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Condensed matter physics in the 21st century: The legacy of Jacques Friedel

One-dimensional physics in the 21st century

La physique unidimensionnelle au XXI^{e} siècle

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ABSTRACT

This paper presents a brief introduction to some of the systems and questions concerning one-dimensional interacting quantum systems. Historically, organic conductors and superconductors – a field extremely active in the "Laboratoire de physique des solides" in Orsay, in a good part thanks to the influence of Jacques Friedel, played a crucial role in this field. I will describe some of the aspects of this physics and also review some of the very exciting theoretical and experimental developments that took place in the 1D world in the last 15 years or so.

RÉSUMÉ

Cet article constitue une courte introduction à une sélection de questions et de systèmes expérimentaux ayant trait à la physique des systèmes quantiques en interaction. Historiquement, les conducteurs et supraconducteurs organiques – un domaine extrêmement actif au sein du Laboratoire de physique des solides à d'Orsay, en grande partie grâce à l'influence de Jacques Friedel, ont joué un rôle crucial dans ce domaine de recherche. Je décrirai certains des aspects de cette physique et épasserai également en revue certains des développements, tant du point de vue théorique qu'expérimental, qui se sont produits dans le monde des unidimensionnels pendant la dernière quinzaine d'années.

1. Introduction

Solids exhibit strong quantum properties, and are in fact quantum systems one can touch! This is due to the extremely high energy scales of the Fermi energy that allow the system to remain degenerate even at ambient temperatures [1]. A remarkable consequence is for example the existence of the Friedel oscillations [2,3] occurring even for free electrons, and which shows that even a local disturbance of the electron fluid leads to long-range density oscillations.

These effects are considerably amplified when interactions are present. In "high" dimension (read three-, two- being a very special case), one can find an escape route by incorporating the effects of interactions in the parameters (such as the mass) of new excitations that "resemble" the free electrons. This is the famous Landau Fermi liquid theory [4–6] that has

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been the cornerstone of our understanding of solids for a good part of the 20th century. With it we can eliminate the main complicated term in the many-body interacting electron problem and bring back the solid in the reassuring realm of “free” particles. This allows us to study much smaller perturbations – such as, e.g., the electron–phonon interaction – with a chance of success and to understand instabilities such as superconductivity and charge density waves.

However, the situation is radically different when the system is one-dimensional. In that case, there is no way for the electrons to avoid each other and it is easy to see (an experience that anyone stuck in a waiting line has already made) that there are no individual excitations possible, but only collective motion of all the particles in the system. The properties of the Fermi surface that led to Friedel excitations in the first place are at their utmost in one dimension, and as a consequence the amplification of the quantum effects by interactions at its best (or worst). This makes the one-dimensional system have a physics of their own, and a remarkable laboratory to study the effects of interactions.

This physics has of course been extensively studied both theoretically and experimentally in a large variety of systems and at a large number of places. Unravelling the secrets of the one-dimensional quantum world was a considerable challenge, both from the fundamental point of view but also in view of potential new materials exhibiting these remarkable properties. In the beginning of the 1980s, under the influence of Jacques Friedel, the “Laboratoire de physique des solides” in Orsay was a paradise in that field. In particular, among other remarkable lines of research, superconductivity was discovered in a novel class of organic materials, synthesized by K. Bechgaard, by D. Jérome and collaborators [7]. It was thus a fantastic place to learn about this field, especially in contact with exceptional young theorists such as H.J. Schulz. I caught there during my PhD the one-dimensional virus and thus it is quite natural for me to discuss in this article some aspects of this remarkable physics of one-dimensional systems.

The plan of this article is as follows. In Sec. 2 I will give an extremely brief reminder of the Tomonaga–Luttinger liquid, the generic model describing one-dimensional interacting quantum systems. In Sec. 3 I will discuss the organic superconductors. I will not focus too much on the organic superconductors per se, since they will be covered in much more details in the articles written by D. Jérome and J.-P. Pouget in this issue, but will start with them as a remarkable example of one-dimensional systems. I will especially point out the open issues that remain in my opinion with these materials. In Sec. 4 I will discuss the large number of breakthroughs that were accomplished in the last 15 years or so in the one-dimensional world both on the theoretical side and on the experimental one. Finally, some conclusion will be put in Sec. 5. As a disclaimer, this article cannot and do not pretend to be exhaustive. It aims to present a few selected highlights whose choice is somewhat subjective and corresponds to my own interests in the subject.

