



Phononic crystals / Cristaux phononiques

## Harvesting vibrations via 3D phononic isolators

*Extraction d'énergie vibrationnelle par des isolants phononiques à trois dimensions*Ioannis E. Psarobas<sup>a,\*</sup>, Vassilios Yannopoulos<sup>b</sup>, Theodore E. Matikas<sup>a</sup><sup>a</sup> Dept. of Materials Science & Engineering, University of Ioannina, 45110 Ioannina, Greece<sup>b</sup> Dept. of Physics, National Technical University of Athens, 15780 Athens, Greece

## ARTICLE INFO

## Article history:

Available online 7 March 2016

## Keywords:

Phononic crystals  
Phononic shields  
Phononic diode  
Trapping vibrations

## Mots-clés:

Cristaux phononiques  
Boucliers phononiques  
Diode phononique  
Piégeage de vibrations

## ABSTRACT

We report on the existence of unidirectional phononic band gaps that may span over extended regions of the Brillouin zone and can find application in trapping elastic (acoustic) waves in properly designed multilayered 3D structures. Phononic isolators operate as a result of asymmetrical wave transmission through a slab of a crystallographic phononic structure with broken mirror symmetry. Due to the use of lossless materials in the crystal, the absorption rate is dramatically enhanced when the proposed isolator is placed next to a vibrational harvesting cell.

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## R É S U M É

Nous rapportons l'existence de bandes interdites phononiques unidirectionnelles qui peuvent se déployer sur des régions étendues de la zone de Brillouin et peuvent être appliquées au piégeage d'ondes (acoustiques) élastiques dans des structures multicouches tridimensionnelles convenablement conçues. Les isolants phononiques opèrent par transmission asymétrique d'ondes à travers une structure phononique cristallographique présentant une symétrie miroir brisée. Du fait de l'utilisation de matériaux sans perte dans le cristal, le taux d'absorption est considérablement accru quand l'isolant proposé est placé près d'une cellule extractrice d'énergie vibratoire.

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## 1. Introduction

In electronics, a diode is a well-known device that allows electrical current to pass in one direction but not in the opposite one. The development of the acoustic counterpart of the electronic diode opens up exciting opportunities for novel

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applications in different research areas ranging from biomedical ultrasound imaging to environmental noise reduction. The acoustic diode (structures that transmit acoustic waves only in one direction) was originally conceived by the invention of the thermal diode [1,2], which rectifies heat flow due to phonons and works for lattice vibrations of all frequencies. A one-dimensional (1D) acoustic rectifier was then realized as a prototype of a one-way acoustic mirror consisting of two segments, a phononic crystal (PnC) [3] and a nonlinear acoustic medium [4]. Nevertheless, such a concept restricts the PnC component to a passive filter and not really an active source of unidirectionality. A more interesting approach using linear materials has been introduced by means of a two-dimensional (2D) PnC consisting of a square lattice of steel rods in contact with a diffraction structure [5]. This diode operates on the concept of interchanging rough and smooth surfaces. The main advantage of the linear acoustic diode is that sound waves do not change their frequencies as they pass through the structure. Also, its easy fabrication method opens up opportunities for on-chip planar acoustic circuits with applications in logical sound processing [6]. In our investigation, we are going to propose PnC structures of low symmetry that can sustain highly asymmetric transmission along with strong multiple scattering effects. To that end, acoustic isolation can be achieved along with efficient wave trapping in a confined region of space.

There are many similarities between photonics and phononics, in the sense that one field can provide guiding principles to the other. Therefore in the quest for acoustic diodes or isolators, we can borrow certain concepts and ideas, most popular nowadays, from Optics. For example, optical isolators are fundamental components in all-optical circuitry, as they constitute the optical analog of electronic diodes. To this end, there have been several proposals in the past for realizing optical isolators such as 1D graded random amplifiers [7] or nonlinear waveguides [8]. Other efforts have focused on systems supporting magneto-optical effects wherein time-reversal symmetry is explicitly broken, leading to nonreciprocal response [9]. Along these lines, we have presented a linear, passive design of a purely dielectric 3D photonic crystal (PC) that possesses unidirectional (one-way) photonic band gaps spanning over wide regions in both the Brillouin zone and electromagnetic spectrum [10]. The respective unit cell of the PC possess a degree of chirality while, at the same time, consists of different material scatterers, which give rise to unidirectional wave propagation and frequency gaps. We have proceeded a step further with this PC design and explored its potential for unidirectional light propagation with real materials simulating the response of colloidal photonic-crystal heterostructures [11].

