Energy and radiosciences / Énergie et radiosciences

Fundamental energetic limits of radio communication systems ☆

Limites énergétiques fondamentales des radiocommunications

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A B S T R A C T

The evaluation of the energy consumption of a radiocommunication requires to analyse the life cycle of the elements used. However, this analysis does not specify the energetic limits. Theoretical approaches allow one to draw these limits, which are known in multiple cases of information transmission. However, the answers are not always satisfactory, in particular in the case of time-varying channels. After a brief presentation of the notion of energetic limits of a radiocommunication, and beginning with a global approach, we show that, contrary to the published results, the energetic limits always differ from zero if the physical constraints are correctly expressed.

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R É S U M É

L'évaluation de la consommation d'énergie d'une radiocommunication nécessite l'analyse du cycle de vie des éléments utilisés. Cependant, cette analyse ne décrit pas précisément de limite énergétique. Les approches théoriques permettent d'évaluer ces limites qui sont connues dans plusieurs cas de transmission de l'information.

Cependant, les réponses ne sont pas toujours satisfaisantes, particulièrement dans le cas de canaux variants dans le temps. Après une rapide présentation des notions d'énergie limite d'une radiocommunication, et en commençant par une approche globale du problème, nous montrons que, contrairement aux résultats publiés, les limites énergétiques sont toujours différentes de zéro si les contraintes physiques sont correctement exprimées.

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1. Introduction

The energetic evaluation of a radiocommunication system, and its limits, can be done following two main approaches: a pragmatic approach based on practical evaluation, and a theoretical approach [1]. Both approaches can provide inputs on

the digital energy balance that remains to assess, and inputs to reach efficient green radio services and systems, top issues of the topic *energy in radiosciences*.

The pragmatic approach is technological in the sense that it is related to a practical realization and consumption. It involves energy consumption evaluation, which is a complex problem when it understands total consumption. This means evaluating the grey energy associated with the products or services using the life-cycle assessment, also known as life-cycle analysis (LCA) [2]. The conventional life-cycle consists of five stages, including all processes implemented during the life of communication devices

(1) raw resource extraction,
(2) production, transformation, assembly,
(3) transportation, distribution, logistics,
(4) use, operation maintenance, reuse,
(5) disposal, recycling.

The measure of the energy flow at each stage is then useful to quantify energy consumption. Combined with Moore's law, or enhanced and derived laws [3], future consumptions, improvements or trends can be predicted. However, these predictions fail to instigate the fundamental consumption lower bound at the present time, because they strongly depend on technological processes and do not take into account possible technological divide, without considering the controversial technological singularity [4]. If the successive technologies are designed to outperform the previous ones, the deduced or expected consumption limits remain strongly dependent on the processes [5]. Furthermore, the stages of the life cycle can change, especially in the case of technological divide leading to new production processes, for example.

This paper deals with the second approach, providing the fundamental energetic limits of a radio communication system. In this context, fundamental means that the limits are related to the laws of physic and do not depend on realization. Even if the stages in LCA would remain valid in the future, which is questionable, the question of the fundamental limits of the energy flow at each stage is still open. For example, what are the energetic limits of raw extraction, of device distribution, or of operational maintenance? The theoretical approach also fails to evaluate the consumption limits using the LCA model. We then reduce our first ambitions to a fundamental limit analysis based on a minimalist model. We particularly focus on the transmitted power, taking different channel information levels into account. Our analysis is based on the channel capacity measure. The channel capacity has been determined with different levels of channel state information (CSI) at the transmitter and receiver sides in the asymptotic [6] or in the non-asymptotic regime [7], leading to energetic evaluation in some cases [8,9]. With linear time-varying (LTV) channels, channel estimates are typically causal in practice and an optimal coding strategy can be derived to the upper and lower bounds of the channel capacity [10]. The feedback path between the receiver and the transmitter, which allows adaptive techniques to increase capacity, is required when channel estimation is assumed at the receiver side. A feedback path can also be used in a Gaussian channel as a control channel to improve the efficient use of the resources, and especially the energy consumed by the system [11]. Low-power regime, widely used in this paper to derive the limits, has also been studied in the absence of CSI at the transmitter side [12,13].

Whereas the capacity is addressed in the case of LTV channel with CSI at both the transmitter and receiver sides, the minimum energy per bit is not yet addressed in this case. We obtain that the conventional channel models used in communication system studies lead to a wrong zero energetic limit. We show how the optimization yields a wrong result and an impossible communication. By introducing the right physical constraint on the received power, we obtain a non-zero energetic limit. The consequence is that the value of the energetic limit already known in many cases [14] is now widely extended. The drawback is that this constraint leads to a strong modification of the channel models.

