



ELSEVIER

Contents lists available at ScienceDirect

## Comptes Rendus Physique

www.sciencedirect.com



The *Comptes rendus de l'Académie des sciences* throughout history / Les *Comptes rendus de l'Académie des sciences* à travers l'histoire

## Poincaré, the dynamics of the electron, and relativity

*Poincaré, la dynamique de l'électron et la relativité*

Thibault Damour

Institut des hautes études scientifiques, 35, route de Chartres, 91440 Bures-sur-Yvette, France

## ARTICLE INFO

Article history:  
Available online 18 October 2017

Keywords:  
Relativity  
Lorentz transformations  
Electron model  
Cosmological constant

Mots-clés:  
Relativité  
Transformations de Lorentz  
Modèle de l'électron  
Constante cosmologique

## ABSTRACT

On 5 June 1905 Poincaré presented a Note to the Académie des Sciences entitled *Sur la dynamique de l'électron* ("On the dynamics of the electron"). After briefly recalling the context that led Poincaré to write this Note, we comment its content. We emphasize that Poincaré's electron model consists in assuming that the interior of the worldtube of the (hollow) electron is filled with a *positive cosmological constant*. We then discuss the several novel contributions to the physico-mathematical aspects of Special Relativity that are sketched in the Note, though they are downplayed by Poincaré, who describes them as having only completed the May 1904 results of Lorentz *dans quelques points de détail* ("in a few points of detail").

© 2017 Académie des sciences. Published by Elsevier Masson SAS. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

## R É S U M É

Le 5 juin 1905, Poincaré présenta à l'Académie des sciences une Note intitulée «*Sur la dynamique de l'électron*». Après avoir brièvement rappelé le contexte qui conduisit Poincaré à écrire cette Note, nous commentons son contenu. Nous soulignons que le modèle de l'électron proposé par Poincaré consiste à supposer que l'intérieur du tube d'univers de l'électron (creux) est rempli par une constante cosmologique positive. Nous exposons ensuite les contributions novatrices aux aspects physico-mathématiques de la relativité restreinte qui sont esquissées dans la Note. Ces contributions sont minimisées par Poincaré, qui décrit ses résultats comme ayant seulement complété les résultats de mai 1904 de Lorentz «*dans quelques points de détail*».

© 2017 Académie des sciences. Published by Elsevier Masson SAS. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

E-mail address: [damour@ihes.fr](mailto:damour@ihes.fr).

<https://doi.org/10.1016/j.crhy.2017.10.006>

1631-0705/© 2017 Académie des sciences. Published by Elsevier Masson SAS. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

## 1. Context

In order to apprehend the meaning, and importance, of Poincaré's Note, in the 5 June 1905 issue of the *Comptes rendus* [1], it is necessary to recall its context.<sup>1</sup> For an access to Poincaré's works and archives, see [7]. See also the 2012 commemorative colloquium of the centenary of Poincaré's death at the French Academy of Sciences [8].

Poincaré gave sets of lectures on *Électricité et Optique*, at the Sorbonne, in 1888, 1890 and 1899. These lectures were part of his duty as holder of a chair of “Mathematical Physics and Probability Calculus” (*Physique mathématique et calcul des probabilités*). In his 1899 lectures (published as the last part of the book [9]) he expounded, in particular, Lorentz' approach to electrodynamics, which he considered as the most satisfactory one. Lorentz' approach had been developed a few years before, notably in Refs. [10,11]. Lorentz' 1892 paper [10] already implicitly contains the exact form (involving  $1/\sqrt{1-v^2/c^2}$  factors) of the Lorentz transformation.<sup>2</sup> On the other hand, Lorentz' 1895 paper [11] works most of the time only to first order in  $v/c$ , but expounds in clearer physical terms the usefulness of defining what Lorentz called there the “local time”, namely

$$t' \equiv t - \frac{1}{c^2} \mathbf{v} \cdot \bar{\mathbf{x}}, \quad (1)$$

where

$$\bar{\mathbf{x}} = \mathbf{x} - \mathbf{v}t, \quad (2)$$

denotes the usual (Galilean-transformed) spatial coordinates in a moving frame. In addition, in paragraphs 89–92, Lorentz recalls his earlier (1892) hypothesis (invented to explain the negative result of the Michelson–Morley experiment) according to which solid bodies moving with respect to the ether get contracted (in the direction of the motion) by a factor of  $\simeq 1 - \frac{1}{2} \frac{v^2}{c^2}$ .

Both in his 1899 lectures [9], and in his invited review talk on the *Relations entre la physique expérimentale et la physique mathématique* at the “Congrès international de physique” that took place in Paris in 1900, Poincaré expresses his dissatisfaction at Lorentz' approach, which is based on an accumulation of disconnected hypotheses (famously referred to by Poincaré as *coups de pouce*, i.e. “nudges”; see citation below). In particular, he writes about the Lorentz(–Fitzgerald) contraction hypothesis that (p. 536 of [9]):

Cette étrange propriété semblerait un véritable « coup de pouce » donné par la nature pour éviter que le mouvement de la Terre puisse être révélé par des phénomènes optiques. Ceci ne saurait me satisfaire et je crois devoir dire ici mon sentiment : je considère comme très probable que les phénomènes optiques ne dépendent que des mouvements relatifs des corps en présence, sources lumineuses ou appareils optiques et *cela non pas aux quantités près de l'ordre du carré ou du cube de l'aberration, mais rigoureusement*. À mesure que les expériences deviendront plus exactes, ce principe sera vérifié avec plus de précision. Faudra-t-il un nouveau *coup de pouce*, une hypothèse nouvelle à chaque approximation ? Évidemment non : une théorie bien faite devrait permettre de démontrer le principe d'un seul coup dans toute sa rigueur. La théorie de Lorentz ne le fait pas encore. De toutes celles qui ont été proposées, c'est elle qui est le plus près de le faire. On peut donc espérer la rendre parfaitement satisfaisante sous ce rapport sans la modifier trop profondément.<sup>3</sup>

Let us also mention that, in his paper *La théorie de Lorentz et le principe de réaction* [12], written in 1900 at the occasion of the 25th anniversary of Lorentz' thesis, Poincaré discusses (as emphasized by O. Darrigol) the effect of an overall translation, at some speed  $v$ , on the synchronization of clocks by the exchange of electromagnetic signals. More precisely, he works only to *first order* in  $v$ , and notes that, if moving observers synchronize their watches by exchanging optical signals, and if they correct these signals by the transmission time under the (incorrect) assumption that the signals travel at the same speed in both directions, their watches will indicate not the “real time”, but the “apparent time”, say (denoting  $\bar{x} \equiv x - vt$ )

$$\tau = t - \frac{v\bar{x}}{c^2} + O(v^2). \quad (3)$$

His main point is that the “apparent time”  $\tau$  coincides with the formal mathematical variable  $t' \equiv t - \frac{v\bar{x}}{c^2} + O(v^2)$  introduced by Lorentz in 1895 under the name of “local time” (and used by him to show the invariance of Maxwell's theory under uniform translations, to first order in  $v$ ).

