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Cosmic inflation / Inflation cosmique

Issues on the inflationary magnetogenesis

Inflation et production d'un champ magnétique cosmologique pendant l'inflation

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ABSTRACT

Recent observations of high-energy photons from blazars suggest that there exist magnetic fields with typical amplitude around 10^{-15} G ubiquitously even in void regions. This being the case, it is natural to invoke them to explain the processes occurring during inflation in the early universe. We provide a list of models of magnetogenesis during inflation and consider several problems associated with them.

RÉSUMÉ

L'observation récente de photons ultra-énergétiques en provenance des blazars suggère l'existence de champs magnétiques cosmologiques, d'amplitude typique 10^{-15} G, omniprésents, y compris au sein des vides cosmiques. Il est dès lors naturel de les associer à la physique inflationnaire dans l'univers primordial. Nous donnons une liste de modèles de génération d'un champ magnétique pendant l'inflation et discutons un certain nombre de problèmes qu'ils posent.

1. Introduction

Magnetic fields play important roles in various astrophysical objects. In this article we focus on magnetic fields on cosmological scales. The amplitude of magnetic fields in galaxies and clusters of galaxies have been measured as $B \sim 10^{-5} - 10^{-6}$ G and $10^{-6} - 10^{-7}$ G, respectively (see, e.g., [1–3]). The origin of the galactic magnetic field is usually attributed to the dynamo mechanism, which, however, requires a nonvanishing seed field, although it may be as tiny as $10^{-22} - 10^{-16}$ G [4,5].

For a long time, only upper bounds had been obtained for the amplitude of magnetic fields outside galaxies and clusters or on larger scales. Recently, however, a new method to probe magnetic field was proposed, combining observations of TeV

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photons by HESS [6] and GeV photons by Fermi LAT [7] from blazars, which may be interpreted as setting a lower bound on the field amplitude on relatively large scales [8].

The existence of magnetic fields on scales well above the galactic scale and in particular in void regions [9] would motivate us to attribute their origin to the processes during to the inflationary period in the early universe [10–13]. Hence here we review and consider several issues on magnetogenesis during inflation, starting with a review on the observational results in Section 2.

In Section 3 we present an incomplete list of proposed models for inflationary magnetogenesis. Then in Section 4 we describe a model to produce magnetic fields by modifying the kinetic term of the gauge field. In Section 5, we summarize the constraints imposed on such a model mainly by the excessive electric field generated during inflationary magnetogenesis. Next we consider the effect of Schwinger's process [15], which may take place due to the strong induced electric field. Finally Section 7 is devoted to our conclusion.

2. Observational evidence of large-scale magnetic field in empty space

According to the unified model of the accretion disks, we identify them as blazars if they are observed from the polar directions along which jets of high-energy TeV photons are emitted. These photons of energy E_{γ_0} scatter extragalactic background light (EBL) [16], consisting mainly of optical or infrared photons to create electron–positron pairs [17]. This occurs at a typical distance

$$D_\gamma(E_{\gamma_0}, z) = \frac{40\kappa}{(1+z)^2}\left(\frac{E_{\gamma_0}}{20\ \text{TeV}}\right)^{-1}\ \text{Mpc} \tag{1}$$

which is not close to the blazar. Here κ is a numerical parameter ranging between 0.3 and 3. The electron–positron pairs thus created with energy E_e scatter off cosmic microwave background (CMB) radiation with energy E_CMB to create GeV energy photons with typical energy

$$E_\gamma = \frac{4E_\text{CMB}E_\text{e}^2}{3\,(1+z_{\gamma\gamma})} \simeq 0.32\left(\frac{E_{\gamma_0}}{20\ \text{TeV}}\right)^2\ \text{TeV} \tag{2}$$

with $E_\text{e} = E_{\gamma_0}/2$ and the mean free path

$$\begin{aligned} D_\text{e} &\simeq 10^{23}(1+z_{\gamma\gamma})^{-4}\left(\frac{E_\text{e}}{10\ \text{TeV}}\right)^{-1}\ \text{cm} \\ &\simeq 33\left(\frac{E_\text{e}}{10\ \text{TeV}}\right)^{-1}\ \text{kpc} \end{aligned} \tag{3}$$

Thus the high-energy photons travel a long cosmological distance before encountering an extragalactic background photon, but once they create electron–positron pairs, subsequent cascade processes occur almost locally.