2. A zest of theory – the Tomonaga–Luttinger liquid

I will not here recall all the properties of one-dimensional systems, since extensive literature exists on the subject (see, e.g., [8–10]). I will just give here a brief digest of the salient properties, pertinent in the context of this chapter and refer the reader to the existing literature, either on fermions or bosons, on the subject for more details and references.

As briefly mentioned above, the individual excitations do not exist in one dimension. Thus the description of, e.g., fermionic systems in terms of the standard Landau quasiparticles is failing. Instead one has a fully different set of properties that went globally by the name of Tomonaga–Luttinger liquid [11]. The main characteristic of such a TLL are as follows.

1. **Collective excitations:** Only collective excitations exist. These excitations are analogous to sound waves, i.e. density oscillations, but the oscillating density is the electronic density or some other density (e.g., spin density). They are characterized by a velocity u. For noninteracting fermionic particles, u corresponds to the Fermi velocity v_F, but assumes a different (renormalized) value in an interacting system. Remarkably, the existence of sound-like excitations is quite general, and applies also to physical systems of bosons (cold atoms) and to insulating magnetic materials as well.
2. **Quasi-long-range order:** A one-dimensional system is poised at the verge of an instability but, because quantum fluctuations usually prevent the breaking of a continuous symmetry, the system is usually only having *quasi*-long-range order, even at zero temperature, with power-law-decreasing correlation functions. For example, for antiferromagnetic spin chains with an anisotropy between the z axis and the xy plane (so-called XXZ spin chain), the correlation functions read [8] in the absence of external magnetic field:

$$\begin{aligned}\langle S^z(x)S^z(0)\rangle &= \frac{1}{x^2} + (-1)^x A_z \left(\frac{1}{x}\right)^{2K} \\ \langle S^-(x)S^+(0)\rangle &= (-1)^x A_x \left(\frac{1}{x}\right)^{\frac{1}{2K}} + B_x \left(\frac{1}{x}\right)^{\frac{1}{2K}+2K}\end{aligned} \tag{1}$$

Similar formulas are valid for fermions and bosons. The number K is a dimensionless number that depends on the interactions. The amplitudes A, B are non-universal numbers depending on the precise microscopic model and the interactions.
3. **Tomonaga–Luttinger liquid:** The above constitute the essence of a Tomonaga–Luttinger liquid that describes the generic behavior of an interacting one-dimensional system [11]. Its asymptotic properties (i.e. the ones for large distances and/or large time differences) are fully determined by the knowledge of the two numbers: the velocity u and the Luttinger parameter K. This last number is dimensionless and controls the decay of *all* the correlation functions as is obvious

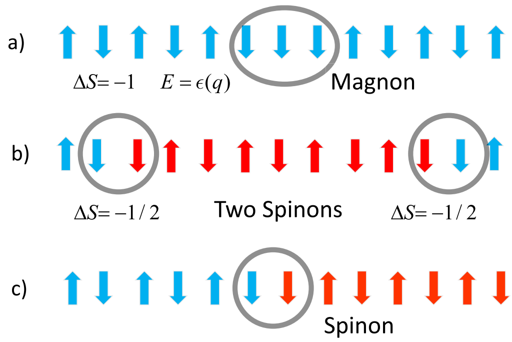

Fig. 1. Fractionalization of excitations: a) Flipping one single spin is the simplest excitations possible. In high dimension this excitation (magnon) would carry a spin 1 and have a well defined relation between its energy $E(q)$ and its momentum q. b) In one dimension this excitation fractionalize and breaks into two elementary excitations (spinons) which carry a spin 1/2. As a consequence the relation between the energy E and momentum of the magnon is not univalued but leads to a continuum of excitations since the energy and momentum can be spread between the two spinons. c) A single spinon excitation: this is a non-local excitation in which every spin on the right of one given point have been flipped.

from (1), as well as the thermodynamics quantities. There has been procedures developed to compute these numbers, as well as the amplitude with great accuracy using either Bethe ansatz or numerical methods.

