C. R. Physique 17 (2016) 778–787

Contents lists available at ScienceDirect

Comptes Rendus Physique

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Quantum microwaves / Micro-ondes quantiques

Cat-qubits for quantum computation ☆

Qubits de chat pour le calcul quantique

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ARTICLE INFO

Article history:
Available online 26 July 2016

Keywords:
Universal quantum computation
Quantum memory
Quantum error correction
Schrödinger cat states
Quantum superconducting circuits

Mots-clés :
Calcul quantique universel
Mémoire quantique
Correction des erreurs quantiques
États du chat de Schrödinger
Circuits quantiques supraconducteurs

ABSTRACT

The development of quantum Josephson circuits has created a strong expectation for reliable processing of quantum information. While this progress has already led to various proof-of-principle experiments on small-scale quantum systems, a major scaling step is required towards many-qubit protocols. Fault-tolerant computation with protected logical qubits usually comes at the expense of a significant overhead in the hardware. Each of the involved physical qubits still needs to satisfy the best achieved properties (coherence times, coupling strengths and tunability). Here, and in the aim of addressing alternative approaches to deal with these obstacles, I overview a series of recent theoretical proposals, and the experimental developments following these proposals, to enable a hardware-efficient paradigm for quantum memory protection and universal quantum computation.

RÉSUMÉ

Le développement des circuits quantiques Josephson a généré de grands espoirs pour le traitement fiable de l'information quantique. Alors que ces progrès se sont accompagnés de diverses expériences de principe sur des systèmes quantiques de petites tailles, il faut désormais franchir l'étape importante du passage à l'échelle supérieure en nombre de qubits pour les protocoles. Le calcul tolérant aux erreurs avec des qubits logiques protégés est cependant habituellement envisagé au prix d'un significatif surcoût en ressources matérielles. Chacun des qubits physiques impliqués devra par ailleurs toujours disposer de caractéristiques optimales (temps de cohérence, force de couplage et accordabilité). Ici, et dans le but d'explorer des approches alternatives pour dépasser ces obstacles, je passe en revue un ensemble de propositions théoriques récentes et les premières expériences

☆ This paper, written in March 2015, is an overview of recent proposals and experiments for encoding, protecting and manipulating quantum information in so-called Schrödinger cat states of a quantum harmonic oscillator. The author acknowledges the collaboration and discussions with Zaki Leghtas, Michel H. Devoret, Robert J. Schoelkopf, and Liang Jiang, as well as many other collaborators at Yale University, the group of Benjamin Huard at the École Normale Supèrieure and the Quantronics group at CEA Saclay.

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http://dx.doi.org/10.1016/j.crhy.2016.07.011

correspondantes, qui rentrent dans un paradigme de protection de mémoire quantique et de calcul quantique universel qui reste peu gourmand en ressources matérielles.

1. Introduction

Since the first demonstration of a superconducting quantum bit (qubit) a decade and a half ago [1], the coherence time of superconducting circuits has witnessed a tremendous increase by about five orders of magnitude, reaching now more than 100 μs in the best systems [2,3]. The powerful assets of these systems over their atomic physics counterparts lie in the flexibility in the design of the Hamiltonian of a complex system composed of many parts. This flexibility leads to a very rich set of functionalities that could be performed by such devices. Furthermore, contrarily to the quantum optics systems, no major physical limitations have been observed on various parameters of these Hamiltonians (e.g., coupling strengths or nonlinearity of the field modes). Microwaves, unlike light signals, are well controlled using commercial electronics. They are deep in the quantum regime when cooled at milli Kelvin temperatures. These properties have rendered the field of quantum superconducting circuits (QSC) a very promising framework for quantum information processing (QIP) [4].

This rapid progress has already led to various proof-of-principle experiments on small scale quantum systems (few interacting degrees of freedom). Indeed, many earlier experiments within the contexts of NMR-based quantum information processing, trapped ions or cavity quantum electrodynamics (QED) with Rydberg atoms, have been successfully replicated on these systems. In many of these experiments, the properties of QSC such as strong coupling and nonlinearity together with reasonable coherence times have allowed one to achieve better performances than earlier atomic physics and quantum optics experiments. Furthermore, the constant progress of the coherence time for these systems make us very confident that these performances will keep improving through the following years.

Despite all these achievements, a major scaling step is required towards many-qubit protocols. Indeed, the next, critical stage in the development of QIP is most certainly the active quantum error correction (QEC) [5–9]. Through this stage one designs, possibly using many physical qubits, an encoded logical qubit which is protected against major decoherence channels and hence admits a significantly longer effective coherence time than a physical qubit. The usual approach for the realization of QEC is to use many qubits to obtain a larger Hilbert space of the qubit register [10,11]. By redundantly encoding quantum information in this Hilbert space of larger dimension one makes the QEC tractable: different error channels lead to distinguishable error syndromes. There are two major drawbacks in using multi-qubit registers. The first, fundamental, drawback is that with each added physical qubit, several new decoherence channels are added. Because of the exponential increase of the Hilbert's space dimension versus the linear increase in the number of decay channels, using enough qubits, one is able to eventually protect quantum information against decoherence. However, multiplying the number of possible errors requires measuring more error syndromes. The second, more practical, drawback is that it is still extremely challenging to build a register of more than on the order of 10 qubits where each of the qubits is required to satisfy close to the best achieved properties: these properties include the coherence time, the coupling strengths and the tunability. Indeed, building such a register is not merely only a fabrication task but rather, it requires to look for architectures such that, each individual qubit can be addressed and controlled independently from the others. One is also required to make sure that all the noise channels are well-controlled and uncorrelated for the QEC to be effective.

We have recently introduced a new paradigm for encoding and protecting quantum information in a quantum harmonic oscillator (e.g., a high-Q mode of a 3D superconducting cavity) instead of a multi-qubit register [12]. The infinite dimensional Hilbert space of such a system can be used to redundantly encode quantum information. The power of this idea lies in the fact that the dominant decoherence channel in a cavity is photon damping, and no extra decay channels are added if we increase the number of photons we insert in the cavity. Hence, only a single error syndrome needs to be measured to identify if an error has occurred or not.

In this scheme, the logical qubit is encoded in a four-component Schrödinger cat state. Repeated quantum non-demolition (QND) monitoring of a single physical observable, consisting of photon number parity, enables then the tractability of single photon jumps. We obtain therefore a first-order quantum error correcting code using only a single high-Q cavity mode (for the storage of quantum information), a single qubit (providing the non-linearity needed for controllability) and a single low-Q cavity mode (for reading out the error syndrome). As sketched in Fig. 1, this leads to a significant hardware economy for realization of a protected logical qubit.

