b. It is clear from Fig. 2a that for long times and in the presence of a strong magnetic field, the ratio $m(t)/\varepsilon(t)$ increases with an increase in the magnetic interaction number, and the Joule dissipation is responsible for the major part of the total dissipation rate. This result is in good agreement with

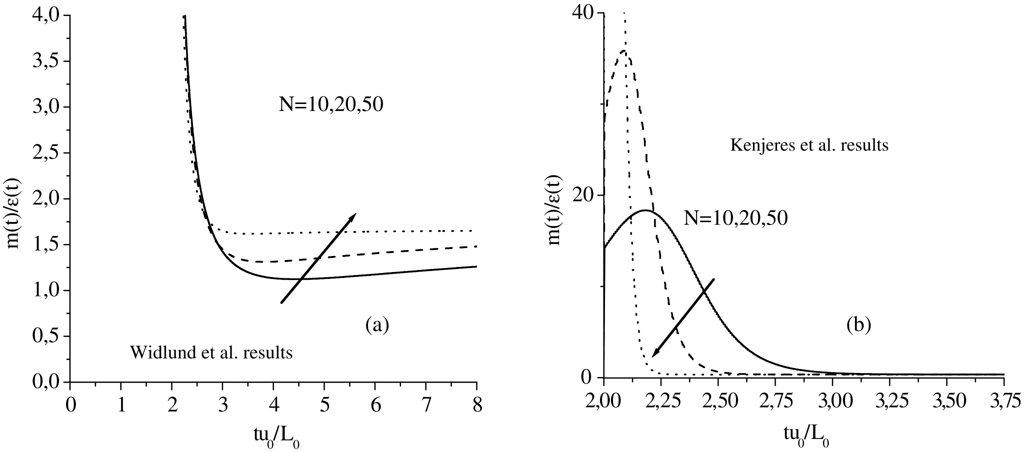

Fig. 2. Time evolution profile of the Joule dissipation to viscous dissipation ratio.

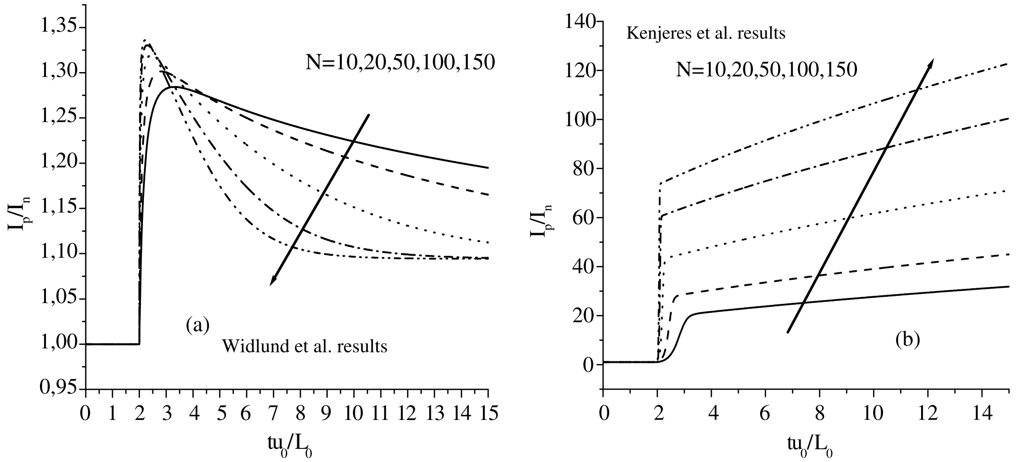

Fig. 3. Time evolution profile of the parallel turbulence intensity to normal turbulence intensity ratio.

the DNS of Vorobev et al. [1]. Fig. 2b shows that the Kenjeres et al. model yields bad results for the time evolution of this ratio. We note that the Joule dissipation rate becomes negligible for short times compared with the viscous dissipation rate. It thus seems that the Joule dissipation time is badly estimated by this model.