<sup>1</sup> For wider, and more detailed, historical studies of Poincaré's contributions to electrodynamics, see [2–6]. See also Olivier Darrigol's text *Faut-il réviser l'histoire de la relativité ?* in the *Lettre de l'Académie des sciences* n° 14, hiver 2004, and in the *Bulletin de la SFP* (150) July–August 2005; available on <http://www.academie-sciences.fr/archivage-site/activite/archivage/dossiers/Einstein/Einstein-pdf/Darrigol-amp.pdf>. See also Scott Walter's web page, <http://scottwalter.free.fr/>, for studies of several aspects of Poincaré's life and work, as well as Galina Weinstein's blog (<https://myalberteinstein.com/2012/07/04/centenary-of-the-death-of-poincare-einstein-and-poincare-2012/>), and her papers, notably: “Poincaré's Dynamics of the Electron – A Theory of Relativity?” <https://arxiv.org/pdf/1204.6576>, and “A Biography of Henri Poincaré – 2012 Centenary of the Death of Poincaré” <https://arxiv.org/pdf/1207.0759>.

<sup>2</sup> See paragraph 138, pages 141–142 in [10] (one should only replace Lorentz' variable  $t'$  by  $t'_{\text{new}} = \sqrt{1 - v^2/c^2} t'$  to get the exact Lorentz transformation).

<sup>3</sup> The italics are Poincaré's.

In addition, Poincaré mentions, in his 1902 book *La science et l'hypothèse* [13], as one of the principles of physics, *le principe du mouvement relatif* (see notably chapter VII), which he also refers to, at the end of chapter XIII, as *le principe de relativité*, and writes in chapter XIV (now attributing to Lorentz, what he was, in 1900, reproaching Lorentz not to take seriously enough), about the issue of whether experimental results might, one day, allow one to determine the absolute motion of the Earth:

Lorentz ne l'a pas pensé; il croit que cette détermination sera toujours impossible; l'instinct commun de tous les physiciens, les succès éprouvés jusqu'ici le lui garantissent suffisamment. Considérons donc cette impossibilité comme une loi générale de la nature; admettons-la comme postulat. Quelles en seront les conséquences? C'est ce qu'a cherché Lorentz...<sup>4</sup>

On 27 May 1904, Lorentz publishes his breakthrough paper: “Electromagnetic phenomena in a system moving with any velocity smaller than that of light” [14].<sup>5</sup> In the Introduction of his paper, Lorentz explicitly mentions, as a motivation for extending his previous results, the discontent expressed by Poincaré in his review talk at the 1900 “Congrès international de physique” in Paris. In his 1904 paper, Lorentz defines some auxiliary variables, denoted  $(x', y', z', t')$ , that are defined in terms of the space and time coordinates  $(x, y, z, t)$  measured in the ether frame, by the formulas

$$\begin{aligned} x' &= \gamma \ell \bar{x}, \\ y' &= \ell y, \\ z' &= \ell z, \\ t' &= \frac{\ell}{\gamma} t - \gamma \ell \frac{v}{c^2} \bar{x}. \end{aligned} \tag{4}$$

Here,  $\bar{x}$  (implicitly) denotes (as above) the Galilean-transformed  $x$ -coordinate,  $\bar{x} \equiv x - vt$ , corresponding to a Galilean reference frame moving with the considered system, namely with the velocity  $v$  in the  $x$  direction. In addition,  $\gamma$  denotes  $\gamma \equiv 1/\sqrt{1 - v^2/c^2}$  (which is actually denoted  $\beta$  by Lorentz), while  $\ell(|v|^2)$  denotes an *a priori* undetermined rescaling factor, assumed to be some function of the squared modulus of the velocity  $v$ .

Lorentz shows that Maxwell's equations in vacuum are rigorously invariant under the change of variables (4), provided the electric and magnetic fields in the primed system are appropriately transformed. He also shows that the inhomogeneous Maxwell(-Lorentz)'s equations are approximately invariant when changing the charge and current densities by a transformation he writes down.

Then Lorentz makes two further assumptions:

- (A1) “that the electrons, which I take to be spheres of radius  $R$  in the state of rest, have their dimensions changed by the effect of a translation, the dimensions in the direction of the motion becoming  $\gamma \ell$  times, and those in perpendicular directions  $\ell$  times smaller”; and
- (A2) “that the forces between uncharged particles, as well as those between such particles and electrons, are influenced by a translation in quite the same way as the electric forces in an electrostatic system.”

Lorentz then computes the “electromagnetic momentum” of a uniformly moving electron (assuming this accounts for the full linear momentum of an electron, i.e. that the “true”, or ‘material’ mass” of the electron vanishes) as being

$$\mathbf{p}^{\text{Lor}} = \frac{4}{3} \frac{E_{\text{em}}}{c^2} \gamma(v^2) \ell(v^2) \mathbf{v}, \tag{5}$$

where

$$E_{\text{em}} = \int_{r>R} d^3x \frac{1}{2} \mathbf{E}^2 = \frac{e^2}{8\pi} \int_R^\infty \frac{dr}{r^2} = \frac{e^2}{8\pi R}, \tag{6}$$

denotes the electrostatic energy of the field generated by a spherical, hollow electron, of radius  $R$ , at rest (the electric field is equal to  $E = e/(4\pi r^2)$  outside the electron, i.e. for  $r > R$ , and vanishes inside the spherical electron).<sup>6</sup> The factor  $\frac{4}{3}$  in (5) will be further commented below.

Requiring that the force law  $d\mathbf{p}^{\text{Lor}}/dt = \mathbf{F}$  [with  $\mathbf{F} = e(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$  being the (Lorentz) force] leads to consistent accelerations (using the transformation (4)) in the ether (rest) frame and in the moving frame, Lorentz derives the condition

<sup>4</sup> In which one can particularly note the sentences: *Admettons-la comme postulat. Quelles en seront les conséquences?*, i.e. “let us admit the impossibility of detecting the absolute motion of the Earth as a postulate; and let us study the consequences of this postulate.” Though he attributes this idea to Lorentz.

<sup>5</sup> Poincaré (probably informed by Lorentz) is soon aware of this paper.

<sup>6</sup> Like Lorentz and Poincaré, we use here Heaviside units.

$$\frac{d(\gamma(v^2)\ell(v^2)v)}{dv} = \gamma^3(v^2)\ell(v^2), \quad (7)$$

which implies

$$\frac{d(\ell(v^2))}{dv} = 0, \quad (8)$$

and therefore [using  $\ell(v^2) = 1 + O(v^2/c^2)$ ]

$$\ell(v^2) = 1. \quad (9)$$

With his definitions and his assumptions, Lorentz is then able to show the following theorem of “corresponding states”: “If, in the system without translation, there is a state of motion in which, at a definite place, the components of [the polarization]  $\mathbf{P}$ ,  $\mathbf{E}$  and  $\mathbf{B}$  are certain functions of the time, then the same system after it has been put in motion (and therefore deformed) can be the seat of a state of motion in which, at the corresponding place, the components of  $\mathbf{P}'$ ,  $\mathbf{E}'$  and  $\mathbf{B}'$  are the same functions of the local time.” In other words, there is an *active map* (given by Eqs. (4), together with the field-transformation laws derived by Lorentz) between a physical system at rest (in the ether) and a corresponding physical system moving with velocity  $v$ , such that the functions describing the electromagnetic field generated by the system at rest,  $\mathbf{E}(x, y, z, t)$  and  $\mathbf{B}(x, y, z, t)$ , are equal to the corresponding transformed fields considered as functions of the transformed variables,  $\mathbf{E}'(x', y', z', t')$  and  $\mathbf{B}'(x', y', z', t')$ . Note that Lorentz systematically emphasizes that  $t'$  is not the “true time”, but rather the auxiliary “local time” defined in terms of the true time by the last equation (4).

