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Non-linear couplings, from the early to the late time universe

Couplages non linéaires, de l'univers primordial à l'univers récent

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A B S T R A C T

Deciphering the mechanisms at play in the formation and evolution of the large-scale structure of the universe is part of the scientific goals of many projects of observational cosmology. In particular, large-scale structure observations can be used to infer mode-coupling effects, whether they come from the physics of the early universe or from its late time evolution. Specificities of such couplings are presented, noting that in principle they can be directly detected through bispectra of the cosmic microwave background temperature anisotropies or density in the local universe. The existence of such couplings have however more far-reaching consequences for the growth of the structure. Those are sketched as well as their possible observational impacts.

R É S U M É

La compréhension fine des mécanismes en jeu au cours de la formation des structures à grande échelle de l'univers est l'un des objectifs scientifiques communs à de nombreux projets de cosmologie observationnelle. Les observations des grandes structures permettent de révéler les effets des couplages de modes, qu'ils soient associés à des processus physiques dans l'univers primordial ou à l'évolution plus tardive de ces structures. Les propriétés de ces couplages sont décrites, en soulignant qu'en principe ils peuvent être directement détectés grâce au bispectre des anisotropies de température du fond diffus cosmologique ou du champ de densité dans l'univers local. L'existence de tels couplages a toutefois des conséquences plus profondes pour la croissance des structures. Celles-ci sont esquissées, ainsi que leurs possibles implications observationnelles.
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1. Introduction

The large-scale structure (LSS) of the universe and the cosmic microwave background (CMB) offer unique windows on the physics of the early Universe, in particular on inflationary models thought to be at the origin of the large-scale structure

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of the universe. The statistical properties of the density and velocity perturbations for the former and of the temperature anisotropies and polarization for the latter depend both on the inflationary period during which they were created and on the physics at play after horizon crossing, during recombination time and during the subsequent stages of the evolution of the universe. The angular power spectrum of the CMB anisotropies has been extensively used to set constraints on the basic cosmological parameters and the shape of the inflationary potentials (see of course the Planck results in [1] and this volume).

At linear order in metric perturbations, physical processes at play affect the metric perturbations by a multiplicative transfer function. The characteristic features observed in the large-scale cosmological observations in general, such as spectra, originate therefore from the metric and fluid couplings that this transfer function encodes, whereas the overall amplitude of the initial (that is super-Hubble) metric perturbation and its scale dependences are determined by the inflationary phase. All these aspects, from the identification of modes in the early universe, to the late time growth of structure, are now fully understood (and detailed in standard reference textbooks, [2–7]) at least at the linear level. In such calculations, the metric perturbations are linearized so that the nonlinear couplings that are inherently present in the evolution equations are ignored. An important consequence in models that predict Gaussian initial metric fluctuations is that all cosmic fields will follow Gaussian statistical properties. This is a priori the case for generic models of inflation to be contrasted with models with active topological defects—such as textures or cosmic strings—that have soon been recognized as models that could produce large primordial non-Gaussianities (see the early works in [8–11]).

In the case of single-field standard slow-roll inflation, it has been unambiguously shown in [12] that it can produce only very weak non-Gaussian signals. It has however been realized that some models of inflation might produce significant deviation from Gaussianity, whether in the context of non-standard single-field inflation (such as DBI inflation,[1] see [14]), in the context of the curvaton model [15,16] or in the context of multiple-field inflation [17–19]. The question of the observation of primordial non-Gaussianities is then largely open. In general, however, primordial deviations from Gaussianity are in competition with the couplings induced during the non-linear evolution of the cosmic fields. This is the issue we would like to present in this short review paper, whether it is at the level of the CMB observations—in the observation of the CMB bispectrum in particular—or at the level of the LSS observations. The second section will be devoted to the general formalism; the third and fourth ones explore respectively the consequences of such couplings for CMB and LSS observations. In the last section, a general scheme that one may want to employ in order to extract cosmological information out of LSS observations is sketched.