The paper is organized as follows. In Section 2, we define and discuss the communication model without any technological consideration. With the defined model, we recall some known limits in Section 3. The rest of the paper is dedicated to the consumption limit related to information transmission through a channel. The limit, firstly given in the case of a noisy channel in Section 3, is derived in the case of the LTV channel in Section 4 and for a general probability density function (p.d.f.) of the link budget, which can capture all channel models. Two cases are considered: without CSI at the transmitter side and with CSI at the transmitter side. The CSI is considered to be known at the receiver side in both cases. We show in Section 5, and it is the main contribution of this paper, that the conventional channel models lead to wrong energetic limit with CSI at the transmitter side. Using correct physical assumptions, we derive the energetic limit in Section 6 for LTV channels and extend the result in the case of independent and parallel channels in Section 7. In Section 8, we come back to pragmatic aspects by comparing some system consumptions to the obtained fundamental limits. Finally, Section 9 concludes the paper with an open discussion and problem.

2. Communication model

As the LCA model fails to provide fundamental energetic limits, one needs models allowing the derivative of these limits. However, we should still bear in mind that the chosen model fits into a conceptual framework that can become outdated or can compete with other conceptual frameworks. It is then essential to clearly identify the assumptions. If we focus on the objective of a radiocommunication, that is the transmission of information through a radio wireless channel, the minimalist or reasonable assumptions we use are as follows.
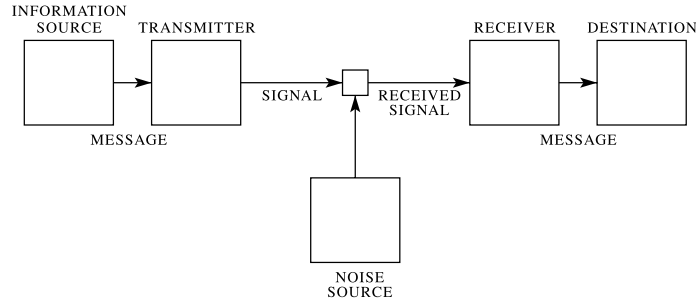
Fig. 1. Shannon–Weaver model of communication, in [17].

Assumption 2.1. A radiocommunication needs to send the information.

Assumption 2.2. A radiocommunication needs to adapt to the channel, to transform and receive the information.

The first assumption is a translation of the goal of a communication that is to make an unknown information available at a receiving position different from the transmitting one. With the second assumption, we assume that the information does not come from an electromagnetic source and needs transformations to be propagated through the radiocommunication channel.

With these assumptions, the Shannon–Weaver model recalled in Fig. 1 is of practical interest. The communicated message coming from an information source needs a transmitter, a channel of transmission, and a receiver. The questions of the energetic limits can be formulated as follows: what is the minimum energy to process the information at the receiver and transmitter sides, and what is the minimum energy to send the information? By combining processing at the transmitter and receiver sides, one obtains the energy E_{BB} related to the baseband signal transformations, or computing, which is distinguished from the energy E_{RF} related to the radiofrequency transmission. In this model, nothing is said about how to realize the transformations, the computation, and the transmission. There is no technological and no LCA considerations. We have then reduced our first ambitions, with the holistic approach of energy consumption based on LCA, to a fundamental limit analysis based on a minimalist model. The model that we choose is completely different from the information–friction one developed in [15]. The information–friction model considers frictional losses associated with moving information bits. This is another modelling not considered in our case. However, and following the ideas developed in [16], it is not clearly established that the energy required in a message has to be dissipated.

By considering the computation and transmission parts, the resulting power or energy consumption model is then an additive one. The total energy is simply the sum of the energy consumed at each part of the model, which is the energy dedicated to the information or signal processing and the energy transmitted with the electromagnetic wave.

Note that this paper focuses on radiocommunication, but the result also applies to wireline communication and it is not only restricted to electromagnetic waves.

3. Known limits

Two power-information expressions allow us to evaluate the power or the consumed energy to process and to send one bit of information.

The first one is the Shannon–von Neumann–Landauer expression. It gives the minimum energy needed to process one bit in an irreversible operation [18]. Let E_{BB} be the energy needed to process one bit. According to the Landauer's principle, its minimum value ε_{BB} is [19]

$$\varepsilon_{BB} = \min E_{BB} = kT \ln 2 \tag{1}$$

where k is the Boltzmann constant and T the temperature (in Kelvins). At room temperature, this energy is around $3 \cdot 10^{-21}$ J/bit.

The second expression comes from the Shannon–Hartley theorem [17], which sets the capacity $C(P)$ in bit per second of a channel disturbed by a Gaussian noise

$$C(P) = \Delta w \log_2 \left(1 + \frac{P}{\Delta w \, k T} \right) \tag{2}$$

where P is the received power, Δw is the channel bandwidth and $\Delta w \, k T$ is the noise power. In the information point of view, the capacity is reached with codewords of infinite size and with an error probability as low as desired [20]. If all the transmitted power is received, which means that the energy is conserved during the transmission, the needed energy to send one bit is