Poincaré (probably directly informed by Lorentz) appreciated the “extreme importance” (as Poincaré wrote to Lorentz in 1905) of Lorentz’ work and alluded to it (though not in detail) in his 24 September 1904 invited talk, on “The principles of mathematical physics”, at the International congress of arts and science in St. Louis (USA). His talk was delivered in French, and later translated [15]. Among the “five or six<sup>7</sup> general principles of physics”, Poincaré lists (in fourth position):

The principle of relativity, according to which the laws of physical phenomena should be the same, whether for an observer fixed, or for an observer carried along in a uniform movement of translation; so that we have not and could not have any means of discerning whether or not we are carried along in such a motion.<sup>8</sup>

Note in this respect that I could not figure out whether Poincaré was the first to use (in this sense) the expression “principle of relativity” (which he had already used, but only in passing, in his 1902 book *Science and Hypothesis*).<sup>9</sup> By contrast, not only Lorentz does not use this expression, but he does not seem to believe in the exact unobservability of a common translation. Indeed, he writes in his 1904 memoir [14]: “It would be more satisfactory if it were possible to show by means of certain fundamental assumptions and without neglecting terms of one order of magnitude or another, that many electromagnetic actions are entirely independent of the motion of the system.”<sup>10</sup> Later in his talk, Poincaré mentions that Lorentz’ “local time” is the (apparent) time indicated by the watches of two moving observers (“station A” and “station B”) when they are synchronized by exchanging light signals and by (‘wrongly’ but conventionally) assuming the isotropy of the speed of light in the moving frame, *i.e.* the equality between the transmission times during the two exchanges  $A \rightarrow B$  and  $B \rightarrow A$ .<sup>11</sup> Finally, Poincaré ends his St. Louis discussion of the principle of relativity (and of the related synchronization via light signals) by asking the question:

What would happen if one could communicate by non-luminous signals whose velocity of propagation differed from that of light? If, after having adjusted the watches by the optical procedure, one wished to verify the adjustment by the aid of these new signals, then would appear divergences which would render evident the common translation of the two stations. And are such signals inconceivable, if we admit with Laplace that universal gravitation is transmitted a million times more rapidly than light?

As we see, Poincaré’s reading of Lorentz’ 1904 memoir had (in particular) induced Poincaré to think about the connection between “the principle of relativity” and gravitation.<sup>12</sup>

<sup>7</sup> This ambiguity comes from Poincaré’s last suggested general principle: “I would add the principle of least action.”

<sup>8</sup> This is the only principle that Poincaré defines in detail.

<sup>9</sup> Lorentz writes in [16]: “Poincaré, on the contrary, obtained a perfect invariance of the equations of electrodynamics, and he formulated the ‘postulate of relativity’, terms which he was the first to employ.”

<sup>10</sup> Note that Lorentz is not aiming at deriving an *exact* principle of relativity, but only a *partial* one (“many”). I think that he had in mind the fact that the auxiliary time variable  $t'$  differed (by a factor  $1/\gamma$ ; see last equation (4)) from the “true” time, so that the unobservability was limited (as he says at the end of his paper) to non time-related experiments, such as optical experiments “in which the geometrical distribution of light and darkness is observed”, or in which “intensities in adjacent parts of the field of view are compared”.

<sup>11</sup> However, he does not write down any equations, so that it is not clear whether he is alluding to his previous *first-order-in-v* result, (3), or to an all-order result.

<sup>12</sup> See, however, also the reference to Langevin’s St. Louis talk below.

## 2. The main new results of Poincaré's 5 June 1905 Note to the *Comptes rendus*

In the spring of 1905, Poincaré started to study in detail Lorentz' memoir. His study led him to improve, and generalize, Lorentz' results. He announced some of his results in his 5 June Note [1], reserving a detailed exposition to a long paper sent for publication to the *Rendiconti del Circolo Matematico di Palermo* on 23 July 1905 [17]. This choice of medium of publication did not help to publicize the novelty of Poincaré's results. On the one hand, the Note, as we shall now discuss, is too short, too modest, and too incomplete to convey a clear idea of Poincaré's achievements. On the other hand, the Palermo memoir is written in a rather obscure way, which hides some of the most important new results of Poincaré amidst very technical derivations. As a consequence, it seems that Poincaré's achievements remained essentially unnoticed until Minkowski studied them, extracted their essential core, and generalized them, in 1908 [18–20].<sup>13</sup> Here, we shall limit ourselves to commenting the content of Poincaré's June 1905 Note, emphasizing both its importance, and its shortcomings.

### 2.1. The first important result (according to Poincaré himself): a dynamical derivation of the Lorentz contraction of moving electrons

The first point I wish to make concerns the title of Poincaré's Note, namely *Sur la dynamique de l'électron* ("On the dynamics of the electron"). This title is quite different from the title used by Lorentz ("Electromagnetic phenomena in a system moving with any velocity smaller than that of light"). It is also quite different from the title of Einstein's paper on Relativity [23] ("On the electrodynamics of moving bodies").<sup>14</sup> Though Poincaré's text does not make it so clear, I think that this title indicates that Poincaré considers that his main new result consists in "dynamically deriving" one of the key assumptions of Lorentz, namely assumption (A1) above, stating (as an ad hoc hypothesis) that a moving (hollow) spherical electron gets Lorentz-contracted into an ellipsoidal shape. Let me also note that the only explicit article citation in Poincaré's Note is Lorentz' 1904 memoir. [In addition, Poincaré cites the names of Michelson, Langevin, Kaufmann, Abraham, and Laplace.]

Poincaré's dynamical derivation of the contraction of each electron is initially based on a physical reasoning: because of the electrostatic self-repulsion, a hollow spherical electron of radius  $R$  needs to be stabilized by some counteracting force, holding together the charge distribution on the shell of radius  $R$ . Poincaré assumes that this counteracting force is *une sorte de pression constante extérieure dont le travail est proportionnel aux variations du volume* ("a kind of constant exterior pressure whose work is proportional to the variations of volume").<sup>15</sup> Alternatively, one can (as Poincaré does) consider that there exists a *negative internal pressure* (i.e. an internal tension) that holds together the electron. Poincaré's derivation is quite involved and is not even sketched in his Note. He only says that his derivation is based on "an application of the principle of least action". His derivation was given in the long, follow-up Palermo article [17], submitted on 23 July 1905 (and published in January 1906). Let us sketch here the essence of Poincaré's derivation, anachronistically reformulated in modern notation, and terminology.<sup>16</sup>

Poincaré assumes that the interior (labeled int) of the electron worldtube is filled with a *positive cosmological constant*  $\Lambda$ , corresponding to an action contribution (we use  $c = 1$  like Poincaré, except when it is physically clarifying to use physical units)

$$S_{\Lambda} = - \int_{\text{int}} d^4x \Lambda. \quad (10)$$

This action contribution is clearly Lorentz invariant. The total action for the electron then contains two terms:

$$S_{\text{tot}} = S_{\text{em}} + S_{\Lambda}, \quad (11)$$

where the first term,  $S_{\text{em}}$ , is the electromagnetic action. When considered as a functional of both the electromagnetic 4-potential  $A_{\mu}$  and the sources (i.e. the 4-current  $j^{\mu}$ ), the electromagnetic action reads

$$-\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} + \int d^4x A_{\mu} j^{\mu}, \quad (12)$$

where  $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ . However, as emphasized in Ref. [24], Poincaré uses an action which is an (implicit) functional of the sources. The latter "Fokker action" would generally be written (after replacing  $A_{\mu}$  by its functional expression in terms of  $j^{\mu}$ , and using suitable integrations by parts) as  $S_{\text{em}}[j] = \frac{1}{2} \int d^4x A_{\mu} j^{\mu}$ . However, Poincaré writes it in the equivalent (modulo integration by parts) form

<sup>13</sup> See, e.g., [21,22] for assessments of Minkowski's debt towards Poincaré.