2. The gravity-induced mode coupling structure

Hereafter, for the sake of our presentation, we assume that on super-Hubble scales, the only significant scalar perturbations are *adiabatic*—they correspond to only one scalar degree of freedom—and that they obey a nearly Gaussian statistics (observations anyhow exclude strongly non-Gaussian initial metric perturbations). To be more precise, they can be described in the Fourier space by a single scalar field[2] $\zeta(\mathbf{k})$ defined in such a way that it is constant at super-Hubble scales (see [20] and also [12]), $\mathbf{k}$ being a comoving wave-number, that satisfies

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle_h = P_\zeta(k_1) \tag{1}$$

and

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle_h = 2\, f_{\rm NL}^{\zeta}(\mathbf{k}_1,\mathbf{k}_2) P_\zeta(k_1) P_\zeta(k_2) + \text{sym.} \tag{2}$$

where $\langle\ldots\rangle_h$ stands for the ensemble average of product of Fourier modes after the $\delta_{\rm D}(\sum \mathbf{k}_i)$ factor that always appear in such ensemble averages (due to statistical homogeneity) has been dropped and where "sym." stands for the two other terms obtained by permutation of the wave-numbers. This defines the primordial power spectrum $P_\zeta(k)$ and the primordial mode coupling amplitude[3] $f_{\rm NL}^{\zeta}$. Considering an observable quantity θ related to the perturbation variables, the effect of evolution can generically be recapped[4] as

$$\theta(\mathbf{k}) = \mathcal{T}_\theta^{(1)}(k)\,\zeta_0(\mathbf{k}) + \int_{\mathbf{k}_1\ldots\mathbf{k}_p} \mathcal{T}_\theta^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\,\zeta_0(\mathbf{k}_1)\zeta_0(\mathbf{k}_2) + \ldots \tag{3}$$

[1] These models of inflation can be captured with an effective theory model as shown initially in [13].

[2] Note that the choice of ζ as the primordial field is not unique and one could have chosen the Bardeen potential. With such a choice, however, the $f_{\rm NL}^{\zeta}$ incorporates only the inflation-dependent couplings—$f_{\rm NL}^{\zeta}$ is proportional to the slow-roll parameter in single-field inflation, for instance. The other coupling terms induce by the change of variable can be incorporated into $\mathcal{T}^{(2)}$; see Eq. (5) below.

[3] This is the expression for the bispectrum obtained assuming ζ could be expanded as $\zeta = \zeta_G + f_{\rm NL}^{\zeta}\zeta_G\zeta_G$, where ζ_G is assumed to obey Gaussian statistics. This is not however a valid description when the bispectrum originates from multiple-field couplings or from quantum calculation. The formal expression (2) is always valid though; see Refs. [21,22].

[4] Things are actually slightly more complicated since usually observables cannot be decomposed into 3D Fourier modes. The functions $\mathcal{T}_\theta^{(1)}(k)$ and $\mathcal{T}_\theta^{(2)}(\mathbf{k}_1,\mathbf{k}_2)$ should then be thought as projection operators. This is in particular the case for temperature anisotropies and polarizations. This does not affect, however, the general point we want to make in this introductory section.

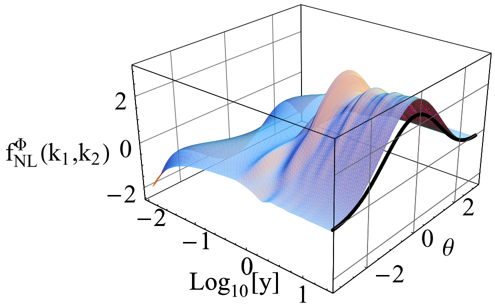

Fig. 1. (Color online.) The second-order potential computed from a full treatment of the field couplings (including CDM, baryons and photons) as a function of time variable y, the expansion factor in units of its value at equality, and of θ, angle between wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. The figure corresponds to $k_1 = 6k_{eq}$ and $k_2 = 12k_{eq}$, where k_{eq} is the horizon wave mode at equality. It is compared to its expected late-time behavior (6). It is to be noted that the convergence toward this solution is extremely rapid and takes place as soon as equality is reached, e.g. $y = 1$. The difference in the amplitude of the function is due to the fact that the baryon component has been neglected in the derivation of (6).
Figure taken from [23].