$$E_{RF} = \frac{P}{C(P)} \tag{3}$$

Since it is a concave and non-decreasing function with respect to P, the minimum value is reached when P goes down to 0. Since it is also a convex and non-increasing function with respect to Δw, the minimum value is reached when Δw goes up to infinity. Then the former assertion leads to low spectral efficiency and to a transmission at low bit rate in the asymptotic regime, whereas the latter leads to high bit rate transmission, as mentioned in [21]. The minimum value ε_{RF} of transmitted energy, or energy needed to send one bit, is given by the well known result that dates back to [22]

$$\varepsilon_{RF} = \min E_{RF} = \lim_{\frac{P}{\Delta w} \to 0} E_{RF} = kT\ln 2 \tag{4}$$

where the convergence rate is in $\mathcal{O}\left(\frac{P}{\Delta w}\right)$. Surprisingly, the same minimum energy is needed to process or to send one bit. This is because the noise considered in the later case is the conventional one-side additive white Gaussian noise generally considered in communication system, with thermal noise density kT.

The minimum energy per bit ε_{RF} in (4) is a lower bound obtained in the case of asymptotic continuous transmission. The number of transmitted bits has to be infinite. With packet or finite blocklength transmissions widely encountered in communication systems, the number of bits is finite, and the lower bound can not be reached [8]. The result in (4) needs the following assumption.

Assumption 3.1. The asymptotic regime is assumed with an infinite number of transmitted bits.

However, a blocklength transmission of 10^6 bits leads to a minimum energy per bit only 0.02 dB higher than the asymptotic limit ε_{RF} [8].

The capacity $C(P)$ in (2) is expressed in bits per second, its inverse is then the bit duration time. Let Δt be this duration time, the energy per bit is

$$E_{RF} = P\Delta t = \left(2^{\frac{1}{\Delta t \Delta w}} - 1\right)\Delta t \Delta w kT \tag{5}$$

and

$$\varepsilon_{RF} = \lim_{\Delta w \Delta t \to \infty} E_{RF} = kT\ln 2 \tag{6}$$

with a convergence rate in $\mathcal{O}\left(\frac{1}{\Delta w \Delta t}\right)$. The minimum energy is reached in the wideband regime, $\Delta w \to \infty$, as well as in the widetime regime [23], $\Delta t \to \infty$. The widetime regime leads to low bit rate transmission as the low-power regime does. The low duty cycle on–off signaling is an example of widetime regime. Note that the minimum energy in (4) and (6) is not reached if the variables P, Δw and Δt are dependant. For example, if $\Delta t = \Delta w^{-1}$, then the energy E_{RF} in (5) does not depend on the two parameters, and the minimum energetic limit is not ε_{RF} in (6).

Both minimum energies in (1) and (4), or (6), depend on the temperature. By using superconductors and cryogenics in communication systems [24], one can expect low or very low values of ε_{BB} and ε_{RF}. If a theoretical zero minimum energy can be expected, it remains to evaluate the fundamental limits of superconductor or cryogenic systems in the terrestrial environment. Furthermore, it is not established that the limit of the limit is zero. Other fundamental physical constants than the Boltzmann's one can interfere, disabling zero-power computing [5,25,26] or zero-power transmitting [27].

Note that both minimum energies are bandwidth independent. This is in contrast with practical models used to evaluate the energy consumption of wireless communicating devices where E_{BB} has a bandwidth-dependent part [28].

The latter energetic limit (4) is given in the case of a Gaussian channel which models a fixed point-to-point communication. Hereafter, we consider the mobility in radiocommunication systems with LTV channels to evaluate the minimum value of E_{RF}.

4. Linear time-varying channels

With this channel model, the received power depends linearly on the channel gain γ and on the transmitted power P. In the general case, the transmitted power can also depend on γ. The received power is then $\gamma P(\gamma)$. This channel model is a narrow-band channel model without frequency selective fading, i.e. with flat fading. For calculus convenience, we use the continuous channel model, but we consider the independent and identically distributed memoryless block-fading model [29]. Note that the colored Gaussian noise channel model is equivalent to the model used here.

The ergodic capacity of this channel model is [6]

$$C(P) = \max_{P(\gamma)} \int_{D_{f_\gamma}} \Delta f \log_2 \left(1 + \gamma \frac{P(\gamma)}{\Delta w kT}\right) f_\gamma(\gamma)\,d\gamma \tag{7}$$

The transmitted power satisfies the constraint

$$\int_{D_{f_\gamma}} P(\gamma) f_\gamma(\gamma)\,d\gamma = P \tag{8}$$

where $D_{f_\gamma} = \{\gamma | f_\gamma(\gamma) > 0\} \in \mathbb{R}_+$ is the support of f_γ, with f_γ the p.d.f. of γ. It is simplified by using D_f when there is no confusion. The results can also be extended to non-ergodic measure through outage capacity. The problem is now to evaluate the minimum value of E_{RF}, with E_{RF}, always defined by (3) but now using (7) and (8).

The transmitted power can or can not depend on the channel gain γ. In the former case, we consider that the CSI is known only at the receiver side, while in the latter case the CSI is known at both the transmitter and receiver sides.