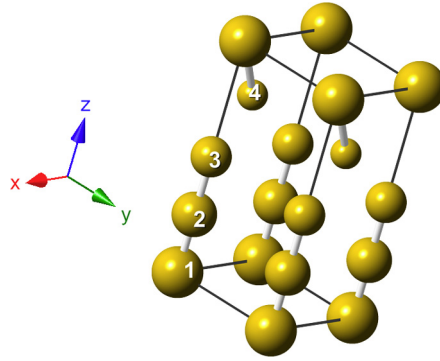
Low-symmetry 3D PnC structures [12,13] have been recently investigated and the results presented within the framework of the layer-multiple-scattering (LMS) [14,15] revealed striking resemblances with the behavior of PCs with respective low-symmetry aspects. In what follows, starting from the asymmetry of a 3D PnC's unit cell along with the strong multiple scattering of acoustic waves taking place within the crystal, we demonstrate in detail the presence of one-way frequency gaps. Besides the obvious operation of the proposed system for phononic isolation, its one-way functionality can be also applied in sound trapping and attenuation enhancement by a properly designed vibration harvesting setup. Finally, its characteristic length (lattice constant) can be chosen so that sound, phonon or heat trapping is effective in a desired spectral region.

## 2. Structure of the 3D phononic crystal

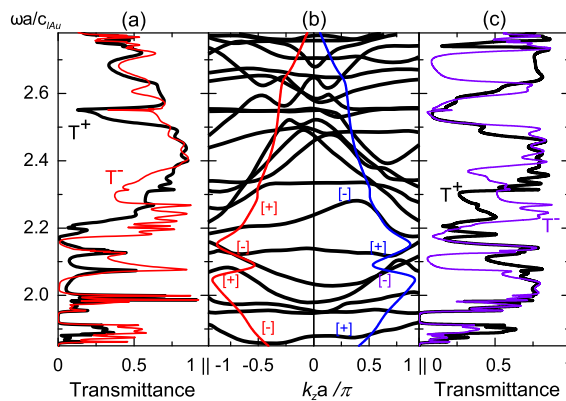
Our investigation will concentrate on a 3D PnC consisting of a four-point basis of different size spheres on a monoclinic lattice. The topology of the structure is carefully chosen, so that it has only one mirror plane [say the  $xz$  plane, i.e., the (010) crystallographic surface] and has therefore a well-developed asymmetry. In particular, the chosen low-symmetry system, besides the lack of spatial inversion symmetry, exhibits chirality to a certain degree [13]. Evidently, wave transmission through a finite slab of such a crystal would be different for incidence from the two opposite faces of the slab, if the slab is conceived as a stack of planes parallel to a crystallographic direction other than normal to the (010) surface. The crystal is viewed as a succession of planes of spheres parallel to the  $xy$  plane of the same 2D periodicity defined by the primitive vectors  $\mathbf{a}_1 = (a, 0, 0)$  and  $\mathbf{a}_2 = (0, a, 0)$  [the (001) crystallographic surface]. Each plane of spheres forms the same square lattice with lattice vectors  $\mathbf{R}_n = n_1\mathbf{a}_1 + n_2\mathbf{a}_2$ ,  $n_1, n_2 = 0, \pm 1, \pm 2, \dots$  and corresponding reciprocal lattice vectors  $\mathbf{g} = m_1\mathbf{b}_1 + m_2\mathbf{b}_2$ ,  $m_1, m_2 = 0, \pm 1, \pm 2, \dots$ , where  $\mathbf{b}_i \cdot \mathbf{a}_j = 2\pi\delta_{ij}$ , with  $i, j = 1, 2$ . A unit layer of the crystal consists of four nonprimitive planes of spheres at  $(0, 0, 0)$ ,  $(0, 0, a/2)$ ,  $(0, 0, a)$ , and  $(0.3a, 0, 3a/2)$ . The specifics regarding the size of the spheres in each nonprimitive plane is clearly illustrated in Fig. 1. Finally, the unit layer is obtained by the primitive translation given by  $\mathbf{a}_3 = (-0.3a, 0, 2a)$ .