where $\int_{\mathbf{k}_1\ldots\mathbf{k}_p}$ stands for a convolution, $\int_{\mathbf{k}_1\ldots\mathbf{k}_p} = \int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_p\,\delta_D(\mathbf{k} - \sum_i \mathbf{k}_i)$, $\mathcal{T}_\theta^{(1)}(k)$ is the linear transfer function and $\mathcal{T}_\theta^{(2)}(\mathbf{k}_1, \mathbf{k}_2)$ is the second-order transfer function. When computing for instance the bispectrum of θ, there will be a contribution from the mode couplings induced by the second-order transfer function $\mathcal{T}^{(2)}$ and possible initial non-Gaussianities,

$$\langle\theta(\mathbf{k}_1)\theta(\mathbf{k}_2)\theta(\mathbf{k}_3)\rangle_h = 2\left[f_{NL}^{\zeta} + f_{NL}^{\theta}\right]\mathcal{T}_\theta^{(1)}(k_1)\mathcal{T}_\theta^{(1)}(k_2)\mathcal{T}_\theta^{(1)}(k_3)\;P_\zeta(k_1)P_\zeta(k_2) + \text{sym.} \tag{4}$$

where f_{NL}^{θ} is related to the second-order transfer function by

$$\mathcal{T}_\theta^{(2)}(\mathbf{k}_1, \mathbf{k}_2) \equiv f_{NL}^{\theta}(\mathbf{k}_1, \mathbf{k}_2)\,\mathcal{T}_\theta^{(1)}(|\mathbf{k}_1 + \mathbf{k}_2|) \tag{5}$$

In such observables, there will then be a competition between the initial coupling structure encoded by f_{NL}^{ζ} and the one coming from the nonlinear evolution of the fields, f_{NL}^{θ}. Regarding the first one some specific forms have been put forward, such as $f_{NL}^{\text{loc.}}$ which corresponds to a pure local coupling and which is independent of the wave modes, $f_{NL}^{\text{eq.}}$ which peaks for equilateral configuration and f_{NL}^{orth} for yet other configurations. More specifically, these templates allow to explore different extensions of single-field slow-roll inflation and should be constrained—if not detected—independently. For instance local non-Gaussianity is a sensitive probe of multi-field models (due to the single-field consistency relations) whereas equilateral non-Gaussianity is a probe of non-slow roll dynamics through self-interactions of the perturbations. Finally the shape of f_{NL}^{θ} is eventually to be compared to those forms to asses how much it can mimic primordial coupling effects.

The derivation of the full details of $\mathcal{T}_\theta^{(2)}$, however, is, for any realistic θ, a fantastic task. It requires an understanding of the metric fluctuations behavior at second order, from radiation-dominated super-Hubble scales to the matter-dominated era at sub-Hubble scale, as well as a comprehension of the physics of the recombination—through the Boltzmann equation—at a similar order. Such a task has been undertaken by several groups[5] for CMB temperature bispectra, and the multitude of effects at play have now been sorted out. On the other hand, at sub-horizon scales and for a pressure-less fluid, mode coupling due to gravitational clustering is, by far, not a novel subject. It can be traced back to early works by Peebles [2], where the function $\mathcal{T}^{(2)}$ for the non-linear sub-Hubble evolution of cold dark-matter field (CDM) during a matter-dominated era was derived. General mode-coupling effects, within the same regime, have been extensively studied in the 1980s and 1990s, when a whole corpus of results has been obtained (see, e.g., Ref. [29] for an exhaustive review). For instance, on sub-horizon scales, the second-order mode-coupling function for the gravitational potential reads:

$$f_{NL}^{\Phi}(\mathbf{k}_1, \mathbf{k}_2) = \frac{k_1^2 k_2^2}{\frac{3}{2}H^2 a^2 |\mathbf{k}_1 + \mathbf{k}_2|^2}\left(\frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_2^2} + \frac{2}{7}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2 k_2^2}\right) \tag{6}$$

in the particular case of an Einstein–de Sitter universe (here a is the scale factor, H the Hubble constant). This well-established result proved to be useful for observational cosmology. This is illustrated in Fig. 1, where one can observe the behavior of the f_{NL}^{Φ} parameter for a specific geometry and as a function of the time parameter y defined as $y = a/a_{eq}$, where a_{eq} is the time of equivalence, i.e. the time at which the energy densities in the non-relativistic species and relativistic species are the same (so that for $y > 1$ the universe become matter dominated). The figure clearly shows that the form (6), corresponding to the solid line, is very robust.

[5] Early derivations are to be found in [24–26]; a somewhat more rigorous and comprehensive calculation—including a proper derivation of the Boltzmann coupling terms and taking into account the polarization effects—is to be found in Refs. [27,28] in the context of CMB observations.

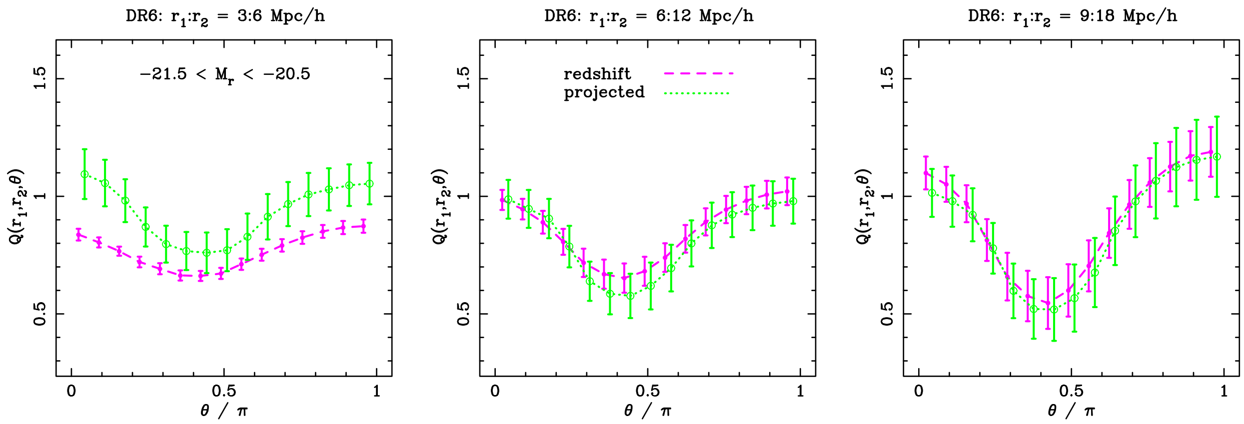

Fig. 2. (Color online.) Example of measured reduced three-point correlation function (from the data release 6 of the Sloan Digital Sky galaxy Survey) showing both redshift space and measurements in the projected catalogue. The filled symbols are redshift measurements, and the hollow symbols are projected. The selected galaxy population corresponds to limited magnitudes $-21.5 < M_r < -20.5$. The three panels are different scales, specified by the first side of the triangle (r_1) representing the smallest scale measured. Error bars denote 1σ uncertainties calculated from 30 jackknife samples.
Figure taken from [30].