4.1. Side information at the receiver side

In this case, power adaptation is not possible and $P(\gamma) = P$ for all γ.

Lemma 4.1. *For all p.d.f. f_γ, E_{RF} is an increasing function with respect to $\frac{P}{\Delta w}$.*

Proof. See Appendix A. □

From a physical point of view, this quite trivial result says that capacity grows more slowly than power. This first result is useful for the following proposition.

Proposition 4.1. *The minimum energy per bit to send information through ergodic, LTV channel without CSI at the transmitter side is*

$$\min E_{RF} = \frac{k T \ln 2}{E_\gamma[\gamma]}$$

Proof. See Appendix B. □

The minimum energy per bit is proportional to the inverse of the mean value of γ and proportional to $k T \ln 2$. With normalized channel gain $E_\gamma[\gamma] = 1$, this result gives the well-known energetic limit of transmission ε_{RF} in (4). While the ergodic capacity in the LTV channel is lower than the capacity in the Gaussian channel, as shown in [30] in the case of the Rayleigh fading channel, the energetic limit remains the same. This means that the slope of the curves of $C(P)$ with respect to P is the same in the Gaussian and in the LTV flat fading channels when P goes down to zero [14].

With a non-ergodic channel, the capacity is a random variable, and the outage probability and the outage capacity are used to quantify the channel [29]. The minimum energy is also a random variable depending on γ. An outage probability can be defined as the probability that the minimum energy $E_{RF}(\gamma) = \frac{k T \ln 2}{\gamma}$ is lower than a given threshold. Similarly to the outage capacity, also called ϵ-capacity, an outage minimum energy can be defined.

4.2. Side information at the transmitter and receiver sides

In this case, the transmitter can adapt the power to the channel state to increase the capacity. The adaptation is done by means of resource allocation algorithms and the transmitter needs CSI knowledge. This knowledge, in turn, needs strategies to be realized using backward channels, which consumes energy. In the case of duplex transmission, CSI can be obtained using the reverse communication channel without any additional cost of energy. Nevertheless, we consider that CSI at the transmitter side has zero additional energetic cost. With this assumption, we are sure that the derived limit can not be outperformed by a genius strategy.

The channel capacity of the LTV channel with CSI at the transmitter side is derived in [6] using convex optimization tools. It is given by

$$C(P) = \int_{\gamma \geq \gamma_0} \Delta w \log_2 \frac{\gamma}{\gamma_0} f_\gamma(\gamma) \, d\gamma \qquad (9)$$

for all p.d.f. f_γ, where the cutoff value γ_0 satisfies the power constraint

$$\Delta w \, k \, T \int_{\gamma \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma} \right) f_\gamma(\gamma) \, d\gamma = P \qquad (10)$$

The calculation of the cutoff value γ_0 depends on the p.d.f. f_γ. However, the knowledge of the p.d.f. is not required for minimum energy calculus; only the first moment or the maximum value is needed, as we will see later in this paper.

The energy per bit E_{RF} is now expressed using (9) and (10). We first derive the following result with the new notations.

Lemma 4.2. *For all f_γ, E_{RF} is a decreasing function with respect to γ_0, when the CSI is know at both the transmitter and receiver sides.*

Proof. See Appendix C. □

As for Lemma 4.1, this result says that the capacity grows more slowly than γ_0. This lemma is of practical interest for the following theorem.

Theorem 4.1. *The minimum energy per bit to send information through ergodic, LTV channel with CSI at both the transmitter and receiver sides is*

$$\min E_{\mathrm{RF}} = \frac{k\,T \ln 2}{\max\limits_{D_f} \gamma}$$

Proof. See Appendix D. □

The energetic limit is proportional to $k\,T \ln 2$. This theorem looks like Proposition 4.1, where the minimum energy is inversely proportional to $\max_{D_f} \gamma$, in Theorem 4.1, instead of $E_\gamma[\gamma]$ in Proposition 4.1. This comparison means that increasing the amount of available knowledge in the communication system then reduces the minimum energy needed to send an information bit. The definition of D_f is the critical point here and it is discussed in the following sections.

In a non-ergodic channel, the full statistic of the channel coefficient is not experienced during one communication, and neither $E_\gamma[\gamma]$ nor $\max_{D_f} \gamma$ can be evaluated. Due to the causal channel, only the current value of the channel coefficient can be used. The cutoff value γ_0 is then reduced to γ, and the minimum energy writes $E_{\mathrm{RF}}(\gamma) = \frac{k\,T \ln 2}{\gamma}$, as in the case of CSI at the receiver side only. In other words, $E_\gamma[\gamma]$ and $\max_{D_f} \gamma$ are reduced to the same value γ in a nonergodic channel.

5. Defective conventional channels

Conventional models of radiocommunication channels use p.d.f. with $D_f = \mathbb{R}_+$ as with Rayleigh, Rice, Nakagami or Hoyt distributions, for example. The support of the distribution is then unbounded. This is not a drawback to evaluate the channel capacity with or without CSI at the transmitter side, as it has been done in [30] or [6] in the case of a Rayleigh block fading channel, or in [14] in the case of more general fading channels. However, this unbounded support has major consequences.