<sup>14</sup> Note that Einstein's paper was received by the *Annalen der Physik* on 30 June 1905.

<sup>15</sup> Note that this is one of the very rare sentences italicized by Poincaré, thereby confirming its central role.

<sup>16</sup> We directly consider the case of interest leading to the Lorentz contraction, rather than to the electron models of Abraham and Langevin. For more details and references on classical electron models, the "4/3" issue, and Poincaré's Palermo memoir, see <https://en.wikipedia.org/wiki/Electromagnetic-mass>; notably: M. Janssen and M. Mecklenburg, (2004) "Electromagnetic Models of the Electron and the Transition from Classical to Relativistic Mechanics" preprint available on <http://philsci-archiv.pitt.edu/id/eprint/1990>.

$$S_{\text{em}} = +\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} = \int dt d^3x \frac{1}{2} (-\mathbf{E}^2 + \mathbf{B}^2), \quad (13)$$

where he (seemingly) considers that the electromagnetic field is expressed as a functional of the electric sources.<sup>17</sup> Note that the overall sign in (13) is the opposite of the usual field action  $S[A_\mu] = \int d^4x \frac{1}{2} (\mathbf{E}^2 - \mathbf{B}^2)$ .<sup>18</sup>

Working with the standard sign, the total action for the electron used by Poincaré reads:

$$S_{\text{tot}} = S_{\text{em}}[j_\mu] + S_\Lambda = +\frac{1}{4} \int d^4x F^{\mu\nu}[j] F_{\mu\nu}[j] - \int_{\text{int}} d^4x \Lambda. \quad (14)$$

When considering the dynamics of a single electron, Poincaré considers a four-current  $j^\mu$  localized on the surface of the hollow electron worldtube, and reducing to a uniform electric charge density on a sphere of radius  $R$  in the rest-frame of the electron, when the latter is in equilibrium.<sup>19</sup>

Given  $\Lambda$ , the value of the electron radius  $R$  is determined (as emphasized by Poincaré) by extremizing  $S_{\text{tot}}$  with respect to  $R$ . In view of the formal relativistic invariance of  $S_{\text{tot}}$ , it is enough to work in the rest frame of the electron. The corresponding value of the total Lagrangian is clearly

$$L_{\text{tot}}(R) = -\frac{1}{2} \int d^3x \mathbf{E}^2 - \Lambda \frac{4\pi}{3} R^3, \quad (15)$$

where the electric field is  $\mathbf{E} = \mathbf{er}/(4\pi r^3)$  for  $r > R$ , and vanishes for  $r < R$ . This yields

$$L_{\text{tot}}(R) = -E_{\text{em}}(R) - \Lambda \frac{4\pi}{3} R^3, \quad (16)$$

where  $E_{\text{em}}(R)$  denotes the electric field energy,  $E_{\text{em}}(R) = \frac{e^2}{8\pi R}$ , as given in Eq. (6). Extremizing  $L_{\text{tot}}(R)$  with respect to  $R$  then yields the condition

$$0 = R \frac{dL_{\text{tot}}(R)}{dR} = +E_{\text{em}}(R) - \Lambda 4\pi R^3, \quad (17)$$

i.e. an equilibrium radius  $R_*(\Lambda)$  satisfying

$$\Lambda 4\pi R_*^4 = \frac{e^2}{8\pi}. \quad (18)$$

We see that  $\Lambda$  must be positive. As the equilibrium value  $R_*$  of  $R$  corresponds to maximizing  $L_{\text{tot}}(R)$ , i.e. minimizing the corresponding (rest-frame) Hamiltonian  $H_{\text{tot}}(R) = -L_{\text{tot}}(R)$ , the Poincaré electron model is stable under spherical perturbations. Surprisingly, Poincaré did not seem to worry about the more delicate issue of stability under non-spherical perturbations.

Following the paragraph 6 of Poincaré's Palermo article [17] (whose results and notation we follow, except that we do not use primes for rest-frame quantities), it is easy to consider an ellipsoidally deformed electron (behaving like a hollow conductor) having, in its rest-frame, a volume  $\frac{4\pi}{3} \bar{R}^3$  and an ellipticity  $\theta$  ( $\theta = R_y/R_x = R_z/R_x$ ). The rest-frame Hamiltonian ( $H_{\text{tot}} = H' + F'$  in Poincaré's notation) depends on the volume (i.e. on  $\bar{R}$ ) and on the ellipticity  $\theta$  as

$$H_{\text{tot}}(\bar{R}, \theta) = -L_{\text{tot}}(\bar{R}, \theta) = E_{\text{em}}(\bar{R}) \bar{\varphi}(\theta) \theta^{2/3} + \Lambda \frac{4\pi}{3} \bar{R}^3, \quad (19)$$

where we denoted  $\bar{\varphi}(\theta) = \varphi(\theta)/\varphi(1)$ . The latter function is defined (from the Abraham electron model) by Eq. (5) in paragraph 6 of [17], i.e. as the analytic continuation in  $\theta$  (starting from the interval  $0 < \theta < 1$ ) of

$$\bar{\varphi}(\theta) = \left[ \frac{1}{2\varepsilon} \ln \frac{1+\varepsilon}{1-\varepsilon} \right]_{\varepsilon=\sqrt{1-\theta^2}}. \quad (20)$$

One then finds that though the necessary condition of equilibrium of the ellipsoidally-deformed electron (namely that the function  $H_{\text{tot}}(\bar{R}, \theta)$  has an extremum at  $(\bar{R}, \theta) = (R_*, 1)$ ) is satisfied, this equilibrium is unstable because the extremum of  $H_{\text{tot}}(\bar{R}, \theta)$  is a saddle point: a local minimum with respect to the  $\bar{R}$  axis, but a local maximum with respect to the  $\theta$  axis.<sup>20</sup> This shows the instability of Poincaré's electron model with respect to shape variations (under his Abraham-like assumption that the electric charge is distributed on the surface of the electron as if it were a conductor). Lorentz, who had

<sup>17</sup> However, Poincaré does not specify the dynamics determining the four-current.

<sup>18</sup> Somewhat confusingly, Poincaré systematically works with quantities that he calls "actions", which have the opposite sign to that of the usual actions, so that he ends up with an electron "action" equal to  $+m_{\text{electron}} c^2 \sqrt{1 - \frac{v^2}{c^2}}$  instead of the standard, opposite definition (Eq. (21) below).

<sup>19</sup> When discussing an ellipsoidally deformed electron in [17], Poincaré assumes that it behaves as a conducting shell.

<sup>20</sup> The constraint  $[\varphi'(\theta)/\varphi(\theta)]_{\theta=1} = -2/3$  derived by Poincaré is equivalent to extremizing  $\psi(\theta) \equiv \bar{\varphi}(\theta) \theta^{2/3}$  at  $\theta = 1$ . However, this is a local maximum.

worried about the stability of Poincaré’s electron under shape variations, had also found its instability under the assumption that the electric charge is uniformly distributed, keeping fixed the total area (see note 80 in [25]). This ellipsoidal instability problem of the Poincaré electron model is shared by the Bucherer–Langevin model [26], as well as by the Dirac “extensible model of the electron” [27], in which Poincaré’s volumic interior negative pressure is replaced by the surface tension of an elastic, charged conducting membrane (with vanishing electromagnetic field inside) [28].