If there exists a magnetic field, electron trajectories are bent with the Larmor radius

$$R_L = \frac{E_\text{e}}{eB} \simeq 3\cdot10^{28}\left(\frac{B_0}{10^{-18}\ \text{G}}\right)^{-1}\left(\frac{E_\text{e}}{10\ \text{TeV}}\right)\text{cm} \tag{4}$$

Hence if the correlation length of the magnetic field, λ_B, satisfies $\lambda_B \gg D_\text{e}$, the deflection angle is given by

$$\delta = \frac{D_\text{e}}{R_L} \simeq 3\cdot10^{-6}(1+z_{\gamma\gamma})^{-2}\left(\frac{B_0}{10^{-18}\ \text{G}}\right)^{-1}\left(\frac{E_\text{e}}{10\ \text{TeV}}\right)^{-2} \tag{5}$$

If it is much smaller than D_e, the net deflection angle is given by the result of random walks as

$$\delta = \frac{\sqrt{D_\text{e}\lambda_B}}{R_L} \simeq 5\cdot10^{-7}(1+z_{\gamma\gamma})^{-1/2}\left(\frac{B_0}{10^{-18}\ \text{G}}\right)\left(\frac{E_\text{e}}{10\ \text{TeV}}\right)^{-3/2}\left(\frac{\lambda_B}{1\ \text{kpc}}\right)^{1/2} \tag{6}$$

As a result, GeV photons created by the scatter of electron and positron arrive at the observer with an opening angle $\Theta_\text{ext} \cong D_\gamma\delta/D_\theta$, where D_θ is the distance between the blazar and the observer. The opening angle is given by

$$\Theta_\text{ext} \simeq \frac{0.5^\circ}{(1+z)^2}\left(\frac{\tau_\theta}{10}\right)^{-1}\left(\frac{E_\gamma}{0.1\ \text{TeV}}\right)^{-1}\left(\frac{B_0}{10^{-14}\ \text{G}}\right) \tag{7}$$

for $\lambda_B \gg D_\text{e}$ with $\tau_\theta \equiv D_\theta/D_\gamma$, and

$$\Theta_\text{ext} \simeq \frac{0.07^\circ}{(1+z)^{1/2}}\left(\frac{\tau_\theta}{10}\right)^{-1}\left(\frac{E_\gamma}{0.1\ \text{TeV}}\right)^{-3/4}\left(\frac{B_0}{10^{-14}\ \text{G}}\right)\left(\frac{\lambda_B}{1\ \text{kpc}}\right)^{1/2} \tag{8}$$

for $\lambda_B \ll D_\text{e}$. These are to be compared with the point spread function of Fermi LAT, given by

$$\Theta_{PSF} \simeq \begin{cases} 2^\circ \left(\frac{E_\gamma}{1\ \text{GeV}}\right)^{-0.8} & \text{for } E_\gamma < 1\ \text{GeV}, \\ 0.2 & \text{at } E_\gamma = 10\ \text{GeV} \end{cases}$$

If the magnetic field satisfies the inequality $B \geq B_{PSF}$ with

$$B_{PSF} \simeq 6{\cdot}10^{-17} \frac{D_\theta}{D_\gamma} \left(\frac{E_{\gamma\,\min}}{10\ \text{GeV}}\right)\ \text{G} \tag{9}$$

for $\lambda_B > D_e$ and

$$B_{PSF} \simeq 8{\cdot}10^{-16} \frac{D_\theta}{D_\gamma} \left(\frac{E_{\gamma\,\min}}{10\ \text{GeV}}\right)^{3/4} \left(\frac{\lambda_B}{1\ \text{kpc}}\right)^{1/2}\ \text{G} \tag{10}$$

for $\lambda_B < D_e$, we find $\Theta_{ext} > \Theta_{PSF}$ for $E_\gamma < E_{\gamma\,\min}$ so that such photons are invisible as a point source.

Assuming the spectrum of photons from a blazar takes the form $dN_\gamma/dE \propto E^{-\Gamma} e^{-E/E_{cut}}$, Neronov and Vovk [8] calculated processed photon spectra to match the HESS observation of three blazars and found that the amount of resultant GeV photons for two of the blazars were in contradiction with the upperbounds obtained by Fermi LAT without assuming the presence of extragalactic magnetic fields. Using various values of Γ and E_{cut}, they have found that for blazar 1ES 0229+200 to be compatible with Fermi data, a magnetic field of strength $B = 3{\cdot}10^{-16}$ G with a coherent length larger than ∼0.1 Mpc is required [8]. If the correlation length is smaller, a larger magnetic field is required. Tavecchio et al. [18] obtained a lower bound $B \gtrsim 5{\cdot}10^{-15}$ G for the collimation angle of the blazar 1ES0229 + 200 $\theta_c = 0.1$.