As mentioned in the literature, the turbulent intensities are inhibited by the presence of a magnetic field. This inhibition is more pronounced in the direction normal to the magnetic field. So the turbulence is dominated by the velocity component parallel to the magnetic field. This can be seen clearly in Figs. 3a and 3b, which display the time evolution of the ratio I_p/I_n between the turbulent intensities in the directions parallel and normal to the magnetic field predicted by the two models. These figures show that for $t < 2$, I_p/I_n is equal to 1; this means that the turbulence is isotropic for the time interval $[0, 2]$. After the imposition of the magnetic field, this ratio deviates from its isotropic value and becomes larger than 1. So when the magnetic field is imposed, the turbulence transits to an anisotropic state. This anisotropy is generated by Joule dissipation and its degree depends on the strength of the magnetic field. In general, the Reynolds stress anisotropy is captured here by both models. This observation agrees with the numerical experiences of Shumann [4] and Vorobev et al. [1]. In the time evolution of the ratio I_p/I_n given by the Widlund et al. model (Fig. 3a), three phases are distinguished. In the first one ($t < 2$), where the magnetic field is absent, the turbulence is dominated by the nonlinear effects which maintain it in an isotropic state. However, when a strong magnetic field is applied, the turbulence is developed to a strongly anisotropic state for $t = [2, 4]$. In this region, the nonlinear effects decrease with increasing the magnetic interaction number and become dominated by the linear effects of the Lorentz force which generate the anisotropy of the turbulence. It is also clear from Fig. 3a that the degree of anisotropy of the turbulence, in this phase, increases with increasing the initial magnetic interaction number. As confirmed by several works, anisotropy is primarily reflected in an increase of the integral length scale in the direction of the magnetic field. Vorobev et al. show that, when the anisotropy is significant, the magnetic interaction number should include the longitudinal integral length scale L_p and propose the formula: $N = \frac{\sigma B^2 L^3}{\rho \nu L_p^2} = N_0(L/L_p)^2$. L is the integral length scale of the isotropic turbulent flow. So when L_p grows, the interaction number decreases and the nonlinear effects due to the inertia force take place again. These effects make the turbulence approach a nearly isotropic state for high magnetic interaction numbers. This phenomenon is very well presented by the Widlund et al. model. As one can remark, for the last phase the magnitude of the anisotropy decreases and the ratio I_p/I_n approaches its isotropic value. It is also observed from these results that eventually the effects of the Lorentz force become weak. There is no important difference between the cases $N = 100$ and $N = 150$. In this case, the Lorentz force vanishes. According to relation (2), this means that the velocity gradients in the direction of the magnetic field are eliminated. So the turbulence transits to a two-dimensional state independent of the z-coordinate. This picture of the decaying homogeneous MHD turbulence for strong magnetic field is a result of the interaction between linear and nonlinear effects. This interaction, which is controlled by the magnetic field, is well predicted here by the Widlund et al. model. When we observe the results plotted in Fig. 3b, we remark that the Kenjeres et al. model yields extremely bad results. The return of the turbulence to its isotropic state for high magnetic interaction numbers and for long times is not observed in this figure. These predictions are not surprising because in this model, the Lorentz term does not appear explicitly in the pp-component of the Reynolds stress tensor, which is inconsistent with the Navier–Stokes equations for a free MHD turbulent flow.

4. Conclusion

In this paper, the second-order models of Widlund et al. and Kenjeres et al. are retained to study the effects of strong imposed magnetic field on the decaying homogeneous MHD turbulence. Five simulations in which $N = 10$, 20, 50, 100, 150 are investigated, and the results are compared qualitatively with theoretical analysis and numerical experience data. The computational results show that the Kenjeres et al. model yields poor predictions of the time evolution of a decaying MHD turbulence. The results of DNS indicate that the action of a strong magnetic field is twofold. First the turbulent velocity fluctuations are dissipated due to the Joule dissipation. Second, the turbulence becomes anisotropic, its structure being elongated in the direction of the magnetic field. The limiting case is a two-dimensional state independent of the coordinate direction aligned with the imposed magnetic field. This picture of the flow transformation for a strong magnetic field is well presented by the model of Widlund et al.

References

[1] A. Vorobev, O. Zikanov, A. Davidson, B. Knaepen, Anisotropy of magnetohydrodynamic turbulence at low magnetic Reynolds number, Phys. Fluids 17 (2005) 125105.
[2] K. Moffat, On the suppression of turbulence by a uniform magnetic field, J. Fluid Mech. 28 (1967) 571.
[3] A. Davidson, Magnetohydrodynamics in materials processing, Annu. Rev. Fluid Mech. 31 (1999) 273.
[4] U. Shumann, Numerical simulation of the transition from three-to-two-dimensional turbulence under a uniform magnetic field, J. Fluid Mech. 74 (1976) 31.
[5] O. Zikanov, A. Thess, Direct numerical simulation as a tool for understanding MHD liquid metal turbulence, Appl. Math. Model. 28 (2004) 1.
[6] A. Votsish, B. Kolesnikov, Investigation of transition from three-dimensional turbulence to two-dimensional under a magnetic field, Magnetohydrodynamics 3 (1976) 141.
[7] A. Alemany, R. Moreau, L. Sulem, U. Frisch, Influence of external magnetic field on homogeneous MHD turbulence, J. Mech. 18 (1979) 280.
[8] S.C. Kassinos, W.C. Reynolds, Structure-based modeling for homogeneous MHD turbulence, in: Center of Turbulence Research, Annu. Res. Briefs, 1999.
[9] J. Sommeria, R. Moreau, Why, how and when MHD turbulence becomes two-dimensional, J. Fluid Mech. 118 (1982) 507.
[10] O. Widlund, S. Zaharai, H. Bark, Development of a Reynolds stress closure for modeling of homogeneous MHD turbulence, Phys. Fluids 10 (1998) 1987.
[11] S. Kenjeres, K. Hanjalic, D. Bal, A direct–numerical–simulation-based second-order–moment closure for turbulent magnetohydrodynamic flows, Phys. Fluids 16 (2004) 1229.
[12] M. Kinet, P. Burattini, D. Carati, B. Kanaepen, Decay of passive scalar fluctuations in homogeneous magnetohydrodynamic turbulence, Phys. Fluids 20 (2008) 075105.
[13] A. Davidson, Turbulence: An Introduction for Scientists and Engineers, Oxford University Press, 2004.
[14] A. Davidson, On the decay of Saffman turbulence subject to rotation, stratification or an imposed magnetic field, J. Fluid Mech. 663 (2010) 268.
[15] B. Gatski, G. Speziale, On explicit algebraic stress models for complex turbulent flows, J. Fluid Mech. 254 (1993) 59.
[16] H. Noguchi, Y. Ohtsubo, N. Kasagi, DNS database of turbulence heat transfer, ftp.thtlab.tu–tokyo.ac.jp/dns, 1998.
[17] N. Okamoto, A. Davidson, Y. Kaneda, The decay of magnetic Reynolds number turbulence in an imposed magnetic field, J. Fluid Mech. 651 (2009) 295.