Through the next section, I will briefly review our theoretical/experimental results [13,12,14,6] on the encoding and protecting of quantum information in such Schrödinger cat states, exploring the strong dispersive coupling regime [15] of a transmon qubit [16] and a high-Q cavity mode. These methods perform the encoding/protection tasks through entangling the cavity photon states to the qubit degrees of freedom. I will explain how this limits the performance of the QEC process through exposing quantum information to undesired noise channels. Next, in Section 3, I will review a more recent proposal on the possibility of encoding, protecting and performing protected logical operations by engineering a highly non-standard interaction of a high-Q cavity mode to the environment [17,18]. Finally in Section 4, I will discuss some possible improvements and future directions.

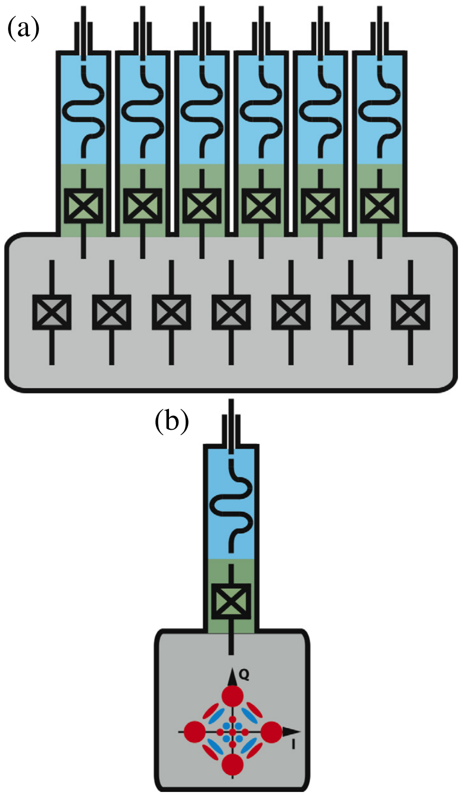

Fig. 1. (a) A protected logical qubit consisting of a register of many qubits: here, we see a possible architecture for the Steane code [11] consisting of 7 qubits requiring the measurement of 6 error syndromes. In this sketch, 7 transmon qubits in a high-Q resonator and the measurement of the 6 error syndromes is ensured through 6 additional ancillary qubits with the possibility of individual readout of the ancillary qubits via independent low-Q resonators. (b) Minimal architecture for a protected logical qubit, adapted to circuit quantum electrodynamics experiments. Quantum information is encoded in a Schrödinger cat state of a single high-Q resonator mode and a single error syndrome is measured, using a single ancillary transmon qubit and the associated readout low-Q resonator.

2. Coupling with a transmon qubit and encoding/protecting quantum information in a cavity mode

The idea consists in mapping the qubit state $c_0|0\rangle + c_1|1\rangle$ into a superposition of four coherent states of a quantum harmonic oscillator (e.g., a high-Q mode of a 3D superconducting cavity) $|\psi_\alpha^{(0)}\rangle = c_0|0\rangle_L + c_1|1\rangle_L = c_0|\mathcal{C}_\alpha^{(0\mathrm{mod}4)}\rangle + c_1|\mathcal{C}_\alpha^{(2\mathrm{mod}4)}\rangle$, where

$$
\begin{aligned}
|\mathcal{C}_\alpha^{(0\mathrm{mod}4)}\rangle &= \mathcal{N}_0(|\alpha\rangle + |-\alpha\rangle + |\mathrm{i}\alpha\rangle + |-\mathrm{i}\alpha\rangle),\\
|\mathcal{C}_\alpha^{(1\mathrm{mod}4)}\rangle &= \mathcal{N}_2(|\alpha\rangle - |-\alpha\rangle - i|\mathrm{i}\alpha\rangle + i|-\mathrm{i}\alpha\rangle),\\
|\mathcal{C}_\alpha^{(2\mathrm{mod}4)}\rangle &= \mathcal{N}_1(|\alpha\rangle + |-\alpha\rangle - |\mathrm{i}\alpha\rangle - |-\mathrm{i}\alpha\rangle),\\
|\mathcal{C}_\alpha^{(3\mathrm{mod}4)}\rangle &= \mathcal{N}_3(|\alpha\rangle - |-\alpha\rangle + i|\mathrm{i}\alpha\rangle - i|-\mathrm{i}\alpha\rangle).
\end{aligned}
$$

Here, $\mathcal{N}_0 \approx \mathcal{N}_1 \approx \mathcal{N}_2 \approx \mathcal{N}_3 \approx 1/2$ are normalization factors, and $|\alpha\rangle$ denotes a coherent state of complex amplitude α. For α large enough, $|\alpha\rangle$, $|-\alpha\rangle$, $|\mathrm{i}\alpha\rangle$ and $|-\mathrm{i}\alpha\rangle$ are quasi-orthogonal (note that for $\alpha = 2$, $|\langle\alpha|\mathrm{i}\alpha\rangle\rangle|^2 < 10^{-3}$) and therefore the normalization constants $\mathcal{N}_k$ are well-approximated by 1/2. The two states $|\mathcal{C}_\alpha^{(0\mathrm{mod}4)}\rangle$ and $|\mathcal{C}_\alpha^{(2\mathrm{mod}4)}\rangle$, playing the roles of logical 0 and 1 of the qubit, are orthogonal for any α. Note furthermore that, whenever developed in the Fock states basis, the state $|\mathcal{C}_\alpha^{(k\mathrm{mod}4)}\rangle$ is a superposition of photon number states $|n\rangle$ where $n \equiv k \pmod 4$, which explains the choice of notation $|\mathcal{C}_\alpha^{(k\mathrm{mod}4)}\rangle$.