One may wonder if sufficiently strong magnetic fields localized in filaments may explain the above deficits in Fermi data. To answer this question, Dolag et al. [9] performed a number of numerical simulations with various profiles of the extragalactic magnetic field. For example, they calculated the photon spectrum assuming that a strong magnetic field with strength 10^{-10} G exists in filamentary regions with separation 10 Mpc each and estimated the minimal filling factor of such regions to suppress GeV photons below the upper bounds obtained by Fermi. They have found that the strong magnetic field must fill at least 80% of the space. More sophisticated simulations based on MHD follow also shown the same tendency; in particular, they have shown that $B_0 = 10^{-17}$ G in void regions is too small to account for Fermi data.

These observations and considerations strongly suggest there exist magnetic fields ubiquitously in extragalactic space including void regions. One should remember, however, that this line of argument, namely, putting a lower bound on magnetic fields from NON-observation of GeV photons, can apply if and only if we fully understand the underlying physics.

3. Magnetogenesis during cosmic inflation

If indeed there exist magnetic fields with coherent length larger than ∼ 1 Mpc or in void regions, where no astrophysical generation mechanisms are known to operate, then we must seek their origin in the early universe, especially during inflation, when a large coherent scale was easily achieved by virtue of the quasi-exponential expansion.

Unlike the inflaton, which is a minimally coupled nearly massless scalar field in the standard slow-roll models [13], and the tensor perturbations, the electromagnetic field is conformally invariant, as one can easily see from the electromagnetic action in the spatially flat Robertson–Walker metric $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$,

$$\begin{aligned} S_{EM} &= -\int \sqrt{-g}\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}\, d^4x \\ &= -\int a^4(\eta)\frac{1}{4} a^{-4}(\eta)\eta^{\mu\alpha}\eta^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}\, d^4x \\ &= -\int \frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}\, d^4x \end{aligned} \tag{11}$$

Thus it behaves in the same way as in the Minkowski space. Hence no long-wave quantum fluctuations are generated during inflation unless the conformal invariance is broken in an appropriate manner. The study of magnetogenesis during inflation needs therefore modifying (11) to break the conformal invariance appropriately.

The first attempt along this line was done by Turner and Widrow [14], who introduced couplings to spacetime curvatures. Their first model was

$$S_{R1} = \int \sqrt{-g}\, d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{c_1}{2} R A_\mu A^\mu - \frac{c_2}{2} R_{\mu\nu} A^\mu A^\nu\right) \tag{12}$$

which also breaks gauge invariance. This Lagrangian, however, suffers from the ghost problem [19,20] and cannot serve as a realistic model.

The second model of Turner and Widrow [14] was to couple $F_{\mu\nu}$ with curvature tensors without breaking gauge invariance.

$$S_{R2} = \int \sqrt{-g}\, d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{c_3}{4m^2} R F_{\mu\nu} F^{\mu\nu} - \frac{c_4}{4m^2} R_{\mu\nu} F^{\mu\alpha} F^\nu_\alpha - \frac{c_5}{4m^2} R_{\mu\nu\alpha\beta} F^{\mu\alpha} F^{\nu\beta}\right) \tag{13}$$

which predicted too weak field strength on cosmological scales.

Ratra [21] introduced a coupling with the inflaton field, which was extended to an arbitrary scalar field in [22,23]. This class of model was studied in more detail in [24]; its action reads

$$S_1 = -\int \sqrt{-g}\, \mathrm{d}^4x \frac{1}{4} f^2(\Phi) F_{\mu\nu} F^{\mu\nu} \tag{14}$$

Here Φ is a scalar field that may or may not be the inflaton, but that dynamically evolves during inflation. In this model, $1/f(\Phi)$ gives a time-dependent gauge coupling that induces a problem, as we will see later.

Another type of gauge-scalar coupling has been considered in the literature [25–27],

$$S_2 = -\int \sqrt{-g}\, \mathrm{d}^4x \frac{1}{4} h^2(\phi) F_{\mu\nu} \tilde{F}^{\mu\nu}$$
$$\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \tag{15}$$

Here $\varepsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor. This means that time derivative is always associated with spacial derivative. In Fourier space, conformally noninvariant terms have an extra k factor, which tends to suppress long-wave fluctuations compared with the model (14), which we will call fFF hereafter.