4. **Fractionalization of excitations:** In one dimension since individual (local) excitations can be converted into collective ones this offers the possibility of obtaining excitations that have lower quantum numbers than should be normally possible. For example, in a magnetic chain, one would imagine that the lowest possible quantum number for a magnetic excitation is $\Delta S^z = 1$, since one has to flip at least *one* spin 1/2. This is what normally happens in higher dimensions where this excitation, a magnon, can propagate as a particle, with a well-defined relation between its energy $E(q)$ and its momentum q. In one dimension, the magnon decomposes into two elementary *non-local* excitations that are shown in Fig. 1. The fact that the magnon is now the sum of two excitations means that instead of a well-defined dispersion $E(q)$, one has in general a continuum of energies for a given momentum q.
5. **Non-local order parameters:** As is obvious when looking at the spinon in Fig. 1, one can find in one dimension orders that are essentially non-local and that have easily a topological character. This has important consequences. Such types of order are notoriously difficult to probe since many of the probes that we have at our disposal are local ones. Fortunately, as we will see novel systems also allow now the probing of such properties.

The above points constitute a very partial description of some of the special properties of one-dimensional systems. Let us now see how these theoretical considerations apply to the real world.

3. Organics and TLL

Organic conductors, and in particular the so-called Bechgaard salts, have provided one of the first examples of one-dimensional conductors and superconductors. They have been extensively studied and reviewed [12–14], and I will not repeat the extensive analyzes done here, but concentrate on how they position themselves towards within the framework discussed in the previous section.

First the hopping in various direction are $t_a \sim 3000$ K along the chains, $t_b \sim 300$ K in the intermediate direction, $t_c \sim 30$ K in the weakest one. So, at very low temperatures, these materials are three dimensional. To probe one-dimensional physics, it is thus important to be in an energy (temperature T, frequency ω, etc.) which is above the effective coupling towards the first transverse direction, so something of the order of 300 K. It is thus difficult to reach this regime by changing the temperature (although some renormalization of the transverse hopping by interactions helps in that respect), but much more easy to do it with, e.g., frequency.

Thus one very successful experiment in observing the power laws associated with TLL physics was the optical conductivity as shown in Fig. 2. This behavior was confirmed in several other experiments (see, e.g., [14]). The dependence under pressure (which reinforces the kinetic energy compared to the interactions) was also studied [17]. Thus in addition to their many other remarkable properties, the Bechgaard salts provided the first experimental realization of an itinerant TLL.

Of course, this is not the end of the story for these systems and they present many other challenges. I will refer the reader to the reviews by Denis Jérome and Jean-Paul Pouget. I will just mention three points, which in my opinion deserve a very special attention and whose import goes way beyond this class of compounds.

1. **Commensurate filling: 1/2 or 1/4 filling.** As was shown, one can get Mott insulators for various commensurate fillings. The question is what is the dominant mechanism responsible for the properties of the organics [18]. From a chemical point of view, the compounds are 1/4 filled, but a small dimerization brings the effective filling to 1/2 by opening a dimerization gap in the middle of the band. These different fillings give rise to quite different scattering mechanisms and the optical conductivity seems to favor the 1/4 filling. In order to analyze these effects, a 1/4 compounds (thus without

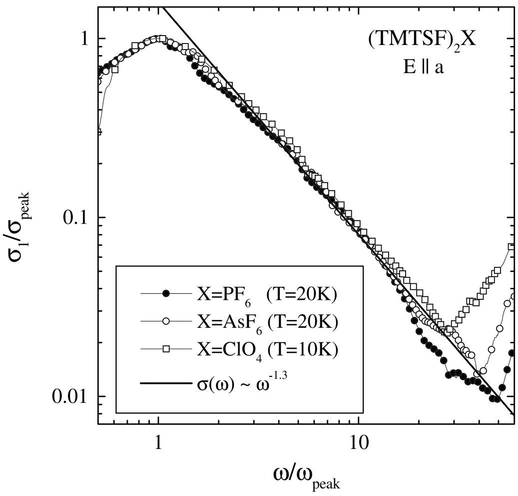

Fig. 2. Optical conductivity for several organic compounds (from [15]). The curves are normalized to the frequency ω_{peak} of the peak in optical conductivity. They show clearly the expected [16] powerlaw of a TLL.