From Eq. (17), Poincaré (implicitly) deduces that the last ( $\Lambda$ -related) contribution to the electron Lagrangian (16) is equal to  $-\frac{1}{3}E_{em}(R)$ , so that, after extremization on  $R$ , the value of the electron Lagrangian in its rest frame is  $-E_{em}(R_*) - \frac{1}{3}E_{em}(R_*) = -\frac{4}{3}E_{em}(R_*)$ .

Finally, by an analysis equivalent to using the relativistic covariance of the action (14), Poincaré deduces that the Lagrangian describing, in the ether frame, the dynamics of electrons moving in a quasi-stationary manner is

$$L_{\text{electron}} = -m_{\text{electron}}c^2\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}, \tag{21}$$

where

$$m_{\text{electron}} = \frac{4}{3} \frac{E_{em}}{c^2}. \tag{22}$$

An alternative way (not available to Poincaré) of getting these results is to combine Einstein’s famous result of September 1905, namely

$$m = \left[ \frac{E_{\text{tot}}}{c^2} \right]_{\text{rest frame}} = \frac{1}{c^2} \left[ \int d^3 T_{\text{tot}}^{00} \right]_{\text{rest frame}}, \tag{23}$$

with von Laue’s well-known (virial) theorem stating that  $\int d^3 T_{\text{tot}}^{ij} = 0$  for a body in a stationary state (in its rest frame) [29].

The total stress-energy tensor corresponding to the action (14) is [we use the signature  $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ ]

$$T_{\text{tot}}^{\mu\nu} = T_{em}^{\mu\nu} + T_{\Lambda}^{\mu\nu} = T_{em}^{\mu\nu} - \Lambda \eta^{\mu\nu} \theta(R - r), \tag{24}$$

where  $\theta(x)$  denotes Heaviside’s step function.<sup>21</sup> Using  $\eta_{\mu\nu} T_{em}^{\mu\nu} = 0$  and  $\int d^3 T_{\text{tot}}^{ii} = 0$  immediately yields  $\int d^3 x T_{em}^{00} = -\int d^3 x T_{\Lambda}^{ii}$ , i.e.

$$E_{em} = \int d^3 x T_{em}^{00} = 3 \int_{r < R} d^3 x \Lambda = 3E_{\Lambda}, \tag{25}$$

in agreement with Eq. (17).

Summarizing Poincaré’s result in modern terms: the condition of dynamical equilibrium between the electrostatic self-repulsion and the inner (negative) pressure  $-\Lambda$  associated with the additional action (10) yields an additional (positive) energy contribution  $E_{\Lambda} = \int_{r < R} d^3 x \Lambda$  related to the electromagnetic rest energy  $E_{em} = \int d^3 x T_{em}^{00}$  by the simple relation

$$E_{\Lambda} = \frac{1}{3} E_{em}. \tag{26}$$

As a consequence, the total rest energy of the electron is

$$E_{\text{tot}} = E_{em} + E_{\Lambda} = \frac{4}{3} E_{em}, \tag{27}$$

which corresponds to the electron mass (22) (implicitly) derived by Poincaré.

Alas, though it is likely that Poincaré had derived Eqs. (21) and (22) when he wrote his short Note, he did not display these results then, but buried them, in the middle of a rather abstruse technical discussion, in his 47-page-long Palermo memoir.

## 2.2. The second important result (according to Poincaré himself): defining the class of relativistically-invariant gravitational force laws

Poincaré writes that it is important to examine in more detail the second assumption of Lorentz, namely assumption (A2) above “that the forces between uncharged particles, as well as those between such particles and electrons, are influenced by a translation in quite the same way as the electric forces in an electrostatic system.” This led him to define, for the first time, the class of relativistically-invariant, action-at-a-distance gravitational force laws, between two (arbitrarily moving) masses.

<sup>21</sup> Note in passing that the pressure corresponding to  $T_{\Lambda, \text{int}}^{\mu\nu} = -\Lambda \eta^{\mu\nu}$  is  $-\Lambda < 0$ , as emphasized by Poincaré.

However, in his 5 June Note, he barely sketches his results, only saying that: (i) he succeeded in constructing such Lorentz-invariant force laws; (ii) he assumed that the “propagation of gravitation is not instantaneous, but takes place at the velocity of light” (which makes Poincaré speak of a “gravitational wave” (*onde gravifique*) leaving the attracting body and propagating towards the attracted one); and (iii) he could arrange his construction so that the fractional deviations from Newton’s non-relativistic  $1/r^2$  law were only of order  $v^2/c^2$ .

Again, he buried the most interesting, and most novel aspects of his results, in the last pages of his Palermo article. Indeed, it is only in his Palermo memoir that Poincaré remarks (in the middle of some technical developments) that:

(R1) the Lorentz transformations leave invariant the quadratic form (with  $c = 1$ )  $x^2 + y^2 + z^2 - t^2$ ;

(R2) the Lorentz transformations can be viewed as “rotations” in a “4-dimensional space” with coordinates “ $x, y, z, t\sqrt{-1}$ ”.<sup>22</sup>

Actually, I find rather likely that Poincaré had not fully understood the latter results (especially the “Wick rotation” (R2)) at the time when he was writing his Note. Indeed, the results (R1) and (R2) just quoted do not enter the Palermo article in the first paragraphs (which display most of the results alluded to in the Note), but only (somehow in passing) at the end of paragraph 4 (for the remark (R1)), and in the middle of paragraph 9 (for the remark (R2)). When reading Poincaré (and especially his Palermo article), I have the impression that Poincaré had not well planned his article, but was finding new results as he proceeded with his technical developments, and was incorporating them without coming back and rewriting the first paragraphs in a more logical manner.

### 2.3. Other important results (as viewed today)

The results discussed above are, in my opinion, the ones that Poincaré himself would have recognized as significant new contributions. Poincaré was known to be very critical, and demanding, when presented with supposedly new scientific results. His usual reaction was to say: *À quoi bon ?* (“What is it good for?”). He applied this demanding criterion to himself. This is why many other results mentioned in his short Note (which we appreciate today as having broken new ground in Relativity) are qualified by Poincaré himself as being only incremental additions to Lorentz’ 1904 memoir. Indeed, at the beginning of his Note, just before starting to write his first equation, Poincaré writes:

Les résultats que j’ai obtenus sont d’accord sur tous les points importants avec ceux de Lorentz; j’ai été seulement conduit à les modifier et à les compléter dans quelques points de détail.

Let me, however, indicate some of these “points of detail” where Poincaré actually went significantly beyond Lorentz. First, and foremost, there is the fact that Poincaré begins his Note by asserting the principle of relativity as being exact:

Il semble que cette impossibilité de démontrer le mouvement absolu soit une loi générale de la nature.

Then Poincaré’s first equation (which he calls the “Lorentz transformation”) differs from Lorentz’ original writing, Eqs. (4) (though it is equivalent to it). Poincaré writes

$$\begin{aligned}x' &= k\ell(x + \varepsilon t), \\y' &= \ell y, \\z' &= \ell z, \\t' &= k\ell(t + \varepsilon x),\end{aligned}\tag{28}$$

with

$$k \equiv \frac{1}{\sqrt{1 - \varepsilon^2}}.\tag{29}$$

Curiously enough, Poincaré never says that  $\varepsilon = -v/c$ , but only says that “ $\varepsilon$  is a constant which defines the transformation.” He also neglects the physical difference (important for Lorentz) between the ether-frame coordinates, and the transformed ones, just saying: “ $x, y, z$  are the coordinates and  $t$  the time before the transformation,  $x', y', z'$  and  $t'$  after the transformation.” These are two examples of his (fruitful) mathematical approach to the problem (which, for example, makes natural for him to define the “velocity after the transformation” as  $dx'/dt', dy'/dt', dz'/dt'$ , while Lorentz was always working with the Galilean-transformed velocity  $v'^x = (dx/dt) - v$ ,  $v'^y = dy/dt$ ,  $v'^z = dz/dt$ ).