4. fFF model

Let us consider the fFF model (14) assuming that Φ is the inflaton that is a minimally coupled scalar field with a potential $V[\Phi]$ following [24]. Taking the Coulomb gauge $A_0 = \partial_i A^i = 0$, we expand the gauge field as

$$A_i(\eta, \boldsymbol{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}} \sum_{\lambda=1}^{2} \epsilon_{i\lambda}(\boldsymbol{k}) \left[a_\lambda(\boldsymbol{k}) A(\eta, k) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + a_\lambda^\dagger(\boldsymbol{k}) A^*(\eta, k) \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} \right] \tag{16}$$

in terms of the annihilation and creation operators, $a_\lambda(\boldsymbol{k})$ and $a_\lambda^\dagger(\boldsymbol{k})$, with $\epsilon_i^\lambda(\boldsymbol{k})$ being the transverse polarization vector. The canonically normalized mode function $\mathcal{A}(\eta, k) \equiv a(\eta) f(\eta) A(\eta, k)$ satisfies the following equation of motion

$$\mathcal{A}''(\eta, k) + \left(k^2 - \frac{f''}{f} \right) \mathcal{A}(\eta, k) = 0 \tag{17}$$

and the normalization condition

$$\mathcal{A}(\eta, k) \mathcal{A}^{*\prime}(\eta, k) - \mathcal{A}'(\eta, k) \mathcal{A}^*(\eta, k) = i$$

Let us consider de Sitter inflation $a(\eta) = -1/(H\eta)$ and assume that f is a function of conformal time η through evolution of $\Phi(\eta)$ as $f(\eta) = (a/a_{\mathrm{end}})^{-\gamma}$. Then we find

$$\frac{f''}{f} = \frac{\gamma(\gamma - 1)}{\eta^2} \tag{18}$$

so that the mode function is given by

$$\mathcal{A}(\eta, k) = \sqrt{\frac{\pi}{4k}} \mathrm{e}^{\frac{\mathrm{i}}{2}\gamma\pi} (-k\eta)^{\frac{1}{2}} H^{(1)}_{\gamma - \frac{1}{2}}(-k\eta) \tag{19}$$

which behaves as

$$\mathcal{A}(k, \eta) \to \frac{1}{\sqrt{2k}} \mathrm{e}^{-\mathrm{i}k\eta} \tag{20}$$

in the short-wavelength limit. Thus the initial vacuum state in the short wave regime coincides with the Minkowski vacuum. In the long-wave regime, the leading contributions to the mode function reads

$$\mathcal{A}(k, \eta) \to \frac{\sqrt{\pi}}{2^{\gamma + 1/2}} \frac{\mathrm{e}^{\mathrm{i}\pi\gamma/2} k^{-1/2} (k\eta)^\gamma}{\Gamma(\gamma + 1/2) \cos(\pi\gamma)} + \frac{\sqrt{\pi}}{2^{-\gamma + 3/2}} \frac{\mathrm{e}^{\mathrm{i}\pi(1-\gamma)/2} k^{-1/2} (k\eta)^{1-\gamma}}{\Gamma(-\gamma + 3/2) \cos[\pi(1 - \gamma)]} \tag{21}$$

where the amplitude of the two modes has been written such that the symmetry $\gamma \to 1 - \gamma$ be manifest. The limit changes depending on whether $\gamma > 1/2$ or $\gamma < 1/2$.

Now the magnetic field B_i and the electric field E_i are given by

$$B_i = \epsilon_{ijk} \frac{\partial_j A_k}{a} \text{ and } E_i = -\frac{A_i'}{a} \tag{22}$$

respectively, and their energy density due to the quantum fluctuations of the gauge field are given by

$$\frac{\mathrm{d}\rho_E(\eta,k)}{\mathrm{d}\ln k} = \frac{f^2(\eta)}{2\pi^2}\frac{k^4}{a^4}\left|\left(\frac{\mathcal{A}(\eta,k)}{f}\right)'\right|^2 \tag{23}$$

$$\frac{\mathrm{d}\rho_B(\eta,k)}{\mathrm{d}\ln k} = \frac{1}{2\pi^2}\frac{k^4}{a^4}|\mathcal{A}(\eta,k)|^2 \tag{24}$$

per each logarithmic wavenumber interval.

Using the above equalities we find

$$\frac{\mathrm{d}\rho_B(\eta,k)}{\mathrm{d}\ln k} = \frac{\mathcal{F}(1-\gamma)H^4}{2\pi^2}\left(\frac{k}{aH}\right)^{-2\gamma+6} \tag{25}$$

for $\gamma > 1/2$ and

$$\frac{\mathrm{d}\rho_B(\eta,k)}{\mathrm{d}\ln k} = \frac{\mathcal{F}(\gamma)H^4}{2\pi^2}\left(\frac{k}{aH}\right)^{2\gamma+4} \tag{26}$$

for $\gamma < 1/2$, with

$$\mathcal{F}(\delta) \equiv \frac{\pi}{2^{2\delta+1}\Gamma^2(\delta+1/2)\cos^2(\pi\delta)} \tag{27}$$