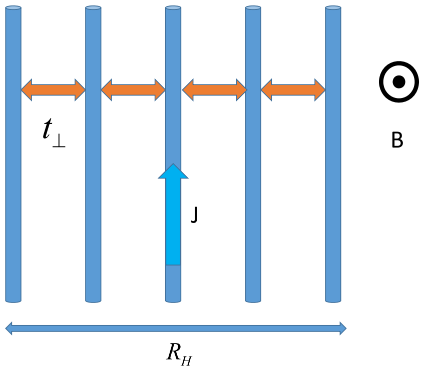

Fig. 3. Typical geometry for the Hall effect in a quasi-one dimensional system. The one-dimensional chains along which a current is passed, are weakly connected by a transverse hopping $t_\perp$. A magnetic field B is perpendicular to the plane. The Hall resistance R_H is zero if $t_\perp = 0$. In the presence of $t_\perp$ the interactions along the chains and the corresponding TLL behavior, as well as the various scattering mechanisms and states (such as Mott insulator for example), can deeply influence the Hall effect. The value of the Hall resistance thus would be quite different from the classical one.

dimerization) has been synthesized [19] and turned indeed to be a very nice insulator. However, no superconductivity has been seen, leaving open the question on how much this new compound and the Bechgaard salts are or not in the same universality class and clearly deserve more investigations.

2. **Dimensional crossover.** One of the main issues is of course how the crossover between the properties in the one-dimensional regime to the ones in the three-dimensional one (for example by lowering the temperature) takes place. This issue is pertinent not only for the organics [18], but also for new realizations of quasi-one-dimensional systems such as cold atoms (see next section). This is a largely open question and I refer the reader to [20] for more details and references on that point.
3. **Hall effect.** Finally, one last issue that deserves a special mention is the Hall effect. With simple Fermi liquids, the Hall effect is supposedly measuring the number of carriers [1]. This is of course a very restricted answer that changes drastically as soon as interactions are taken into account [21–23]. Computing the Hall effect in strongly correlated materials is still a major challenge from the general point of view. Note that I am not talking here of the quantum Hall effect, but simply of its simpler cousin for small magnetic fields.

   In a quasi-one dimensional system, as shown in Fig. 3 in the absence of hopping between the chains, there would not be any Hall effect. How does the Hall resistance depends on the perpendicular hopping $t_\perp$, the temperature T, the magnetic field B and the interactions is still largely open. Of course, one can expect the answer to depend strongly on the commensurability of the material [24,25]. Naively, one could expect a power law in temperature of the form $R_{\mathrm{H}} \sim T^{\nu}$ as for most quantities reflecting TLL behavior. Actually theoretical calculations that are perturbative in the perpendicular hopping $t_\perp$ show [26,27] more a structure of the form