## 3. Method of calculation

Computations are carried out in terms of the frequency band structure of the crystal and the transmission/absorption response of a finite slab of the crystal by employing the LMS [14] method. The LMS code [15] is ideally suited for the calculation of the transmission, reflection and absorption coefficients of an elastic wave incident on a composite slab consisting of a number of layers which can be either planes of spheres or nonspherical axisymmetric scatterers [16], with the same 2D periodicity or homogeneous plates. For each plane of particles, the method calculates the full multipole expansion of the total multiply scattered wave field and deduces the corresponding transmission and reflection matrices in the plane-wave basis. The transmission and reflection matrices of the composite slab are evaluated from those of the constituent layers. By imposing periodic boundary conditions one can also obtain the complex frequency band structure of an infinite periodic



**Fig. 1.** Unit cell of a monoclinic PnC with a four-point basis. The crystal is viewed as a succession of four nonprimitive planes of spheres parallel to the (001) surface at positions  $(0, 0, 0)$ ,  $(0, 0, a/2)$ ,  $(0, 0, a)$ , and  $(0.3a, 0, 3a/2)$ . The radius of Au spheres for each plane is  $S_1 = 0.30a$ ,  $S_2 = 0.25a$ ,  $S_3 = 0.20a$ , and  $S_4 = 0.15a$ , respectively. The four-point basis of the Bravais lattice is designated by the thick grey links.



**Fig. 2.** Asymmetric transmittance for a compressional elastic wave (a) and a p-polarized shear (c) for oblique incidence at  $\mathbf{k}_{\parallel} = (\pi/4, 0)/a$  on a finite slab of the PnC (see Fig. 1) consisting of 8 unit layers.  $T^+$  and  $T^-$  correspond to the transmittance associated with wave incidence from the left and right (001) faces of the slab, respectively. (b) Frequency band structure of the infinitely periodic PnC for  $\mathbf{k}_{\parallel} = (\pi/4, 0)/a$ .

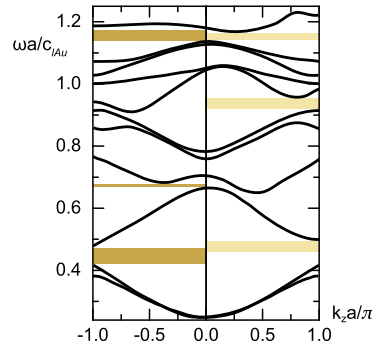
crystal. In such a case, we view the infinite crystal as a sequence of identical parallel slabs (parallel to a given crystallographic plane), extending over all space. The unit slab may consist of a single layer or any number of layers, provided they have the same 2D periodicity. In  $\mathbf{k}$  space therefore, it implies directly that  $\mathbf{k}_{\parallel}$  is a conserved quantity. The reduced zone of the 3D  $\mathbf{k}$  space is chosen to extend over the surface Brillouin zone (SBZ) of the given crystallographic plane. This reduced  $\mathbf{k}$  zone is of course completely equivalent to the commonly used, more symmetrical bulk Brillouin zone, in the sense that a point in one of them lies also in the other or differs from such one by a vector of the 3D reciprocal lattice. In that sense, all relative information is included, in terms of symmetry points, in the corresponding SBZ. A more detailed presentation can be found in Ref. [17].

The method applies equally well to nonabsorbing systems and to absorbing ones. Its main advantage over the other existing numerical methods lies in its efficient and reliable treatment of systems containing strongly dispersive materials.

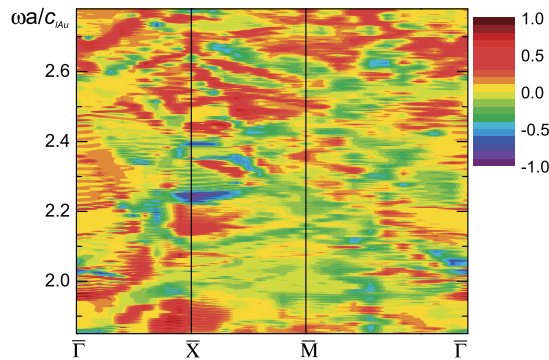
#### 4. Asymmetric phononic transmission