This encoding enables the protection of quantum information against photon-loss events. In order to see this, let us also define $|\psi_\alpha^{(1)}\rangle = c_0|\mathcal{C}_\alpha^{(3\mathrm{mod}4)}\rangle + c_1|\mathcal{C}_\alpha^{(1\mathrm{mod}4)}\rangle$, $|\psi_\alpha^{(2)}\rangle = c_0|\mathcal{C}_\alpha^{(2\mathrm{mod}4)}\rangle + c_1|\mathcal{C}_\alpha^{(0\mathrm{mod}4)}\rangle$ and $|\psi_\alpha^{(3)}\rangle = c_0|\mathcal{C}_\alpha^{(1\mathrm{mod}4)}\rangle + c_1|\mathcal{C}_\alpha^{(3\mathrm{mod}4)}\rangle$. The state $|\psi_\alpha^{(n)}\rangle$ evolves after a photon-loss event to $\mathbf{a}|\psi_\alpha^{(n)}\rangle/\|\mathbf{a}|\psi_\alpha^{(n)}\rangle\| = |\psi_\alpha^{[(n+1)\mathrm{mod}4]}\rangle$, where $\mathbf{a}$ is the harmonic oscillator's annihilation operator. Furthermore, in the absence of jumps during a time interval t, $|\psi_\alpha^{(n)}\rangle$ deterministically evolves to $|\psi_{\alpha(\mathrm{e}^{-\kappa t/2})}^{(n)}\rangle$, where κ is the decay rate of the harmonic oscillator. Now, the photon number parity operator $\Pi = \exp(\mathrm{i}\pi\mathbf{a}^\dagger\mathbf{a})$ can act as a photon jump indicator. Indeed, we have $\langle\psi_\alpha^{(n)} \mid \Pi \mid \psi_\alpha^{(n)}\rangle = (-1)^n$, and therefore the measurement of the photon number parity can indicate the occurrence of a photon-loss event. While the parity measurements keep track of the photon-loss events, the deterministic relaxation of the energy, replacing α by $\alpha(\mathrm{e}^{-\kappa t/2}$, remains inevitable. To overcome this relaxation of energy, we need to intervene before the coherent states start to overlap in a significant manner to re-pump energy into the codeword. This energy repumping in the cat state requires a non-linear interaction with the cavity mode. As explained through the following paragraphs, it can be performed either through the application of a sequence of well-chosen pulses on a qubit-cavity system [12], or through parametric methods [17].

Through this section, we will explore the first direction, i.e. controlling the state of the quantum harmonic oscillator by virtue of its coupling with a physical qubit and through the application of appropriate microwave pulses.

2.1. Encoding quantum information using strong dispersive coupling

By off-resonantly coupling a transmon qubit with a high-Q cavity mode, we can achieve a strong dispersive interaction regime [15] where both the qubit and the resonator transition frequencies split into well-resolved spectral lines corresponding, respectively, to the number of excitations in the resonator and in the qubit. More precisely, the qubit admits different frequencies $\{\omega_q^n\}_{q=0}^n$ ($\omega_q^n = \omega_q^0 - n\chi_{qr}$), corresponding to the qubit frequency when the resonator is in the Fock state $|n\rangle$. Similarly, the resonator admits two different frequencies ω_r^g and ω_r^e ($\omega_r^g - \omega_r^e = \chi_{qr}$), depending on whether the qubit is in the ground or excited state. The strong dispersive coupling regime is achieved when $\chi_{qr} \gg \kappa, \gamma$, where χ_{qr} is the dispersive coupling strength, and κ and γ represent, respectively, the line-widths of the resonator and the qubit.

In such a strong dispersive coupling regime modeled by the interaction Hamiltonian $H_{\text{int}} = -\chi_{qr}|e\rangle\langle e|\mathbf{a}^\dagger\mathbf{a}$, one can apply a long-enough pulse with carrier frequency ω_q^n to rotate the qubit state only if the resonator is in the Fock state $|n\rangle$ [19]. These photon-number-selective qubit pulses could be used to entangle the resonator and the qubit. It is also possible to apply a short pulse with carrier frequency $\omega_r = (\omega_r^g + \omega_r^e)/2$ to coherently displace the cavity state independently of the qubit state: in this aim, the length of the pulse is required to be shorter than the inverse of χ_{qr} to ensure the unconditionality of the operation.

In [13,12], we proposed to employ the above photon-number-selective qubit pulses and the unconditional cavity displacements to encode quantum information in a Schrödinger cat state of the quantum harmonic oscillator. While leaving the reader to follow the steps to encode a qubit state $c_0|g\rangle + c_1|e\rangle$ into the four-component Schrödinger cat state $c_0|\mathcal{C}_\alpha^{(0\text{mod}4)}\rangle + c_1|\mathcal{C}_\alpha^{(2\text{mod}4)}\rangle$ through [12], here we briefly overview the steps to perform the simpler task of transferring this superposition to a two-component Schrödinger cat state $c_0|\alpha\rangle + c_1|-\alpha\rangle$. We start with the qubit in the state $c_0|g\rangle + c_1|e\rangle$ and the cavity in the vacuum state $|0\rangle$ and we apply a short unconditional cavity pulse of appropriate phase and amplitude to prepare the joint qubit-cavity state $c_0|g,\alpha\rangle + c_1|e,\alpha\rangle$. We wait for $T_{\text{wait}} = \pi/\chi$ and under the effect of the interaction Hamiltonian $H_{\text{int}} = -\chi_{qr}|e\rangle\langle e|\mathbf{a}^\dagger\mathbf{a}$, the qubit gets entangled to the cavity, generating the joint state $c_0|g,\alpha\rangle + c_1|e,-\alpha\rangle$. We apply a second unconditional cavity pulse of the same amplitude and phase, bringing the joint state to $c_0|g,2\alpha\rangle + c_1|e,0\rangle$. Now, we apply a long photon-number selective qubit pulse at frequency ω_q^0. Taking α to be large enough so that the overlap of the coherent state $|2\alpha\rangle$ with the vacuum state is small enough, we can rotate the qubit whenever the cavity is in the vacuum state, leaving it untouched when the cavity is in the coherent state $|2\alpha\rangle$. An appropriate choice of the amplitude of the pulse brings the entangled joint state to an un-entangled state of the form $c_0|g,2\alpha\rangle + c_1|g,0\rangle$. Now we apply a final unconditional cavity displacement pulse, bringing the cavity state to $c_0|\alpha\rangle + c_1|-\alpha\rangle$ while leaving the qubit in its ground state.

Such an encoding of quantum information on a Schrödinger cat state with up to 100 average number of photons, as well as the extension to three or four component Schrödinger cat states, were successfully implemented using a device similar to the sketch of Fig. 1(b) [14]. The performance (achieved fidelity) of the encoding protocol is however limited, mainly because of the T_1 and T_2 decay times of the transmon qubit. In order to achieve the performances required for quantum information algorithms, one needs to significantly improve the qubit's coherence time or rather consider an encoding procedure which is less susceptible to the qubit's decoherence. The approach of Section 3 should enable such a qubit-independent quantum information encoding approach.