$$R_{\mathrm{H}} = R_{\mathrm{H}}^0 - g\,T^{\nu} \tag{2}$$

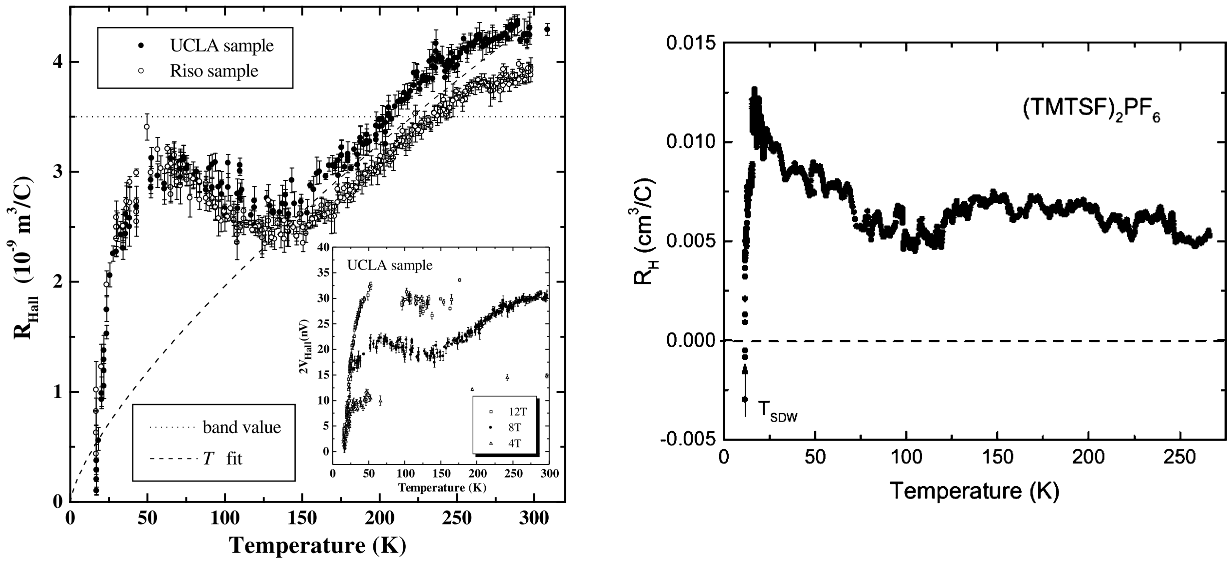

Fig. 4. Hall effect in Bechgaard salts. Left from [28], right from [29]. Hall effect is measured on the same system $(TMTSF)_2PF_6$, albeit in two different orientations where the weak and intermediate directions have been exchanged. At the temperatures of the experiments this should in principle not affect the temperature dependence. However there is a very different observed temperature dependence. The left experiment would be consistent with the TLL formula (2), while the right one is more consistent with a temperature independent Hall effect.

where the *deviations* from the classical (i.e. band) value of the Hall resistance reflect the TLL behavior (ν depends on the TLL parameter K). Confirming or challenging this behavior is of course crucial. From this point of view experiments did not really help since, at the pure experimental level, measuring the Hall effect in two different groups gave two different answers (see Fig. 4). Several other experiments [30,31] measured the Hall effect in compounds with different anions or in the parent insulating compounds (Fabre salts), but the full explanation of the Hall effect in these materials remains to be given.

4. 2001 – a one-dimensional odyssey

In many ways, one could imagine that by the end of the 20th century the field of one-dimensional systems was essentially complete for the one-dimensional part, or simply stuck with problems of a complexity comparable to the understanding of all strongly correlated systems in arbitrary dimension, and thus that additional progress to address some of the problems mentioned in the context of the organic conductors would be moving at a geological pace.

Such a view has largely turned out to be incorrect and one (and quasi-one) dimensional system(s) has (have) known remarkable progress in the last 15 years or so. This is due in large part to important achievements on the theoretical side, both analytical and numerical, as well as the discovery of several families of experimental systems in which the 1D physics can be probed in an extremely controlled way. This interplay between theory and experiments has thus stimulated progress in both fields at an even higher rate. I will in this section briefly recall some of these progress and systems.

4.1. On the experimental side

On the experimental side, relatively few one-dimensional systems existed, most of which are described in [8]. Three major classes of systems appeared with extremely strong promises [32].

1. **Cold atoms:** The first and foremost class of systems was given by artificial systems realized in cold atomic gases [33]. Such systems allow one to trap neutral atoms in an artificial lattice made by interfering lasers. Controlling the power of the lasers thus allows a direct control of the hopping parameters in each direction. This allowed us to realize one- and quasi-one-dimensional structures, similar to the organic materials for both bosonic particles and fermionic ones. Recently, artificial gauge fields similar to real magnetic fields for electrons [34] could be applied to those systems, giving access to questions such as ladders [35], Hall effect [36], etc. There is an extensive literature on the subject so I will not comment further on this remarkable class of materials.
2. **Edge states:** It was known since already some time that the physics at the edge of materials was very different from the physics in the bulk. The first paradigm of such a situations was certainly the edge states of the quantum Hall effect [37]. Since then this physics at the edge has been extensively explored. In the last 10 years, the realization that other phenomena at the edge were of the utmost importance appeared. This is in particular the case in presence of a spin–orbit coupling that leads to effects similar to those of a magnetic field and the existence of topological insulators