As for the electric field, we find

$$\frac{\mathrm{d}\rho_E(\eta,k)}{\mathrm{d}\ln k} = \frac{\mathcal{G}(-\gamma)H^4}{2\pi^2}\left(\frac{k}{aH}\right)^{-2\gamma+4} \tag{28}$$

for $\gamma > -1/2$ and

$$\frac{\mathrm{d}\rho_E(\eta,k)}{\mathrm{d}\ln k} = \frac{\mathcal{G}(1+\gamma)H^4}{2\pi^2}\left(\frac{k}{aH}\right)^{2\gamma+6} \tag{29}$$

for $\gamma < -1/2$, with

$$\mathcal{G}(\iota) \equiv \frac{\pi}{2^{2\iota+3}\Gamma^2(\iota+3/2)\cos^2(\pi\iota)} \tag{30}$$

In the case $\gamma = 0$, for which gauge coupling is constant, we find both scale k^4 as they should.

In the above simple setup, spectrum of the energy densities in magnetic and electric fields take a power-law form, and in order to obtain appreciable amplitude of magnetic fields on cosmological scale, we need a nearly scale-invariant power spectrum for magnetic fields to satisfy both upper and lower bounds on large scales. Models with $f(\eta)$ evolving according to more complicated functions have been considered in [29].

The exactly scale-invariant spectrum is realized if $\gamma = 3$ or -2.

For $\gamma = -2$ we find that long-wave energy density in the electric field is suppressed in proportion to $(k/aH)^2$, and so only the magnetic field is relevant on large scales. In this case, however, $f(\eta)$ rapidly increases in time during inflation, which means that, in the beginning of inflation, the effective gauge coupling constant or the electric charge was exponentially large [22], so that perturbative calculation was not possible there [20]. This strong coupling problem may be ameliorated if coupling between gauge fields and matter fields also have time dependence in such a way that the effective gauge coupling constant remain small [28].

For $\gamma = 3$, there is no strong coupling problem. On the other hand, the energy spectrum of the electric field has a severely blue spectrum

$$\begin{aligned}\frac{\mathrm{d}\rho_E(\eta,k)}{\mathrm{d}\ln k} &= \frac{\mathcal{G}(-3)H^4}{2\pi^2}\left(\frac{k}{aH}\right)^{-2} \\ &\sim \left(\frac{k}{aH}\right)^{-2}\frac{\mathrm{d}\rho_B(\eta,k)}{\mathrm{d}\ln k}\end{aligned} \tag{31}$$

and, in fact, the factor $(k/aH)^{-2}$ in the last expression is quite common in magnetogenesis models without the strong coupling problem [30]. This means that we inevitably obtain much larger electric fields than magnetic fields on astrophysical and cosmological scales. When inflation ends and charged particles are created, the electric conductivity rises up to eliminate the electric field [21]. Hence it is harmless after inflation. For the moment, let us assume that this is the case and also that $f(\eta)$ becomes unity at the end of inflation and remains constant thereafter. Then the energy density of the magnetic field created during inflation starts to dissipate in proportion to a^{-4} just in the same way as radiation. During inflation, however, the amplitude of the electric field imposes nontrivial constraints on the magnetogenesis model, which we discuss in the next section.

5. Constraints on the energy scale of inflation in the fFF model

First of all, its energy density may not exceed that of the inflaton to sustain inflation. For $\gamma = 3$, we find that the energy scale of inflation must satisfy $\rho_{\rm inf}^{1/4} < 10^7$ GeV [24].

A more stringent constraint is obtained by the requirement that the extra energy density should not perturb the space-time excessively. Let us consider how the curvature perturbation is affected by the existence of a magnetic field by solving the evolution equation for the curvature perturbation, ζ, on the uniform density surface. On superhorizon scales $k/(aH) \ll 1$, its evolution equation is derived from the Einstein equation as

$$\ddot{\zeta} + 3H\dot{\zeta} + \frac{1}{a^3}\left(\frac{a^3 H}{\rho + P}\delta P_{\rm rel}\right)^{\cdot} - \frac{8\pi G}{3}\Pi = 0 \tag{32}$$

where ρ, P, and Π represent the background energy density, pressure, and the anisotropic stress, respectively [31,32]. Here

$$\delta P_{\rm rel} \equiv \delta P_{\rm em} - \frac{\dot{P}}{\dot{\rho}}\delta\rho_{\rm em} \tag{33}$$

represents the nonadiabatic pressure perturbation due to the relative entropy perturbation, where $\delta\rho_{\rm em}$ and $\delta P_{\rm em}$ are the energy density and pressure perturbations of the electromagnetic field, respectively, but since the electromagnetic field does not have any homogeneous mode, they are identical with the energy density and the pressure themselves, respectively.