We now consider the elastodynamic response of the crystal given in Fig. 1, assuming the constituent materials to be elastically isotropic and lossless, which is a reasonably good approximation in our case. We use a mass density  $\rho_{\text{Au}} = 19.3 \text{ g/cm}^3$  for Au, with the respective compressional and shear speed of sound to be  $c_{\text{tAu}} = 3376 \text{ m/s}$  and  $c_{\text{tAu}} = 1482 \text{ m/s}$ . The embedding epoxy matrix has  $\rho_e = 1.142 \text{ g/cm}^3$ ,  $c_{\text{le}} = 2570 \text{ m/s}$ , and  $c_{\text{te}} = 1138 \text{ m/s}$ . In Fig. 2 we present how asymmetric transmission can be observed in a slab of the PnC consisting of 8 unit layers, where each layer contains four square lattices (planes) arranged as shown in the unit cell of Fig. 1. Size variation of the Au spheres is an essential part of our investigation, in order to enhance asymmetric transmission. Our calculations have also shown that low-volume filling fractions favor asymmetric transmission, as well. With  $T^+$  we define the transmittance of an acoustic wave incident from the left (001) face of the PnC slab, i.e., a wave propagating from left to right, whereas  $T^-$  is associated with a wave propagating from right to left. Evidently, there are spectral regions where  $T^+$  and  $T^-$  differ remarkably, with some small exceptions.

In Fig. 2(b) we also show the frequency band corresponding to  $\mathbf{g} = \mathbf{0}$ , in order to account for the sign of the group velocity of acoustic beams. The sign of the group velocity is crucial to be presented here (shown in square brackets), because



**Fig. 3.** Frequency band structure of the infinitely periodic PnC at a different direction from Fig. 2, defined by  $\mathbf{k}_{\parallel} = (\pi/8, \pi/8)/a$ . Asymmetry is manifested through the formation of unidirectional frequency gaps.



**Fig. 4.** Transmission difference, recorded as  $\Delta T = T^+ - T^-$ , for a  $p$ -polarized shear wave incident from the left ( $T^+$ ) and right ( $T^-$ ) faces of an 8-layer slab of the PnC of Fig. 1, projected on the SBZ of the (001) surface along its symmetry points.

it explains why a wave incident from the left couples with the  $k_z < 0$  branch and not the  $k_z > 0$  as expected and vice versa. The rest of the frequency bands shown in Fig. 2(b) correspond to  $\mathbf{g} \neq \mathbf{0}$ , giving rise to diffracted beams. An acoustic beam which is incident on a finite PnC slab couples almost exclusively with the  $\mathbf{g} = \mathbf{0}$  frequency band unless the incident wave vector is large enough to lie outside the SBZ. This means that the rest of the bands, i.e., those associated with  $\mathbf{g} \neq \mathbf{0}$ , do not couple with the incident beams considered in our case and are therefore irrelevant in the above analysis regarding the sign of the group velocity [14].

In Fig. 3, the frequency band structure, at a different direction from Fig. 2(b), is presented. Here the asymmetry is manifested more clearly through the formation of unidirectional gaps. In this case, which is a lower spectral region, asymmetric scattering merely contributes. Directional partial gaps dominate the scene and functioning as spectral diodes.

In order to have a full picture of the one-way behavior of the PnC, in Fig. 4, we show the transmission difference from left to right and the opposite for all incident wave vectors  $\mathbf{k}_{\parallel}$  within the SBZ associated with the four square lattices of the PnC unit cell. Fig. 4 records the full response of the system for a  $p$ -polarized incident shear wave. It can be seen that there exist wide regions both in  $\mathbf{k}_{\parallel}$  and frequency ( $\omega a/c_{\text{IAu}}$ ) where one-way band gaps are observed. Finally, at this point we stress the fact that the unidirectional band gaps reported here, which have direct application in acoustic isolation, are based on linear and passive materials and can be more easily realized.

## 5. A phononic isolator

A step further is to introduce the above unidirectional PnC as an isolation component in a vibration-harvesting setup, since it operates by itself solely with lossless materials. This means that the proposed 3D phononic isolator can find application as a trap of elastic waves (in general) next to a vibration-harvesting device or an energy conversion apparatus (mostly for heat extraction or acquisition). In Fig. 5, we present schematically a sequence of a phononic isolator, an absorbing slab (AS) and an acoustic mirror, in a similar way to that introduced for optical applications [10,18,19]. The endgame of such an attempt is to place an AS atop a mirror substrate next to a lossless phononic isolator, so that absorbance can be enhanced. This concept is expected to work due to the existence of the trapping vibrations mechanism: after the elastic waves penetrate the PnC isolator from the left, they are partially absorbed by the absorbing material. However, after being reflected by the mirror substrate and partially reabsorbed by the slab, it is reflected back to the AS as it impinges on the isolator from the right face. This way, the time during which vibrations are confined within the absorbing slab increases drastically and therefore the corresponding rate of absorbance. In testing the above concept we use an AS with

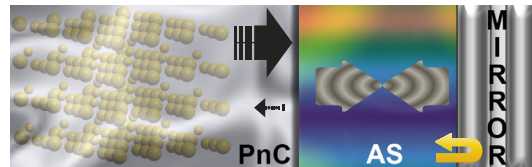


Fig. 5. Harvesting vibrations via a 3D PnC, an absorbing slab (AS) and an acoustic mirror.