It is to be stressed that the angular modulation it exhibits has eventually been observed in actual data sets (see, e.g., Refs. [31,32,30]), as shown for instance in Fig. 2. It is needless to say that such measurements are quite difficult to do in practice and compared to the two-point correlation function, or its counterpart in Fourier space, progress to overcome the systematic error associated with such measurements has been very slow. This is probably unfortunate, as such measurements provide a direct test of the current large-scale structure paradigm through the measurement of the coupling structure. It is also of crucial importance if one wants to have a good understanding of the biasing properties of the galaxy field, that is how galaxies trace, or not, the underlying density field as we will discuss in the last section.[6]

What these explorations show is that mode couplings are inevitable as soon as gravity starts to shape the structures. Consequences of such couplings are then ubiquitous and multifold. They are to be found in the early universe and in the properties of the low-z large-scale structure of the universe. It will induce mode-coupling effects not only in the 3D galaxy fields, but also in the CMB observations. The most direct way to actually grasp the importance of such effects is to consider the CMB bispectrum. This is the topic of the next section.

3. Consequences for the temperature CMB bispectrum

The direct consequences of the existence of gravitational couplings during the phases of recombination is indeed that it induces a non-vanishing bispectrum that can be superimposed on the one originating from the primordial mode-coupling effects. The calculation of these effects proved cumbersome and was painstakingly developed over the previous years [35–41]. It finally culminated with the results presented in [34], where all numerical difficulties were shown to be under control. Results of calculations are illustrated in Fig. 3.

The expected signal-to-noise ratio for such detection is however small, of the order of 1 at best, for *Planck*-like observations. Its intrinsic detection is therefore beyond reach for the current data sets, but these effects should nonetheless be taken into account when one wishes to put constraints on possible primordial couplings (usually parametrized with the $f_{\rm NL}^{\rm loc}$, $f_{\rm NL}^{\rm eq}$ and $f_{\rm NL}^{\rm orth}$ templates introduced before). The current constraints obtained from Planck[7] are, $f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8$, $f_{\rm NL}^{\rm eq} = -42 \pm 75$ and $f_{\rm NL}^{\rm orth} = -25 \pm 39$ [43] where intrinsic mode-coupling effects have been taken into account.

These are truly impressive results! Note that the fact that the constraints are of the order of a few units does not mean that the departure from Gaussianity can be significant. It actually means that the departure from Gaussianity (as it would be measured from a reduced skewness for instance) is of the order of the amplitude of the metric fluctuations, that is of order 10^{-5} (it can be inferred from Eq. (2)). Departure from Gaussianity in the early universe can therefore only be extremely small. As a consequence, it can be treated as a small perturbation throughout the late-time evolution of the universe, which is an advantage, but it makes its detection all the more challenging. And finally note that although the current constraints have a real discriminatory power, LSS observations would be extremely useful were we able to put more stringent upper bounds on these quantities. This is partly the challenge the community faces with the analysis and the scientific exploitation of the upcoming LSS surveys.

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[6] The determination of the bias parameters in some specific prescriptions have in particular been done in [33].
[7] Note that similar constraints have been placed on trispectrum templates from LSS observations as in [42].