We find the following inhomogeneous solution

$$\zeta(t) = -\int_{t_*}^{t} dt_1 \frac{H(t_1)}{\rho(t_1) + P(t_1)}\delta P_{\rm rel}(t_1) + \frac{8\pi G}{3}\int_{t_*}^{t}\frac{dt_1}{a^3(t_1)}\int_{t_*}^{t_1} dt_2\, a^3(t_2)\Pi(t_2) \tag{34}$$

starting from $\zeta(t_*) = 0$ at some initial time t_* that may be identified with the horizon crossing time [33]. Since $P \simeq -\rho$ during inflation, the first term is larger than the second one in general. Hence we concentrate on the first term hereafter.

During inflation, we find $\delta P_{\rm rel} \simeq 4\delta\rho_{\rm em}$, and during radiation domination, we find $\delta P_{\rm rel} = 0$, because the magnetic field dissipates in the same way as radiation. We therefore find

$$\zeta(t) \simeq -\frac{2\mathcal{N}}{\epsilon}\frac{\delta\rho_{\rm em}}{\rho_{\rm inf}} \tag{35}$$

where $\epsilon \equiv -\dot{H}/H^2$ is the slow-roll parameter and $\mathcal{N}$ is the number of e-fold measured from the time when the mode of interest crossed the horizon to the end of inflation. Thus as a conservative bound we may adopt

$$\rho_{\rm em} < \epsilon\zeta\rho_{\rm inf} \tag{36}$$

which is more stringent than the first constraint by a factor of $\sim 10^{-5}\epsilon$.

Extending this line of thoughts to generic cases where the ratio of the electric field to the magnetic field is enhanced by a factor of e^N with N being the number of e-folds of inflation corresponding to the scale we measure it, it has been found that the scale of inflation must satisfy $\rho_{\rm inf}^{1/4} < 30$ GeV to realize $B \gtrsim 10^{-15}$ G on a megaparsec scale today [30].

Further constraints may be imposed by the calculation of non-Gaussianity of curvature perturbations due to the electric field, which obeys a chi-square distribution rather than a Gaussian one [34].

6. Schwinger effects in a de Sitter space

So far we have seen that in fFF type models of inflationary magnetogenesis we encounter much stronger electric fields than magnetic fields and they are tightly constrained by the excessive strength of electric fields. If one could effectively remove electric fields during inflation, one may obtain a successful scenario of magnetogenesis during inflation. This motivates us to study the Schwinger effect [15] during inflation or in a de Sitter space [35–38].

If this process operates efficiently to produce charged particles abundantly, electric conductivity may take a large enough value to eliminate the induced electric field. Calculation of the pair production rate in a curved background is a difficult problem and analytic solution is possible only in a simplified case, where the gauge field is treated as a homogeneous external back ground as discussed below.

Let us consider pair production of a charged scalar field φ with mass m in a de Sitter background $a(\eta) = 1/(1 - H\eta) = e^{Ht}$. Here the domain of the conformal time is taken as $-\infty < \eta < H^{-1}$, so that we can easily take the Minkowski limit by taking the Hubble parameter $H \longrightarrow 0$ when η reduces to the physical time t. To allow an analytic solution, we consider the case when the proper electric field is constant along the z-direction in spite of the exponential expansion of the background space, taking the gauge field as $A_\mu = (0, 0, 0, A_z)$ with

$$A_z(\eta) = -\frac{E}{H}(a(\eta) - 1) \tag{37}$$

which satisfies the Lorenz and Coulomb gauge conditions, $A_0 = \partial_i A_i = 0$. We expand a conformally rescaled scalar field $\chi \equiv a\varphi$ as

$$\chi(\eta, \boldsymbol{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} (\chi_{\boldsymbol{k}}(\eta) b_{\boldsymbol{k}} + \chi^*_{\boldsymbol{k}}(\eta) d^\dagger_{-\boldsymbol{k}}) \tag{38}$$

The commutation relations for the creation and annihilation operators take the form,

$$[b_{\boldsymbol{k}}, b^\dagger_{\boldsymbol{k}'}] = [d_{\boldsymbol{k}}, d^\dagger_{\boldsymbol{k}'}] = (2\pi)^3 \delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}')$$
$$[b_{\boldsymbol{k}}, b_{\boldsymbol{k}'}] = [b^\dagger_{\boldsymbol{k}}, b^\dagger_{\boldsymbol{k}'}] = [d_{\boldsymbol{k}}, d_{\boldsymbol{k}'}] = [d^\dagger_{\boldsymbol{k}}, d^\dagger_{\boldsymbol{k}'}] = 0$$