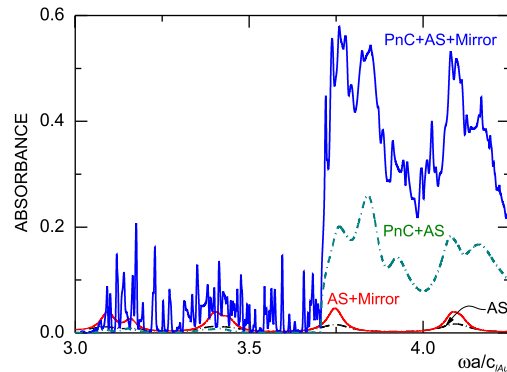


Fig. 6. Absorbance for a compressional elastic wave incident at  $\mathbf{k}_{\parallel} = (\pi/4, 0)/a$  on different types of slab sequences. AS: absorbing slab [ $c_{IAS} = 2c_{le}(1 - i10^{-3})$ ] of thickness  $d = 10a$ . AS + Mirror: absorbing slab on top of a perfect mirror. PnC + AS: 16-layer PnC slab on top of the absorbing slab. PnC + AS + Mirror: 16-layer PnC slab on top of the absorbing slab and perfect mirror.

$\rho_{AS} = 2\rho_e$ ,  $c_{IAS} = 2c_{le}(1 - i10^{-3})$ ,  $c_{tAS} = 3c_{te}$ , and width  $d = 10a$ . The PnC isolator is a 16-layer slab of the crystal shown in Fig. 1. The absorbance spectra of a compressional elastic wave incident on the left of the PnC slab at an angle defined by  $\mathbf{k}_{\parallel} = (\pi/4, 0)/a$  is presented in Fig. 6. The following cases are calculated in order to estimate the efficiency of the presence of the PnC isolator: *i*) the AS alone (AS), *ii*) the AS slab atop a perfect mirror (AS + mirror), *iii*) the AS slab covered by the PnC isolator (PnC + AS), and *iv*) lastly, the AS sandwiched between the PnC isolator and the mirror (PC + AS + mirror). First of all, it is obvious that the addition of a mirror supporting the AS improves drastically the absorbance. When a phononic isolator is placed on top of the AS + mirror, the isolator reflects vibrations that have escaped absorption twice and forces them to pass through the AS slab once again. Of course, this procedure can continue an infinite number of times, leading to a *multiple-scattering process* which eventually increases the rate of absorption.

## 6. Conclusion

We have designed a 3D phononic isolator based on a PnC structure that has broken mirror symmetry, which is clearly seen in the frequency band structure of the crystal (see Figs. 2 and 3). The presence of unidirectional phononic band gaps leads to highly asymmetric transmission of elastic waves. The PnC lattice is a monoclinic lattice with a four-point basis of lossless spherical scatterers of different sizes within the unit cell. Due to the inclusion of lossless materials in the PnC, when the proposed phononic isolator is placed next to a vibration harvesting cell, it can enhance dramatically the absorption rate via a trapping mechanism. The structure is by no means optimized in terms of the width of the unidirectional gaps in both the vibrational spectrum and wave vector space (SBZ). We therefore believe that the absorption in a vibration-harvesting prototype containing a 3D PnC-based isolator can be much more enhanced within a broader spectral region from what is reported here (which is at least five times stronger than the case without the PnC).

The present theoretical study served as a proof of concept, which has shown the importance of utilizing 3D phononic structures as means to enhance interesting physical phenomena through strong multiple scattering processes.

## Acknowledgements

This research has been co-financed by the European Union (European Regional Development Fund – ERDF) and Greek national funds through the Operational Program Education and Lifelong Learning 2007–2013 of the National Strategic Reference Framework (NSRF 2007–2013), Action ARISTEIA II, under Grant Number GSRT-4412 (<http://phononix-des.project.uoi.gr>).

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