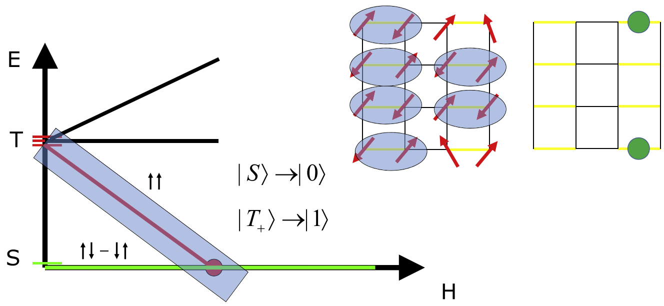

Fig. 5. Spectrum of a spin system made of weakly coupled dimers. Left: in the absence of coupling between the dimers (only the yellow bonds exist on the lattice on the right), the ground state of each dimer is made of a singlet $|S\rangle$ and a triply degenerate triplet $|T_{+,0,-}\rangle$. In the presence of a magnetic field one of the triplets $|T_+\rangle$ disperses down. There is thus a quantum phase transition when this level crosses the one of the singlet (red dot). One can map this problem onto a boson problem for which the singlet is the absence of a boson on the rung, and the triplet is the presence of a boson. In the presence of exchange coupling between the dimers (the black lines on the lattice on the right), the triplet disperses which means that the bosons can hop from one rung to the next (see right part of the figure). The triplet line disperses (blue box on the left). There are thus two quantum phase transitions corresponding to the injection of the first triplet in the system and then to the filling of the band of triplets. For the bosons it means going from zero bosons per site (rung) to one boson per rung. These systems can thus be used as quantum simulators of itinerant boson systems. Right: the system of dimers and its equivalent in terms of bosons (green dots). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

[38,39]. In a similar way, topological states at the edge of one-dimensional materials got intensively studied. A prototype of this type of physics was provided by the spin 1/2 edge states that exist at the end of a spin 1 Haldane spin chain [40–42], for which the bulk is a spin liquid with only gapped excitations but essentially free spin 1/2 exist at the edge.

3. **Magnetic insulators:** Magnetic insulators have proven to be remarkable systems to tackle in a controlled way the one-dimensional physics of *itinerant systems*. The key to this mapping is to recognize that a two-state system (such as a spin 1/2) can be mapped onto a hard core boson [43,44]. A spin down is the absence of a boson on a site, and a spin up can be mapped to the presence of the boson. The bosons need to have a hard core constraint to limit the Hilbert space. A spin chain problem is thus strictly equivalent to mobile bosons hopping on a lattice. Actually doing it directly with spins can have certain drawback and it has proven very useful to use dimers [45–47]. The spectrum of a ladder is shown in Fig. 5. Under a magnetic field, the singlet $|S\rangle$ and the lowest triplet $|T_+\rangle$ can be used for mapping:

$$\begin{aligned} S^+_{i,k} &\to \frac{1}{\sqrt{2}}(-1)^{i+k} b^\dagger_i \\ S^z_{i,k} &\to \frac{1}{4}\left[1 + 2\left(b^\dagger_i b_i - \frac{1}{2}\right)\right] \end{aligned} \tag{3}$$

where $k = 1, 2$ labels the spins of a dimer and i the position of the dimers. A detailed derivation can be found, e.g., in [45]. These systems have several advantages:

(a) since they are Mott insulators, the complicated problem of the screened Coulomb interaction, which we usually know extremely poorly, is irrelevant. The system is characterized by a small number of superexchange constants J, which can be either inferred from measurements such as magnetization or directly measured in, e.g., experiments. As a result the *microscopic* model is extremely well known. The "hopping amplitudes" and interactions between the bosons are thus perfectly known microscopically. These experiments are thus quantum simulators of a controlled microscopic model [48]. It means that they can be compared to the theoretical predictions without any chance that a mismatch could come from the need to improve the Hamiltonian;

(b) the magnetic field acts as a chemical potential for the bosons. This allows us to use the magnetic field to control very precisely the density of bosons, which is in turn directly measured by measuring the magnetization. These systems allow a doping from zero bosons per site (only singlets) to one boson per site (fully polarized system). The fact that the density of particles can be tuned at will makes them ideal realizations for probing quantum critical physics, allowing us to tune through a quantum critical point by controlling density and temperature [49];

(c) many probes exist for magnetic systems. When translated in boson language, it means that one can easily measure both the single particle correlation functions and the density ones

$$\langle S^-_j(t) S^+_0(0)\rangle \to (-1)^j \langle b_j(t) b^\dagger_0(0)\rangle \tag{4}$$

as well as the density–density correlation function

$$\langle S^z_j(t) S^z_0(0)\rangle \to \langle \rho_j(t) \rho_0(0)\rangle \tag{5}$$

where $\rho_j(t) = b^\dagger_j(t) b_j(t)$ is the density operator of the bosons.