The Klein–Gordon equation for each mode reads

$$\left(\partial_\eta^2 + \frac{\alpha}{(1-H\eta)^2} + \frac{\beta}{1-H\eta} + \gamma\right)\chi_k = 0$$
$$(\alpha, \beta, \gamma) = \left(m^2 - 2H^2 + l^2,\ -2lp_z,\ p^2\right) \tag{39}$$

where $\boldsymbol{p}$ is the shifted wavenumber vector

$$\boldsymbol{p} = (k_x, k_y, k_z + l) \text{ with } l \equiv eE/H$$

Its solution is expressed in terms of the Whittaker functions $W_{\kappa,\mu}(z)$ and $M_{\kappa,\mu}(z)$ with

$$\kappa \equiv -\mathrm{i}L\frac{p_z}{p}, \quad \mu \equiv \sqrt{\frac{9}{4} - M^2 - L^2}$$
$$z \equiv -2\mathrm{i}p\left(\frac{1}{H} - \eta\right) \tag{40}$$

where $M \equiv m/H$ and $L \equiv l/H$.

The appropriate positive frequency mode at the in-regime $\eta \longrightarrow -\infty$ is given by

$$\chi_p^{\mathrm{in}}(\eta) = \frac{\mathrm{e}^{\mathrm{i}\kappa\pi/2}}{\sqrt{2p}} W_{\kappa,\mu}(z) \tag{41}$$

The positive frequency mode in the asymptotic future $\eta \longrightarrow H^{-1}$ is well defined if and only if μ is pure imaginary as

$$\chi_p^{\mathrm{out}}(\eta) = \frac{\mathrm{e}^{\mathrm{i}|\mu|\pi/2}}{\sqrt{4|\mu|p}} M_{\kappa,\mu}(z) \tag{42}$$

Using the relation between the two Whittaker functions

$$W_{\kappa,\mu}(z) = \frac{\Gamma(-2\mu)}{\Gamma(1/2-\mu\kappa)} M_{\kappa,\mu}(z) + \frac{\Gamma(2\mu)}{\Gamma(1/2+\mu\kappa)} M_{\kappa,-\mu}(z) \tag{43}$$

the Bogoliubov coefficients can be calculated as

$$\alpha_{\boldsymbol{k}} = \frac{\sqrt{2|\mu|}\Gamma(-2\mu)}{\Gamma(1/2-\mu-\kappa)} \mathrm{e}^{\mathrm{i}(\kappa-\mu)\pi/2} \tag{44}$$
$$\beta_{\boldsymbol{k}} = \frac{\sqrt{2|\mu|}\Gamma(2\mu)}{\Gamma(1/2+\mu-\kappa)} \mathrm{e}^{\mathrm{i}(\kappa+\mu-1)\pi/2} \tag{45}$$

Thus the number of particles observed in the asymptotic future is given by

$$N_{\boldsymbol{k}} = |\beta_{\boldsymbol{k}}|^2 = \frac{\mathrm{e}^{2\pi L\cos\theta} + \mathrm{e}^{-2\pi|\mu|}}{\mathrm{e}^{2\pi|\mu|} - \mathrm{e}^{-2\pi|\mu|}}, \quad \cos\theta \equiv \frac{p_z}{p} \tag{46}$$

One can easily confirm that taking the limit $H \longrightarrow 0$ reproduces the well-known result of Schwinger, which gives a null result for $\cos\theta < 0$. What is interesting here is that in a de Sitter space, the creation of particles with its momentum anti-parallel to the electric field is also possible.

From this result, Kobayashi and Afshordi [38] obtain the number density of φ particles during inflation as

$$n = \frac{(|\mu|^2 + 1/4)^{3/2}}{3\sinh(2|\mu|\pi)}\left[\frac{1}{L}\sinh(2\pi L) + 2\pi\mathrm{e}^{-2|\mu|\pi}\right]\frac{H^3}{(2\pi)^3} \tag{47}$$

which is constant, even though the background space is expanding exponentially, because this calculation is based on a toy model where the electric field is not diluted by the cosmic expansion.