2.2. Tracking photon-loss events by repeated quantum non-demolition photon-number parity measurements

As mentioned at the beginning of this section, once quantum information is encoded in a logical state of the form $c_0|0\rangle_L + c_1|1\rangle_L = c_0|\mathcal{C}_\alpha^{(0\text{mod}4)}\rangle + c_1|\mathcal{C}_\alpha^{(2\text{mod}4)}\rangle$, repeated quantum non-demolition photon number parity measurements will let us track the photon-loss events and therefore undo the effect of decoherence induced by such an error channel. Ideally, one could imagine a perfect measurement process, indicating the photon-number parity of the cavity state in a measurement time of τ_{M}. For such a perfect measurement, a photon-loss event during the measurement process would potentially lead to an erroneous measurement result, but a subsequent measurement should reveal the photon jump event. More precisely, the only events that might not be captured by such a perfect measurement correspond to the case where two or more photon-loss events happen during a single measurement step. For a Schrödinger cat state with a mean photon number $\bar{n}$, such an event happens with a probability given by $\frac{(\bar{n}\kappa\tau_{\text{M}})^2}{2}$, where κ is the cavity's decay rate. This means that, similar to any first order error correcting code, after tracking these single photon-loss events, we can decrease the decay rate from $\bar{n}\kappa$ to

$$\kappa_{\text{eff}} = \frac{(\bar{n}\kappa\tau_{\text{M}})^2}{2\tau_{\text{M}}} = \frac{(\bar{n}\kappa\tau_{\text{M}})}{2}\bar{n}\kappa. \tag{1}$$

While a discussion on possible sources of imperfection for such a measurement is left to the next subsection, here we briefly overview a first tentative [6] based on proposals within the framework of cavity QED with Rydberg atoms [20,21]. The idea consists in employing a single physical qubit coupled dispersively with the cavity mode (possibly the same qubit employed for the encoding process) as the meter for the parity measurement. Under the effect of the interaction Hamiltonian $H_{\text{int}} = -\chi_{qr}|e\rangle\langle e|\mathbf{a}^\dagger\mathbf{a}$, and in the rotating frame of the cavity mode, the Fock states accumulate a qubit-dependent phase of $\phi = \chi_{qr}t\mathbf{a}^\dagger\mathbf{a}$ when the qubit is in the excited state. Waiting for a time of $t = \pi/\chi_{qr}$, this realizes a unitary operation of the form $|g\rangle\langle g| \otimes I + |e\rangle\langle e| \otimes e^{i\pi\mathbf{a}^\dagger\mathbf{a}}$. Therefore, initializing the qubit in the state $(|g\rangle + |e\rangle)/\sqrt{2}$, such an interaction enables a photon number parity measurement by mapping the even parity cavity states to the final qubit state $(|g\rangle + |e\rangle)/\sqrt{2}$ and odd parity ones to the orthogonal state $(|g\rangle - |e\rangle)/\sqrt{2}$. A final $\pi/2$-pulse on the qubit and its subsequent measurement would reveal the photon number parity for the cavity state.

As illustrated through the experiment of [6], such a measurement is QND, in the sense that the parity itself is not perturbed by the measurement process. Therefore, by combining this parity tracking with the Schrödinger cat state encoding of [14], we should be able to achieve a quantum memory protected against the major decay channel of single photon loss.

Through the next subsection, we will study various limitations of this approach. These limitations mainly concern the existence of other decay channels against which the system is not protected.

2.3. Limitations due to un-protected decay channels

The above cat encoding and continuous photon-number parity measurements only protect quantum information against the photon loss channel of the storage cavity mode. Therefore, a dephasing channel for the cavity mode, caused for instance by fluctuations in its resonance frequency, would potentially lead to the loss of quantum information. However, the decay due to this dephasing channel is usually considered to be significantly slower than the dominant photon loss channel, and thus such a phenomena does not represent the preliminary limitation of the above approach. The main limitations are caused by the invasive use of the ancillary transmon qubit and the necessity in entanglement of the cavity state with the qubit. More precisely, each time the encoded cavity state gets entangled to the qubit, we expose quantum information to the loss channels of the qubit which are not protected by the scheme.

Through the above scheme, such an entanglement is clearly used for the encoding and decoding process and also the recurrent energy re-pumping step. Indeed, while the repeated photon-number parity measurements let us keep track of the photon-loss events, they do not compensate for the deterministic energy relaxation in the Schrödinger cat state. In particular, even in the case where the parity measurements do not indicate any photon-loss event, the amplitude of the coherent states encoding the Schrödinger cat state will deterministically decay with the cavity's decay rate κ. This will eventually lead to a significant overlap of the coherent components and a loss of quantum information due to the non-orthogonality of the encoding coherent states. In [12], we have proposed an energy re-pumping process similar to the encoding–decoding ones, which explores once again the strong dispersive coupling to a transmon qubit. This process, however, also utilizes the possibility of entangling the cavity state to the qubit and therefore exposes the encoded information to the un-protected decay channels of the qubit.

Another place, where such a qubit-cavity entanglement is extensively used is through the photon-number parity measurement itself. Indeed, as explained through the previous subsection, the measurement protocol in [6] is based on the fact that the Fock states accumulate a qubit-dependent phase of $\phi = \chi_{qr}t\mathbf{a}^\dagger\mathbf{a}$, which for $t = \pi/\chi_{qr}$ leads to a unitary operation $|g\rangle\langle g| \otimes I + |e\rangle\langle e| \otimes e^{i\pi\mathbf{a}^\dagger\mathbf{a}}$. While this unitary operation does not affect a given parity state (odd or even), the qubit gets entangled to such a state during the evolution. For instance, starting with an even cat state $|\mathcal{C}_\alpha^{(0,2\text{mod}4)}\rangle$, and at intermediary times $0 < s < \pi/\chi_{qr}$, the qubit-cavity state is given by the entangled state $(|g, \mathcal{C}_\alpha^{(0,2\text{mod}4)}\rangle + |e, \mathcal{C}_{\alpha e^{i\chi_{qr}s}}^{(0,2\text{mod}4)}\rangle)/\sqrt{2}$. Noting that $|\mathcal{C}_{-\alpha}^{(0,2\text{mod}4)}\rangle = |\mathcal{C}_\alpha^{(0,2\text{mod}4)}\rangle$, the qubit and cavity get disentangled after a time $t = \pi/\chi_{qr}$, but a qubit decay during the evolution can affect the measurement result and/or corrupt the encoded information. Indeed, as discussed through the supplementary material of [6], such qubit-induced errors can be divided into two categories: misinterpreting photon jumps due to qubit's pure dephasing T_ϕ (and possibly the qubit's readout inaccuracy), and dephasing of the cat state due to the relaxation of the ancilla qubit T_1. The first process is usually less important as it only leads to a loss of efficiency in the parity measurement and does not cause a complete loss of the encoded information. Indeed, this measurement inefficiency can be compensated via multiple repeated measurements and a majority vote. The second type of error is however much more detrimental. A T_1 relaxation happening at a random time s during the measurement process sends the above entangled state $(|g, \mathcal{C}_\alpha^{(0,2\text{mod}4)}\rangle + |e, \mathcal{C}_{\alpha e^{i\chi_{qr}s}}^{(0,2\text{mod}4)}\rangle)/\sqrt{2}$ to the unentangled state $|g, \mathcal{C}_{\alpha e^{i\chi_{qr}s}}^{(0,2\text{mod}4)}\rangle$ and the evolution in the phase space freezes as the qubit is in the ground state. Therefore, after the full measurement time $t = \pi/\chi_{qr}$, the cat state has acquired a random phase $e^{i\chi_{qr}s}$ in the phase space. Thus the qubit T_1 decay leads to cat states dephasing that would be impossible to recover from, without an auxiliary correction protocol.