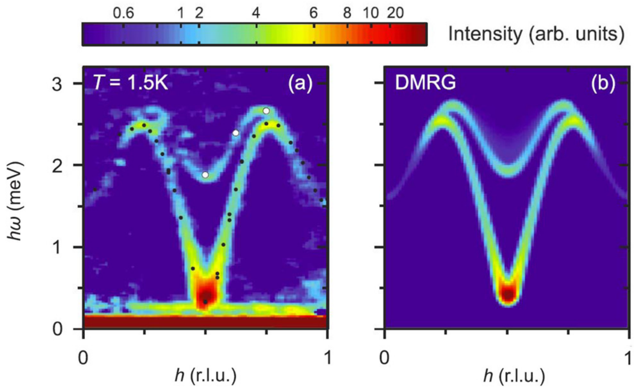

Fig. 6. Dynamical structure factor of neutron scattering, corresponding to the spin–spin correlation at the frequency ω, and momentum $h = q/(2\pi)$: $\text{Im}S(\omega, q)$ for a two-leg, weak rung spin ladder (so-called DIMPY compound). On the left is the experimentally measured one, while on the right is the computed one using time-dependent DMRG. As can be seen, the agreement between the two methods is excellent, showing the ability to compute and to measure dynamical correlation functions reliably. The spectrum shows that a two-spinon bound state does exist in this material, in agreement with theoretical predictions (from [64]).

Such systems thus allow one to probe various remarkable phenomena connected with bosons. In high dimensions, they have showed Bose–Einstein condensation [46,47]. In one dimension, they have allowed one to test *quantitatively* for the first time the physics of the TLL [50–52]. They have allowed us to see other features expected for one-dimensional systems such as the fractionalization of excitations [53] and the frequency–temperature scaling expected in a TLL [54]. They certainly constitute one of the very promising lines of research in connection with one-dimensional physics. Disorder effects and the existence of a Bose glass phase [55,56] have started to be probed in these materials [57–59,48].

4.2. On the theoretical side

In parallel with the progress on the experimental side and clearly stimulated by those, the theoretical aspects of one-dimensional physics have also known spectacular developments. I will of course not cover in details those developments, but briefly mention some of them.

1. **DMRG and calculation of correlation functions and dynamics:** The biggest breakthrough in recent years in one dimension has been the spectacular development of the numerical techniques and in particular of the Density Matrix Renormalization Group (DMRG) [60], originally invented by S. White [61]. Although this technique was already a remarkable tool to compute zero-temperature static correlation functions, its application range has been incredibly extended in recent years. It is now able to compute dynamics correlation functions and finite-temperature ones. Moreover, it can also deal with out-of-equilibrium situations in real time without any need for uncontrolled analytic continuations.
This has of course opened a host of possibilities since it allows us now to obtain essentially exactly the correlation functions from a given Hamiltonian. This has allowed one to make direct comparison with experiments for which the field theory description would be too limited since the energy at which the system is probed is too high. It has also allowed a convenient route to extract the important non-universal parameters to inject in the field theory [62,63]. As a result, quantitative tests of the TLL theory could be made, and direct comparison with the experiments such as neutrons, NMR, ESR could be made. This is only the tip if the iceberg and this method is clearly exploding at the moment. In practice, a rule of thumb is that it allows one to explore energy scales down to $J/20$ without going into prohibitive calculations, which is low enough to allow for a comfortable contact with the field theory description. An example is shown in Fig. 6.
2. **Calculation of correlation functions by BA:** In a similar way, Bethe ansatz has also known important developments. There is a long way between knowing that a model is exactly solvable by BA and extracting anything useful from this knowledge. For a long time, BA has only be able to provide the spectrum of excitations and some elements of thermodynamics. This was already invaluable and allowed one to prove for the first time the existence of a Mott gap in the Hubbard model or to study the spectrum of 1D bosons [65,66] or by using a trick to obtain the TLL parameters that one needs to inject in the field theory [67,68]. In recent years, it has been possible to compute also *correlation functions* from the BA. This is in particular the case for the XXZ chain and the Lieb–Lininger model of interacting bosons in one dimension [69]. This exact solution of correlation functions, which is now gradually extended to finite temperatures as well was of course invaluable to understand finer points of their behavior and also as a benchmark of other methods to compute them, such as the DMRG one.
3. **Non-Luttinger liquid corrections:** Finally, even at the conceptual level, progress was done on the applicability of the TLL model. Although this model is clearly the correct fixed point at low energy, there are many situations in which we are interested by a behavior at finite energy. For example, as indicated in Fig. 7, the absorption at a $q \neq \pi$ in a spin