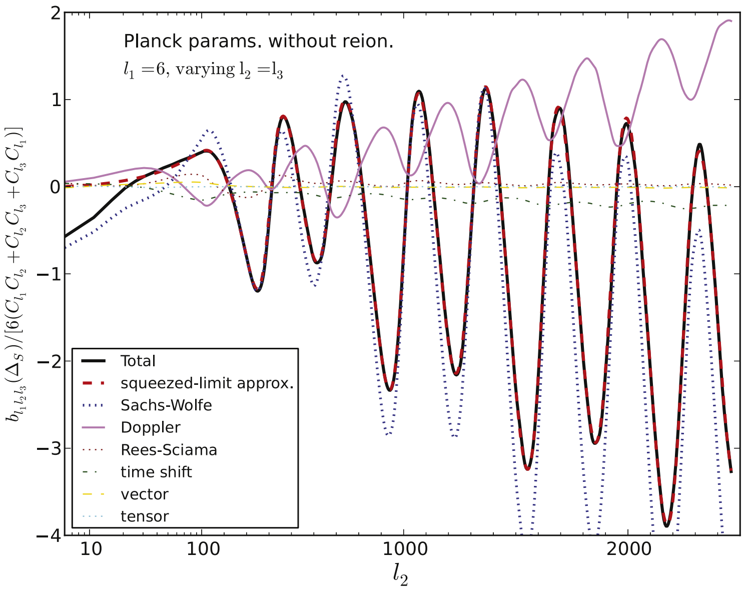

Fig. 3. (Color online.) Thick solid lines: reduced bispectrum $b(l_1, l_2, l_3)$, for $l_1 = 6$ as a function of $l_2 = l_3$, decomposed in its different contributions (thin lines) and compared to the squeezed-limit formula (dashed line). The horizontal axis uses logarithmic (linear) scale for $l \leq 100$ ($l > 100$). Figure taken from [34].

4. Mode couplings effects on the LSS of the universe

LSS observations offer a wealth of information on the dynamical processes that shape the way the matter is distributed across the universe. In particular, it is clear that LSS can be used to probe, directly or indirectly, the whole coupling structure of the matter field as it emerged from the dynamical evolution of the cosmic fields.

At the sub-horizon scale, for a pressure-less fluid—such as one dominated by cold dark matter—the whole coupling structure expected for the density field has been explored in details. In particular, it is possible to compute any order of the density field and how it is related on the linear solution. It can easily be shown that the p-th order expression of the density field is of the order of the power p of the linear density field. In other words, there exist kernels functions F_p such that

$$\delta(\mathbf{k}, \eta) = \int\limits_{\mathbf{k}_1 \ldots \mathbf{k}_p} F_p(\mathbf{k}_1, \ldots, \mathbf{k}_p; \eta)\delta_+(\mathbf{k}_1, \eta) \ldots \delta_+(\mathbf{k}_p, \eta) \tag{7}$$

where $\delta_+(\mathbf{k}_i, \eta)$ is the linear growing mode for wave modes $\mathbf{k}_i$ and where $F_p(\mathbf{k}_1, \ldots, \mathbf{k}_p; \eta)$ are dimensionless functions of the wave modes. These functions are a priori time dependent. For an Einstein–de Sitter background, however, their time dependence drops. In general, they depend only very weakly on time—henceforth on the cosmological parameters. For instance, it is easy to show that

$$F_2 = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} + \frac{2}{7}\frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2 k_2^2} \tag{8}$$

for an Einstein–de Sitter universe, a functional form that can be recognized in the right-hand side of Eq. (6). The consequences of these coupling terms are dramatic for LSS observations: as soon as the density contrasts get comparable to unity in amplitude, non-linear effects change completely the shape and amplitude of spectra or correlators. At variance to CMB observations, where most of what can be observed can be accurately computed with linear theory, a large part of the LSS observations can only be interpreted—and exploited—after the fully developed non-linear evolution of the cosmic fields is taken into account. Even at large scale such as those of the so-called Baryonic Acoustic Oscillations (BAO), for which the density contrast is modest, non-linear effects should still be taken into account. As such scales, however, it is possible to develop controlled Perturbation Theory (PT) calculations.

The forms (7) have indeed been used to develop full PT calculations applied to observables such as spectra and correlators. It is easy to see that, taking into account such expansions, power spectra can be computed at successive orders. This is particularly simple for Gaussian initial conditions as it leads to contributions at linear, one-loop, two-loop orders, etc., which are respectively linear, quadratic, and cubic in the initial power spectrum.[8] Over the last few years, alternative formulations

[8] The loop number refers to the number of loops that appear in the diagrammatic representation of each one of the contributing terms.