Note that such a measurement-induced decay is not particular to the above QEC protocol and is related to the concept of fault-tolerance for the error syndrome measurements. Indeed, even for the standard multi-qubit codes, one needs to ensure a protection of the quantum system playing the role of the meter to avoid the propagation of its errors to the encoded information. We will get back to this through Section 4. Here, we note that, even with a non-fault-tolerant parity measurement, it is possible to achieve an improvement of the coherence time for a quantum memory. Indeed, as discussed through the supplementary material of [6], considering perfect measurements (in particular qubit $T_\phi = \infty$) of duration τ_M, one needs to consider an appropriate waiting time τ_W between two subsequent parity measurements, to achieve an effective decay rate for the error-correcting code of

$$\kappa_{\text{eff}} = \left[\frac{(\bar{n}\kappa)^2(\tau_M + \tau_W)^2}{2} + \epsilon_{T_1}\right]\frac{1}{\tau_M + \tau_W},$$

where $\epsilon_{T_1} \sim \tau_M/T_1$ denotes the loss of cat fidelity due to T_1 errors during a measurement step. A simple calculation illustrates that the optimal choice for the waiting time τ_W is given by

$$\tau_W = \frac{\sqrt{2\epsilon_{T_1}}}{\bar{n}\kappa} - \tau_M,$$

leading to an optimal decay rate of

$$\kappa_{\text{eff}} = (\bar{n}\kappa)\sqrt{2\epsilon_{T_1}}. \tag{2}$$

Therefore, we should be able to observe an improvement of the coherence time with respect to the non-corrected cat state, as soon as $\epsilon_{T_1} < 1/2$ (this is already the case with the experimental parameters in [6]). Note, however, that this improvement in the coherence time is most probably much less significant than the case of a fault-tolerant parity measurement, leading to an effective rate given by (1).

3. Confining the dynamics to a quantum manifold of protected states by engineered dissipation

In the sequel to the above proposals for encoding and protecting cat qubits, we recently proposed an alternative approach that does not require an invasive use of an ancillary transmon qubit [17]. In this approach, Josephson circuits are merely used to provide the non-linearity required for a non-classical manipulation of the cavity state. More precisely, we avoid the entanglement of the cavity state (where the quantum information is stored) to the qubit degree of freedom.

The idea consists in engineering the coupling of the quantum harmonic oscillator with a bath in such a way that it significantly enhances the probability of exchanging simultaneously multiple photons (here four photons) with the environment. In other words, in a fixed time interval, the probability of exchanging four photons with the bath should significantly dominate the probability of exchanging any other number of photons (and in particular a single photon as is the case of a regular driven damped harmonic oscillator). By confining the dynamics to a quantum manifold spanned by a number of coherent states (here four of them) whose superpositions are used to encode quantum information, such a dissipative process will protect the encoded state against photon dephasing decoherence channel. Together with the continuous QND monitoring of the photon number parity operator, this also ensures the protection against the dominant single-photon decay channel. In order to design circuits that enable such a non-classical interaction between a quantum harmonic oscillator and its bath, we exploit the ideas behind recently developed quantum multi-wave mixing parametric devices [22–24].

3.1. Multi-photon driven dissipative process for stabilization of a quantum manifold

In a recent experiment [18], a two-photon driven dissipative process was realized to confine the dynamics to the manifold of quantum states spanned by two coherent states $|\pm\alpha\rangle$ [25]. The setup, similar to the sketch of Fig. 1(b), was comprised of two microwave cavities coupled through a Josephson junction. One of the cavity modes (with resonance frequency ω_s) is high-Q and plays the role of the storage mode and the other one (with resonance frequency ω_r) strongly dissipates to a transmission line and mediates a non-standard interaction of the high-Q mode to the environment. Indeed, by exploring the four-wave mixing property of a Josephson junction and off-resonantly pumping the low-Q resonator at frequency $\omega_p = 2\omega_s - \omega_r$, one engineers an effective interaction Hamiltonian well approximated by

$$\frac{\mathbf{H}_{\text{int}}}{\hbar} = g_{rs}(\mathbf{a}_r^2\mathbf{a}_s^\dagger + \mathbf{a}_r^{\dagger 2}\mathbf{a}_s) + \chi_{rs}(\mathbf{a}_r^\dagger\mathbf{a}_r)(\mathbf{a}_s^\dagger\mathbf{a}_s) + \frac{\chi_{rr}}{2}(\mathbf{a}_r^\dagger\mathbf{a}_r)^2 + \frac{\chi_{ss}}{2}(\mathbf{a}_s^\dagger\mathbf{a}_s)^2,$$

where $\mathbf{a}_r$ and $\mathbf{a}_s$ are respectively the field mode operators for the low-Q and high-Q cavity modes. In this Hamiltonian, the first term, of strength g_{rs}, provides the non-standard interaction between the high-Q mode and the low-Q one. The second term, of strength χ_{rs}, is the cross-Kerr between the two cavity modes and the last two ones are the self-Kerr (or anharmonicities) of the two modes, all induced by their coupling with the nonlinear Josephson junction [26].

The coupling strength g_{rs} is given by (ϵ_p being the pump amplitude)

$$g_{rs} = \frac{\epsilon_p}{\omega_p - \omega_r}\frac{\chi_{rs}}{2},$$

and can therefore be tuned via the pump power. It should however be noted that, as illustrated in the experiment of [18], increasing infinitely the pump power will eventually lead to undesired interactions reducing the efficiency of the manifold confinement scheme. Indeed, an important direction for the improvement of this experiment is to design new circuits enabling us to achieve higher coupling strengths g_{rs} while the induced undesired interactions remain limited.