The first new result of Poincaré is to say that the transformations (28) (together with spatial rotations) “must form a group”, and that this requires the unknown function  $\ell(v^2)$  to be equal to 1. He adds that “this is a consequence that Lorentz had obtained by another route.”

<sup>22</sup> The latter remark is used by Poincaré to find all the relativistic invariants constructible with what we would call today the two spacetime points of the attracted and attracting bodies,  $x^\mu, x_1^\mu$ , and their 4-velocities  $dx^\mu/ds, dx_1^\mu/ds_1$ .



Actually, it seems to me that, here, Poincaré is (fruitfully) betraying Lorentz' approach. Indeed, for Lorentz, the set of Lorentz transformations does not need to form a group, because these transformations must always transform the ether coordinates (modulo spatial rotations and time shifts) into some auxiliary variables attached to a moving frame parameterized by  $\mathbf{v}$ . It makes no sense (for Lorentz) to compose two Lorentz transformations,  $(x, y, z, t) \rightarrow (x', y', z', t') \rightarrow (x'', y'', z'', t'')$ , because the only physically meaningful transformations are  $(x, y, z, t) \rightarrow (x', y', z', t')$  and  $(x, y, z, t) \rightarrow (x'', y'', z'', t'')$ . We have here an example of a (fruitful) contradiction between Poincaré, the mathematician, and Poincaré, the physicist. If asked, Poincaré the physicist would have agreed with Lorentz that only  $t$  (in the ether frame) measures the “real time”, while  $t'$  measures some (fictitious) “ideal time” (see below). On the other hand, Poincaré the mathematician considers all the different variables  $(x, y, z, t); (x', y', z', t'); (x'', y'', z'', t'')$  as being on an equal footing, which allows him to consider the group composition  $(x, y, z, t) \rightarrow (x', y', z', t') \rightarrow (x'', y'', z'', t'')$ .<sup>23</sup>

Note in passing that, though Poincaré used in his derivations the relativistic law of composition of velocities, both for composing a boost velocity with the electron velocity, and for composing two boost velocities, e.g.,

$$\varepsilon'' = \frac{\varepsilon + \varepsilon'}{1 + \varepsilon\varepsilon'}, \tag{30}$$

he does not bother to mention it in his Note.<sup>24</sup>

The second new result of Poincaré (which actually corrects a very important shortcoming of Lorentz' paper) is to give, for the first time, the correct transformation law for the electromagnetic four-current  $j^\mu$ . In a modern notation, this law reads

$$\begin{aligned} j'^x &= \frac{k}{\ell^3} (j^x + \varepsilon j^t), \\ j'^y &= \frac{1}{\ell^3} j^y, \\ j'^z &= \frac{1}{\ell^3} j^z, \\ j'^t &= \frac{k}{\ell^3} (j^t + \varepsilon j^x), \end{aligned} \tag{31}$$

and Poincaré certainly understood that  $(j^x, j^y, j^z, j^t)$  varied, under a Lorentz transformation (when  $\ell = 1$ ), like  $(x, y, z, t)$  – i.e. that they are both 4-vectors. Armed with his definition (31), Poincaré showed (for the first time) the (exact) relativistic invariance of the inhomogeneous Maxwell–Lorentz equations.

Let me also mention that, in his Palermo article, Poincaré obtains further important results on the mathematics of Relativity. Notably, he computes the Lie algebra of the Lorentz group, showing in particular that the commutator of two boosts is a rotation:

$$[t \partial_x + x \partial_t, t \partial_y + y \partial_t] = x \partial_y - y \partial_x. \tag{32}$$

He understands and uses the fact that (in the Lorenz gauge) the 4-potential  $A^\mu$  transforms as  $x^\mu$ . In addition, he discusses many invariants of the (Poincaré and) Lorentz group, such as the “Minkowski” scalar product of two four-vectors, and the electromagnetic invariants  $\mathbf{E}^2 - \mathbf{B}^2$  and  $\mathbf{E} \cdot \mathbf{B}$ .

### 3. Conclusions

In conclusion, Poincaré's 5 June 1905 Note announces important mathematical and physical advances in what we would call today, Special Relativity, relativistic electrodynamics and relativistic gravitation (see the partial summary above). One can, however, regret that this Note did not explicitly report some of the most important advances made by Poincaré, and only published in the follow-up Palermo memoir, such as:

- (1) the relativistic law of addition of velocities (30);
- (2) the relativistic electron Lagrangian (21), (22);
- (3) the understanding of Lorentz transformations as “rotations” in a “4-dimensional space” with coordinates “ $x, y, z, t\sqrt{-1}$ ”, and the associated method of constructing relativistic invariants;
- (4) the Lie algebra of the Lorentz group (32);
- (5) a more explicit description of the class of relativistically-invariant, action-at-a-distance gravitational force laws, between two (arbitrarily moving) masses, and of the associated, inter-body “gravitational wave” propagation effects.

<sup>23</sup> By contrast, it was natural for Einstein the physicist to compose transformations, because he was really considering all inertial coordinates as being physically on the same footing [23].

<sup>24</sup> It will, however, be explicitly used several times in his Palermo memoir.

My feeling is that Poincaré wrote his Note too soon, before starting the writing of his long Palermo memoir, during which he finalized, as he proceeded, some of his most important advances [such as (3) and (4) above]. If Poincaré had written, say in July 1905, a Note containing a more complete summary of his Palermo article, with technical indications of the results (1)–(4) above, this Note might have been included in the booklet on the Principle of Relativity, edited by Sommerfeld, collecting the important original papers on Relativity [30]. One can understand why Sommerfeld decided to include in his collection neither the June Note (whose relativistic content is too slim), nor the Palermo memoir (from which one cannot easily extract a self-contained account of its scattered relativistic content), but resorted instead to adding some notes after the Minkowski Cologne lecture [19] in which he mentioned several (though not all<sup>25</sup>) of Poincaré’s relevant achievements.

It seems that neither Poincaré’s Note, nor his follow-up Palermo article, attracted much attention at the time. The earliest citation of Poincaré’s Note that I found is in the 2 March 1906 paper by the experimental physicist (of electron-dynamics fame) Walter Kaufmann [31]. What attracted Kaufmann’s attention was Poincaré’s introduction of a pressure to account for the internal constitution of the electron. Concerning the dynamics of electrons, he speaks of the “Lorentz–Einstein basic assumption”.

On 8 March 1906, Lorentz belatedly acknowledges the reception of the Palermo memoir, which he claims to “have studied with the greatest interest” (*Inutile de vous dire que je l’ai étudié avec le plus grand intérêt, et que j’ai été très heureux de voir mes conclusions confirmées par vos considérations.*). However, he does not seem to have taken a full measure of the new contributions brought by Poincaré, some of which were already mentioned in his June 1905 Note. Indeed, in the set of lectures he gave in March and April 1906 in Columbia University, New York, Lorentz [25] (who cites the Palermo memoir but not the June 1905 Note) only credits Poincaré for having introduced a constant “normal stress  $S$ ” that “makes much clearer” how a spherical electron at rest can Lorentz-contract and remain in equilibrium when moving at any velocity  $v < c$ . Though he borrowed Poincaré’s notation  $k$  for the Lorentz “ $\gamma$  factor”, and uses the “modern” writing (28) for the Lorentz transformation, Lorentz attributes only to Einstein several of the results already displayed in the June 1905 Note, notably the correct transformation law (31) for the electromagnetic four-current  $j^\mu$ .