We thus expect that charged particles may be ambiently produced during inflationary magnetogenesis with an enhanced electric field, and hope to proceed to the calculation of the electric conductivity, again using the same toy model, which can be done by calculating the vacuum expectation value of the current

$$J^{\mu} = -\mathrm{i}\, e \left[\varphi^{\dagger} D^{\mu}\varphi - (D^{\mu}\varphi)^{\dagger}\varphi\right] \tag{48}$$

with $D_{\mu} = \partial_{\mu} + \mathrm{i}\, e\, A_{\mu}$. It is formally given by

$$\langle J_i \rangle \equiv \langle 0|J_i|0\rangle = \frac{2e}{a^2}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\,(k_i + eA_i)\,|\chi_{\boldsymbol{k}}^{\mathrm{in}}(\eta)|^2 \tag{49}$$

Since this is a divergent quantity, we must regularize it using, say, the adiabatic regularization scheme as

$$\langle J_z \rangle_{\mathrm{reg}} = \langle J_z \rangle - \langle J_z \rangle\Big|^{(A=n)} \tag{50}$$

where the second term in the right-hand side is calculated in the same way as in (49), but with $\chi_{\boldsymbol{k}}^{\mathrm{in}}(\eta)$ replaced by a mode function calculated up to the n-th adiabatic expansion. It is given in the WKB form:

$$\chi_{\boldsymbol{k}} = \frac{1}{\sqrt{2\Omega_{\boldsymbol{k}}(\eta)}}\mathrm{e}^{-i\int^{\eta}\mathrm{d}\eta'\,\Omega_{\boldsymbol{k}}(\eta')} \tag{51}$$

with $\Omega_{\boldsymbol{k}}(\eta) = \Omega_{\boldsymbol{k}}^{(0)}(\eta) + \Omega_{\boldsymbol{k}}^{(2)}(\eta) + \Omega_{\boldsymbol{k}}^{(4)}(\eta) + \cdots$. Here $\Omega_{\boldsymbol{k}}^{(n)}(\eta)$ is composed of terms containing n time derivatives.

It is known that, in order to regularize the energy-momentum tensor, we need to use adiabatic expansion up to $n = 4$, which is explicitly given by

$$\Omega_{\boldsymbol{k}}(\eta) = \omega(\eta) + \left(\frac{3}{8}\frac{\dot{\omega}^2}{\omega^3} - \frac{1}{4}\frac{\ddot{\omega}}{\omega^2}\right) + \frac{1}{8}\left(-\frac{13}{4}\frac{\ddot{\omega}^2}{\omega^5} + \frac{99}{4}\frac{\dot{\omega}^2\ddot{\omega}}{\omega^6} - 5\frac{\dot{\omega}\omega^{(3)}}{\omega^5} + \frac{1}{2}\frac{\omega^{(4)}}{\omega^4} - \frac{297}{16}\frac{\dot{\omega}^4}{\omega^7}\right) + \cdots \tag{52}$$

where $\omega^2 \equiv \alpha(1 - H\eta)^{-2} + \beta(1 - H\eta)^{-1} + \gamma$.

Kobayashi and Afshordi [38] reported the regularized current in this setting for the first time. They have obtained a finite result using the adiabatic expansion only up to the second order. Their result, however, has exhibited a strange behavior that the current takes a negative value when m is much smaller than H for a range of L. We have found that the situation may not be improved even if we take the fourth-order effect into account as we must do in the calculation of the regularized energy momentum tensor. The induced current is proportional to the external electric field only in the regime it is small, where the conductivity σ is well defined. For generic cases, one must solve the field equation for the gauge field taking the induced current into account.

7. Outlook

While the calculation of the Schwinger effects as well as the induced current by the quantum processes is difficult to perform in more realistic situations, one can imagine what would happen in case charged particles are abundantly produced during inflation to enhance the electric conductivity effectively to eliminate the electric field.

The situation in which the conductivity rises up to a large and constant value compared with the Hubble parameter has been studied by Ratra [21], who has shown the electric field indeed vanishes and the magnetic field is frozen. This means that the amplitude of the magnetic field starts to decrease in proportion to a^{-2} apart from the effect of the time dependence of the coupling function $f(\eta)$.

As we have seen in the magnetogenesis scenario without the strong coupling problem, one generically finds that the magnetic field is suppressed by a factor of k/aH compared with the electric field. Hence in case the Schwinger effect is absent, we find the amplitude of the magnetic field is smaller than the electric field by a factor of e^N on the current horizon scale, where $N \sim 60$ is the number of e-folds of inflation after that scale left the Hubble radius.

If the Schwinger effect starts to operate at some time t_{cond} during inflation to make the effective conductivity large enough to eliminate the electric field and freeze the magnetic field, the final amplitude of the magnetic field becomes smaller than the case without the Schwinger effect by a factor of $\mathrm{e}^{N_{\mathrm{cond}}}$ where N_{cond} is the number of e-folds of inflation after the conductivity has become large. Hence the situation is expected to become even worse, although the electric field may be eliminated earlier.

Thus although the Schwinger effect during inflation is an interesting topic of study in relation with the magnetogenesis without the strong coupling problem, it may not provide us with a way out to solve the mystery of the origin of the large-scale magnetic field.

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