This interaction Hamiltonian, together with the strong dissipation of the mode $\mathbf{a}_r$, leads, after an adiabatic elimination of the mode $\mathbf{a}_r$ (see [27] and supplementary material of [18]), to a strong two-photon dissipation channel for the mode $\mathbf{a}_s$ modeled by the Lindblad operator $\mathbf{a}_s^2$. Additionally, driving the low-Q mode $\mathbf{a}_r$ at its resonance frequency, the above non-standard interaction translates into a two-photon drive on the storage mode. Consequently, one achieves an effective master equation of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = \left[\epsilon_{2\mathrm{ph}}\mathbf{a}_s^{\dagger 2} + \epsilon_{2\mathrm{ph}}^*\mathbf{a}_s^2, \rho\right] + \kappa_{2\mathrm{ph}}\mathcal{D}\left[\mathbf{a}_s^2\right]\rho,$$

where $\mathcal{D}[L]\rho = L\rho L^\dagger - 1/2L^\dagger L\rho - 1/2\rho L^\dagger L$. Considering a dissipation rate κ_r exceeding significantly the interaction strength g_{rs}, the parameters of this effective model are given by

$$\epsilon_{2\mathrm{ph}} = \frac{2\epsilon_r g_{2\mathrm{ph}}}{\kappa_r}, \qquad \kappa_{2\mathrm{ph}} = \frac{4g_{2\mathrm{ph}}^2}{\kappa_r},$$

where ϵ_r denotes the amplitude of the drive at resonance with the low-Q resonator. Despite the above-mentioned limitations on the strength of $g_{2\mathrm{ph}}$, it was shown in [18] that with the state-of-the-art coherence times for high-Q resonators, it is possible to make the two-photon decay rate $\kappa_{2\mathrm{ph}}$ exceed the natural single-photon dissipation κ_s of the storage mode. This is the regime required to confine and manipulate the dynamics in the degenerate manifold spanned by two coherent states $|\pm\alpha\rangle$ where $\alpha = \sqrt{\epsilon_r/g_{2\mathrm{ph}}}$. In particular, it was illustrated that starting with the vacuum state, such a two-photon process drives the system towards a transient Schrödinger cat state before it decays (due to the single-photon decay) to a classical mixture of $|\alpha\rangle$ and $|-\alpha\rangle$. In order to observe a high-fidelity Schrödinger cat state before its decay, one needs to ensure a larger separation between the time-scales $\kappa_{2\mathrm{ph}}$ and κ_s. For this to happen, one needs to either improve the coherence time of the storage cavity significantly or as explained earlier, to design new circuits that enable a significant increase in $g_{2\mathrm{ph}}$ and therefore in $\kappa_{2\mathrm{ph}}$.

Note however that, for this two-photon process, even if the separation of the time-scales is significantly improved, no protection against the single-photon-loss events can be considered. Indeed, under these confinement conditions, the eventual single photon jumps translate to phase flip errors in the basis $\{|\alpha\rangle, |-\alpha\rangle\}$: the application of the annihilation operator $\mathbf{a}_s$ sends $\{|\alpha\rangle, |-\alpha\rangle\}$ to $\{|\alpha\rangle, -|-\alpha\rangle\}$. In order to keep track of this dominant single-photon loss errors, we need to go back to an encoding in a four-fold degenerate manifold. As proposed in [17], this could be done by engineering a four-photon-driven dissipative process. In principle, if we are able to engineer an interaction Hamiltonian of the form $g_{4\mathrm{ph}}(\mathbf{a}_s^4\mathbf{a}_r^\dagger + \mathbf{a}_s^{\dagger 4}\mathbf{a}_r)$, the same type of adiabatic elimination as for the two-photon case leads to an effective master equation of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = \left[\epsilon_{4\mathrm{ph}}\mathbf{a}_s^{\dagger 4} + \epsilon_{4\mathrm{ph}}^*\mathbf{a}_s^4, \rho\right] + \kappa_{4\mathrm{ph}}\mathcal{D}\left[\mathbf{a}_s^4\right]\rho,$$

which stabilizes the manifold spanned by $\{|\pm\alpha\rangle, |\pm\mathrm{i}\alpha\rangle\}$ with $\alpha = \left(2\epsilon_{4\mathrm{ph}}/\kappa_{4\mathrm{ph}}\right)^{1/4}$. Then, in a similar fashion to that in the previous section, by encoding a qubit in the basis $\{|\mathcal{C}_\alpha^{0\mathrm{mod}4}\rangle, |\mathcal{C}_\alpha^{2\mathrm{mod}4}\rangle\}$, it is possible to keep track of photon-loss events by continuous QND monitoring of photon-number parity. Furthermore, by stabilizing the above manifold, the four-photon process will also compensate for the deterministic energy relaxation.

The main question would therefore be an efficient engineering of the interaction Hamiltonian $g_{4\mathrm{ph}}(\mathbf{a}_s^4\mathbf{a}_r^\dagger + \mathbf{a}_s^{\dagger 4}\mathbf{a}_r)$. One might think of extending the above two-photon process by applying a pump drive of frequency $\omega_\mathrm{p} = 4\omega_s - \omega_r$ and benefiting from the six-wave mixing property of the Josephson junctions Hamiltonian. While such a six-wave mixing has never been experimentally implemented yet, it turns out that it is impossible to make the desired interaction strong compared to all other parasitic interactions such as cross and self Kerr terms. This will significantly limit the efficiency of such a 4-photon process. In [17], we suggested that a design similar to that of the Josephson parametric converter [23,24] consisting of a ring of Josephson junctions shunted by a cross of 4 linear inductances can overcome such limitations in the strength of the 4-photon conversion. Such a device would however also depend on a six-wave mixing, and its fabrication and the analysis and optimization of its properties, such as tunability and bandwidth, will require a significant theoretical/experimental investigation.

3.2. Towards universal quantum computation: quantum Zeno dynamics

As proposed in [17], the multi-photon-driven dissipative processes can be combined with additional (weaker) Hamiltonians acting only on the Hilbert space of the storage mode to perform various logical gates required for universal quantum computation. Three kinds of gates on a logical qubit were presented in [17].

– *Arbitrary rotations around the X axis.* The idea consists in applying a squeezing Hamiltonian $\epsilon_X\mathbf{a}_s^{\dagger 2} + \epsilon_X^*\mathbf{a}_s^2$ in the presence of the driven dissipative process. Note that, as this Hamiltonian can only imply exchanges of photons in pairs with the storage mode, the photon-number parity remains intact. Also, in the case where $\epsilon_X \ll \kappa_{4\mathrm{ph}}$, the four-photon process keeps the system in the vicinity of the manifold spanned by $\{|\pm\alpha\rangle, |\pm i\alpha\rangle\}$. Therefore, starting with a superposition of the qubit states $|\mathcal{C}_\alpha^{0\text{mod}4}\rangle$ and $|\mathcal{C}_\alpha^{2\text{mod}4}\rangle$, the above squeezing Hamiltonian should necessarily lead to an operation inside this two-dimensional space. Indeed, applying a Zeno-type argument, the first-order (with respect to $\epsilon_X/\kappa_{4\text{ph}}$) effect of the above Hamiltonian can be represented by its projection on this space:

$$\Pi_{|\mathcal{C}_\alpha^{0,2\text{mod}4}\rangle}\left(\epsilon_X \mathbf{a}_s^{\dagger 2} + \epsilon_X^* \mathbf{a}_s^2\right)\Pi_{|\mathcal{C}_\alpha^{0,2\text{mod}4}\rangle} = (\epsilon_X \alpha^{*2} + \epsilon_X^* \alpha^2)\left(|\mathcal{C}_\alpha^{0\text{mod}4}\rangle\langle\mathcal{C}_\alpha^{2\text{mod}4}| + |\mathcal{C}_\alpha^{2\text{mod}4}\rangle\langle\mathcal{C}_\alpha^{0\text{mod}4}|\right).$$