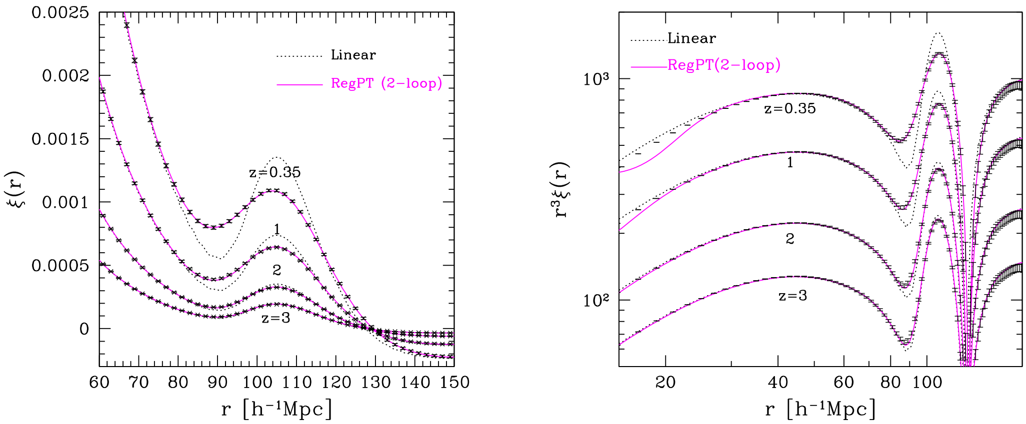

Fig. 4. Comparison of two-point correlation function between N-body and RegPT results at $z = 3$, 2, 1, and 0.35 (from bottom to top). In each panel, magenta solid, and black dotted lines represent the prediction from RegPT and linear theory calculations, respectively. Left panel focuses on the behavior around baryon acoustic peak in linear scales, while right panel shows the overall behavior in a wide range of separation in logarithmic scales. Note that in right panel, the resulting correlation function is multiplied by the cube of the separation for illustrative purpose. For interpretation of references to color in this figure caption, the reader is referred to the web version of this article.
Figure taken from [44].

have been put forward that take into account partial re-summations, as in the RPT formulation [45] or, more recently, the RegPT proposition [44]. We illustrate the quality of the resulting analytic predictions in Fig. 4, where results computed at two-loop order are compared to results of numerical simulations. They show how dramatic the improvement compared to the linear theory can be. It demonstrates that the shape of the correlation functions at the BAO scale—an important observational goal—is now fully under control. Note that such calculations can be undertaken not only for the power spectrum, but also for any polyspectra.

For all these effects, however, mode couplings due to gravity are by far the dominant contributions. How then could one have a handle on the primordial non-Gaussianities? It seems hopeless in general, but one can take advantage of the different k dependence of the mode-coupling effects. This is the essence of the result found in [46], which shows that at large scale, it must be possible to disentangle the effect of primordial non-Gaussianities from the nonlinear evolution of the fields. The key result that has been obtained takes the form,

$$\delta_h(k) = \left(b_1 + f_{\rm NL}^{\zeta} \frac{3\Omega_{\rm m} H_0^2}{c^2 k^2 T(k) D(z)} \frac{\partial \log n}{\partial \log \sigma_0} \right) \delta(k) \tag{9}$$

It is a bit beyond the scope of this paper to comment all the terms and factors that appear in this expression. It relates how the density contrast δ_h in some specific halo population is expected to be related to the underlying matter density contrast $\delta(k)$. The relation is written in the Fourier space. What is to be taken is that b_1 is the standard bias factor expected to be k independent at large scale is complemented by a second term induced by a non-zero initial local bispectrum, whose amplitude is characterized by $f_{\rm NL}^{\zeta}$. It can be noted that at large scale (low k), the transfer function $T(k)$ being constant, one expects the corrective term of scale like $1/k^2$, a striking dependence that can be exploited. This is so far what has allowed to put the most reliable constrain on $f_{\rm NL}^{\zeta}$ from LSS observations. The first constraints on $f_{\rm NL}^{\zeta}$ were indeed shortly given after it was identified as a signature of primordial non-Gaussianity in [47]. It was found that $-29 < f_{\rm NL}^{\zeta} < +70$ (at 95% confidence) from the clustering statistics of a variety of biased tracers: photometric luminous red galaxies (LRGs) from SDSS Data Release 6 (DR6), spectroscopic LRGs from SDSS DR4, photometric quasars from SDSS DR6, and NVSS radio galaxies (in cross-correlation with the CMB). The constraints obtained from the CMB bispectrum with *Planck* are now much more stringent, but the number of available modes is much larger in LSS surveys compared to CMB observations.