Theorem 4.1 leads to the straightforward corollary.

Corollary 5.1. *The minimum energy per bit to send information is null with unbounded D_f and with CSI at both the transmitter and receiver sides.*

This result, already suggested in [14], obviously contradicts the well-known limit of $k\,T \ln 2$, which is valid in a wide variety of communication channels. Let us now analyse how this zero minimum energy is possible.

First, the minimum mean received power is

$$\lim_{\gamma_0 \to \infty} \int_{\gamma \geq \gamma_0} \gamma\, P(\gamma) f_\gamma(\gamma)\, \mathrm{d}\gamma = 0 \tag{11}$$

and it goes down to 0 as the transmitted one

$$\lim_{\gamma_0 \to \infty} P = 0 \tag{12}$$

The mean received power can also be written as

$$\int_{D_f} \gamma\, P(\gamma) f_\gamma(\gamma)\, \mathrm{d}\gamma = \Delta w k\,T \int_{\gamma \geq \gamma_0} \left(\frac{\gamma}{\gamma_0} - 1 \right) f_\gamma(\gamma)\, \mathrm{d}\gamma \tag{13}$$

This power is lower than the transmitted one, given in (10), if and only if $f_\gamma(\gamma) = 0$ when $\gamma \geq \gamma_0 > 1$, which contradicts the hypothesis that D_f is unbounded. In other word, as $\gamma_0 \to \infty$, then for low values of P there exists $M > 1$ such as

$$\forall \gamma \geq \gamma_0 \geq M,\ \gamma\, P(\gamma) > P(\gamma) \tag{14}$$

which means that the received power is higher than the transmitted one, contradicting the physical behaviour of passive channels. The same reasoning can be made in the case of the wideband regime.[1] However, this contradiction is not really

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[1] From a theoretical point of view, the channel should remain a flat fading channel also in the wideband regime to ensure convergence. This is not the case in practical mobile channels.

a problem in the case of Proposition 4.1 or in the case of the capacity measure in (9), because the p.d.f. vanishes with γ. The high values of γ do not contribute to the increase of the measure. The problem with CSI at both the transmitter and receiver sides is that the cutoff value γ_0 goes up to infinity, as stated by Lemma 4.2. This means that the power is only allocated when the channel gain goes to infinity, even if the probability to have this gain vanishes. Then, the needed energy per bit also vanishes and goes down to 0. The limit of energy per bit to send information is strongly related to the support used to model channel fadings.

Allowing a model with the received power higher than the transmitted one is not a physical model, but it is of practical interest to evaluate many communication system measures as a performance evaluation. However, it leads to wrong minimum energy evaluation in the case of a LTV channel with CSI at both the transmitter and receiver sides. To evaluate this performance, we then need channel models compatible with physical phenomena, where the received power or the received energy is lower than the transmitted one.

6. Models with limited channel gain

With a limited channel gain, the problem becomes how to truncate the distribution to obtain a finite support. This problem has been addressed in [31] in terms of minimum value of electromagnetic field reception, for example. However, even if truncation is small, it has a non-negligible impact [32]. To solve this problem, we handle a general case with the simple constraint

$$\gamma P(\gamma) \leq P(\gamma) \tag{15}$$

without any other constraint than the distribution. The consequence is that the support D_f is bounded, but we do not say anything about the shape or the truncation of the p.d.f. In this case, the following corollary is directly deduced from Proposition 4.1 and Theorem 4.1.

Corollary 6.1. *The minimum energy per bit to send information is not lower than $k\,T \ln 2$ for a received power lower than the transmitted one, with and without CSI at transmitter side.*

The minimum energy $\varepsilon_{\mathrm{RF}}$ (4) is then recovered with the right constraint (15). The only constraint on the channel gain distribution is that its support is bounded. Instead of an instantaneous constraint in (15), an average constraint can be used:

$$\int_{D_f} \gamma P(\gamma) f_\gamma(\gamma)\,\mathrm{d}\gamma \leq \int_{D_f} P(\gamma) f_\gamma(\gamma)\,\mathrm{d}\gamma \tag{16}$$

If the resulting support, D_f is unbounded, the minimum energy will be zero, regardless of the decreasing of f_γ with respect to the high γ value. However, the following proposition translates the average constraint (16) in terms of distribution support.

Proposition 6.1. *In the case of the low-power regime, if the power is lower than the transmitted one in average, then the support D_f is bounded.*

Proof. It is straightforward by writing the average constraint (16) with (10) and (13) in the low-power regime. □

By contraposition and in the low-power regime, Proposition 6.1 says that if the support D_f is not bounded, then the received power is higher than the transmitted one in average. As this statement can not be justified in a passive channel, it is a physical justification of Proposition 6.1 by the principle of excluded middle.