This corresponds to the σ_x Pauli matrix in the logical qubits basis. Therefore, selecting the argument of ϵ_X such that $(\epsilon_X \alpha^{*2} + \epsilon_X^* \alpha^2) \neq 0$ (the optimal choice is ϵ_X in quadrature with respect to α^2 or equivalently $\epsilon_{4\text{ph}}$), the above squeezing Hamiltonian induces a Rabi oscillation around the X axis of the effective qubit. It has been illustrated, via numerical simulations, that a modest separation of time scales of $|\kappa_{4\text{ph}}/\epsilon_X| = 20$ leads to gate fidelity in excess of 99.9% [17].

An important property of this gate is its tolerance to the photon-loss events during the operation. Indeed, an eventual single-photon-loss event would change the qubit basis to $|\mathcal{C}_\alpha^{3,1\text{mod}4}\rangle$, while the squeezing Hamiltonian will continue to induce a Rabi oscillation in this new basis. Therefore, a photon-number parity measurement after the operation will reveal this photon jump event and no quantum information is lost. This could be interpreted as fault-tolerance of the gate with respect to the decoherence of the cavity.

– *Two-qubit entangling gates.* Assume two effective qubits achieved by four-photon-driven dissipative processes applied on two cavity modes $\mathbf{a}_1$ and $\mathbf{a}_2$. Similarly to the previous gate, one can apply an interaction Hamiltonian of the form $\epsilon_{XX}\left(\mathbf{a}_1^2\mathbf{a}_2^{\dagger 2} + \mathbf{a}_2^2\mathbf{a}_1^{\dagger 2}\right)$ to produce an effective Hamiltonian of the form

$$2|\alpha|^4\epsilon_{XX}\left(|\mathcal{C}_\alpha^{0\text{mod}4}\rangle\langle\mathcal{C}_\alpha^{2\text{mod}4}| + |\mathcal{C}_\alpha^{2\text{mod}4}\rangle\langle\mathcal{C}_\alpha^{0\text{mod}4}|\right)^{\otimes 2}$$

which is equivalent to the entangling Hamiltonian $2|\alpha|^4\epsilon_{XX}\sigma_x^1 \otimes \sigma_x^2$ for the logical qubits. Once again, a modest separation of time scales of $|\kappa_{4\text{ph}}/\epsilon_{XX}| = 20$ leads to gate fidelity in excess of 99.5% [17]. This gate is also tolerant to single-photon-loss events happening on one or both of the cavity modes. Indeed, a parallel photon-number parity measurement on the two modes after the operation will reveal such jump events and no information will be lost.

– *$\pi/2$-rotation around the Z axis.*

Such a single qubit gate, which is needed to achieve a complete set of universal gates, is perhaps the hardest to be performed in a fault-tolerant way. This is due to the fact that, by rendering each of the coherent states $|\pm\alpha\rangle, |\pm i\alpha\rangle$ stable points, the four-photon process protects the system against the transition between the two states $|+_x\rangle = \mathcal{N}(|\alpha\rangle + |-\alpha\rangle)$ and $|-_x\rangle = \mathcal{N}(|i\alpha\rangle + |-i\alpha\rangle)$. In [17], we proposed an approach that consists in turning off the four-photon process and make use of the induced Kerr effect on the storage Hamiltonian. Indeed, applying an effective Hamiltonian of the form $-\chi_{\text{Kerr}}\mathbf{a}_s^{\dagger 2}\mathbf{a}_s^2$ for a time duration of $\pi/8\chi_{\text{Kerr}}$, one achieves an effective $\pi/2$-rotation around the Z axis of the cat qubit [28]. It was discussed in [17] that, as soon as the photon-number parity measurements are performed at a much faster rate than the Kerr effect, this gate is fault-tolerant with respect to single-photon loss channel. Indeed, noting the commutation relation

$$\mathbf{a}_s e^{it\chi_{\text{Kerr}}\mathbf{a}_s^{\dagger 2}\mathbf{a}_s^2} = e^{2it\chi_{\text{Kerr}}\mathbf{a}_s^\dagger\mathbf{a}_s} e^{it\chi_{\text{Kerr}}\mathbf{a}_s^{\dagger 2}\mathbf{a}_s^2}\mathbf{a}_s,$$

a single-photon-loss event during the Kerr effect would lead to an extra rotation in the phase space by an angle of $2t\chi_{\text{Kerr}}$, where t is the random time at which the loss event happened. However, by continuously monitoring the photon-number parity (feasible as the parity observable commutes with the Kerr Hamiltonian), we can keep track of the times at which these loss events happen and therefore also the accumulated phase (with a resolution given by the parity measurement duration divided by the gate duration). This fault-tolerance however requires the operation to be much slower than the measurement duration. Also note that assuming the cavity dephasing rate to be much weaker than the Kerr strength, turning on the four-photon process after the gate would approximately correct for the phase noise accumulated throughout the gate duration.

In [29], an alternative approach was proposed, which avoids the necessity of turning off the four-photon process during the operation. The idea consists in reducing adiabatically towards zero the strength of the drive at resonance with the low-Q cavity (this reduction should be slow with respect to the four-photon-decay rate) so that the four components of the Schrödinger cat state collide with each other near the zero drive strength. Next, one re-pumps the Schrödinger cat state by increasing the strength of the same drive, changing however the phase of the drive (shifting it by ϕ). It was shown in [29] that this is equivalent to a rotation by an angle of 2ϕ around the Z axis of the qubit. While this gate benefits from the advantage of not requiring the four-photon decay to be turned off, it is not fault-tolerant with respect to the single-photon decay channel. Indeed, during the time the components of the Schrödinger cat state overlap, quantum information is extremely sensitive to this decay channel, and a single photon loss can erase the superposition. In order to see this, we note that at zero drive strength the cat states $|\mathcal{C}_\alpha^{0\text{mod}4}\rangle$ and $|\mathcal{C}_\alpha^{2\text{mod}4}\rangle$ respectively converge to the two Fock states $|0\rangle$ and $|2\rangle$. A single photon-loss event then would project any superposition of these Fock states onto the Fock state $|1\rangle$, erasing the encoded information.