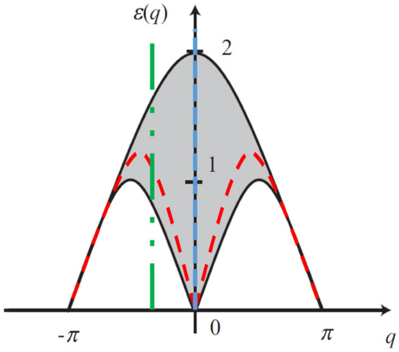

Fig. 7. Schematic spectrum of a two spinon excitations, and associated crossover and threshold energies. The spectrum indicates the boundaries of the dynamical spin–spin response. The TLL theory predicts reliably the behavior at low energy (so for example around the points $q = 0, 2\pi$ and $q = \pi$). So a scan along the blue dashed line will be fully described (when the energy is much smaller than the exchange J) by TLL theory and will show the TLL exponents. At finite energy, "irrelevant" operators can drastically change the behavior. There is thus a crossover line (red dashed line) below which these operator will change drastically the exponents. So between the redline and the threshold, the exponents are modified compared to the ones given be the TLL, in the same way as an X-ray edge singularity modifies the exponents close to the threshold of absorption in a Fermi liquid. A scan along the green dash-dotted line would thus show as the energy is lowered, the TLL exponents (provided the energy is low enough compared to J), and then the new exponents. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

chain near the threshold of excitations is *not* a low-energy process. Although one can compute it directly from the TLL model as well (as long as the threshold is not too close to J), it is legitimate to ask whether additional terms, that would be totally irrelevant close to $q = \pi$, could lead to a different behavior. This is indeed the case [70,71] and the behavior close to the threshold is characterized by an exponent that is *different* from the one that TLL theory would predict. This is due to particle–hole excitations that produce *close-to-the-threshold* additional singularities. The situation is very similar to the case of a X-ray edge singularity in a Fermi liquid: the absorption close to the threshold would give a step function reflecting the nearly energy-independent density of states in a Fermi liquid. However, coupling with a deep core state produces additional singularities that change the exponent. In the TLL, the deep core state is particle–hole excitations. As a result, there is, as indicated in Fig. 7, a new exponent appearing. This of course is *not* in contradiction with the TLL theory, but builds on it in an intelligent way.

In a similar way, although two component systems are known to be TLL with a spin–charge separation behavior [8, 72], the situation has proven to be quite different when there is only one atom in one of the species. In that case, the behavior is very different, and the minority species acts as a mobile impurity plunged in the bath of the majority one [73]. Due to the peculiarities of the one-dimension case, this leads to results quite different from those obtained with a standard polaron picture. A similar "non-TLL" behavior appears when the energy scale for the charge mode and spin mode are very separate and the temperature lies between them. In that case, the charge can be coherent, while the spin is incoherent. This is the so-called "spin incoherent TLL" [74–76]. I will not elaborate more on it here and refer the reader to the literature.

5. Conclusions and perspectives

This paper was a very brief tour of one-dimensional physics. This physics has been definitely impulsed on the experimental side by the remarkable physics of the organics, and is now striving in several novel directions. Building on the understanding that was developed during the 20th century, this field has known remarkable developments in the last 15 years or so. This is due to the appearance of remarkable new experimental systems, ranging from edge states, cold atoms, magnetic insulators that have allowed us to realize this physics with an unprecedented level of control and with new physical effects. On the theoretical side, the appearance of effective numerical methods, the progress in Bethe ansatz have given access to the dynamical correlation functions beyond the low-energy asymptotic limit that the field theory was allowing before. Finally some effects that go beyond the TLL description are now under control and even non-TLL behaviors have been explored.

There is no doubt that this blossoming of physics will not stop there and that we will see in the coming years other new remarkable physics emerging from the bizarre one-dimensional world.

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This paper is dedicated to Jacques Friedel, who was not only an exceptional physicist but also a true gentleman. In addition to his own scientific achievements, his vision and influence has provided France with a unique environment in solid-state physics. To have had the chance to meet him and discuss with him or exchange letters, first as a young graduate student and then for the next thirty years was a real honor and pleasure.

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