It would be interesting to study who else cited Poincaré’s Note in the early years of Special Relativity. But it seems clear that Poincaré’s short and cryptic Note [1], and its long and opaquely written follow-up paper [17] (published in January 1906 in a mathematics journal), had difficulties competing with the very transparently written, conceptually novel, quasi-simultaneous paper of Einstein [23] (published on 26 September 1905 in a very well-read physics journal).

I emphasized above how Poincaré seems to downplay the novelty of some of his own results. Poincaré was certainly over-modest in repeatedly stating that he was only bringing incremental additions to Lorentz’ results. On the other hand, when focusing one’s attention on Poincaré’s contribution (as we did above), without studying in detail the state of the art of the field around 1905, one runs the risk of over-appreciating some of Poincaré’s results. For instance, while preparing this text, I had a brief look at some of the contemporary articles of Langevin, notably his lecture [32] at the 1904 St. Louis international conference,<sup>26</sup> and a Note to the *Comptes rendus* published by Langevin in 1905 [34], just before Poincaré’s Note.

Langevin’s St. Louis talk [32] reviews, in particular, Lorentz’ 1904 results and mentions the issue of whether gravitation might not transform as assumed by Lorentz and thereby allow one to observe one’s motion with respect to the ether. In addition, Langevin emphasizes the problem of finding which extra forces might ensure the stability of the electron: “It seems necessary to admit something else in its structure than its electric charge, an action which maintains the unity of the electron and prevents its charge from being dissipated by the mutual repulsions of the elements which constitute it.” He even mentions the possibility that gravitation “acts at the interior of the electrons in order to insure their stability”, though he is aware that gravity is much weaker than electromagnetism.<sup>27</sup>

In addition, Langevin’s 1905 Note [34] (which is entitled “On the physical impossibility to observe [*mettre en évidence*] the translatory motion of the Earth”) reports the result that the Lagrangian of any electrified system transforms under a Lorentz boost as  $L' = L\sqrt{1 - \frac{v^2}{c^2}}$ . This might explain why Poincaré felt that his result (21) might not be a novelty [in addition, Poincaré certainly knew that Lorentz’ result (5) implied the Lagrangian (21) (modulo an additive constant)].

It is probable that the first person to fully grasp the importance of Poincaré’s results concerning the mathematical aspects of Special Relativity, and relativistic gravitation, was Minkowski. In his first (November 1907) work [18] (which is a digest of Poincaré’s results), Minkowski cites six times Poincaré’s name (and only twice Einstein’s name) [see [22] for a detailed discussion]. On the other hand, Minkowski decided *not to cite at all* Poincaré when he delivered (and then wrote up) his famous September 1908 Cologne lecture on “Raum und Zeit” (“Space and Time”).<sup>28</sup> As already mentioned, Sommerfeld completed the reprinting of Minkowski’s article in his booklet [30] by mentioning some of Poincaré’s results. It seems that people fully recognized Poincaré’s contributions to Relativity only after his death, see notably the scientific obituaries of

<sup>25</sup> In particular, Sommerfeld does not mention Poincaré’s use of a “4-dimensional space” with coordinates “ $x, y, z, t\sqrt{-1}$ ”.

<sup>26</sup> Both Poincaré and Langevin gave invited talks there; moreover, they spent a week together “in the vast plains of North America”, on their way back from St. Louis, which gave them ample time to discuss [33].

<sup>27</sup> Let me mention in passing that Langevin also writes: “A very simple calculation shows also that the stock of energy represented by the electric and magnetic fields surrounding the electrons contained in an atom is sufficiently great to supply for ten million years the evolution of heat discovered by Curie in the radium salts.” In view of the then classic result (22), we see that Langevin probably had in mind the pre-Einsteinian energy  $E_{em} = \frac{3}{4}m_{electron}c^2$ .

<sup>28</sup> For discussions of Minkowski’s attitude towards Poincaré, see, e.g., [21,22].

Langevin [33] and Lorentz [16]. Let me also mention the fair, and complete, account of Poincaré's contributions to Relativity given by the young Pauli (prompted by the old Klein who insisted on citing the contributions of Poincaré) in his 1921 Encyclopedia article [35].

To end this account, let me recall the well-known fact that, in spite of Poincaré's repeated pleads (especially in his non-technical lectures) for the importance of the "Principle of Relativity", in spite of his (pre-Einstein) discussions of clock synchronization by means of electromagnetic signals, and in spite of his important technical results concerning the mathematical aspects of Relativity, it seems that he never abandoned his conviction that there exists an absolute time and an absolute space.<sup>29</sup> He also, apparently, never cited, nor probably appreciated, Einstein's contributions to Relativity. Indeed, a few months before his death, in a lecture given on 4 May 1912 at the University of London [37], after recalling what we would call today the "Poincaré–Minkowski" picture of "time as a fourth dimension of space", with rotations among " $x$ ,  $y$ ,  $z$ ,  $t\sqrt{-1}$ ", etc., he concludes his lecture by pleading for "keeping one's old habits":

Quelle va être notre position en face de ces nouvelles conceptions? Allons-nous être forcés de modifier nos conclusions? Non certes : nous avons adopté une convention parce qu'elle nous semblait commode, et nous disions que rien ne pourrait nous contraindre à l'abandonner. Aujourd'hui certains physiciens veulent adopter une convention nouvelle. Ce n'est pas qu'ils y soient contraints ; ils jugent cette convention nouvelle plus commode, voilà tout ; et ceux qui ne sont pas de cet avis peuvent légitimement conserver l'ancienne pour ne pas troubler leurs vieilles habitudes. Je crois, entre nous, que c'est ce qu'ils feront encore longtemps.

Moreover, in his last lectures (July 1912) "on the dynamics of the electron" [38], he insists on saying that the Lorentz transformation maps "a *real* phenomenon which takes place in  $x$ ,  $y$ ,  $z$ , at the instant  $t$ , into an *ideal* phenomenon which is its image, and which takes place in  $x'$ ,  $y'$ ,  $z'$ , at the instant  $t'$ " (with italics added by me on the words "réel" and "idéal"). By contrast, let me recall that Einstein once said that his main new insight was to realize that the auxiliary time variable  $t'$  used by Lorentz was "time, pure and simple".

Let us finally mention that Lorentz (in spite of his high appreciation of Einstein), also always kept his "old habits" about absolute space and absolute time. One has an insight on how Lorentz (and probably also Poincaré) thought about the issue of the "principle of relativity" through Lorentz' comments (in his book [25]) on "the many highly interesting applications that Einstein made of this principle [of Relativity]". In particular, Lorentz writes (p. 230):

[...] the chief difference being that Einstein simply postulates what we have deduced, with some difficulty and not altogether satisfactorily, from the fundamental equations of the electromagnetic field.

It is probable that Poincaré also felt that Einstein was simply postulating what had to be proven from underlying microscopic dynamical considerations. I think that when Poincaré was using the word "principle" he had (mostly) in mind a general physical property that is rooted in (and provable from) some microscopic dynamics (see, e.g., the citation from [9] in the first section above: *une théorie bien faite devrait permettre de démontrer le principe d'un seul coup dans toute sa rigueur*). By contrast, Einstein used his Principle of Relativity as a primitive symmetry requirement restricting the laws of physics.

## Acknowledgement

I thank Olivier Darrigol for informative email exchanges.