A third direction, avoiding the necessity of turning off the four-photon process, while ensuring the fault-tolerance with respect to the major decay channels, would be to engineer an effective Hamiltonian of the form $\chi_{\text{mod}4}\cos(\pi\mathbf{a}_s^\dagger\mathbf{a}_s/2)$. Through a Zeno type dynamics, this Hamiltonian would directly lead to a phase accumulation of rate $2\chi_{\text{mod4}}$ between the two states $|\mathcal{C}_\alpha^{0\text{mod}4}\rangle$ and $|\mathcal{C}_\alpha^{2\text{mod}4}\rangle$:

$$\chi_{\text{mod4}}\Pi_{\mathcal{C}_\alpha^{0,2\text{mod}4}}\cos(\pi\mathbf{a}_s^\dagger\mathbf{a}_s/2)\Pi_{\mathcal{C}_\alpha^{0,2\text{mod}4}}=\chi_{\text{mod4}}(|\mathcal{C}_\alpha^{0\text{mod}4}\rangle\langle\mathcal{C}_\alpha^{0\text{mod}4}|-|\mathcal{C}_\alpha^{2\text{mod}4}\rangle\langle\mathcal{C}_\alpha^{2\text{mod}4}|).$$

Engineering a quantum circuit leading to such an effective Hamiltonian is a current research topic and remains to be done.

Before finishing this section, let us mention that the design and the implementation of quantum circuits that provide the above required Hamiltonians in a controlled manner (with the possibility of switching them on and off) lead to a significant theoretical/experimental project on parametric multi-wave mixing methods.

4. Future directions and conclusion

In this last section, we briefly introduce a few important directions for further improvements.

One main topic in this regard concerns the fault-tolerance of the parity measurement protocol. Indeed, a QND photon-number parity measurement, which is less sensitive to the T_1 decay of the qubit, playing the role of the meter, would greatly enhance the expected performance of the QEC protocol in both sections 2 and 3. Indeed, while dealing with a quantum memory, as illustrated in Subsection 2.3, this would improve the expected effective decay rate from $\bar{n}\kappa\sqrt{2\tau_M/T_1}$ (for the current non- fault-tolerant protocol) to $\bar{n}\kappa(\bar{n}\kappa\tau_M/2)$ (for the fault-tolerant case). For the parameters of the experiment [6], this improvement is a factor of about 20–30, i.e. under a fault-tolerant parity measurement the error-corrected memory would admit a lifetime that is 20–30 times longer than that of the non-fault-tolerant protocol. With the continuous increase of the coherence times in these circuits, this improvement will be even more drastic in the future. Indeed, while the effective decay rate in the case of the non- fault-tolerant parity measurement will decrease only as the square-root of the inverse of the coherence time of the qubit, the effective decay rate for the fault-tolerant case will decrease linearly with the inverse of the coherence time of the storage cavity. Note moreover that the recent improvement in coherence times of microwave resonators with respect to superconducting qubits further intensifies this difference between the performance of fault-tolerant and non-fault-tolerant schemes. In practice, one possibility to ensure such a fault-tolerance of the error syndrome measurement is to actively protect the meter (here the transmon qubit) against the problematic decay channel (here T_1 decay). Remembering that phase-flip errors of the transmon qubit do not imply an erasure of quantum information, and potentially lead to erroneous syndrome measurements that could be corrected for by subsequent measurements (see Subsection 2.3), one only needs to take care of bit-flip errors. Thus replacing the transmon qubit by a three-qubit bit-flip code [10] is, in principle, enough to protect the system against the propagation of the errors of the meter. Finally, noting that one does not even need to fully correct or even detect such bit-flip errors, but merely replace them with phase-flip errors, we should be able to further simplify this protection protocol. Such a photon-number parity measurement, together with an experimental realization of the four-photon driven dissipative process and the associated logical gates, based on Zeno-type dynamics, provide a full set of fault-tolerant (with respect to the decoherence channels of the involved quantum systems) gadgets for universal quantum computation.

Another important direction concerns the extension to higher-order error correcting codes. The above protocol, based on repeated monitoring of photon-number parity observable, enables the protection against single photon losses during a measurement step. Indeed, two quantum jumps during a measurement step lead to an effective bit-flip error in the code space: it sends the state $|\mathcal{C}_\alpha^{0\text{mod}4}\rangle$ to $|\mathcal{C}_\alpha^{2\text{mod}4}\rangle$ and vice versa. By remaining in the code space, such an error is neither tractable by parity measurements nor by any other type of measurements. One therefore needs to think of another encoding of quantum information to correct for multiple jumps. Throughout the supplementary material of [12], we provided some preliminary ideas based on encoding quantum information in a superposition of $2n$ coherent states to achieve an nth order code. This consists in encoding the information in two logical qubit states of the form

$$|0_L\rangle=|\mathcal{C}_\alpha^{0\text{mod}(2n)}\rangle=\mathcal{N}(\sum_{k=0}^{2n-1}|e^{ik\pi/n}\alpha\rangle),$$
$$|1_L\rangle=|\mathcal{C}_\alpha^{n\text{mod}(2n)}\rangle=\mathcal{N}(\sum_{k=0}^{2n-1}|(-1)^k e^{ik\pi/n}\alpha\rangle).$$

A repeated monitoring of the observable corresponding to the number of photons modulo n would then enable us to track up to $(n-1)$ quantum jumps during a single measurement step. Such an encoding, however, comes at the expense of increasing the required average number of photons (to avoid the overlap of the coherent components). Indeed, it turns out that when considering such an encoding in Schrödinger cat states with much larger number of components, the protection stops improving the coherence time and this coherence time will even start to decrease for very large values of n. Here a possible direction to bring these ideas closer to a real nth order correction, leading perhaps to a threshold theorem, would be to consider a few coupled cavity modes, instead of a single one, to avoid the requirement for encoding in Schrödinger cat states of very large size.

I have overviewed a series of recent theoretical proposals to achieve hardware-efficient quantum computation with protected qubits encoded in Schrödinger cat states of a single superconducting cavity. The preliminary, but very significant, experiments in this regard have illustrated the great promise that such an approach provides for a fast development of the field of quantum information processing within this framework. These proposals can be extended and improved in various directions. Design and implementation of fault-tolerant photon-number parity measurements, of a device enabling the four-photon driven dissipative process, and the required Hamiltonians for the fault-tolerant gates are some of these directions, which require a significant investigation of both theoretical and experimental aspects. Furthermore, extensions of the protocols towards more efficient fault-tolerant gates (in particular rotations around the Z axis of the logical qubit) and towards higher-order codes require an intensive research on the theoretical side.

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