7. Parallel transmission

Parallel transmission is widely encountered in communication systems, as in orthogonal frequency division multiplexing (OFDM) or multiple-input–multiple-output (MIMO) systems. Multiple links with different wireless technologies or multiple links in a network can also be modelled with parallel transmission. This parallel transmission model is used as a classical decomposition technique to calculate the capacity of the frequency-selective channels [33], splitting one frequency channel in multiple flat fading channels.

With parallel transmission, the power constraint can be a peak power constraint or a sum, also call average, power constraint.[2] With peak power constraint, there is no power sharing between the channels. If P is the total power budget,

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[2] In the seminal paper by Shannon [17], the peak constraint is an instantaneous power constraint. The peak power constraint used here would be related to an average power constraint per channel in [17], and not to an instantaneous one.

the peak constraint for each one of the n parallel channels is P/n. The minimum energy can be independently calculated for each channel, and the fundamental energetic limit is the same as in the single channel case.

The capacity and the power constraint in the general case of sum power constraint are

$$C(P) = \max_{\{P_i(\gamma_i)\}_{i \in [1,n]}} \sum_{i=1}^{n} \int_{D_{f_{\gamma_i}}} \Delta w_i \log_2 \left(1 + \gamma_i \frac{P_i(\gamma_i)}{\Delta w_i \, k \, T} \right) f_{\gamma_i}(\gamma_i) \, d\gamma_i \tag{17}$$

and

$$\sum_{i=1}^{n} \int_{D_{f_{\gamma_i}}} P_i(\gamma_i) f_{\gamma_i}(\gamma_i) \, d\gamma_i = P \tag{18}$$

The energy per bit E_{RF} given in (3) is now expressed using (17) and (18). Both CSI cases are now analysed.

7.1. Side information at receiver

Corollary 7.1. *The minimum energy per bit to send information through n parallel, independent, LTV channels without CSI at the transmitter side is*

$$\min E_{RF} = \frac{k \, T \ln 2}{\max_{i \in [1,n]} E_i[\gamma_i]}$$

Proof. See Appendix E. □

This corollary says that the minimum energy is reached when the best mean channel is used, i.e. the channel with the lower mean attenuation. However, using only one channel when several ones are available is not optimal from the time transmission point of view. This point is discussed in [23].

The use of the best channel needs to know and to compare the mean channel gains. If this information is not a prior knowledge, it can not be obtained in a causal channel. In this case, the optimal power sharing is the uniform one and the minimum energy is simply

$$\min E_{RF} = \frac{1}{n} \sum_{i=1}^{n} \frac{k \, T \ln 2}{E_i[\gamma_i]} \tag{19}$$

This value is higher than the one given in Corollary 7.1, which needs more information. Once more, the information increase decreases the minimum energy needed for transmission.

7.2. Side information at transmitter and receiver

We now consider CSI at both the transmitter and receiver sides.

Corollary 7.2. *The minimum energy per bit to sent information through n parallel, independent, LTV channels with CSI at both the transmitter and receiver sides is*

$$\min E_{RF} = \frac{k \, T \ln 2}{\max_{\bigcup_{i \in [1,n]} D_{f_{\gamma_i}}} \gamma}$$

Proof. See Appendix F. □

It is important to emphasize that there are no more constraints on the p.d.f. f_{γ_i}, nor on the bandwidths Δw_i. Specifically, the p.d.f. and the bandwidth can be different for each channel i. The result in Corollary 7.2 then holds for a very general parallel transmission cases, not restricted to OFDM or MIMO ones. It can also be used to minimize the energy in a heterogeneous mesh network.

Another important fact is that the maximum values have to be known and compared, instead of the means values in Corollary 7.1. This knowledge can then be considered in an operational communication system and, at least, γ can not be lower than 1 in passive channels.

8. How far are we from the limits?

We have extended the well-known limit of minimum energy needed to send one information bit, $\varepsilon_{RF} = kT\ln 2$, to a broad range of communication channels, especially in the case of CSI at both the transmitter and receiver sides. The minimum values ε_{BB} and ε_{RF} can be used to evaluate the energy efficiency of operational communication systems.

Let P_c be the total power consumed in practice to send and receive information with a bit-rate R. Its value can be compared to the minimum values ε_{BB} and ε_{RF} as follows

$$P_c = \eta_{BB}^{-1} R\, \varepsilon_{BB} + \eta_{RF}^{-1} R\, \varepsilon_{RF} \tag{20}$$

where η_{BB} and η_{RF} are the coefficients of energy efficiencies in computing and in transmission, respectively. If P_c reaches its minimum value, then the system is efficient and $\eta_{BB} = \eta_{RF} = 1$. Otherwise, both efficiencies are lower than 1.