5. Conclusions: extracting cosmological parameters from LSS observations

Large-scale structure data sets consist in angular or redshift space catalogs of a variety of tracers, from galaxy of all types and colors to galaxy clusters, and also cosmic shear maps constructed from various background objects—from distant galaxies taken in different redshift bins to CMB temperature and polarization maps. Such observations will then be affected by a number of various and diverse systematics, such as instrumental systematics, selection biases, intrinsic alignments in case of cosmic shear observations, etc. Extracting the best of such observations will therefore require the identification of a number of relevant discovery channels where such effects can be sorted out. In the previous section, we mentioned in particular that the expected power spectra amplitude and shape are affected by mode-coupling effects and are therefore computable from first principles only in a limited range of scale and with a limited precision. This is one of the major difference between LSS and CMB observations since in the latter case, theoretical predictions could be obtained with negligible systematics. Sorting out theoretical and observational systematics for any specific observation is then of crucial importance!

In case of cosmic shear observation, propositions in this direction include multi-plane observations, nulling methods, etc. (see, for instance, Refs. [48–52]). For redshift space catalogs, the fact that observables are directly related to specific astrophysical objects adds an extra layer of difficulty. The full discriminatory power of most of such observations will only be obtained once one disposes of robust descriptions of the tracer properties and in particular of biasing, that is in general how a given population of tracers is related to the underlying matter density field. Early ideas suggested that the local galaxy density contrast, δ_h, smoothed at some scale, could be a mere function of the local density δ that can be Taylor expanded [53]. However simple and appealing such a scheme is for data analysis purposes, it does not stand any serious analysis. It cannot be valid beyond linear order. For instance, such a local bias scheme is not preserved by the field evolution, which inevitably induces non-local couplings between halo and matter density fields. More precisely, taking into account the general symmetry properties of the fields, such as the expected Galilean invariance of the field evolution, the δ_h–δ relation is expected to make intervene so-called non-local operators[9] of the orm [55],

$$\mathcal{G}_2 = (\partial_{ij}\Phi)^2 - (\partial_{ii}\Phi)^2 \tag{10}$$

$$\mathcal{G}_3 = (\partial_{ii}\Phi)^3 + 2\partial_{ij}\Phi\partial_{jk}\Phi\partial_{ki}\Phi - 3(\partial_{ij}\Phi)^2\partial_{ii}\Phi \tag{11}$$

when written up to third order, where Φ is the proportional to the local gravitational potential and is such that $\partial_{ii}\Phi = \delta$. This leads the functional forms between the halo density field and the matter density contrast to read,

$$\delta_{\rm h} = b_1\delta + \left[\frac{b_2}{2}\delta^2 + \gamma_2\mathcal{G}_2\right] + \left[\frac{b_3}{3}\delta^3 + \alpha\delta\mathcal{G}_2 + \gamma_3\mathcal{G}_3\right] + \ldots \tag{12}$$

where b_1, b_2, γ_2, γ_3 and α are order unity coefficients. Those coefficients are a priori dependent on the nature, mass, luminosity, etc., of the considered catalogue of objects. That such a description is correct for actual tracers is yet to be demonstrated and is probably not valid for all types of tracers.

Extracting cosmological parameters will then be obtained by a joint determination of the biasing factors and the basic cosmological parameters, with possible initial non-Gaussianities, using as many different channels as possible, cosmic shear, galaxy catalogs in angular or redshift space, etc. But it is needless to say that the completion of such a work program is still a long way to go!

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