## References

- [1] H. Poincaré, Sur la dynamique de l'électron, C. r. hebdomadaire des séances Acad. Sci. Paris 140 (1905) 1504–1508 (séance du 5 juin).
- [2] O. Darrigol, Henri Poincaré's criticism of fin de siècle electrodynamics, Stud. Hist. Philos. Mod. Phys. 26 (1995) 1–44.
- [3] O. Darrigol, The electrodynamic origins of relativity theory, Hist. Stud. Nat. Sci. 26 (1996) 241–312.
- [4] O. Darrigol, Electrodynamics from Ampère to Einstein, Oxford University Press, Oxford, UK, 2000.
- [5] O. Darrigol, The genesis of the theory of relativity, in: T. Damour, O. Darrigol, B. Duplantier, V. Rivasseau (Eds.), Einstein 1905–2005: Poincaré Seminar 2005, Birkhäuser Verlag, Basel, Switzerland, 2006, pp. 1–31, see also [www.bourbaphy.fr/](http://www.bourbaphy.fr/).
- [6] P. Galison, Einstein's Clocks, Poincaré's Maps; Empires of Time, Norton, New York, 2003.
- [7] <http://poincare.univ-lorraine.fr/fr/fonds-et-archives>, which gives links to many works, and documents, of Poincaré, notably his correspondence, to which we shall sometimes allude below.
- [8] <http://www.academie-sciences.fr/fr/Colloques-conferences-et-debats/henri-poincare.html>.
- [9] H. Poincaré, Électricité et optique : la lumière et les théories électrodynamiques, edited by J. Blondin and E. Néculcéa, Carré et Naud, Paris, 1901.
- [10] H.A. Lorentz, La théorie électromagnétique de Maxwell et son application aux corps mouvants, E. J. Brill, Leiden, The Netherlands, 1892.
- [11] H.A. Lorentz, Versuch einer Theorie der Elektrischen und Optischen Erscheinungen in Bewegten Körpern, 1895, Leiden, also in: Collected Papers [9 vols.], The Hague, 1934–1936, vol. 5, pp. 1–139.
- [12] H. Poincaré, La théorie de Lorentz et le principe de réaction, Archives néerlandaises des sciences exactes et naturelles, deuxième série 5 (1900) 252–278. Reprinted in Poincaré's Complete Works, vol. 9, pp. 464–488.
- [13] H. Poincaré, La science et l'hypothèse, Flammarion, Paris, 1902.
- [14] H.A. Lorentz, Electromagnetic phenomena in a system moving with any velocity smaller than light, Proc. Roy. Acad. Amsterdam 6 (1904) 809–831; also in Lorentz' Collected Papers, vol. 5, pp. 172–197.

<sup>29</sup> See, e.g., the discussion around Eq. (1.6) in [36].

- [15] H. Poincaré, The principles of mathematical physics, in: Congress of Arts and Science, Universal Exposition, St. Louis, 1904, Philosophy and Mathematics, vol. 1, Houghton, Mifflin & Co., Boston, MA, USA, 1905, pp. 604–622.
- [16] H.A. Lorentz, Deux Mémoires de Henri Poincaré sur la Physique Mathématique, *Acta Math.* 38 (1921) 293–308 [but written in 1914]; this text is reprinted in *Œuvres de Henri Poincaré*.
- [17] H. Poincaré, Sur la dynamique de l'électron, *Rend. Circ. Mat. Palermo* 21 (1906) 129–176.
- [18] H. Minkowski, Das Relativitätsprinzip, *Ann. Phys. (Berlin)* 47 (1915) 927–938. [Text of a lecture given by Minkowski in Göttingen, on 5 November 1907; edited posthumously by A. Sommerfeld.]
- [19] H. Minkowski, Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern, *Königliche Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse. Nachrichten* (1908) 53–111.
- [20] H. Minkowski, Raum und Zeit, *Phys. Z.* 10 (Jahrgang 1909) 104–115.
- [21] S. Walter, Minkowski, mathematicians, and the mathematical theory of relativity, in: H. Goenner, J. Renn, J. Ritter, T. Sauer (Eds.), *The Expanding Worlds of General Relativity (Einstein Studies 7)*, Birkhäuser, Boston, 1999, pp. 45–86, see also <http://scottwalter.free.fr/>.
- [22] T. Damour, What is missing from Minkowski's 'Raum und Zeit' lecture, *Ann. Phys. (Berlin)* 520 (2008) 619, arXiv:0807.1300 [physics.hist-ph].
- [23] A. Einstein, On the electrodynamics of moving bodies, *Ann. Phys. (Berlin)* 17 (1905) 891; *Ann. Phys. (Berlin)* 14 (2005) 194.
- [24] C. Bracco, J.-P. Provost, De l'électromagnétisme à la mécanique : le rôle de l'action dans le mémoire de Poincaré de 1905, *Rev. Hist. Sci.* 62 (2009) 457–493.
- [25] H.A. Lorentz, *The Theory of Electrons*, second edition, Teubner, Leipzig, 1916 (Lectures delivered in Columbia University, New York, in March and April 1906).
- [26] P. Ehrenfest, Zur Stabilitätsfrage bei den Bucherer–Langevin–Elektronen, *Phys. Z.* 7 (1906) 302–303.
- [27] P. Gnädig, Z. Kunszt, P. Hasenfratz, J. Kuti, Dirac's extended electron model, *Ann. Phys.* 116 (1978) 380.
- [28] P.A.M. Dirac, An extensible model of the electron, *Proc. R. Soc. Lond. A* 268 (1962) 57.
- [29] M. Laue, Zur Dynamik der Relativitätstheorie, *Ann. Phys. (Berlin)* 35 (1911) 524–542.
- [30] H.A. Lorentz, A. Einstein, H. Minkowski, H. Weyl, *The Principle of Relativity*, Dover, New York, 1952, with Notes by A. Sommerfeld, translated by W. Perrett and G.B. Jeffery [First (German) edition: 1913].
- [31] W. Kaufmann, Über die Konstitution des Elektrons, *Ann. Phys. (Berlin)* 324 (3) (1906) 487–553.
- [32] P. Langevin, The relations of physics of electrons to other branches of science, in: Congress of Arts and Science, Universal Exposition, vol. 4, St. Louis, 1904, Houghton, Mifflin & Co., Boston, MA, USA, 1906, pp. 121–156.
- [33] P. Langevin, L'œuvre d'Henri Poincaré : le physicien, *Rev. Métaphys. Morale* 21 (5) (1913) 675–718.
- [34] P. Langevin, Sur l'impossibilité physique de mettre en évidence le mouvement de translation de la Terre, *C. r. hebd. séanc. Acad. Sci. Paris* 140 (1905) 1171–1173.
- [35] W. Pauli, *Relativitätstheorie*, *Encyklopädie der Mathematischen Wissenschaften*, Teubner, Leipzig, 1921, pp. 539–775; W. Pauli, *Theory of Relativity*, Dover, New York, 1981.
- [36] T. Damour, Poincaré, relativity, billiards and symmetry, in: P. Gaspard, M. Henneaux, F. Lambert (Eds.), *Symposium Henri Poincaré, Proceedings*, vol. 2, Solvay Workshops and Symposia, 2004, International Solvay Institutes, Brussels, 2007, pp. 149–173, arXiv:hep-th/0501168.
- [37] H. Poincaré, L'espace et le temps, *Scientia (Rivista di Scienza)* 12 (1912) 159–170, reprinted in H. Poincaré, *Dernières pensées*, Flammarion, Paris, 1913 (réédition 1963).
- [38] H. Poincaré, La dynamique de l'électron, in: M. Viard (Ed.), *Supplément aux Annales des postes, télégraphes et téléphones*, Dumas, Paris, March 1913.