In [34], for example, a short range of wireless communications in a wireless network-on-chip has been realized. A bit rate of 16 Gbit/s and a transmitted power of 0.9 mW ensure a reliable communication with a bit error rate lower than 10^{-12}. The corresponding energy efficiency η_{RF} is then around 10^{-8}. The overall energy consumption of 0.25 pJ/bit leads to the energy efficiency η_{BB} also around 10^{-8}. This energy efficiency η_{BB} can be compared to the capability of high-performance computers [35]. The supercomputer expected value of 50 GFLOPS/W leads to an energy efficiency of 10^{-8}. This indicates that it remains 8 orders of magnitude to reach the minimum value of the computing energy.

Energy efficiency has been deeply studied in long-range wireless communication systems, as in [36], for example. The overall energy consumption per delivered bit lies between 10^{-2} and 1 mJ/bit [37]. These very bad energetic performance compared to that of short-range communication is caused by various factors, and leads to the lowest energy efficiency, $\min\{\eta_{BB}, \eta_{RF}\}$, less than 10^{-16}. Concerning the E_{RF} part of the energy in a mobile network, there is a lack of ability to send the transmitted energy to the receiving antenna: we can say that all of the transmitted energy is then wasted, excepted a very small part! This is one objective of the cognitive green radio concept "to decrease the electromagnetic level by sending the right signal in the right direction with the optimal power, only when it is necessary" [38]. The possible gains are enormous, with at most 16 orders of magnitude of improvement.

These examples of energetic performance concern only the operational part of the life-cycle of the communication systems. The LCA of electronic devices indicates that less than 1 % of the grey energy is used for transmission in a wireline network [39]. The energetic efficiency previously drawn is then reduced by two more orders of magnitude in an holistic approach of the energy consumption.

9. Conclusion

In a theoretical evaluation of the energetic limit of a radiocommunication system, two power-information expressions gives the minimum energy needed to compute and to send an information bit. We have shown in this paper how this well-known minimum energy per bit $kT\ln 2$ can be applied in a wide range of wireless channels if the physical constraints are correctly applied. We mainly derived the energetic limit of the transmitted energy in the low-power regime, but we have shown that wideband and widetime regimes can also be used. However, the mobile wireless channel is not a double-flat fading channel in the frequency and in the time domains. The wideband or widetime regime can not be applied to reach the minimum value, as it has been done in the Gaussian channel case, which requires that the channel remains flat in the wide dimension. Then, it remains the low power regime where the power goes down to zero to reach the minimum energetic consumption. This means that we have implicitly made the following assumption.

Assumption 9.1. The Shannon's capacity model and expressions remain valid at very low power regime.

With very low power regime, quantum phenomenas can occur that are not taken into account in this paper. However, the minimum energetic limit is much higher than the energy of a photon in Terrestrial environment. Assumption 9.1 remains a reasonable one in our common environment of radiocommunication.

Appendix A. Proof of Lemma 4.1

Let F be the function such as $E_{RF} = kT\ln 2\, F(u)$ with $u = \frac{P}{\Delta w\, kT} \geq 0$, then

$$F(u) = \frac{u}{\int_{D_f} \ln(1 + \gamma u) f_\gamma(\gamma)\, d\gamma} \tag{21}$$

The first derivative of F is

$$F'(u) = \frac{\displaystyle\int_{D_f} \left(\ln(1 + \gamma u) - \frac{\gamma u}{1 + \gamma u} \right) f_\gamma(\gamma)\, \mathrm{d}\gamma}{\left(\displaystyle\int_{D_f} \ln(1 + \gamma u) f_\gamma(\gamma)\, \mathrm{d}\gamma \right)^2} \tag{22}$$

As $\ln(1 + x) - \frac{x}{1+x} > 0$ for all $x > 0$, then $F'(u) > 0$ and F is an increasing function.

Appendix B. Proof of Proposition 4.1

Using Lemma 4.1, the minimum value is

$$\min E_{\mathrm{RF}} = \lim_{\frac{P}{\Delta w} \to 0} E_{\mathrm{RF}} \tag{23}$$

Let $u = \frac{P}{\Delta w k T}$, then

$$\min E_{\mathrm{RF}} = \lim_{u \to 0} \frac{k T \ln 2}{\displaystyle\int_{D_f} \frac{\gamma}{1 + \gamma u} f_\gamma(\gamma)\, \mathrm{d}\gamma} = \frac{k T \ln 2}{\displaystyle\int_{D_f} \gamma f_\gamma(\gamma)\, \mathrm{d}\gamma} \tag{24}$$

where the first equality is obtained using L'Hôspital's rule applied to (21) and by differentiating with respect to u.

Appendix C. Proof of Lemma 4.2

Let F be the function such as $E_{\mathrm{RF}} = k T \ln 2\, F(\gamma_0)$, then

$$F(\gamma_0) = \frac{\displaystyle\int_{\gamma \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma} \right) f_\gamma(\gamma)\, \mathrm{d}\gamma}{\displaystyle\int_{\gamma \geq \gamma_0} \ln \frac{\gamma}{\gamma_0} f_\gamma(\gamma)\, \mathrm{d}\gamma} \tag{25}$$

The first derivative with respect to γ_0 is

$$F'(\gamma_0) = \frac{\displaystyle\int_{\gamma \geq \gamma_0} f_\gamma(\gamma)\, \mathrm{d}\gamma \times \int_{\gamma \geq \gamma_0} \left(\ln \frac{\gamma_0}{\gamma} + 1 - \frac{\gamma_0}{\gamma} \right) f_\gamma(\gamma)\, \mathrm{d}\gamma}{\left(\gamma_0 \displaystyle\int_{\gamma \geq \gamma_0} \ln \frac{\gamma}{\gamma_0} f_\gamma(\gamma)\, \mathrm{d}\gamma \right)^2} \tag{26}$$

As $\ln u + 1 - u < 0$ for all $u \in [0, 1)$, then $F'(\gamma_0) < 0$ and F is a decreasing function.

Appendix D. Proof of Theorem 4.1

Using Lemma 4.2, the minimum value is

$$\min E_{\mathrm{RF}} = \lim_{\gamma_0 \to \max_{D_f} \gamma} E_{\mathrm{RF}} \tag{27}$$

As in Appendix B, L'Hôspital's rule is applied to (25) by differentiating with respect to γ_0

$$\min E_{\mathrm{RF}} = k T \ln 2 \lim_{\gamma_0 \to \max_{D_f} \gamma} \frac{\displaystyle\int_{\gamma \geq \gamma_0} \frac{1}{\gamma_0^2} f_\gamma(\gamma)\, \mathrm{d}\gamma}{\displaystyle\int_{\gamma \geq \gamma_0} \frac{1}{\gamma_0} f_\gamma(\gamma)\, \mathrm{d}\gamma} \tag{28}$$

and then

$$\min E_{\mathrm{RF}} = \frac{k\,T \ln 2}{\max_{D_f} \gamma} \tag{29}$$

Appendix E. Proof of Corollary 7.1

To derive the result, we consider that all the p.d.f. f_{γ_i}, given by the used channel model, are known at the transmitter side, whereas the CSI are not known. The capacity is given by the water-filling solution, which shares the power between the parallel channels [20]. In low-power regime, $\exists \epsilon$ such as $P \le \epsilon$ and only the best channel has to be used. In this case, the "water level" is too low to allow power splitting. Sharing becomes adverse, and the capacity is given by the best channel, which depends on Δw_i and on the p.d.f. f_{γ_i}. Then $\forall P \le \epsilon$, the energy per bit in the low-power regime is

$$E_{\mathrm{RF}} = \frac{P}{\max_i \int_{D_{f_{\gamma_i}}} \Delta w_i \log_2 \left(1 + \gamma_i \frac{P}{\Delta w_i k\,T}\right) f_{\gamma_i}(\gamma_i)\,\mathrm{d}\gamma_i} \tag{30}$$

and Proposition 4.1 finishes the proof.

Note that the same result is derived in the wideband regime. However, the wideband regime, which is obtained when $\sum_i \Delta w_i \to \infty$, leads to two opposite wideband models. Let Δw be such as $\Delta w_i = \alpha_i \Delta w$, the two wideband models are

1. $\Delta w \to \infty$ and n is constant,
2. $n \to \infty$ and Δw is constant.

The minimum energetic limit is derived using the first asymptotic model, assuming that the channels remain flat over each Δw_i, which remains questionable in wireless fading channels, but can be applied in multiple Gaussian link transmissions. With the second model, the energy is given by (30) regardless of the value of n. The energetic limit is then not reached with $n \to \infty$.

Appendix F. Proof of Corollary 7.2

Let $D_f = \bigcup_i D_{f_{\gamma_i}}$ and f_i such as

$$f_i(\gamma) = \begin{cases} f_{\gamma_i}(\gamma), & \forall \gamma \in D_{f_{\gamma_i}}, \\ 0, & \forall \gamma \notin D_{f_{\gamma_i}} \end{cases} \tag{31}$$

The channel capacity is

$$C(P) = \max_{\{P_i(\gamma)\}_i} \int_{D_f} \sum_i \Delta w_i \log_2 \left(1 + \gamma \frac{P_i(\gamma)}{\Delta w_i k\,T}\right) f_i(\gamma)\,\mathrm{d}\gamma \tag{32}$$

Applying the Lagrangian multiplier and the Karush–Khun–Tucker condition to the $C(P)$ maximization problem under a power constraint, as in [6], the energy per transmitted bit is

$$E_{\mathrm{RF}} = k\,T \ln 2 \frac{\sum_i \Delta w_i \int_{\gamma \ge \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_i(\gamma)\,\mathrm{d}\gamma}{\sum_i \Delta w_i \int_{\gamma \ge \gamma_0} \ln \frac{\gamma}{\gamma_0} f_i(\gamma)\,\mathrm{d}\gamma} \tag{33}$$

With this notation, Theorem 4.1 finishes the proof.

As in Appendix E, only one of the two wideband regimes yields the same energetic limit.

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